

Analytic approach to the study of the electric-magnetic asymmetry of the dimension 2 condensate*

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Recent work by Chernodub and Ilgenfritz has uncovered non-trivial temperature dependence in the electric-magnetic asymmetry in the dimension 2 condensate. This asymmetry measures the difference between the spatial and the temporal components of the $\langle A_\mu^2 \rangle$ condensate. Lattice computations have shown very interesting phenomena. The asymmetry shows a jump at the deconfinement phase transition, beyond which it approaches its perturbative value. At temperatures lower than the critical temperature, it shows an exponential behavior with in the exponent a mass smaller than the lowest glueball mass. In this talk we present research done on this asymmetry, using a generalization of analytical methods developed to study $\langle A_\mu^2 \rangle$. The purpose is to shed more insight on the findings of Chernodub and Ilgenfritz.

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1. Introduction

Recent years have witnessed a great deal of interest in the possible existence of mass dimension two condensates in gauge theories, see for example [1, 2, 3, 4, 6, 7, 5, 8, 9, 10, 11, 13, 14, 12] and references therein for approaches based on phenomenology, operator product expansion, lattice simulations,

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an effective potential and the string perspective. There is special interest in the operator

$$A_{\min}^2 = \min_{U \in SU(N)} \mathcal{V}^{-1} \int d^4x (A_\mu^U)^2, \quad (1)$$

which is gauge invariant due to the minimization along the gauge orbit. It should be mentioned that obtaining the global minimum is delicate due to the problem of gauge (Gribov) ambiguities [15]. As is well known, local gauge invariant dimension two operators do not exist in Yang-Mills gauge theories. The nonlocality of (1) is best seen when it is expressed as [16]¹

$$A_{\min}^2 = \int d^4x \left[A_\mu^a \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} \right) A_\nu^a - gf^{abc} \left(\frac{\partial_\nu}{\partial^2} \partial A^a \right) \left(\frac{1}{\partial^2} \partial A^b \right) A_\nu^c \right] + \mathcal{O}(A^4). \quad (2)$$

The relevance of the condensate $\langle A_\mu^2 \rangle_{\min}$ was discussed in [1, 2], where it was shown that it can serve as a measure for the monopole condensation in the case of compact QED.

All efforts so far have concentrated on the Landau gauge $\partial_\mu A_\mu = 0$. The preference for this particular gauge fixing is obvious since the nonlocal expression (2) reduces to a local operator, more precisely

$$\partial_\mu A_\mu = 0 \Rightarrow A_{\min}^2 = A_\mu^2. \quad (3)$$

In the case of a local operator, the Operator Product Expansion (OPE) becomes applicable, and consequently a measurement of the soft (infrared) part $\langle A_\mu^2 \rangle_{\text{OPE}}$ becomes possible. Such an approach was followed in e.g. [8] by analyzing the appearance of $1/q^2$ power corrections in (gauge variant) quantities like the gluon propagator or strong coupling constant, defined in a particular way, from lattice simulations. Let us mention that already three decades ago attention was paid to the condensate $\langle A_\mu^2 \rangle$ in the OPE context [17].

Recently, Chernodub and Ilgenfritz [12] have considered the asymmetry in the dimension two condensate. They performed lattice simulations, computing the expectation value of the electric-magnetic asymmetry in Landau gauge, which they defined as

$$\Delta_{A^2} = \langle g^2 A_0^2 \rangle - \frac{1}{d-1} \sum_{i=1}^{d-1} \langle g^2 A_i^2 \rangle. \quad (4)$$

¹ We will always work in Euclidean spacetime.

At zero temperature, this quantity must, of course, be zero due to Lorentz invariance². Necessarily it cannot diverge as divergences at finite T are the same as for $T = 0$, hence this asymmetry is, in principle, finite, and it can be computed without renormalization, for all temperatures. At high temperatures, general thermodynamic arguments predict a polynomial behavior $\propto T^2$, and this is also what the authors of [12] found³. For the low-temperature behavior, however, one would expect an exponential fall-off with the lowest glueball mass in the exponent, $\Delta \sim e^{-m_{\text{gl}}T}$. Instead, they found an exponential with a mass m significantly smaller than m_{gl} .

2. $\langle A_\mu^2 \rangle$ and Δ_{A^2} in the LCO formalism

In order to get more insight in the behavior of the asymmetry, we have investigated it using the formalism presented in [3]. A meaningful effective potential for the condensation of the *Local Composite Operator* (LCO) A_μ^2 was constructed by means of the LCO method. This is a nontrivial task due to the compositeness of the considered operator. We consider pure Euclidean SU(N) Yang–Mills theories with action

$$S_{\text{YM}} = \int d^4x \frac{1}{4} (F_{\mu\nu}^a)^2 + S_{\text{gf}}, \quad (5)$$

$$S_{\text{gf}} = \int d^4x (b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu \mathcal{D}_\mu^{ab} c^b).$$

We couple the operator A_μ^2 to the Yang–Mills action by means of a source J :

$$S_J = S_{\text{YM}} + \int d^4x \left(\frac{1}{2} J (A_\mu^a)^2 - \frac{1}{2} \zeta J^2 \right). \quad (6)$$

The last term, quadratic in the source J , is necessary to kill the divergences in vacuum correlators like $\langle A^2(x) A^2(y) \rangle$ for $x \rightarrow y$, or equivalently in the generating functional $W[J]$, defined as

$$e^{-W[J]} = \int [\text{fields}] e^{-S_J}. \quad (7)$$

The presence of the LCO parameter ζ ensures a homogenous renormalization group equation for $W[J]$. Its arbitrariness can be overcome by making

² We shall deliberately use the term Lorentz invariance, though we shall be working in Euclidean space throughout this paper.

³ A perturbative computation gives a positive proportionality constant, in contrary to what is erroneously [18] found in [12]. The lattice computations for $T < 6 T_c$ find a negative proportionality constant, so one would expect the real high-temperature behavior to start yet later.

it a function $\zeta(g^2)$ of the strong coupling constant g^2 , allowing one to fix $\zeta(g^2)$ order by order in perturbation theory in accordance with the renormalization group equation.

In order to access the electric-magnetic asymmetry, a second source $K_{\mu\nu}$ is coupled to the traceless part of $A_\mu^a A_\nu^a$. This second operator will not mix with A_μ^2 itself, which allows control over the renormalization group of these two operators. Again a term quadratic in the new source must be added, introducing a second parameter $\omega(g^2)$ which can, again, be fixed order by order in accordance with the renormalization group equation. We have proven the all-order perturbative renormalizability of this extension of the formalism using the algebraic method based on the Ward identities [19].

In order to recover an energy interpretation, the term $\propto J^2$ can be removed by employing a Hubbard–Stratonovich transformation

$$\begin{aligned} 1 &= \int [d\sigma] e^{-\frac{1}{2\zeta} \int d^d x \left(\frac{\sigma}{g} + \frac{1}{2} A_\mu^2 - \zeta J \right)^2}, \\ 1 &= \int [d\varphi_{\mu\nu}] e^{-\frac{1}{2\omega} \int d^d x \left(\frac{1}{g} \varphi + \frac{1}{2} A_\mu A_\nu - \omega k_{\mu\nu} \right)^2}, \end{aligned} \quad (8)$$

with $\varphi_{\mu\nu}$ a traceless field, leading to the action

$$\begin{aligned} S = S_{\text{YM}} + \int d^d x \left[\frac{1}{2\zeta} \frac{\sigma^2}{g^2} + \frac{1}{2\zeta g} \sigma A_\mu^2 + \frac{1}{8\zeta} (A_\mu^2)^2 \right. \\ \left. + \frac{1}{2\omega} \frac{\varphi_{\mu\nu}^2}{g^2} + \frac{1}{2\omega g} \varphi_{\mu\nu} A_\mu A_\nu + \frac{1}{8\omega} (A_\mu^a A_\nu^a)^2 \right]. \end{aligned} \quad (9)$$

Starting from this, it is possible to compute the effective potential $V(\sigma, \varphi_{\mu\nu})$, where the correspondences

$$\langle \sigma \rangle = -\frac{g}{2} \langle A_\mu^2 \rangle, \quad \langle \varphi_{\mu\nu} \rangle = -\frac{g}{2} \left\langle A_\mu A_\nu - \frac{\delta_{\mu\nu}}{d} A_\lambda^2 \right\rangle, \quad (10)$$

hold.

Now we determine the values of ζ and ω from the renormalization group equations for the sources J and $K_{\mu\nu}$. For this, some anomalous dimensions and renormalization factors have to be computed up to one loop order higher than the intended loop order we are interested in. We have done this using the MINCER algorithm. The final result is up to one-loop order:

$$\begin{aligned} \zeta &= \frac{N^2 - 1}{16\pi^2} \left[\frac{9}{13} \frac{16\pi^2}{g^2 N} + \frac{161}{52} \right], \\ \omega &= \frac{N^2 - 1}{16\pi^2} \left[\frac{1}{4} \frac{16\pi^2}{g^2 N} + \frac{73}{1044} \right]. \end{aligned} \quad (11)$$

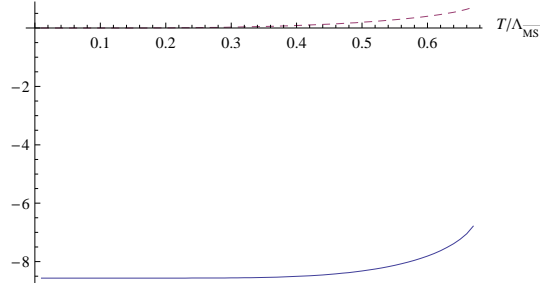


Fig. 1. The $\langle g^2 A_\mu^2 \rangle$ condensate (full line) and the asymmetry Δ_{A^2} (dashed line) as functions of the temperature, in units $\Lambda_{\overline{\text{MS}}}$.

3. Computation and minimalization of the action

The effective potential $V(\sigma, \varphi_{\mu\nu})$ can now be computed using standard techniques. We have taken the background fields σ and $\varphi_{\mu\nu}$ to have space-time independent vacuum expectation values and $\varphi_{\mu\nu}$ to be the traceless diagonal matrix $\text{diag}(A, -\frac{1}{d-1}A, \dots, -\frac{1}{d-1}A)$.

Computing the effective action up to one-loop order at zero temperature, yields only the minimum found in [3], which is what we expect. For finite but still not too high temperatures, the potential can be minimized numerically. The result is depicted in Fig.1. We see that the asymmetry rises at low temperatures, which agrees qualitatively with the findings of [12]. The low-temperature expansion of Δ_{A^2} reads

$$\Delta_{A^2} = (N^2 - 1) \frac{g^2 \pi^2}{30} \left(1 - \frac{85}{1044} \frac{g^2 N}{(4\pi)^2} \right) \frac{T^4}{m^2}, \quad (12)$$

and there is no correction to $\langle A_\mu^2 \rangle$ at this order. Remark that we find a polynomial behavior $\propto T^4/m^2$ instead of an exponential. This does not agree with the lattice results, but in [12] the lowest temperatures reached were $T = 0.4 T_c$, where our expansion is not valid anymore.

At temperatures higher than $0.67\Lambda_{\overline{\text{MS}}}$, the minimum disappears. This signals a phase transition to the perturbative vacuum. In order to access this regime, it is possible to expand the effective potential for high temperatures, which yields

$$\langle A_\mu^2 \rangle = (N^2 - 1) \frac{T^2}{4}, \quad \Delta_{A^2} = (N^2 - 1) \frac{T^2}{12}. \quad (13)$$

This is the perturbative result. In order to compute higher-order corrections to this, it is necessary to perform a Hard Thermal Loop resummation, as a

mere one-loop result leads to an imaginary part coming from the tachyonic mass caused by the condensate. After resummation, a positive mass should remain. Work in that direction is forthcoming [20].

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