# The Boundary Multiplet of $N$=4 SU(2)⊗U(1) Gauged Supergravity on Asymptotically-AdS$_5$

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**Abstract**

We consider $N$=4 SU(2)⊗U(1) gauged supergravity on asymptotically-AdS$_5$ backgrounds. By a near-boundary analysis we determine the boundary-dominant components of the bulk fields from their partially gauge-fixed field equations. Subdominant components are projected out in the boundary limit and we find a reduced set of boundary fields, constituting the $N$=2 Weyl multiplet. The residual bulk symmetries are found to act on the boundary fields as four-dimensional diffeomorphisms, $N$=2 supersymmetry and (super-)Weyl transformations. This shows that the on-shell $N$=4 supergravity multiplet yields the $N$=2 Weyl multiplet on the boundary with the appropriate local $N$=2 superconformal transformations.

## 1 Introduction

Supergravities on Anti-de Sitter (AdS) spaces play a prominent role in the AdS/CFT correspondence [1], which – in the weakest form of the conjecture – relates classical ten-dimensional supergravity on the near-horizon limit of $p$-brane backgrounds to strongly-coupled superconformal quantum field theories (SCFT) on $p$+1-dimensional flat space. The near-horizon geometry of the $p$-brane solutions is typically given by a product of AdS space and a compact manifold, on which one may perform a Kaluza-Klein expansion. Gauged supergravities on AdS spaces are then employed to describe the Kaluza-Klein expanded ten-dimensional theory truncated to a finite number of Kaluza-Klein modes, and consequently also for a dual description of the corresponding SCFT sector [2, 3]. The explicit AdS/CFT duality relation is given by interpreting the boundary values of the supergravity fields as sources for the dual operators of the SCFT [4], and it has been applied to describe a variety of phenomena in strongly-coupled Quantum Field Theories (QFT) [5].

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In this work we consider five-dimensional half-maximally supersymmetric gauged supergravity. The general gauged matter-coupled $N$=4 supergravities in five dimensions were constructed in [6, 7], and it was noted in [6] that AdS ground states are only possible if the gauge group is a product of a one-dimensional Abelian factor and a semi-simple group. We focus on the $N$=4 SU(2)⊗U(1) gauged supergravity constructed by Romans [8], the only gauging of the pure supergravity without additional matter multiplets which admits an AdS vacuum. Solutions of this theory can be lifted to solutions of the IIB supergravity [9] where they correspond to product geometries involving $S^5$, and also to warped-product solutions of IIA supergravity and the maximal $d$=11 supergravity [10]. We restrict the configuration space to asymptotically-AdS$_5$ geometries with an arbitrary four-dimensional boundary metric. By an analysis of the asymptotic field equations we determine the multiplet of boundary fields, and from the local bulk symmetries we obtain the boundary symmetries with the induced representation on the boundary fields. This limiting procedure does not involve the AdS/CFT conjecture and does not rely on the choice of boundary conditions. We find the $N$=2 Weyl multiplet with local $N$=2 superconformal transformations. Similar calculations have previously been carried out for bulk theories in $d$=3, 6, 7 dimensions and for $N$=2 supergravity in $d$=5 [11].

The relevance of these results is twofold. Firstly in the AdS/CFT context, where the bulk theory is understood as dual to four-dimensional $N$=2 SCFTs[1] [3, 12]. With the boundary fields and symmetries obtained here one may calculate the holographic counterterms needed to render the on-shell bulk action finite, which have been given for the pure-gravity case in [13], for the full $N$=4 supergravity. Then, one can extend the calculation of the purely gravitational part of the conformal anomaly of the dual CFT [14] to the full result including matter-field contributions for $N$=2 SCFTs[2]. Secondly, a duality relation of QFTs on AdS space and on its conformal boundary has been formulated and proven in [16] in the framework of algebraic QFT. In contrast to the AdS/CFT correspondence, gravity does not seem to play a dedicated role in the algebraic holography. In particular, the constructions in [17] suggest that a gravitational theory is induced on the conformal boundary by a gravitational bulk theory. A similar result was obtained in [18] by deforming the AdS/CFT correspondence. It was shown there that changing the Dirichlet boundary conditions to Neumann or mixed boundary conditions promotes the boundary metric to a dynamical field. In this context our construction yields the kinematics of the boundary theory, for which we thus expect an $N$=2 conformal supergravity.

The paper is organized as follows. In Section 2 we review the $N$=4 SU(2)⊗U(1) gauged supergravity [8] to fix notation. In Section 3 the notion of an asymptotically-AdS$_5$ space is introduced and the multiplet of fields induced on the conformal boundary is constructed. We employ Fefferman-Graham coordinates and partial gauge fixing of the local super-, Lorentz and SU(2)⊗U(1) symmetries. The asymptotic scalings of the boundary-irreducible components of the bulk fields are determined in Section 3.1 from the linearized field equations. Subdominant components are projected out in the boundary limit and we find a reduced

[1] For example, the AdS$_5$ vacuum solution of Romans' theory lifted to $d$=11 supergravity is found to describe the near-horizon limit of a semi-localized M5-M5′ brane system, which was discussed in [12] as the AdS/CFT dual of the four-dimensional $N$=2 SCFT on the intersection of the M5-brane stacks.

[2] For the maximally supersymmetric case a discussion of the SCFT effective action, the conformal anomaly and the role of conformal supergravity in AdS/CFT can be found in [15]. Explicit constructions for the boundary of AdS are given there for the metric-dilaton sector.

set of boundary fields, constituting the $N$=2 Weyl multiplet. These results are extended to the nonlinear theory in Section 3.2, where we argue for the consistency of the previous construction with the interaction terms. We also determine those of the subdominant bulk field components which then enter the boundary symmetry transformations. The residual bulk symmetries preserving the gauge fixings, and their action on the boundary fields are determined in Section 3.3. This yields the complete local $N$=2 superconformal transformations of the Weyl multiplet. We conclude in Section 4. Two appendices contain an overview of the conventions and connect our results to the literature on $N$=2 superconformal gravity.

## 2 Romans' N=4 SU(2)⊗U(1) gauged supergravity

In this section we briefly discuss the five-dimensional gauged supergravity [8] in order to fix notation. The theory has $N$=4 supersymmetry (counted in terms of symplectic Majorana spinors) with $R$-symmetry group USp(4), of which an SU(2)⊗U(1) subgroup is gauged. The symplectic metric is denoted by $\Omega$, and exploiting the isomorphism $\mathfrak{usp}(4) \cong \mathfrak{so}(5)$ the Lie algebra generators are given by $\Gamma_{mn} := \frac{1}{2}[\Gamma_m, \Gamma_n]$ with $\mathfrak{so}(5)$ vector indices $m, n$, and $\Gamma_m$ satisfying the five-dimensional Euclidean Clifford algebra relation[3] $\{\Gamma_m, \Gamma_n\} = 2\delta_{mn}\mathbb{1}$. With the obvious embedding of $\mathfrak{su}(2)\oplus\mathfrak{u}(1) \cong \mathfrak{so}(3)\oplus\mathfrak{so}(2)$ into $\mathfrak{usp}(4) \cong \mathfrak{so}(5)$, the vector index $m$ decomposes into $m = (I, \alpha)$ with $I = 1, 2, 3$ and $\alpha = 4, 5$. We consider the theory referred to as $N$=$4^+$ in [8], for which the SU(2) gauge coupling $g_2$ is fixed in terms of the U(1) coupling $g_1$ by $g_2 = +\sqrt{2}g_1 =: g$. For this choice of couplings the theory admits an AdS solution. The bosonic field content is given by the vielbein $e_\mu^a$, two antisymmetric tensor fields $B^\alpha_{\mu\nu}$, the SU(2) and U(1) gauge fields $A^I_\mu$ and $a_\mu$, respectively, and a scalar $\varphi$. The four gravitinos $\psi^i_\mu$ and four spin-$\frac{1}{2}$ fermions $\chi^i$ comprising the fermionic field content are in the spinor $\mathbf{4}$ of $\mathfrak{usp}(4)$, which decomposes as $\mathbf{4} \to \mathbf{2}_{1/2} + \mathbf{2}_{-1/2}$. The vector and tensor fields originate from the vector representation, decomposing as $\mathbf{5} \to \mathbf{3}_0 + \mathbf{1}_1 + \mathbf{1}_{-1}$. The spinors satisfy the symplectic Majorana condition, e.g. $\bar{\chi}^i = \left(\chi^i\right)^T C$ with the conjugate $\bar{\chi}^i := (\chi_i)^\dagger \gamma_0$, the metric is of signature $(+,-,-,-,-)$ and the $\gamma$-matrices are chosen such that $\gamma_{abcde} = \epsilon_{abcde}$ with $\epsilon_{01234} = 1$. For a summary of the conventions see Appendix A. From this point on we denote five-dimensional objects with hat and four-dimensional ones without, e.g. five-dimensional spacetime indices $\hat{\mu} = (\mu, r)$ with $\mu = 0, 1, 2, 3$. The Lagrangian as given up to four-fermion terms in [8] is

$$
\begin{aligned}
\mathcal{L} = &- \frac{1}{4}\hat{e}\hat{\mathcal{R}}(\hat{\omega}) - \frac{1}{2}i\hat{e}\hat{\bar{\psi}}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}\hat{\nu}\hat{\rho}}\hat{D}_{\hat{\nu}}\hat{\psi}_{\hat{\rho}i} + \frac{3}{2}i\hat{e}T_{ij}\hat{\bar{\psi}}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\psi}^j_{\hat{\nu}} - i\hat{e}A_{ij}\hat{\bar{\psi}}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\hat{\chi}^j + \frac{1}{2}i\hat{e}\hat{\bar{\chi}}^i\hat{\gamma}^{\hat{\mu}}\hat{D}_{\hat{\mu}}\hat{\chi}_i \\
&+ i\hat{e}\left(\frac{1}{2}T_{ij} - \frac{1}{\sqrt{3}}A_{ij}\right)\hat{\bar{\chi}}^i\hat{\chi}^j + \frac{1}{2}\hat{e}\hat{D}^{\hat{\mu}}\hat{\varphi}\hat{D}_{\hat{\mu}}\hat{\varphi} + \hat{e}P(\hat{\varphi}) - \frac{1}{4}\hat{e}\,\xi^2\hat{B}^{\hat{\mu}\hat{\nu}\alpha}\hat{B}^{\ \alpha}_{\hat{\mu}\hat{\nu}} \\
&+ \frac{1}{4g_1}\hat{\epsilon}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}\epsilon_{\alpha\beta}\hat{B}^{\ \alpha}_{\hat{\mu}\hat{\nu}}\hat{D}_{\hat{\rho}}\hat{B}^{\ \beta}_{\hat{\sigma}\hat{\tau}} - \frac{1}{4}\hat{e}\,\xi^{-4}\hat{f}^{\hat{\mu}\hat{\nu}}\hat{f}_{\hat{\mu}\hat{\nu}} - \frac{1}{4}\hat{e}\,\xi^2\hat{F}^{\hat{\mu}\hat{\nu}I}\hat{F}^{\ I}_{\hat{\mu}\hat{\nu}} - \frac{1}{4}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}F^{\ I}_{\hat{\mu}\hat{\nu}}\hat{F}^{\ I}_{\hat{\rho}\hat{\sigma}}\hat{a}_{\hat{\tau}} \qquad (1) \\
&+ \frac{1}{4\sqrt{2}}i\hat{e}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} + \frac{1}{\sqrt{2}}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)\hat{\bar{\psi}}^{\hat{\rho}}_i\hat{\gamma}_{[\hat{\rho}}\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\gamma}_{\hat{\sigma}]}\hat{\psi}^{\hat{\sigma}}_j + \frac{1}{2\sqrt{6}}i\hat{e}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} - \sqrt{2}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)\hat{\bar{\psi}}^{\hat{\rho}}_i\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\gamma}_{\hat{\rho}}\hat{\chi}_j \\
&- \frac{1}{12\sqrt{2}}i\hat{e}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} - \frac{5}{\sqrt{2}}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)\hat{\bar{\chi}}_i\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\chi}_j + \frac{1}{\sqrt{2}}i\hat{e}\left(\partial_{\hat{\nu}}\hat{\varphi}\right)\hat{\bar{\psi}}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\nu}}\hat{\gamma}^{\hat{\mu}}\hat{\chi}_i
\end{aligned}
$$

[3]The $\Gamma_m$ can all be chosen hermitian, such that $\Gamma^\dagger_{mn} + \Gamma_{mn} = 0$. With the charge conjugation matrix $C_E$ satisfying $C_E\Gamma_m C_E^{-1} = \Gamma_m^T$, we can identify $\Omega := C_E$ and have $\Omega\Gamma_{mn} + \Gamma^T_{mn}\Omega = 0$, providing the isomorphism $\mathfrak{usp}(4) \cong \mathfrak{so}(5)$.

with $\xi := \exp\sqrt{\frac{2}{3}}\hat{\varphi}$ and the scalar potential $P(\hat{\varphi}) := \frac{1}{8}g^2\left(\xi^{-2}+2\xi\right)$. Antisymmetrization of indices is defined as $X_{[\mu}Y_{\nu]} := \frac{1}{2}(X_\mu Y_\nu - X_\nu Y_\mu)$. Furthermore,

$$\begin{aligned} T^{ij} &:= \frac{g}{12\sqrt{2}}\left(2\xi^{-1}+\xi^2\right)(\Gamma_{45})^{ij}\,, \qquad A^{ij} := \frac{g}{2\sqrt{6}}\left(\xi^{-1}-\xi^2\right)(\Gamma_{45})^{ij}\;, \\ H^{ij}_{\hat{\mu}\hat{\nu}} &:= \xi\left(\hat{F}^I_{\hat{\mu}\hat{\nu}}(\Gamma_I)^{ij} + \hat{B}^\alpha_{\hat{\mu}\hat{\nu}}(\Gamma_\alpha)^{ij}\right)\;, \qquad h^{ij}_{\hat{\mu}\hat{\nu}} := \xi^{-2}\Omega^{ij}\hat{f}_{\hat{\mu}\hat{\nu}}\;. \end{aligned} \tag{2}$$

The covariant derivative on the spinor **4** of $\mathfrak{usp}(4)$ is given by

$$\hat{D}_{\hat{\mu}}v_i = \hat{\nabla}_{\hat{\mu}}v_i + \frac{1}{2}g_1\hat{a}_{\hat{\mu}}\left(\Gamma_{45}\right)_i{}^j v_j + \frac{1}{2}g_2\hat{A}^I_{\hat{\mu}}\left(\Gamma_{I45}\right)_i{}^j v_j\;, \tag{3}$$

with the spacetime-covariant derivative $\hat{\nabla}_{\hat{\mu}}$ and $\Gamma_{IJ} = -\epsilon^{IJK}\Gamma_{K45}$. Acting on a spinor $\hat{\nabla}_{\hat{\mu}} = \partial_{\hat{\mu}} + \frac{1}{4}\hat{\omega}_{\hat{\mu}}^{\hat{a}\hat{b}}\hat{\gamma}_{\hat{a}\hat{b}}$, and the curvatures are defined by

$$\left[\hat{D}_{\hat{\mu}},\hat{D}_{\hat{\nu}}\right]\hat{\epsilon}_i =: \frac{1}{4}\hat{R}^{\hat{a}\hat{b}}_{\hat{\mu}\hat{\nu}}\hat{\gamma}_{\hat{a}\hat{b}}\epsilon_i + \frac{1}{2}g_1\hat{f}_{\hat{\mu}\hat{\nu}}\left(\Gamma_{45}\right)_i{}^j\hat{\epsilon}_j + \frac{1}{2}g_2\hat{F}^I_{\hat{\mu}\hat{\nu}}\left(\Gamma_{I45}\right)_i{}^j\hat{\epsilon}_j\;. \tag{4}$$

On the vector **5** of $\mathfrak{usp}(4)$ the covariant derivative is given by

$$\hat{D}_{\hat{\mu}}v^{I\alpha} = \hat{\nabla}_{\hat{\mu}}v^{I\alpha} + g_1\hat{a}_{\hat{\mu}}\epsilon^{\alpha\beta}v^{I\beta} + g_2\epsilon^{IJK}\hat{A}^J_{\hat{\mu}}v^{K\alpha}\;. \tag{5}$$

The supersymmetry transformations to leading order in the fermionic terms are

$$\begin{aligned} &\delta_{\hat{\epsilon}}\hat{e}^{\hat{a}}_{\hat{\mu}} = i\bar{\hat{\psi}}^i_{\hat{\mu}}\hat{\gamma}^{\hat{a}}\hat{\varepsilon}_i\;, \qquad \delta_{\hat{\epsilon}}\hat{A}^I_{\hat{\mu}} = \Theta^{ij}_{\hat{\mu}}\left(\Gamma^I\right)_{ij}, \qquad \delta_{\hat{\epsilon}}\hat{\varphi} = \frac{1}{\sqrt{2}}i\bar{\hat{\chi}}^i\hat{\varepsilon}_i\;, \\ &\delta_{\hat{\epsilon}}\hat{\psi}_{\hat{\mu}i} = \hat{D}_{\hat{\mu}}\hat{\varepsilon}_i + \hat{\gamma}_{\hat{\mu}}T_{ij}\hat{\varepsilon}^j - \frac{1}{6\sqrt{2}}\left(\hat{\gamma}_{\hat{\mu}}{}^{\hat{\nu}\hat{\rho}} - 4\delta_{\hat{\mu}}{}^{\hat{\nu}}\hat{\gamma}^{\hat{\rho}}\right)\left(H_{\hat{\nu}\hat{\rho}ij} + \frac{1}{\sqrt{2}}h_{\hat{\nu}\hat{\rho}ij}\right)\hat{\varepsilon}^j\;, \\ &\delta_{\hat{\epsilon}}\hat{a}_{\hat{\mu}} = \frac{1}{2}i\xi^2\left(\bar{\hat{\psi}}^i_{\hat{\mu}}\hat{\varepsilon}_i + \frac{2}{\sqrt{3}}\bar{\hat{\chi}}^i\hat{\gamma}_{\hat{\mu}}\hat{\varepsilon}_i\right)\;, \\ &\delta_{\hat{\epsilon}}\hat{\chi}_i = \frac{1}{\sqrt{2}}\hat{\gamma}^{\hat{\mu}}\left(\partial_{\hat{\mu}}\hat{\varphi}\right)\hat{\varepsilon}_i + A_{ij}\hat{\varepsilon}^j - \frac{1}{2\sqrt{6}}\hat{\gamma}^{\hat{\mu}\hat{\nu}}\left(H_{\hat{\mu}\hat{\nu}ij} - \sqrt{2}h_{\hat{\mu}\hat{\nu}ij}\right)\hat{\varepsilon}^j\;, \\ &\delta_{\hat{\epsilon}}\hat{B}^\alpha_{\hat{\mu}\hat{\nu}} = 2\hat{D}_{[\hat{\mu}}\Theta^{ij}_{\hat{\nu}]}\left(\Gamma^\alpha\right)_{ij} - \frac{ig_1}{\sqrt{2}}\epsilon^{\alpha\beta}\left(\Gamma_\beta\right)_{ij}\xi\left(\bar{\hat{\psi}}^i_{[\hat{\mu}}\hat{\gamma}_{\hat{\nu}]}\hat{\epsilon}^j + \frac{1}{2\sqrt{3}}\bar{\hat{\chi}}^i\hat{\gamma}_{\hat{\mu}\hat{\nu}}\hat{\epsilon}^j\right)\;, \end{aligned} \tag{6}$$

where $\Theta^{ij}_{\hat{\mu}} = \sqrt{\frac{1}{2}}i\xi^{-1}\left(-\bar{\hat{\psi}}^i_{\hat{\mu}}\hat{\epsilon}^j + \sqrt{\frac{1}{3}}\bar{\hat{\chi}}^i\hat{\gamma}_{\hat{\mu}}\hat{\epsilon}^j\right)$. The commutator of two supersymmetries is – to leading order in the fermionic fields – given by

$$\left[\delta_{\hat{\epsilon}_2},\delta_{\hat{\epsilon}_1}\right] = \delta_{\hat{X}} + \delta_{\hat{\Sigma}} + \delta_{\hat{\sigma}} + \delta_{\hat{\tau}^I}\;, \tag{7}$$

where $\delta_{\hat{X}}$ denotes a diffeomorphism with $\hat{X}^{\hat{\mu}} = -i\bar{\hat{\epsilon}}^i_1\hat{\gamma}^{\hat{\mu}}\hat{\epsilon}_{2i}$ , $\delta_{\hat{\Sigma}}$ is a local Lorentz transformation with

$$\hat{\Sigma}^{\hat{a}\hat{b}} = \hat{X}^{\hat{\mu}}\hat{\omega}^{\hat{a}\hat{b}}_{\hat{\mu}} + 2i\bar{\hat{\epsilon}}^i_1\left(-\hat{\gamma}^{\hat{a}\hat{b}}T_{ij} + \frac{1}{6\sqrt{2}}\left(\hat{\gamma}^{\hat{a}\hat{b}}{}_{\hat{c}\hat{d}} + 4\delta^{\hat{a}}_{\hat{c}}\delta^{\hat{b}}_{\hat{d}}\right)\left(H^{\hat{c}\hat{d}}_{ij} + \frac{1}{\sqrt{2}}h^{\hat{c}\hat{d}}_{ij}\right)\right)\hat{\epsilon}^j_2\;, \tag{8}$$

and $\delta_{\hat{\sigma}}$ and $\delta_{\hat{\tau}^I}$ denote U(1) and SU(2) gauge transformations, respectively, with

$$\hat{\sigma} = \hat{X}^{\hat{\mu}}\hat{a}_{\hat{\mu}} + \frac{1}{2}i\xi^2\bar{\hat{\epsilon}}^i_1\hat{\epsilon}_{2i}\;, \qquad \hat{\tau}^I = \hat{X}^{\hat{\mu}}\hat{A}^I_{\hat{\mu}} - \frac{1}{\sqrt{2}}i\xi^{-1}\left(\Gamma^I\right)_{ij}\bar{\hat{\epsilon}}^i_1\hat{\epsilon}^j_2\;. \tag{9}$$

## 3 Local N=2 superconformal symmetry on the boundary of asymptotically-AdS configurations

We now restrict the configuration space of the theory discussed in the previous section to geometries which are asymptotically $AdS_5$, and discuss the fields and symmetries induced on the conformal boundary. We give a brief discussion of asymptotically-AdS spaces in the following, and refer to [19] for more details. The metric signature and curvature conventions are those of Section 2 and [8], i.e. AdS has positive curvature.

A metric $\hat{g}$ on the interior of a compact space $X$ is called conformally compact if, for a defining function $r$ of the boundary $\partial X$ (meaning that $r|_{\partial X} = 0$, $dr|_{\partial X} \neq 0$ and $r|_{\text{int}X} > 0$), the rescaled metric $\bar{g} := r^2 \hat{g}$ extends to all of $X$ as a metric. For such a conformally compact metric $\hat{g}$ the conformal structure $[\bar{g}|_{T\partial X}]$ induced on $\partial X$ and the boundary restriction of the function $|dr|^2_{\bar{g}} := \bar{g}^{-1}(dr, dr)$ are independent of the choice of defining function. The curvature of the metric $\hat{g}$ is given by

$$\hat{\mathcal{R}}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} = -|dr|^2_{\bar{g}} \left(\hat{g}_{\hat{\mu}\hat{\rho}}\hat{g}_{\hat{\nu}\hat{\sigma}} - \hat{g}_{\hat{\mu}\hat{\sigma}}\hat{g}_{\hat{\nu}\hat{\rho}}\right) + \mathcal{O}(r^{-3}) \ , \tag{10}$$

where we denote tangent-space indices on $TX$ with hat, e.g. $\hat{\mu}$, $\hat{\nu}$, and tangent-space indices on $T\partial X$ are denoted without hat. Asymptotically, $\hat{g}$ thus has constant sectional curvature given by $-|dr|^2_{\bar{g}}$, and we call a conformally compact metric $\hat{g}$ an asymptotically-AdS metric if the value of the sectional curvature is positive and constant on the boundary, i.e. $|dr|^2_{\bar{g}} = -1/R^2$ on $\partial X$ for some constant $R$. Note that we do not demand $\hat{g}$ to be Einstein.

A representative metric $g^{(0)}$ of the boundary conformal structure uniquely determines a defining function $r$ such that $g^{(0)} = \frac{r^2}{R^2}\hat{g}\,|_{T\partial X}$ and $|dr|^2_{\bar{g}} = -1/R^2$ in a neighbourhood of $\partial X$. Choosing this defining function as radial coordinate, the metric $\hat{g}$ takes the Fefferman-Graham form

$$\hat{g} = \frac{R^2}{r^2}\left(g_{\mu\nu}dx^\mu \otimes dx^\nu - dr \otimes dr\right), \qquad g_{\mu\nu}(x,r) = g^{(0)}_{\mu\nu}(x) + \frac{r^2}{R^2} g^{(2)}_{\mu\nu}(x) + \ldots \tag{11}$$

with $g$ of signature $(+,-,-,-)$ and the limit $r \to 0$ corresponding to the conformal boundary. The expansion of $g$ in powers of $r$ is justified when $\hat{g}$ satisfies vacuum Einstein equations, which, however, we do not assume here. For the time being we will still use that expansion and refer the discussion of its validity to Section 3.2.

According with the Fefferman-Graham form of the metric, we partially gauge-fix the local Lorentz symmetry such that the vielbein is of the form

$$\hat{e}^a_\mu(x,r) = \frac{R}{r} e^a_\mu(x,r), \qquad \hat{e}^{\underline{r}}_\mu = \hat{e}^a_r = 0, \qquad \hat{e}^{\underline{r}}_r = \frac{R}{r} \ , \tag{12}$$

with $e^a_\mu(x,r) = e^{(0)a}_\mu(x) + r e^{(1)a}_\mu(x) + \ldots$ . We denote Lorentz indices by $\hat{a} = (a, \underline{r})$ with an underline below $r$ to avoid confusion. For the gravitinos and the SU(2)⊗U(1) gauge fields we employ axial gauges $\hat{\psi}_{ri} \equiv \hat{A}^I_r \equiv \hat{a}_r \equiv 0$.

In this setting we construct the fields induced on the conformal boundary in Section 3.1. For the discussion of the induced symmetry tranformations we will be interested in the residual bulk symmetries preserving the gauge-fixing conditions. These are to be determined as solutions to

$$\left(\delta_{\hat{X}} + \delta_{\hat{\Sigma}} + \delta_{\hat{\epsilon}_i} + \delta_{\mathrm{U}(1)} + \delta_{\mathrm{SU}(2)}\right)\{\hat{e}^{\underline{r}}_r, \hat{e}^a_r, \hat{e}^{\underline{r}}_\mu, \hat{a}_r, \hat{A}^I_r, \hat{\psi}_{ri}\} = 0 \ , \tag{13}$$

where $\delta_{\hat{X}}$, $\delta_{\hat{\Sigma}}$, $\delta_{\hat{\epsilon}}$ denote diffeomorphisms, local Lorentz and supersymmetry transformations, respectively. The solutions and their action on the boundary fields will be discussed in Section 3.3.

The spin connection is treated in $1.5^{\text{th}}$-order formalism and fixed by its equation of motion as derived from (1). We split $\hat{\omega}_{\hat{\mu}\hat{a}\hat{b}} = \hat{\omega}_{\hat{\mu}\hat{a}\hat{b}}(\hat{e}) + \hat{\omega}_{\hat{\mu}\hat{a}\hat{b}}(\hat{e},\hat{\psi},\hat{\chi})$ where the torsion-free part $\hat{\omega}_{\hat{\mu}\hat{a}\hat{b}}(\hat{e})$ calculated from (12) has the non-vanishing components

$$\hat{\omega}_{\mu}^{ab}(\hat{e}) = \omega_{\mu}^{ab}(e) , \qquad \hat{\omega}_{\mu}^{a\underline{r}}(\hat{e}) = \frac{1}{r}e_{\mu}^{a} - \frac{1}{2}e^{\rho a}\partial_r g_{\mu\rho} \qquad \hat{\omega}_{r}^{ab}(\hat{e}) = e^{\mu[a}\partial_r e_{\mu}^{b]} , \tag{14}$$

and for the remaining part involving fermions we find

$$\hat{\omega}_{\hat{\mu}\hat{a}\hat{b}}(\hat{e},\hat{\psi},\hat{\chi}) = -\frac{1}{2}i\left(\hat{\bar{\psi}}^{i}_{\hat{a}}\hat{\gamma}_{\hat{\mu}}\hat{\psi}_{\hat{b}i} + 2\hat{\bar{\psi}}^{i}_{\hat{\mu}}\hat{\gamma}_{[\hat{a}}\hat{\psi}_{\hat{b}]i}\right) - \frac{1}{4}i\hat{\bar{\psi}}^{i}_{\hat{\lambda}}\hat{\gamma}_{\hat{\mu}\hat{a}\hat{b}}{}^{\hat{\lambda}\hat{\tau}}\hat{\psi}_{\hat{\tau}i} - \frac{1}{4}i\hat{\bar{\chi}}^{i}\hat{\gamma}_{\hat{\mu}\hat{a}\hat{b}}\hat{\chi}_{i} . \tag{15}$$

Thus, the Lorentz-covariant derivative on spinor fields reads

$$\begin{aligned} \hat{\nabla}_{\mu} &= \nabla^{(e)}_{\mu} + \frac{1}{2r}\gamma_{\mu}\gamma_{\underline{r}} - Z_{\mu} + \frac{1}{4}\hat{\omega}_{\mu}^{\hat{a}\hat{b}}(e,\hat{\psi},\hat{\chi})\hat{\gamma}_{\hat{a}\hat{b}} \quad =: \nabla_{\mu} + \frac{1}{2r}\gamma_{\mu}\gamma_{\underline{r}} , \\ \hat{\nabla}_{r} &= \partial_r - Z_r + \frac{1}{4}\hat{\omega}_{r}^{\hat{a}\hat{b}}(e,\hat{\psi},\hat{\chi})\hat{\gamma}_{\hat{a}\hat{b}} , \end{aligned} \tag{16}$$

where $\hat{\gamma}_{\hat{\mu}} = \hat{e}_{\hat{\mu}}^{\hat{a}}\gamma_{\hat{a}}$ , $\gamma_{\mu} = e_{\mu}^{a}\gamma_{a}$. For notational convenience we defined $\nabla^{(e)}_{\mu} := \partial_{\mu} + \frac{1}{4}\omega_{\mu}^{ab}(e)\gamma_{ab}$ and $Z_{\mu} := \frac{1}{4}\left(\partial_r g_{\mu\rho}\right)\gamma^{\rho}\gamma_{\underline{r}}$ , $Z_r := \frac{1}{4}(\partial_r e_{\mu}^{a})\gamma_{a}{}^{\mu}$ .

### 3.1 Boundary fields

In this section we construct the fields induced on the conformal boundary. Similar to the construction of the induced conformal structure on the boundary, we define the classical boundary field as follows. For a bulk field $\hat{\phi}$ with asymptotic $r$-dependence $\hat{\phi}(x,r) = \mathcal{O}(f(r))$, we define the rescaled field $\phi(x,r) := f(r)^{-1}\hat{\phi}(x,r)$. This rescaled field then admits a finite, nonvanishing boundary limit, which is interpreted as the boundary field[4].

Therefore, to determine the multiplet of boundary fields, we have to fix the asymptotic scaling of the various fields. To this end we consider their equations of motion linearized in all fields but the metric/vielbein and decomposed into boundary-irreducible components, e.g. into four-dimensional chiral components for a bulk spinor field. The leading order in the boundary limit turns out to be an ordinary differential equation in $r$, and is solved by fixing the scalings of the different boundary-irreducible bulk field components. The rescaled field is defined by extracting the asymptotic $r$-dependence of the dominant field component, thereby subdominant components are projected out in the definition of the boundary field. The results obtained in this way on the basis of the linearized field equations are extended to the nonlinear theory in Section 3.2.

We start with the vielbein, for which the asymptotic $r$-dependence is already fixed by (11), (12) and the induced boundary field is given by $e_{\mu}^{a}(x,0)$. As discussed in [8], Einstein's equations as derived from (1) in a pure metric-dilaton background read

$$\hat{\mathcal{R}}_{\hat{\mu}\hat{\nu}} - \frac{1}{2}\hat{g}_{\hat{\mu}\hat{\nu}}\hat{\mathcal{R}} + 2\hat{g}_{\hat{\mu}\hat{\nu}}P(\hat{\varphi}) = 0 , \tag{17}$$

[4]This is the classical analog to the construction for the Wightman field in [17].

and the scalar potential $P(\hat\varphi)$, having exactly one extremum $\big(\hat\varphi, P(\hat\varphi)\big) \equiv \big(0, \frac{3}{8}g^2\big)$, provides a cosmological constant such that $\mathrm{AdS}_5$ is a vacuum solution. Here we do not restrict the theory to the metric-dilaton sector and only demand (17) to be solved at leading order in the boundary limit. From (10) we find that $\hat g$ indeed solves the leading order provided that the asymptotic curvature radius $R$ is fixed in terms of the gauge coupling as $R^2 = 8/g^2$. In Section 3.2 we show that – with the scalings obtained in this section – all other terms in the complete Einstein equations contribute to the subleading orders only. In the following we fix $g = 2\sqrt{2}$ such that $R = 1$.

For the gravitinos, which we consider next, the nonlinear equation of motion reads

$$\begin{aligned}\hat\gamma^{\hat\mu\hat\nu\hat\rho}\hat D_{\hat\nu}\hat\psi_{\hat\rho i} - 3T_{ij}\hat\gamma^{\hat\mu\hat\nu}\hat\psi^j_{\hat\nu} = &-\frac{1}{2\sqrt{2}}\Big(H^{\hat\rho\hat\sigma}{}_i{}^j + \frac{1}{\sqrt{2}}h^{\hat\rho\hat\sigma}{}_i{}^j\Big)\hat\gamma^{[\hat\mu}\hat\gamma_{\hat\rho\hat\sigma}\hat\gamma^{\hat\nu]}\hat\psi_{\hat\nu j} - A_{ij}\hat\gamma^{\hat\mu}\chi^j \\ &-\frac{1}{2\sqrt{6}}\Big(H_{\hat\rho\hat\sigma i}{}^j - \sqrt{2}h_{\hat\rho\hat\sigma i}{}^j\Big)\hat\gamma^{\hat\rho\hat\sigma}\hat\gamma^{\hat\mu}\chi_j + \frac{1}{\sqrt{2}}\left(\partial_{\hat\nu}\hat\varphi\right)\hat\gamma^{\hat\nu}\hat\gamma^{\hat\mu}\chi_i\ .\end{aligned} \tag{18}$$

To fix $T_{ij}$ (see (2)) we note that, since it squares to $-\mathbb{1}$ and is traceless, $\Gamma_{45}$ has eigenvalues $\pm i$, each with multiplicity 2. We choose a $\mathfrak{usp}(4)$ basis where $\Gamma_{45}$ is diagonal $(\Gamma_{45})_i{}^j = i\kappa_i\delta_i{}^j$ and split $i = (i_+, i_-)$ such that $\kappa_{i_\pm} = \pm 1$. Since $\Gamma_{45}$ is diagonal $\{\Omega, \Gamma_{45}\} = 0$, and consequently $\Omega^{i_+j_+} = \Omega^{i_-j_-} = 0$. Defining four-dimensional chirality projectors $P_{\mathrm{L/R}} := \frac{1}{2}\left(1 \pm i\gamma^{\underline{r}}\right)$, the L/R projections of the linearized equation (18) for $\hat\mu = \mu$ read

$$\gamma^{\mu\nu\rho}\nabla^{(e)}_\nu\hat\psi^{\mathrm{R/L}}_{\rho i} - \left(\gamma^{\mu\nu\rho}Z_\nu \pm i\gamma^{\mu\rho}Z_r\right)\hat\psi^{\mathrm{L/R}}_{\rho i} + i\gamma^{\mu\rho}\left(\pm\partial_r \mp \frac{1}{r} + \frac{3\kappa_i}{2r}\right)\hat\psi^{\mathrm{L/R}}_{\rho i} = 0\ . \tag{19}$$

Since the $\hat\psi^{\mathrm{L/R}}_{\mu i_-}$ are related to the conjugates of $\hat\psi^{\mathrm{R/L}}_{\mu i_+}$ by the symplectic Majorana condition, it is sufficient to consider the $i_+$-components. Solving (19) at leading order in $r$ yields the two independent solutions $\hat\psi_{\mu i_+} = r^{-1/2}\psi^{\mathrm{L}}_{\mu i_+} + o(r^{-1/2})$ and $\hat\psi_{\mu i_+} = r^{5/2}\psi^{\mathrm{R}}_{\mu i_+} + o(r^{5/2})$ with $\lim_{r\to 0}\psi^{\mathrm{L/R}}_{\mu i_+}$ finite. Thus, the gravitinos lose half of their components in the boundary limit and the rescaled field $\psi_{\mu i_+} := r^{1/2}\hat\psi_{\mu i_+}$ yields the two chiral gravitinos $\psi^{\mathrm{L}}_{\mu i_+}|_{r=0}$ as boundary fields.

Proceeding with the fermionic fields we now discuss the spin-$\frac{1}{2}$ fermions $\hat\chi_i$. Their equation of motion is given by

$$\begin{aligned}\hat\gamma^{\hat\mu}\hat D_{\hat\mu}\hat\chi_i + T_{ij}\hat\chi^j = &\frac{2}{\sqrt{3}}A_{ij}\hat\chi^j + A_{ij}\hat\gamma^{\hat\mu}\hat\psi^j_{\hat\mu} + \frac{1}{2\sqrt{6}}\Big(H_{\hat\mu\hat\nu i}{}^j - \sqrt{2}h_{\hat\mu\hat\nu i}{}^j\Big)\hat\gamma^{\hat\rho}\hat\gamma^{\hat\mu\hat\nu}\hat\psi_{\rho j} \\ &-\frac{1}{6\sqrt{2}}\Big(H_{\hat\mu\hat\nu i}{}^j - \frac{5}{\sqrt{2}}h_{\hat\mu\hat\nu i}{}^j\Big)\hat\gamma^{\hat\mu\hat\nu}\hat\chi_j + \frac{1}{\sqrt{2}}\left(\partial_{\hat\nu}\hat\varphi\right)\hat\gamma^{\hat\mu}\hat\gamma^{\hat\nu}\hat\psi_{\hat\mu i}\ .\end{aligned} \tag{20}$$

Solving the linearized L/R projections given by

$$\gamma^\mu\nabla^{(e)}_\mu\hat\chi^{\mathrm{R/L}}_i - \left(\gamma^\mu Z_\mu \mp iZ_r\right)\hat\chi^{\mathrm{L/R}}_i - i\left(\pm\partial_r + \frac{\kappa_i \mp 4}{2r}\right)\hat\chi^{\mathrm{L/R}}_i = 0 \tag{21}$$

at leading order for $i = i_+$ we find as dominant solution $\hat\chi_{i_+} = r^{3/2}\chi^{\mathrm{L}}_{i_+} + o(r^{3/2})$. Similarly to the gravitinos, the $\hat\chi_{i_+}$ become chiral in the boundary limit and we have the two lefthanded Weyl fermions $\chi^{\mathrm{L}}_{i_+}|_{r=0}$ as boundary fields.

Coming to the tensor fields $\hat{B}^{\alpha}_{\hat{\mu}\hat{\nu}}$ we define $\hat{C}_{\hat{\mu}\hat{\nu}} := \frac{1}{\sqrt{2}}(\hat{B}^4_{\hat{\mu}\hat{\nu}} - i\hat{B}^5_{\hat{\mu}\hat{\nu}})$ and, with the four-dimensional Hodge dual $\star\hat{C}_{\mu\nu} := \frac{1}{2}e^{-1}\epsilon_{\mu\nu}{}^{\rho\sigma}\hat{C}_{\rho\sigma}$, the (anti-)selfdual parts of $\hat{C}_{\mu\nu}$ are defined as $\hat{C}^{\pm}_{\mu\nu} := \frac{1}{2}(\hat{C}_{\mu\nu} \pm i\star\hat{C}_{\mu\nu})$. The equation of motion reads

$$\frac{i}{g_1}\hat{\epsilon}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}\hat{D}_{\hat{\rho}}\hat{C}_{\hat{\sigma}\hat{\tau}} - \hat{e}\xi^2\hat{C}^{\hat{\mu}\hat{\nu}} = -\frac{1}{2}\hat{e}\xi\Big(\frac{1}{2}J_1{}^{\hat{\mu}\hat{\nu}}_{ij} + \frac{1}{\sqrt{3}}J_2{}^{\hat{\mu}\hat{\nu}}_{ij} - \frac{1}{6}J_3{}^{\hat{\mu}\hat{\nu}}_{ij}\Big)\left(\Gamma_4 - i\Gamma_5\right)^{ij} \ , \tag{22}$$

with $J_1{}^{\hat{\mu}\hat{\nu}}_{ij} = i\bar{\hat{\psi}}{}^{\rho}_i\hat{\gamma}_{[\rho}\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\gamma}_{\sigma]}\hat{\psi}^{\sigma}_j$ , $J_2{}^{\hat{\mu}\hat{\nu}}_{ij} = i\bar{\hat{\psi}}{}^{\rho}_i\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\gamma}_{\rho}\hat{\chi}_j$ and $J_3{}^{\hat{\mu}\hat{\nu}}_{ij} = i\bar{\hat{\chi}}_i\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\chi}_j$. From the $\mu r$-components of the linearized equation $\hat{C}_{\mu r}$ is fixed in terms of $\hat{C}_{\mu\nu}$ by $\hat{C}_{\mu r} = \frac{1}{2}ire^{-1}\epsilon_{\mu}{}^{\rho\sigma\tau}\partial_{\rho}\hat{C}_{\hat{\sigma}\hat{\tau}}$, and is of higher order in $r$. The (anti-)selfdual parts of the linearized $\mu\nu$-components

$$\frac{1}{2}e^{-1}\epsilon_{\mu\nu}{}^{\rho\sigma}\left(\partial_r\hat{C}_{\mu\nu} + 2\partial_{\rho}\hat{C}_{\sigma r}\right) = -\frac{i}{r}\hat{C}_{\mu\nu} \ , \tag{23}$$

then yield the solutions $\hat{C}_{\mu\nu} = r^{-1}C^{-}_{\mu\nu} + o(r^{-1})$ and $\hat{C}_{\mu\nu} = r\,C^{+}_{\mu\nu} + o(r)$. Thus, the anti-selfdual part $\hat{C}^{-}$ is dominant in the boundary limit and the selfdual part $\hat{C}^{+}$ is projected out in the definition of the boundary field.

For the U(1) and SU(2) gauge fields the equations of motion are

$$\begin{aligned}\partial_{\hat{\nu}}\left(\hat{e}\xi^{-4}\hat{f}^{\hat{\mu}\hat{\nu}}\right) &= \frac{1}{4}\hat{e}g_1\left(\Gamma_{45}\right)_i{}^j J_4{}^{\hat{\mu}i}{}_j - \frac{1}{4}\hat{\epsilon}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}\left(\hat{B}^{\alpha}_{\hat{\nu}\hat{\rho}}\hat{B}^{\alpha}_{\hat{\sigma}\hat{\tau}} + \hat{F}^I_{\hat{\nu}\hat{\rho}}F^I_{\hat{\sigma}\hat{\tau}}\right)\\ &\quad + \Omega^{ij}\partial_{\hat{\nu}}\left(\hat{e}\xi^{-2}\left(\frac{1}{4}J_1{}^{\hat{\mu}\hat{\nu}}_{ij} - \frac{1}{\sqrt{3}}J_2{}^{\hat{\mu}\hat{\nu}}_{ij} + \frac{5}{12}J_3{}^{\hat{\mu}\hat{\nu}}_{ij}\right)\right) \ ,\end{aligned} \tag{24}$$

$$\hat{D}_{\hat{\nu}}\left(\hat{e}\xi^2\hat{F}^{I\hat{\mu}\hat{\nu}}\right) = \frac{1}{4}\hat{e}g_2\left(\Gamma_{I45}\right)_i{}^j J_4{}^{\hat{\mu}i}{}_j - \hat{\epsilon}^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}\hat{D}_{\hat{\nu}}\left(\hat{F}^I_{\hat{\rho}\hat{\sigma}}\hat{a}_{\hat{\tau}}\right) + \frac{1}{\sqrt{2}}\hat{D}_{\hat{\nu}}\left(\hat{e}\xi K_I^{\hat{\mu}\hat{\nu}}\right) \ , \tag{25}$$

with $J_4{}^{\hat{\mu}i}{}_j = i\bar{\hat{\chi}}^i\hat{\gamma}^{\hat{\mu}}\hat{\chi}_j - i\bar{\hat{\psi}}{}^i_{\nu}\hat{\gamma}^{\nu\hat{\mu}\rho}\hat{\psi}_{\rho j}$ and $K_I^{\hat{\mu}\hat{\nu}} = (\Gamma_I)^{ij}\left(\frac{1}{2}J_1{}^{\hat{\mu}\hat{\nu}}_{ij} + \frac{1}{\sqrt{3}}J_2{}^{\hat{\mu}\hat{\nu}}_{ij} - \frac{1}{6}J_3{}^{\hat{\mu}\hat{\nu}}_{ij}\right)$. For the ansatz $\hat{a}_{\mu} = r^{\alpha}a_{\mu}$ the leading order of the linearized equation yields $\alpha \in \{0, 2\}$, and similarly for $\hat{A}^I_{\mu}$. Thus, $\hat{a}_{\mu}$ and $\hat{A}_{\mu}$ are itself finite in the boundary limit and define boundary vector fields without rescaling.

It remains to analyze the scalar field $\hat{\varphi}$ with equation of motion

$$\begin{aligned}\Box_{\hat{g}}\hat{\varphi} - P'(\hat{\varphi}) = &- \frac{i}{\sqrt{2}}A_{ij}\bar{\hat{\psi}}{}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}\hat{\nu}}\hat{\psi}^j_{\hat{\nu}} - iA'_{ij}\bar{\hat{\psi}}{}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\mu}}\hat{\chi}^j - \frac{i}{\sqrt{3}}\Big(A'_{ij} + \frac{1}{\sqrt{6}}A_{ij}\Big)\bar{\hat{\chi}}^i\hat{\chi}^j\\ &- \sqrt{\frac{2}{3}}\xi^2\bar{\hat{C}}_{\hat{\mu}\hat{\nu}}\hat{C}^{\hat{\mu}\hat{\nu}} + \sqrt{\frac{2}{3}}\xi^{-4}\hat{f}_{\hat{\mu}\hat{\nu}}\hat{f}^{\hat{\mu}\hat{\nu}} - \frac{1}{\sqrt{6}}\xi^2\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^{I\hat{\mu}\hat{\nu}}\\ &+ \frac{1}{4\sqrt{3}}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} - \sqrt{2}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)J_1{}^{\hat{\mu}\hat{\nu}}_{ij} + \frac{1}{6}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} + 2\sqrt{2}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)J_2{}^{\hat{\mu}\hat{\nu}}_{ij}\\ &- \frac{1}{12\sqrt{3}}\Big(H^{ij}_{\hat{\mu}\hat{\nu}} + 5\sqrt{2}h^{ij}_{\hat{\mu}\hat{\nu}}\Big)J_3{}^{\hat{\mu}\hat{\nu}}_{ij} - \frac{1}{\sqrt{2}}\hat{e}^{-1}\partial_{\hat{\nu}}\left(i\hat{e}\bar{\hat{\psi}}{}^i_{\hat{\mu}}\hat{\gamma}^{\hat{\nu}}\hat{\gamma}^{\hat{\mu}}\hat{\chi}_i\right) \ ,\end{aligned} \tag{26}$$

where $A'^{ij} := -\frac{1}{6}g\left(\xi^{-1} + 2\xi^2\right)\left(\Gamma_{45}\right)^{ij}$. The linearized equation is given by

$$r^2\Box_g\hat{\varphi} - \frac{1}{2}r^2\left(g^{\mu\nu}\partial_r g_{\mu\nu}\right)\partial_r\hat{\varphi} - \mathcal{D}_r^2\hat{\varphi} = 0 \ , \tag{27}$$

with $\mathcal{D}_r = r\partial_r - 2$, and the leading-order part is solved by $r^2\varphi_1(x,r)$ and $r^2\log(r)\,\varphi_2(x,r)$ with $\varphi_{1/2}|_{r=0}$ finite. The boundary scalar field is thus defined by extracting the dominant scaling $\hat{\varphi} =: r^2\log(r)\varphi$ and restricting $\varphi$ to the boundary. In summary, the multiplet of boundary fields is given by $(e^a_{\mu}, \psi^{\mathrm{L}}_{\mu\,i_+}, C^{-}_{\mu\nu}, A^I_{\mu}, a_{\mu}, \chi^{\mathrm{L}}_{i_+}, \varphi)|_{r=0}$.

### 3.2 Nonlinear theory and subdominant components

The splitting into dominant and subdominant components and the scaling of the dominant parts as obtained above from the linearized equations of motion fixes the definition of the boundary fields. It remains to be checked whether the obtained scaling behaviour is consistent in the nonlinear theory. Furthermore, the subdominant components of some of the fields are required for the symmetry transformations to be discussed in Section 3.3. These two points are addressed in the following. Note that this discussion does not include the four-fermion terms which are not spelled out in [8]. However, as we find quite some cancellations taking place to ensure consistency of the previously obtained results at the leading orders in the fermions, we expect that this consistency is not accidental and extends to the four-fermion terms as well.

Since the analysis of Section 3.1 crucially relies on the form of the metric (11) in a neighbourhood of the boundary, the first thing to be checked is the validity of the Fefferman-Graham form. Considering the terms in the Lagrangian (1) with the scaling of the fields as obtained in the previous section, $\hat{e}\hat{\mathcal{R}}(\hat{\omega})$ and the cosmological constant $\hat{e}P(0)$ are $\mathcal{O}(r^{-5})$ while the other terms are $\mathcal{O}(r^{-3})$. Thus, the leading order of Einstein's equations reduces to the form discussed in the previous section and the Fefferman-Graham form of the metric (11) is justified. In particular, since there are no $\mathcal{O}(r^{-4})$ terms in the Lagrangian, there is no $\mathcal{O}(r)$ contribution to $g_{\mu\nu}(x,r)$ and the expansion in (11) is justified. Next, we consider the spin connection (14), (15). With the scaling as obtained before, $\hat{\omega}_{\mu ab}(\hat{e},\hat{\psi},\hat{\chi}) = \mathcal{O}(r^0)$ and the other components of $\hat{\omega}_{\hat{\mu}\hat{a}\hat{b}}(\hat{e},\hat{\psi},\hat{\chi})$ are of $\mathcal{O}(r)$. Therefore, the fermionic terms do not alter the $\mathcal{O}(r^{-1})$ part of the covariant derivative (16), which was relevant for the previous section. For the four-dimensional Lorentz-covariant derivative $\nabla_\mu$ defined in (16) we find $\nabla_\mu = \partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}$ with

$$\omega_{\mu ab}\big|_{r=0} = \omega_{\mu ab}(e) - \frac{1}{2}\left(i\bar{\psi}_a^{\mathrm{L}i_+}\gamma_\mu\psi^{\mathrm{L}}_{bi_+} + 2i\bar{\psi}_\mu^{\mathrm{L}i_+}\gamma_{[a}\psi^{\mathrm{L}}_{b]i_+} + \text{c.c.}\right) . \tag{28}$$

From (3) the four-dimensional gauge and Lorentz covariant derivative acting on a boundary spinor is

$$D_\mu v_{i_+} = \nabla_\mu v_{i_+} + \frac{1}{2}ig_1 a_\mu v_{i_+} + \frac{1}{2}ig_2 A_\mu^I\,(\Gamma_I)_{i_+}{}^{j_+}\,v_{j_+} . \tag{29}$$

For the remaining fields we study the interaction terms of (1) directly in the field equations. They turn out to be subdominant in the equations for the boundary-dominant field components, such that their scaling is not affected. They do, however, alter the subdominant components, some of which are in fact not subdominant but play the role of auxiliary fields on the boundary. We start with the gravitinos, for which the scaling of $\hat{\psi}^{\mathrm{L}}_{\mu i_+}$ was determined from the $P_{\mathrm{L}}$-projection of (18) at $\mathcal{O}(r^{3/2})$. One easily verifies that the interaction terms in (18) are of $\mathcal{O}(r^{5/2})$ and thus the analysis of the previous section is not affected. To determine the subdominant components we consider the $P_{\mathrm{R}}$ projection of the $\hat{\mu}{=}\mu$ components. Noting that $(\Gamma_\alpha)_{i_+}{}^{j_+} = (\Gamma_\alpha)_{i_-}{}^{j_-} = 0$ due to $\{\Gamma_\alpha,\Gamma_{45}\} = 0$, and $\hat{B}^\alpha_{\hat{\mu}\hat{\nu}}\,(\Gamma_\alpha)_{i_+}{}^{j_-} = \sqrt{2}\hat{C}_{\hat{\mu}\hat{\nu}}\,(\Gamma_4)_{i_+}{}^{j_-}$, we find $\hat{\psi}_{\mu i_+} = r^{-1/2}\psi^{\mathrm{L}}_{\mu i_+} + r^{1/2}\Phi^{\mathrm{R}}_{\mu i_+}$ with

$$\Phi^{\mathrm{R}}_{\mu i_+}\Big|_{r=0} = -\frac{1}{2}i\Big(\gamma_\mu{}^{\nu\rho} - \frac{2}{3}\gamma_\mu\gamma^{\nu\rho}\Big)\Big(D_\nu\psi^{\mathrm{L}}_{\rho i_+} - \frac{1}{4}\gamma\cdot C^-_{i_+j_+}\gamma_\nu\psi^{\mathrm{R}j_+}_\rho\Big) , \tag{30}$$

where $\gamma \cdot C := \gamma^{\mu\nu} C_{\mu\nu}$ and $C_{\hat\mu\hat\nu\, i_+ j_+} := C_{\hat\mu\hat\nu}\,(\Gamma_4)_{i_+ j_+}$. Note that $\psi_\mu^{\mathrm{R}i_+} = C\big(\bar\psi_\mu^{\mathrm{L}i_+}\big)^T$ by the symplectic Majorana condition, and a possible $C^+$-contribution drops out due to $\gamma \cdot C^\pm = \gamma \cdot C^\pm P_{\mathrm{R/L}}$. For later convenience we define the quantity

$$R_{\mu\nu\, i_+}(Q) := D_{[\mu}\psi^{\mathrm{L}}_{\nu] i_+} - i\gamma_{[\mu}\Phi^{\mathrm{R}}_{\nu] i_+} - \frac{1}{4}\gamma \cdot C^-_{i_+ j_+}\gamma_{[\mu}\psi^{\mathrm{R}j_+}_{\nu]} \ , \tag{31}$$

and note that it is anti-selfdual $i \star R_{\mu\nu\, i_+}(Q) = -R_{\mu\nu\, i_+}(Q)$ and satisfies $\gamma^\mu R_{\mu\nu\, i_+}(Q) = 0$.

We continue with the tensor field $\hat C_{\hat\mu\hat\nu}$. Using $\frac{1}{2}\left(\gamma_{\mu\nu} \pm i\star\gamma_{\mu\nu}\right) = \gamma_{\mu\nu}P_{\mathrm{R/L}}$ we find the interaction terms subdominant in the anti-selfdual part of (22), which was used to determine the scaling $\hat C^-_{\mu\nu} = r^{-1}C^-_{\mu\nu}$. In the selfdual part of (22) the interaction terms are not subdominant, but rather fix $\hat C^+_{\mu\nu} = r^{-1}C^+_{\mu\nu}$ with

$$C^+_{\mu\nu}\big|_{r=0} = \frac{1}{4} i\,(\Gamma_4)^{i_+ j_+}\,\bar\psi^{\mathrm{R}}_{\rho i_+}\gamma^{[\rho}\gamma_{\mu\nu}\gamma^{\sigma]}\psi^{\mathrm{L}}_{\sigma j_+} \ . \tag{32}$$

Thus, $\hat C^+_{\mu\nu}$ is in fact not subdominant with respect to $\hat C^-_{\mu\nu}$. However, since its boundary value is completely fixed in terms of the other boundary fields, $\hat C^+_{\mu\nu}$ plays the role of an auxiliary field on the boundary. From the $(\hat\mu\hat\nu)$=$(\mu r)$ components we find the subdominant $\hat C_{\mu r} = C_{\mu r}$ with

$$C_{\mu r}\big|_{r=0} = \frac{1}{2} i r e^{-1}\epsilon_\mu{}^{\nu\rho\sigma} D_\nu \hat C_{\rho\sigma} + \bar\psi^{\mathrm{R}}_{\rho i_+}\big(\gamma_\mu{}^{\rho\sigma}\Phi^{\mathrm{R}}_{\sigma j_+} + \frac{1}{\sqrt 3}\gamma_\mu\gamma^\rho\chi^{\mathrm{L}}_{j_+}\big)\,(\Gamma_4)^{i_+ j_+} \ . \tag{33}$$

For the spin-$\frac{1}{2}$ fermions $\hat\chi^{\mathrm{L}}_{i_+} = r^{3/2}\chi^{\mathrm{L}}_{i_+}$ was obtained from the $P_{\mathrm{L}}$ projection of (20) at $\mathcal{O}(r^{3/2})$. The only additional contribution at that order is $\propto \gamma^\rho\gamma\cdot C^+_{i_+ j_+}\psi^{\mathrm{R}j_+}_\rho$ which is a three-fermion term by (32) and we expect it to be cancelled by contributions of four-fermion terms in (1). We conclude that – up to the four-fermion terms not considered here – the obtained scaling for $\hat\chi^{\mathrm{L}}_{i_+}$ is not affected by the interaction terms. The subdominant righthanded part is fixed from the $P_{\mathrm{R}}$-projection of (20) and we find $\hat\chi_{i_+} = r^{3/2}\chi^{\mathrm{L}}_{i_+} + r^{5/2}\log(r)\chi^{\mathrm{R}}_{i_+}$ with

$$\begin{aligned}\chi^{\mathrm{R}}_{i_+}\Big|_{r=0} = i\not{D}\chi^{\mathrm{L}}_{i_+} - \frac{1}{\sqrt 2}\varphi\gamma^\mu\psi^{\mathrm{L}}_{\mu i_+} - \frac{i}{2\sqrt 6}\gamma^\rho\gamma^{\mu\nu}\left(F^I_{\mu\nu}\,(\Gamma_I)_{i_+}{}^{j_+} - \sqrt 2 f_{\mu\nu}\delta_{i_+}{}^{j_+}\right)\psi^{\mathrm{L}}_{\rho j_+} \\ + \frac{i}{2\sqrt 3}\gamma^\rho\gamma\cdot C^-_{i_+ j_+}\Phi^{j_+\mathrm{L}}_\rho - \frac{1}{\sqrt 3}\gamma^\rho\gamma^\mu C_{\mu r\, i_+ j_+}\psi^{j_+\mathrm{R}}_\rho \ .\end{aligned} \tag{34}$$

In the equations for the gauge fields (24), (25) the leading-order terms are those involving $J_4{}^{\hat\mu i}{}_j$ (the gravitino part thereof) and $J_1{}^{\mu\nu}_{ij}$, both of which are of $\mathcal{O}(r^{-3})$. However, since $(\Gamma_I)_{i_+}{}^{j_-} = (\Gamma_I)_{i_-}{}^{j_+} = 0$ due to $[\Gamma_I, \Gamma_{45}] = 0$, their leading-order parts cancel exactly in both equations, such that the previous analysis of the linearized equations is not altered. For the scalar field we have to check that the interaction terms are subdominant with respect to the $\mathcal{O}(r^2)$ and $\mathcal{O}(r^2\log(r))$ parts of (26). Similar to the case of the gauge fields, there are cancellations between different terms at leading order. From (32) the $J^{\mu\nu}_{1ij}$ term and the $\hat{\bar C}_{\mu\nu}\hat C^{\mu\nu}$ term add up to zero at leading order, and also $-iA'_{ij}\hat{\bar\psi}^i_{\hat\mu}\hat\gamma^{\hat\mu}\hat\chi^j$ and $-\frac{1}{\sqrt 2}\hat e^{-1}\partial_{\hat\nu}\left(i\hat e\hat{\bar\psi}^i_{\hat\mu}\hat\gamma^{\hat\nu}\hat\gamma^{\hat\mu}\hat\chi_i\right)$ cancel. The remaining terms are subleading and thus the cancellations justify the analysis of the linearized equations also for $\hat\varphi$. We conclude that the scaling behaviours obtained from the linearized equations of motion with the modifications for the subdominant components discussed here are consistent in the nonlinear theory as given by (1).

### 3.3 Induced boundary symmetries

Having obtained the multiplet of boundary fields in the previous section we now discuss the symmetries on the boundary. To this end we determine the residual bulk symmetries from the constraints (13) and examine their action on the boundary fields, which is defined straightforwardly e.g. $\delta\phi := \lim_{r\to 0} f(r)^{-1}\hat{\delta}\hat{\phi}$ for a boundary field $\phi = \lim_{r\to 0} f(r)^{-1}\hat{\phi}$. Relevant to us are solutions to the constraints (13) which act nontrivially on the boundary fields, and in the following we discuss certain special solutions which generate the general symmetry transformation of the boundary fields.

The constraint that $\hat{e}_r^{\,r}$ and $\hat{e}_r^{\,a}$ be preserved yields that, for an arbitrary $\lambda(x)$,

$$\hat{X}^r = r\lambda(x), \qquad \hat{\Sigma}^a{}_{\underline{r}} = -e^a_\mu \partial_r \hat{X}^\mu \; . \tag{35}$$

We parametrize the U(1) and SU(2) gauge transformations by $\hat{\sigma}(x,r)$ and $\hat{\tau}^I(x,r)$, respectively, and using (35) the remaining constraints are

$$\begin{aligned}
\partial_r \hat{X}^\mu =& g^{\mu\rho}\big(r\partial_\rho\lambda(x) + i\bar{\hat{\psi}}{}^i_\rho \hat{\gamma}^r \hat{\epsilon}_i\big) \; , && \text{(36a)}\\
\partial_r \hat{\sigma} =& \hat{a}_\mu \partial_r \hat{X}^\mu + \frac{1}{\sqrt{3}} i\xi^2 \bar{\hat{\chi}}{}^i \hat{\gamma}_r \hat{\epsilon}_i \; , && \text{(36b)}\\
\partial_r \hat{\tau}^I =& \hat{A}^I_\mu \partial_r \hat{X}^\mu + \frac{1}{\sqrt{6}} i\xi^{-1} \bar{\hat{\chi}}{}^i \hat{\gamma}_r \hat{\epsilon}^j \left(\Gamma^I\right)_{ij} \; , && \text{(36c)}\\
\hat{\nabla}_r \hat{\epsilon}_i + \hat{\gamma}_r T_{ij}\hat{\epsilon}^j =& -\left(\partial_r \hat{X}^\mu\right)\hat{\psi}_{\mu i} + \frac{1}{6\sqrt{2}}\Big(\hat{\gamma}_r{}^{\hat{\nu}\hat{\rho}} - 4\delta_r^{\hat{\nu}}\hat{\gamma}^{\hat{\rho}}\Big)\Big(H_{\hat{\nu}\hat{\rho}ij} + \frac{1}{\sqrt{2}} h_{\hat{\nu}\hat{\rho}ij}\Big)\hat{\epsilon}^j \; . && \text{(36d)}
\end{aligned}$$

Thus, (13) is solved for $\hat{\epsilon} \equiv 0$, $\lambda \equiv 0$ and $\hat{X}^{\hat{\mu}} = (X^\mu(x), 0)$, $\hat{\Sigma}^{\hat{a}}{}_{\hat{c}} = \delta^{\hat{a}}{}_a \delta_{\hat{c}}{}^c\, \Sigma^a{}_c(x)$, $\hat{\tau}^I = \tau^I(x)$ and $\hat{\sigma} = \sigma(x)$, acting as four-dimensional diffeomorphisms $\delta_X$, local Lorentz transformations $\delta_\Sigma$ and SU(2)⊗U(1) gauge transformations $\delta_{\tau^I}$, $\delta_\sigma$, respectively, on the boundary fields.

Furthermore, consider $\hat{\delta}_{\rm w} := \delta_{\hat{X}_{\rm w}} + \delta_{\hat{\epsilon}_{\rm w}} + \delta_{\hat{\Sigma}_{\rm w}} + \delta_{\hat{\sigma}_{\rm w}} + \delta_{\hat{\tau}^I_{\rm w}}$, with nonzero $\hat{X}^r = r\lambda$ accompanied by $\hat{\epsilon}_{{\rm w}i} = \mathcal{O}(r^{3/2})$, by $\hat{X}^\mu_{\rm w}$, $\hat{\sigma}_{\rm w}$, $\hat{\tau}^I_{\rm w}$ of $\mathcal{O}(r^2)$ and by $\hat{\Sigma}^a_{{\rm w}\,b} = 0$, $\hat{\Sigma}^a_{{\rm w}\,\underline{r}} = \mathcal{O}(r)$ to solve (35), (36). All transformations preserve the boundary fields, except for $\delta_{\hat{X}_{\rm w}}$ which acts as a Weyl rescaling. The Weyl weights of the boundary fields are fixed by the scaling of the bulk fields from which they are defined, e.g. for $\phi := \lim_{r\to 0} r^\alpha \hat{\phi}$ we have $\delta_{\rm w}\phi := \lim_{r\to 0} r^\alpha \hat{\delta}_{\rm w}\hat{\phi} = -\alpha\lambda(x)\phi$.

Finally, we set $\lambda \equiv 0$ and consider non-vanishing $\hat{\epsilon}_i$ solving (36d). Similarly to the mass terms in the spinor field equations, the $T_{ij}$-term in (36d) affects a splitting of the chiral components when solving the leading order in $r$. We find the two independent solutions $\hat{\epsilon}_{i_+} = r^{-1/2}\epsilon^{\rm L}_{i_+} + o(r^{1/2})$ and $\hat{\epsilon}_{i_+} = r^{1/2}\epsilon^{\rm R}_{i_+} + o(r^{1/2})$ with $\epsilon^{\rm L/R}_{i_+}|_{r=0}$ finite. $\hat{X}^\mu$, $\hat{\sigma}$ and $\hat{\tau}^I$ of $\mathcal{O}(r^2)$ and $\hat{\Sigma}^a{}_{\underline{r}} = \mathcal{O}(r)$ are fixed by solving the remaining constraints, such that $\delta_{\hat{X},\hat{\Sigma},\hat{\sigma},\hat{\tau}^I}$ transform the subleading modes of the bulk fields only. On the boundary fields we thus have a purely fermionic transformation $\hat{\delta}_{\hat{\epsilon}}$.

We define $\zeta_{i_+} := \epsilon^{\rm L}_{i_+}(x,0)$, $\zeta^{i_+} := \epsilon^{{\rm R}i_+}(x,0)$, such that $\zeta^{i_+}$ is related to $\zeta_{i_+}$ by the symplectic Majorana condition, and similarly $\eta_{i_+} := \epsilon^{\rm R}_{i_+}(x,0)$, $\eta^{i_+} := \epsilon^{{\rm L}i_+}(x,0)$. To leading order in the fermionic fields the $\zeta$-transformations of the boundary fields are

$$\begin{aligned}
\delta_\zeta e_\mu^a &= i\bar\psi_\mu^{\mathrm{L}i_+}\gamma^a\zeta_{i_+} + \text{c.c.}\ , \qquad \delta_\zeta\psi^{\mathrm{L}}_{\mu i_+} = D_\mu\zeta_{i_+} - \frac{1}{4}\gamma\cdot C^-_{i_+j_+}\gamma_\mu\zeta^{j_+}\ ,\\
\delta_\zeta A_\mu^I &= \frac{1}{\sqrt{2}}i\Big(\bar\Phi_\mu^{\mathrm{R}i_+}\zeta_{j_+} - \frac{1}{\sqrt{3}}\bar\chi^{\mathrm{L}i_+}\gamma_\mu\zeta_{j_+}\Big)\left(\Gamma^I\right)_{i_+}{}^{j_+} + \text{c.c.}\ ,\\
\delta_\zeta a_\mu &= \frac{1}{2}i\Big(\bar\Phi_\mu^{\mathrm{R}i_+}\zeta_{i_+} + \frac{2}{\sqrt{3}}\bar\chi^{\mathrm{L}i_+}\gamma_\mu\zeta_{i_+}\Big) + \text{c.c.}\ , \qquad \delta_\zeta\varphi = \frac{1}{\sqrt{2}}i\bar\chi^{\mathrm{R}i_+}\zeta_{i_+} + \text{c.c.}\ ,\\
\delta_\zeta\chi^{\mathrm{L}}_{i_+} &= -\frac{1}{\sqrt{2}}i\varphi\zeta_{i_+} + \frac{1}{2\sqrt{6}}\gamma^{\mu\nu}\left(F^I_{\mu\nu}\left(\Gamma_I\right)_{i_+}{}^{j_+} - \sqrt{2}f_{\mu\nu}\delta_{i_+}^{j_+}\right)\zeta_{j_+} - \frac{1}{\sqrt{3}}i\gamma^\mu C_{\mu r\, i_+j_+}\zeta^{j_+}\ ,\\
\delta_\zeta C^-_{ab} &= 2i\left(\Gamma_4\right)^{i_+j_+}\Big(\bar\zeta_{i_+}\hat R_{ab\, j_+}(Q) + \frac{1}{4}\eta_{ac}\bar\psi^{\mu\mathrm{R}}_{i_+}\gamma^{[\nu}\gamma_{b\mu}\gamma^{c]}\delta_\zeta\psi^{\mathrm{L}}_{\nu j_+}\Big)\ ,
\end{aligned} \tag{37}$$

where $\hat R_{\mu\nu\, i_+}(Q) := R_{\mu\nu\, i_+}(Q) - \frac{1}{2\sqrt{3}}i\gamma_{\mu\nu}\chi^{\mathrm{L}}_{i_+}$. These correspond to $N{=}2$ (Q-)supersymmetry transformations of the boundary fields. The $\eta$-transformations are given by

$$\begin{aligned}
&\delta_\eta e_\mu^a = 0\ , \qquad \delta_\eta\psi^{\mathrm{L}}_{\mu i_+} = -i\gamma_\mu\eta_{i_+}\ , \qquad \delta_\eta a_\mu = \frac{1}{2}i\bar\psi_\mu^{\mathrm{L}i_+}\eta_{i_+} + \text{c.c.}\ ,\\
&\delta_\eta C^-_{ab} = \frac{1}{2}i\left(\Gamma_4\right)^{i_+j_+}\eta_{ac}\bar\psi^{\mu\mathrm{R}}_{i_+}\gamma^{[\nu}\gamma_{b\mu}\gamma^{c]}\delta_\eta\psi^{\mathrm{L}}_{\nu j_+}\ , \qquad \delta_\eta\varphi = 0\ ,\\
&\delta_\eta\chi^{\mathrm{L}}_{i_+} = -\frac{1}{2\sqrt{3}}\gamma\cdot C^-_{i_+j_+}\eta^{j_+}\ , \qquad \delta_\eta A^I_\mu = \frac{1}{\sqrt{2}}i\bar\psi_\mu^{\mathrm{L}i_+}\eta_{j_+}\left(\Gamma^I\right)_{i_+}{}^{j_+} + \text{c.c.}\ ,
\end{aligned} \tag{38}$$

and correspond to special conformal (S-)supersymmetry or super-Weyl transformations. The constrained field components $\Phi^{\mathrm{R}}_{\mu i_+}$, $C^+_{\mu\nu}$ and $C_{\mu r}$ are given by (30), (32) and (33), respectively, and the covariant derivative by (29). With $\chi^{\mathrm{R}}_{i_+}$ as given in (34) the transformation of the scalar field may be rewritten as

$$\delta_\zeta\varphi = \frac{1}{\sqrt{2}}\bar\zeta^{i_+}\gamma^\mu\Big(D_\mu - \delta_\zeta(\psi_\mu) - \delta_\eta(\Phi_\mu)\Big)\chi^{\mathrm{L}}_{i_+} + \text{c.c.}\ , \tag{39}$$

where $\delta_\zeta(\psi_\mu)$ denotes a field-dependend $\zeta$-supersymmetry transformation with parameter $\zeta_{i_+} = \psi^{\mathrm{L}}_{\mu i_+}$, and analogously for $\delta_\eta(\Phi_\mu)$ with $\eta_{i_+} = \Phi^{\mathrm{R}}_{\mu i_+}$.

The commutators of Q- and S-superymmetries can be derived from (7) and we find

$$[\delta_{\zeta_2}, \delta_{\zeta_1}] = \delta_{X_\zeta} + \delta_\Sigma\left(X_\zeta^\mu\omega_\mu^{ab}\right) + \delta_\Sigma\left(2i\bar\zeta_1^{i_+}\zeta_2^{j_+}C^{-\,ab}{}_{i_+j_+} + \text{c.c.}\right) + \delta_{\sigma_\zeta} + \delta_{\tau^I_\zeta}\ , \tag{40a}$$

$$[\delta_\eta, \delta_\zeta] = \delta_{\mathrm{Weyl}}\left(\bar\zeta^{i_+}\eta_{i_+} + \text{c.c.}\right) + \delta_\Sigma\left(-\bar\zeta^{i_+}\gamma^{ab}\eta_{i_+} + \text{c.c.}\right) + \delta_{\sigma_{\eta\zeta}} + \delta_{\tau^I_{\eta\zeta}}\ , \tag{40b}$$

$$[\delta_{\eta_2}, \delta_{\eta_1}] = 0\ , \tag{40c}$$

where in (40a) the diffeomorphism is $X^\mu_\zeta = -i\bar\zeta_1^{i_+}\gamma^\mu\zeta_{2i_+} + \text{c.c.}$ and the gauge transformations are $\sigma_\zeta = X^\mu_\zeta a_\mu$, $\tau^I_\zeta = X^\mu_\zeta A^I_\mu$. The gauge transformations in (40b) are $\sigma_{\eta\zeta} = \frac{1}{2}i\bar\zeta^{i_+}\eta_{i_+} + \text{c.c.}$ and $\tau^I_{\eta\zeta} = \frac{1}{\sqrt{2}}i\left(\Gamma_I\right)_{i_+}{}^{j_+}\bar\zeta^{i_+}\eta_{j_+} + \text{c.c.}$.

Altogether, we find the boundary degrees of freedom with properties as given in Table 1 and with the fermionic symmetry transformations (37), (38). The off-shell degrees of freedom are given as the difference of field components and gauge degrees of freedom, e.g. for the chiral gravitino we count 16 components from which $2\cdot 4$ degrees of freedom are removed for the

| | $e_\mu^a$ | $\psi_{\mu i_+}^L$ | $a_\mu$, $A_\mu^I$ | $\chi_{i_+}^L$ | $C_{\mu\nu}^-$ | $\varphi$ |
|---|---|---|---|---|---|---|
| $w$ | $-1$ | $-\frac{1}{2}$ | $0$ | $\frac{3}{2}$ | $-1$ | $2$ |
| $s$ | $2$ | $\frac{3}{2}$ | $1$ | $\frac{1}{2}$ | $1$ | $0$ |
| $n$ | $5$ | $-8$ | $3$ | $-4$ | $6$ | $1$ |
| $c$ | $0$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $1$ | $0$ |

Table 1: Boundary fields with Weyl weights $w$, spin $s$ and $n$ off-shell degrees of freedom. The fermions are SU(2) doublets and $c$ denotes the U(1) charges.

chiral $\zeta$ and $\eta$ supersymmetry transformations. Likewise, of the 16 vielbein components 4 degrees of freedom are subtracted for diffeomorphisms, 6 for local Lorentz and 1 for Weyl transformations. As seen from Table 1, the total numbers of bosonic and fermionic degrees of freedom, both being 24, match nicely, and the boundary fields fill the $N$=2 Weyl multiplet, see [20, 21]. The bulk SU(2)⊗U(1) gauge symmetry has become the chiral U(2) transformations contained in SU$(2,2|2)$ to close the commutator of Q- and S-supersymmetries.

# 4 Conclusion

In this note we have studied SU(2)⊗U(1) gauged $N$=4 supergravity on asymptotically-AdS$_5$ backgrounds. We have constructed the multiplet of fields induced on the conformal boundary and determined the induced representation of the local bulk symmetries on the boundary fields. This has shown that the boundary degrees of freedom, which are given in Table 1, fill the $N$=2 Weyl multiplet and that the complete local $N$=2 superconformal transformations are induced, with Q- and S-supersymmetry transformations given in (37), (38).

For the constructions we have employed gauge fixings for the bulk symmetries, which were chosen such that they do not cause a fixing of the symmetries induced on the boundary. Different gauge fixings are expected to yield the same boundary fields and symmetries, possibly gauge fixed and/or with additional gauge degrees of freedom. An interesting task is to study this in the BRST approach. Note also that, instead of the definition as boundary limit, one may define the boundary field as pullback of the rescaled bulk field, and that the two definitions agree for the cases discussed here.

There are at least three possible applications of our results. The first does not directly involve duality relations, while the following two are devoted to the AdS/CFT correspondence, and to algebraic holography and deformations of the original AdS/CFT conjecture, respectively.

A common issue in the study of black-hole thermodynamics and in the AdS/CFT correspondence is the regularization of the on-shell action of the gravitational theory. For asymptotically-AdS spaces a well-established method is the holographic renormalization [13], i.e. introducing a cutoff on the radial coordinate and supplementing the action by boundary terms, such that removing the cutoff yields a finite action. Remarkably, as shown in [22], for lower-dimensional theories these holographic counterterms coincide with the boundary terms required by supersymmetry. With the results shown here one can calculate the holographic counterterms for the $N$=4 supergravity, and may check whether the relation to the boundary terms required by supersymmetry also applies in that case.

In the AdS/CFT correspondence the multiplet of boundary fields is coupled to the multiplet of currents of the dual SCFT and, in particular, the expectation value of the stress-energy tensor is calculated from the variation of the renormalized on-shell bulk action with respect to the boundary metric. The conformal anomaly of the dual theory is given by the trace of the stress-energy tensor and the purely-gravitational part was calculated in [14]. With our results one may extend this to a holographic calculation of the full conformal anomaly including the matter-field contributions for $N$=2 SCFTs.

Finally, a duality relation reminiscient of the AdS/CFT correspondence has been formulated and proven in [16] in the context of algebraic QFT. Although it is unclear whether the bulk theory considered here can be fit into the framework of algebraic QFT, we may still try to interpret the results in that context[5]. While the physical interpretation of the boundary theory in [16] is not immediately clear, the constructions in [17], where the boundary Wightman field is constructed as boundary limit of the rescaled AdS Wightman field, suggest that the boundary fields constructed here indeed constitute the field content of the boundary theory. This may also be understood in the context of [18], where it was shown that, replacing the Dirichlet boundary conditions employed in the AdS/CFT correspondence by Neumann or mixed boundary conditions, the boundary metric can be promoted to a dynamical field. An interesting task left for the future is to combine our results with the appropriate boundary conditions to construct a dynamical conformal supergravity on the boundary.

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## A Conventions

In this appendix we give a summary of the conventions for the $\mathfrak{usp}(4)$ generators, which agree with those of [8], and for the spacetime $\gamma$-matrices. All spacetime quantities are five-dimensional, so we omit hats for better readability. The $\gamma$-matrices are chosen such that $\gamma_{abcde} = \epsilon_{abcde}$ with $\epsilon_{01234} = 1$. With the charge conjugation matrix $C$ satisfying

$$C\hat{\gamma}_{\hat{\mu}}C^{-1} = \hat{\gamma}_{\hat{\mu}}^T, \qquad C^T = C^{-1} = -C, \qquad C^\star = C \tag{41}$$

the supercharges and hence all the spinors satisfy the symplectic Majorana condition

$$(\chi_i)^\dagger \gamma_0 =: \bar{\chi}^i = \left(\chi^i\right)^T C \; . \tag{42}$$

Fermionic fields are by convention anticommuting and complex conjugation changes their order. Antisymmetrized indices are defined as $X_{[\mu}Y_{\nu]} := \frac{1}{2}(X_\mu Y_\nu - X_\nu Y_\mu)$.

[5] [16] relies on the precise properties of AdS space, so we may regard the bulk theory as expanded around an AdS background for that purpose.

The $\mathfrak{usp}(4)$ symplectic metric $\Omega$ and its inverse satisfy $\Omega_{ij}\Omega^{jk} = \delta_i{}^k$, $\Omega^{ij} = (\Omega_{ji})^*$ and are used to raise and lower spinor indices via

$$\epsilon^i = \Omega^{ij}\epsilon_j, \qquad \epsilon_i = \Omega_{ij}\epsilon^j \ . \tag{43}$$

The $\mathfrak{so}(5)$ Clifford algebra generators $\Gamma_m$ satisfy $(\Gamma_m)_i{}^k\,(\Gamma_n)_k{}^j + (\Gamma_n)_i{}^k\,(\Gamma_m)_k{}^j = 2\delta_{mn}\delta_i{}^j$, which yields canonical Clifford matrices only for these specific index positions. With the charge conjugation matrix $\Omega$ we have

$$\Omega^{ik}\,(\Gamma_m)_k{}^j =: (\Gamma_m)^{ij} = -\,(\Gamma_m)^{ji} \ . \tag{44}$$

The conjugate is denoted by $(\Gamma_m)_{ij} = \big((\Gamma_m)^{ij}\big)^*$ and the $\mathfrak{so}(5)$ generators satisfy $(\Gamma_{mn})^{ij} = (\Gamma_{mn})^{ji}$. The convention for $\epsilon^{\alpha\beta}$ is $\epsilon_{45} = \epsilon^{45} = 1$.

# B Comparison to [20]

To connect the superconformal transformations (37), (38), (39) to the results obtained in [20] we first redefine the tensor field as $\mathcal{C}_{\mu\nu} := C_{\mu\nu} - i\,(\Gamma_4)^{i_+j_+}\,\bar\psi^{\rm R}_{[\mu\,i_+}\psi^{\rm L}_{\nu]\,j_+}$ such that, to leading order in the fermions,

$$\delta_\zeta \mathcal{C}^-_{ab} = 2i\,(\Gamma_4)^{i_+j_+}\,\bar\zeta_{i_+}\hat R_{ab\,j_+}(Q)\ , \qquad \delta_\eta \mathcal{C}^-_{ab} = 0\ , \tag{45}$$

while the transformations of the other fields change by $C^-\!\to \mathcal{C}^-$ only. With the field redefinitions

$$\begin{aligned}
&\psi^{\rm L}_{\mu i_+} =: \frac{\kappa}{\sqrt2}\Psi_{\mu\iota}\ , \qquad \Phi^{\rm R}_{\mu i_+} + \frac{1}{2\sqrt3}\gamma_\mu\chi^{\rm L}_{i_+} =: \frac{\kappa}{\sqrt8}\varPhi_{\mu\iota}\ , \qquad \hat R_{\mu\nu\,i_+}(Q) =: \frac{\kappa}{\sqrt8}\hat R'_{\mu\nu\,\iota}(Q)\ ,\\
&\zeta_{i_+} =: \sqrt2\zeta'_\iota\ , \qquad \eta_{i_+} =: \frac{1}{\sqrt2}\eta'_\iota\ , \qquad \mathcal{C}^-_{\mu\nu\,i_+j_+} =: \frac{\kappa}{4}T^-_{\mu\nu\,\iota\varsigma}\ , \qquad \chi^{\rm L}_{i_+} =: \sqrt{\frac38}\kappa\chi'_\iota\ ,\\
&a_\mu =: \frac{\kappa}{2}\mathcal{A}_\mu\ , \qquad iA^I_\mu\,(\Gamma_I)_{i_+}{}^{j_+} =: \frac{\kappa}{\sqrt8}V_{\mu\,\iota}{}^\varsigma\ , \qquad \varphi =: \sqrt{\frac38}\kappa\varphi'\ ,
\end{aligned} \tag{46}$$

where $\iota := i_+$, $\varsigma := j_+$, the expressions for the auxiliary fields are

$$\begin{aligned}
\omega_{\mu ab} &= \omega_{\mu ab}(e) - \frac14\kappa^2\left(i\bar\Psi^\iota_a\gamma_\mu\Psi_{b\iota} + 2i\bar\Psi^\iota_\mu\gamma_{[a}\Psi_{b]\iota} + \text{c.c.}\right)\ ,\\
\varPhi_{\mu\iota} &= -\frac12 i\Big(\gamma^{\nu\rho}\gamma_\mu - \frac13\gamma_\mu\gamma^{\nu\rho}\Big)\Big(D_\nu\Psi_{\rho\iota} - \frac{\kappa}{16}\gamma\cdot T^-_{\iota\varsigma}\gamma_\nu\Psi^\varsigma_\rho\Big) + \frac12\gamma_\mu\chi'_\iota\ ,\\
\hat R'_{\mu\nu\,\iota}(Q) &= 2D_{[\mu}\Psi_{\nu]\iota} - i\gamma_{[\mu}\varPhi_{\nu]\iota} - \frac{\kappa}{8}\gamma\cdot T^-_{\iota\varsigma}\gamma_{[\mu}\Psi^\varsigma_{\nu]}\ .
\end{aligned} \tag{47}$$

With the Fierz identity

$$v^i w_j = \frac14\,v^k w_k\,\delta_j{}^i + \frac14\,v^k(\Gamma_m)_k{}^l w_l\,(\Gamma_m)_j{}^i - \frac18\,v^k(\Gamma_{mn})_k{}^l w_l\,(\Gamma_{mn})_j{}^i \tag{48}$$

the transformations (37), (38) to leading order in the fermions are

$$
\begin{aligned}
\delta e_\mu^a &= -i\kappa\bar{\zeta}'^{\iota}\gamma^a\Psi_{\mu\iota} + \text{c.c.}\ , \\
\delta\Psi_{\mu\iota} &= 2\kappa^{-1}D_\mu\zeta'_\iota - \frac{1}{8}\gamma\cdot T^-_{\iota\varsigma}\gamma_\mu\zeta'^{\varsigma} - i\kappa^{-1}\gamma_\mu\eta'_\iota\ , \\
\delta T^-_{ab\,\iota\varsigma} &= 8i\bar{\zeta}'_{[\iota}\hat{R}'_{ab\,\varsigma]}(Q)\ , \\
\delta V_{\mu\,\iota}{}^{\varsigma} &= \left(2\bar{\zeta}'^{\varsigma}\Phi_{\mu\,\iota} - 3\bar{\zeta}'^{\varsigma}\gamma_\mu\chi'_\iota - 2\bar{\Psi}^{\varsigma}_\mu\eta'_\iota - \text{h.c.}\right)_{\text{traceless}}\ , \\
\delta\mathcal{A}_\mu &= -\frac{1}{2}i\bar{\zeta}'^{\iota}\Phi_{\mu\,\iota} - \frac{3}{4}i\bar{\zeta}'^{\iota}\gamma_\mu\chi'_\iota + \frac{1}{2}i\bar{\Psi}^{\iota}_\mu\eta'_\iota + \text{c.c.}\ , \\
\delta\chi'_\iota &= -\frac{1}{12}\gamma\cdot T^-_{\iota\varsigma}\eta'^{\varsigma} - i\varphi'\zeta'_\iota + \frac{i}{12}\gamma\cdot\left(T^-_{\iota\varsigma}\overleftarrow{\not{D}}\right)\zeta'^{\varsigma} - \frac{1}{3}\gamma\cdot R(\mathcal{A})\,\zeta'_\iota - \frac{1}{6}i\gamma\cdot R(V)_\iota{}^{\varsigma}\,\zeta'_\varsigma\ , \\
\delta\varphi' &= \bar{\zeta}'^{\iota}\gamma^\mu\Big(D_\mu - \frac{\kappa}{2}\delta_{\zeta'}(\Psi_\mu) - \frac{\kappa}{2}\delta_{\eta'}(\Phi_\mu)\Big)\chi'_\iota + \text{c.c.}\ ,
\end{aligned}
\tag{49}
$$

where $iF^I_{\mu\nu}(\Gamma_I)_{i_+}{}^{j_+} =: \frac{\kappa}{\sqrt{8}}R_{\mu\nu}(V)_\iota{}^{\varsigma}$ and $f_{\mu\nu} =: \frac{\kappa}{2}R_{\mu\nu}(\mathcal{A})$. These are the results obtained in [20] in Euclidean signature up to differences in the phase factors.

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