Schwarzschild-de Sitter black hole from entropic viewpoint

Ee Chang-Young[a*], Myungseok Eune[b†], Kyoungtae Kimm[a‡], and Daeho Lee[a§]

[a] *Department of Physics and Institute of Fundamental Physics, Sejong University, Seoul 143-747, Korea*

[b] *Basic Science Research Institute, Sogang University, Seoul 121-742, Korea*

Abstract

In a Schwarzschild-de Sitter space, we consider an equipotential surface which consists of two holographic screens. Adapting the Bousso-Hawking's reference point of vanishing force, we divide the space into two regions, which are from the reference point to each holographic screen. These two regions can be treated as independent thermodynamical systems, because the Bousso-Hawking reference point with zero temperature behaves like a thermally insulating wall. The entropy obtained in this way agrees with the conventional results; i) when the holographic screens lie at the black hole and cosmological horizons, ii) in the Nariai limit.

Keywords : Schwarzschild-de Sitter space, Unruh-Verlinde temperature, entropic force

*Email: cylee@sejong.ac.kr
†Email: younms@sogang.ac.kr
‡Email: helloktk@naver.com
§Email: dhleep@gmail.com

1 Introduction

Recently, much attention has been focused on the new idea suggested by Verlinde [1] in which gravity can be explained as an emergent phenomenon originated from the statistical properties of unknown microstructure of spacetime. The essential part of this idea is based on two key ingredients: holographic principle and equipartition rule of the energy. With help of these principles, the Newton's law of gravity was derived by interpreting it as an entropic force i.e., force on a test particle at some point was defined as the product of the entropy gradient and the temperature at that point, and relativistic generalization leads to the Einstein equations. This entropic formulation of gravity has been used to study thermodynamics at the apparent horizon of the Friedmann-Robertson-Walker universe [2], Friedmann equations [3, 4], entropic correction to Newtonian gravity [5, 6, 7], holographic dark energy [8, 9, 10, 11]. There have been many works for the entropic force in cosmological models [12, 13, 14, 15] and the black hole backgrounds [16, 17, 18, 19, 20, 21, 22, 23, 24].

In a spacetime admitting timelike killing vector one can define a gravitational potential, and the holographic screen is given by equipotential surface. In general, the holographic screen can have multiple disconnected parts depending upon the matter distribution. The temperature on the holographic screen is given by Unruh-Verlinde temperature associated with the proper acceleration of a particle near the screen. This prescription works well for spacetime with a single holographic screen, however, there has been no known work for multiple holographic screens so far.

On the other hand, the observational evidence for late-time cosmological acceleration [25, 26] gave much impetus on studying the de Sitter space with black holes. Since Schwarzschild-de Sitter black hole is asymptotically de Sitter space, it has cosmological event horizon in addition to black hole horizon and these horizons can form holographic screens. In fact, the potential of Schwarzschild-de Sitter space has two equipotential surfaces for a given potential value, and the two horizons correspond to equipotential surfaces. In this paper we investigate the entropic formulation in the background geometry of the Schwarzschild-de Sitter space, which provides a model for multiple holographic screens.

In the Verlinde's formalism, two equipotential holographic screens in the Schwarzschild-de Sitter space have different temperatures. Thus the whole system cannot be treated as a thermodynamical system in equilibrium. In Ref. [27], Bousso and Hawking set up a reference point in the radial direction, at which force vanishes. They have pointed out that this reference point can play a role of a point at infinity in an asymptotically flat space. Besides, the temperature at this reference point is zero, and thus no thermal exchange can occur across this point. This makes the reference point behave like a thermally insulating wall. Therefore, we can regard the Schwarzschild-de Sitter space as two thermally independent systems: the inner system in the black hole side and the outer system in the cosmological horizon side. Gibbons and Hawking also considered similar construction in a slightly different context [28]: they constructed two separated thermal equilibrium systems by introducing a perfectly reflecting wall in the Schwarzschild-de Sitter space for the calculation of the Hawking temperatures of black hole and cosmological horizons.

Based on the above consideration, we apply the Verlinde's formalism to each system. In the Schwarzschild-de Sitter case we choose the holographic screen of equipotential surface having spherical symmetry. With this choice of holographic screen we show that the thermodynamic relationship $E = 2TS$ holds for each holographic screen, where E, T, and S are the quasilocal energy given by Komar mass, temperature, and entropy, respectively. We then check this result with the known cases: i) when the holographic screens lie at the black hole and cosmological horizons, ii) in the Nariai limit.

In the following section, we briefly review the Verlinde's formalism of entropic approach to gravitational interaction. In section 3, we apply the Verlinde's formalism to a Schwarzschild-de Sitter space which provides a prototype of multiple holographic screens. Finally, we summarize our results. In this paper, we adopt the convention $c = k_B = \hbar = 1$.

2 Verlinde's entropic formalism

According to the Verlinde's formalism [1], gravity is an entropic force emerging from coarse graining process of information for a given energy distribution. In this process, information is stored on holographic screens. In the nonrelativistic case, the holographic screens correspond to Newtonian equipotential surfaces and the holographic direction is given by the gradient of the potential.

In a curved spacetime with a timelike Killing vector ξ^μ, the generalized Newton's potential is given by

$$\phi = \frac{1}{2}\ln(-\xi^\mu \xi_\mu). \tag{1}$$

This potential can be used to define a foliation of space. For a particle with a four velocity u^μ, its proper acceleration is given by $a^\mu = u^\nu \nabla_\nu u^\mu$. In terms of the potential ϕ and the Killing vector ξ^μ, the velocity and the acceleration can be written as

$$u^\mu = e^{-\phi}\xi^\mu, \tag{2}$$

$$a^\mu = -\nabla^\mu \phi, \tag{3}$$

where the Killing equation has been used to derive Eq. (3). In Eq. (3), the acceleration is normal to holographic screen. The Unruh-Verlinde temperature on the screen is defined as

$$T = \frac{1}{2\pi} e^{\phi} n^{\mu} \nabla_{\mu} \phi, \tag{4}$$

where n^{μ} is the unit outward pointing vector normal to the screen and to the Killing vector. The "outward" indicates that the potential increases along n^{μ}, *i.e.,* the normal vector can be written as

$$n_{\mu} = \frac{\nabla_{\mu} \phi}{\sqrt{\nabla_{\nu} \phi \nabla^{\nu} \phi}}. \tag{5}$$

In Eq. (4), a redshift factor e^{ϕ} is inserted because the temperature is measured with respect to the reference point. For asymptotically flat space this reference point corresponds to spatial infinity. In the Schwarzschild-de Sitter case, we choose this reference point as the Bousso-Hawking reference point [27] to be explained in the next section.

We denote the number of bits on the holographic screen $\mathcal{S}$ by N which is assumed to be proportional to the area of the screen [1],

$$N = \frac{A}{G}. \tag{6}$$

Applying the equipartition rule of the energy, each bit of holographic screen contributes an energy $T/2$ to the system, and the total energy on the holographic screen can be written as

$$E = \frac{1}{2} \oint_{\mathcal{S}} T dN. \tag{7}$$

Note that in the above expression the temperature T on the screen is not constant in general. Substituting Eqs. (4) and (6) into Eq. (7), the energy associated with the holographic screen can be rewritten as

$$E = \frac{1}{4\pi G} \oint_{\mathcal{S}} n^{\mu} \nabla_{\mu} e^{\phi} dA. \tag{8}$$

This expression is the conserved Komar mass associated with timelike Killing vector ξ^{μ}.

3 Schwarzschild-de Sitter black hole

Now, we consider a spherically symmetric Schwarzschild-de Sitter black hole as a model of multiple holographic screens. The Schwarzschild-de Sitter space is described locally by the line element,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2), \tag{9}$$

with

$$f(r) = 1 - \frac{2GM}{r} - \frac{1}{3}\Lambda r^2, \tag{10}$$

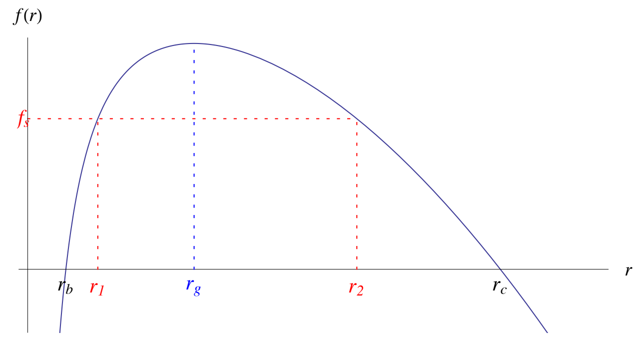

Figure 1: The Schwarzschild-de Sitter space has the event horizon of the black hole at $r = r_b$ and the cosmological event horizon at $r = r_c$. At $r = r_g$, the proper acceleration vanishes. For a given potential value, there exist two screens at $r = r_1$ and $r = r_2$, and each screen has different temperature. Note that the unit normal vectors on both screens direct to the surface $r = r_g$.

where G and M are the gravitational Newton's constant and the mass parameter, respectively, and the cosmological constant will be taken as $\Lambda = 3/\ell^2$. When $0 < M < M_{\rm max} \equiv \ell/(3^{2/3}G)$ static region exists between two horizons with radii r_b and r_c, the black hole and cosmological event horizons. For $M = M_{\rm max}$, the two horizons coincide, the Nariai limit. In the Nariai limit, there exists no timelike Killing vector.

In order to get the potential of the Schwarzschild-de Sitter spacetime, we first consider a timelike Killing vector of Eq. (9), given by

$$\xi^\mu = \gamma \, (\partial/\partial t)^\mu \,, \tag{11}$$

where γ is a normalization constant. If space is asymptotically flat, we may choose the standard Killing vector normalization, $\gamma = 1$. Since Schwarzschild-de Sitter space is not asymptotically flat, we encounter a difficulty in taking the normalization of Killing vector. To avoid this, Bousso and Hawing [27] chose a normalization such that the norm of the Killing vector becomes unity at the region where the force vanishes, the gravitational attraction is exactly balanced out by the cosmological repulsion. Adopting this normalization corresponds to choosing a special observer who follows geodesics.

Since the magnitude of the acceleration of a particle in the Schwarzschild-de Sitter spacetime is obtained as $a = \sqrt{a^\mu a_\mu} = |f'(r)|/\sqrt{2f(r)}$, the geodesic point with no acceleration is given by

$$r_g = (GM\ell^2)^{1/3}. \tag{12}$$

With this normalization, the gravitational potential is obtained from Eq. (1),

$$\phi = \frac{1}{2}\ln(\gamma^2 f) = \frac{1}{2}\ln\frac{f(r)}{f(r_g)}. \tag{13}$$

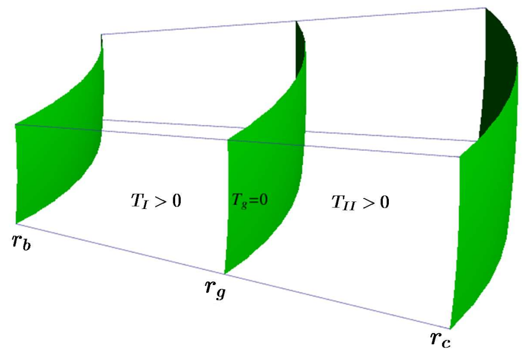

Figure 2: We consider the force free reference point of Bousso and Hawking as a separating boundary dividing the system into two subsystems. Since the temperature of each subsystem is above zero and the boundary between them is maintained at zero, thermal exchange does not occur between the two subsystems.

For a given potential value ϕ_s, there exist two equipotential surfaces at $r = r_1$ and $r = r_2$ as shown in Fig. 1. Then, the Unruh-Verlinde temperature on each screen is given by

$$T = \frac{1}{2\pi} e^{\phi} n^{\mu} \nabla_{\mu} \phi = \gamma \frac{|f'(r)|}{4\pi}, \tag{14}$$

where the unit normal vector n^{μ} is given by $n^{\mu} = \delta^{\mu}_{r} \sqrt{f}$ for $r < r_g$ and $n^{\mu} = -\delta^{\mu}_{r} \sqrt{f}$ for $r > r_g$. Note that the temperature of the holographic screen at $r = r_1$ is different from that of the screen at $r = r_2$. The temperature on each screen is given by

$$T_i = \frac{\gamma}{2\pi} \left| \frac{GM}{r_i^2} - \frac{r_i}{\ell^2} \right|, \tag{15}$$

where $i = 1, 2$.

The temperature becomes zero at the Bousso-Hawking reference point $r = r_g$ from Eq. (14). Now, assume that the region between the black hole and cosmological horizons is separated by a boundary at the reference point $r = r_g$ as shown in Fig. 2. Then the two regions divided by this boundary cannot have thermal exchange between them because the temperature on this boundary is kept at zero always in our static geometry setup. Thus, we can regard this boundary as a thermally insulating wall. Therefore, the two regions separated by the surface at $r = r_g$ can be thought as independent systems: the total system becomes the sum of two independent systems, the inner ($r < r_g$) and outer ($r > r_g$) regions. The concept of thermally insulating wall in our consideration is similar to that of perfectly reflecting wall in the Gibbons-Hawking's work [28]: they constructed two separated thermal equilibrium systems by introducing a perfectly reflecting wall in Schwarzschild-de

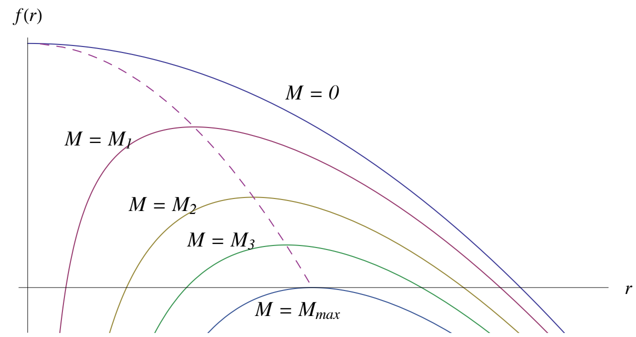

Figure 3: The geodesic point with no acceleration is plotted by the dashed line. The metric approaches the pure dS spacetime as the mass parameter of Schwarzschild-de Sitter space goes to zero. ($0 < M_1 < M_2 < M_3 < M_{\rm max}$)

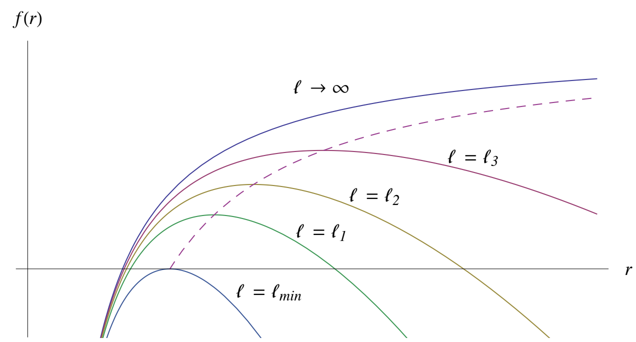

Figure 4: The geodesic point with no acceleration is plotted by the dashed line. The metric approaches the Schwarzschild black hole as the cosmological constant $\Lambda = 3/\ell^2$ goes to zero. ($\ell_{\rm min} < \ell_1 < \ell_2 < \ell_3 < \infty$)

Sitter space for the calculation of the Hawking temperatures of black hole and cosmological horizons.

This can be also understood as follows. The line element (9) approaches the pure de Sitter spacetime when M goes to zero and the pattern of the metric for $r > r_g$ has a similarity to that of the pure de Sitter spacetime (see Fig. 3). And the spacetime approaches the Schwarzschild black hole with asymptotically flat spacetime when Λ goes to zero and the pattern of the metric for $r < r_g$ has the similarity to that of the Schwarzschild black hole (see Fig. 4). This suggests that the whole system has the characteristics of both Schwarzschild black hole and pure de Sitter spacetime.

Plugging the potential (13) into the energy (8) gives the same result from the Komar energy for the Schwarzschild-de Sitter black hole,

$$E = \frac{1}{4\pi G} \oint_{\mathcal{S}} \nabla^\mu \xi^\nu \sigma_\mu n_\nu dA, \tag{16}$$

where σ_μ is the unit normal timelike vector perpendicular to the hypersurface surrounded by the screen $\mathcal{S}$. Since $\sigma_\mu = -\sqrt{f}\,\delta^t_\mu$, the Komar energy (16) becomes

$$E_i = \gamma \frac{r_i^2 |f'(r_i)|}{2G} = \gamma \left| M - \frac{r_i^3}{G\ell^2} \right|, \tag{17}$$

for each screen at $r = r_i$ $(i = 1, 2)$.

If the associated holographic entropy is given by

$$S_i = \frac{A_i}{4G} = \frac{\pi r_i^2}{G}, \tag{18}$$

then with Eqs. (15) and (17) the thermodynamic relation $E_i = 2T_i S_i$ holds for each system. This relation certainly holds for event horizons.[1] When the spacetime is static and spherically symmetric, we can also get this relation directly from Eq. (7) with the relation (18), since the temperature on the holographic screen is constant. Note that the thermodynamic relation $E = 2TS$ does not hold for the whole system, since the energy and entropy are additive and the temperatures on the holographic screens are different.

Now, we check the validity of our formulation in two specific cases. First, we consider the case when the holographic inner and outer screens become the event horizon of black hole and the cosmological horizon, respectively. As the locations of the holographic screens, r_1 and r_2, move to the two roots of $f(r) = 0$, r_b and r_c, as shown in Fig. 1, the inner screen becomes the black hole event horizon and the outer one becomes the cosmological event horizon. The temperatures on the screens seen by an observer located at the Bousso-Hawking reference point are given by

$$T_{b/c} = \frac{1}{\sqrt{1 - (9G^2M^2\Lambda)^{1/3}}} \frac{1}{4\pi} \left| \frac{2GM}{r_{b/c}^2} - \frac{2\Lambda r_{b/c}}{3} \right|. \tag{19}$$

Since the system is composed of the sum of two independent systems, the total entropy is given by the sum of the entropies of subsystems,

$$S = S_1 + S_2. \tag{20}$$

In the present case, S_1 and S_2 correspond to the usual entropy of the black hole and cosmological horizons, respectively. And thus, our result agrees with the previously obtained entropy of Schwarzschild-de Sitter space [28, 31, 32, 33, 34].

Next, we consider the case when the two event horizons, r_b and r_c, approach each other,

[1] In Refs. [29, 30], it was shown that this relation holds when the equipartition rule of energy is assumed for event horizons of stationary spacetimes.

the Nariai limit [35]. In this case, the temperature and the energy on each horizon become

$$T_i \longrightarrow T^{\rm Nariai} = \frac{\sqrt{3}}{2\pi\ell}, \tag{21}$$

$$E_i \longrightarrow E^{\rm Nariai} = \sqrt{3}\left(\frac{M^2\ell}{G}\right)^{1/3}. \tag{22}$$

In this limit, the entropy of each system becomes

$$S_i \longrightarrow \frac{\pi r_g^2}{G}. \tag{23}$$

The total entropy is the sum of the two subsystems', thus it is twice of the above given entropy (23). This agrees with the entropy of the Schwarzschild-de Sitter black hole in the Nariai limit obtained in Refs. [33, 34].

In summary, we apply the Verlinde's entropic formalism of gravity to the Schwarzschild-de Sitter space as a model of multiple holographic screens. Since the Unruh-Verlinde temperature vanishes at the Bousso-Hawking reference point, we can regard two regions separated by zero temperature barrier as thermodynamically isolated systems and thus independently apply the entropic formalism to each region. We confirm that the Verlinde's formalism agrees with the conventional result at least in the following cases; i) when the holographic screens become event horizons, ii) in the Nariai limit.

Acknowledgments

This work was supported by the National Research Foundation (NRF) of Korea grants funded by the Korean government (MEST) [R01-2008-000-21026-0 and NRF-2009-0075129 (E. C.-Y. and K. K.), NRF-2009-351-C00109 (M. E.), and NRF-2009-351-C00111 (D. L.)].

References

[1] E. P. Verlinde, arXiv:1001.0785 [hep-th].

[2] F. W. Shu and Y. Gong, arXiv:1001.3237 [gr-qc].

[3] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **81** (2010) 061501.

[4] T. Padmanabhan, arXiv:1001.3380v1 [gr-qc];

[5] L. Smolin, arXiv:1001.3668 [gr-qc].

[6] P. Nicolini, Phys. Rev. D **82** (2010) 044030.

[7] L. Modesto and A. Randono, arXiv:1003.1998 [hep-th].

[8] M. Li and Y. Wang, Phys. Lett. B **687** (2010) 243.

[9] D. A. Easson, P. H. Frampton and G. F. Smoot, arXiv:1002.4278 [hep-th].

[10] U. H. Danielsson, arXiv:1003.0668 [hep-th].

[11] Y.-F. Cai, J. Liu, H. Li, Phys. Lett. B **690** (2010) 213.

[12] C. Gao, Phys. Rev. D **81** (2010) 087306.

[13] Y. Zhang, Y. G. Gong and Z. H. Zhu, arXiv:1001.4677 [hep-th].

[14] Y. Wang, arXiv:1001.4786 [hep-th].

[15] S. W. Wei, Y. X. Liu and Y. Q. Wang, arXiv:1001.5238 [hep-th].

[16] Y. S. Myung, arXiv:1002.0871 [hep-th].

[17] Y. S. Myung and Y. W. Kim, Phys. Rev. D **81** (2010) 105012.

[18] Y. X. Liu, Y. Q. Wang and S. W. Wei, Class. Quantum Grav. **27** (2010) 185002.

[19] R. G. Cai, L. M. Cao and N. Ohta, Phys. Rev. D **81** (2010) 084012.

[20] Y. Tian and X. Wu, Phys. Rev. D **81** (2010) 104013.

[21] X. Kuang, Y. Ling and H. Zhang, arXiv:1003.0195 [gr-qc].

[22] Ee C.-Y., M. Eune, K. Kimm, and D. Lee, Mod. Phys. Lett. A **25** (2010) 2825.

[23] R. A. Konoplya, Eur. Phys. J. C **69** (2010) 555.

[24] F. Caravelli and L. Modesto, arXiv:1001.4364 [gr-qc].

[25] S. Perlmutter et al., Astrophys. J. **517** (1998) 565.

[26] A. G. Reiss et al., Astron. J. **116** (1998) 1009.

[27] R. Bousso and S. W. Hawking, Phys. Rev. D **54** (1996) 6312.

[28] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15** (1977) 2738.

[29] T. Padmanabhan, Mod. Phys. Lett. A **25** (2010) 1129.

[30] R. Banerjee and B. R. Majhi, Phys. Rev. D **81** (2010) 124006.

[31] D. Kastor and J. Traschen, Class. Quant. Grav. **13** (1996) 2753.

[32] R. G. Cai, J.-Y. Ji, and K.-S. Soh, Class. Quant. Grav. **15** (1998) 2783.

[33] K.-S. Cha, B.-H. Lee, C. Park, J. Korean Phys. Soc. **42** (2003) 735.

[34] R. Garattini, Entropy **2** (2000) 26.

[35] R. Bousso, arXiv:hep-th/0205177.