Beyond cusp anomalous dimension from integrability in SYM_4

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Abstract

We study the first sub-leading correction $O((\ln s)^0)$ to the cusp anomalous dimension in the high spin expansion of finite twist operators in $\mathcal{N}=4$ SYM theory. This term is still governed by a linear integral equation which we study in the weak and strong coupling regimes. In the strong coupling regime we find agreement with the string theory computations.

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1 Aims, motivations and tools

Our aim is to compute anomalous dimensions γ of linear combinations of twist L scalar operators in planar $\mathcal{N}=4$ SYM,

$$\mathcal{O}(x) = \sum_{k_1+...+k_L=s} c_{k_1,...,k_L} \mathrm{Tr}\left(\mathcal{D}_+^{k_1}\phi(x)...\mathcal{D}_+^{k_L}\phi(x)\right) , \tag{1.1}$$

which diagonalise the dilatation operator

$$\hat{D}\,\mathcal{O} = \Delta\,\mathcal{O} = (L+s+\gamma(g,s,L))\mathcal{O} . \tag{1.2}$$

We indicate with $\lambda = 8\pi^2 g^2$ the 't Hooft coupling and, for given s, L, we focus on the linear combination of operators with minimal anomalous dimension $\gamma(g,s,L)$. Moreover, we go to the high spin ($s \to \infty$) limit: in such a limit the anomalous dimension is proportional to $\ln s$,

$$\gamma(g,s,L) = f(g)\ln s + f_{sl}(g,L) + O\left(\frac{1}{\ln s}\right) ,$$

the scaling function $f(g)$ being called cusp anomalous dimension.

The main motivation of our work comes from the AdS/CFT correspondence [1]. This is a remarkable strong/weak coupling duality which, in particular, equates the spectrum of anomalous dimensions of composite operators in $\mathcal{N}=4$ SYM to the energy spectrum of states in type IIB superstring theory in $\mathrm{AdS}_5\times\mathrm{S}^5$. Another reason of interest for twist two operators (1.1) resides in the connection between elements of their anomalous dimension matrix and the Mellin transforms of splitting kernels of DGLAP equations. In particular, $f_{sl}(g,2)$ coincides with the coefficient of the $\delta(1-z)$ term, i.e. the virtual scaling function.

In order to compute anomalous dimensions in planar limit, we will use the powerful tool of integrability. Integrability was first found in one loop planar QCD: on the one hand, scattering of Reggeised gluons was shown to be described by spin 0 Heisenberg chain [2]; on the other hand, the spectrum of anomalous dimensions of aligned-helicity twist operators was found to coincide with the spectrum of the spin -1/2 Heisenberg chain [3]. In planar $\mathcal{N}=4$ SYM integrability exists not only at one loop [4], but at all loops and in all the gauge theory sectors [5]. In brief, every composite operator can be thought of as a state of a 'spin chain', whose Hamiltonian is the dilatation operator itself: the large size (*asymptotic*) spectrum is described by certain Asymptotic Bethe Ansatz (ABA) equations (the so-called Beisert-Staudacher equations, cf. [5, 6] and references therein). Unfortunately, this works only for infinitely long operators: anomalous dimensions of operators with finite length depend not only on ABA data, but also on finite size 'wrapping' corrections [7]. Recent progress has shown that a set of Thermodynamic Bethe Ansatz (TBA) equations [8] provides exact (any length at any coupling) predictions on anomalous dimensions of planar $\mathcal{N}=4$ SYM. Yet, $f(g)$ and $f_{sl}(g,L)$ are wrapping-free, cf. below.

2 From Bethe equations to linear integral equations

We now briefly review how ABA helps in computing anomalous dimensions. Restricting to operators $\mathcal{O}(x)$ and supposing that wrapping correction can be neglected, anomalous dimensions $\gamma(g,s,L)$ can be expressed as

$$\gamma(g,s,L) = \frac{ig^2}{2}\sum_{k=1}^{s}\left[\left(\frac{1}{x^+(u_k)}\right) - \left(\frac{1}{x^-(u_k)}\right)\right],$$

with u_k solutions of 'Bethe' equations,

$$\left(\frac{u_k+\frac{i}{2}}{u_k-\frac{i}{2}}\right)^L \left(\frac{1+\frac{g^2}{2x_k^{-2}}}{1+\frac{g^2}{2x_k^{+2}}}\right)^L = \prod_{\substack{j=1\\ j\neq k}}^{s} \frac{u_k-u_j-i}{u_k-u_j+i}\left(\frac{1-\frac{g^2}{2x_k^+x_j^-}}{1-\frac{g^2}{2x_k^-x_j^+}}\right)^2 e^{2i\theta(u_k,u_j)}, \tag{2.3}$$

where $x_k^{\pm} = x(u_k \pm i/2)$, $x(u) = \frac{u}{2}\left[1+\sqrt{1-\frac{2g^2}{u^2}}\right]$ and $\theta(u,v)$ is the dressing phase [5, 6]. It is convenient to study (2.3) in the high spin limit. In such a limit, indeed, wrapping corrections are reduced: perturbative computations [9] show that they affect $O\left(\frac{(\ln s)^2}{s^2}\right)$ terms. In addition, we showed [10, 11] that, at high spin, the nonlinear Bethe equations (2.3) can be equivalently rewritten as linear integral equations, the nonlinear terms appearing at the order $O\left(\frac{1}{s^2}\right)$. Putting together these two facts, we deduce that both $f(g)$ and $f_{sl}(g,L)$ can be obtained from solutions of linear integral equations for the density of Bethe roots $\sigma(u)$, descending from the ABA equations (2.3). In specific, when $s\to\infty$, we can split the density of Bethe roots as $\sigma(u) = \ln s\ \sigma^{(1)}(u) + \sigma^{(0)}(u) + O\left(\frac{1}{\ln s}\right)$. At leading order $\ln s$ one has the BES equation [6]

$$\hat\sigma^{(1)}(k) = F^{(1)}(k) - \frac{g^2k}{\sinh\frac{k}{2}}\int_0^{+\infty} dte^{-\frac{t}{2}}\hat K(\sqrt{2}gk,\sqrt{2}gt)\hat\sigma^{(1)}(t),$$

with

$$F^{(1)}(k) = \frac{4g^2\pi k}{\sinh\frac{k}{2}}\hat K(\sqrt{2}gk,0)$$

and 'kernel'

$$\hat K(t,t') = \frac{2}{tt'}\left[\sum_{n=1}^{\infty} nJ_n(t)J_n(t') + 2\sum_{k=1}^{\infty}\sum_{l=0}^{\infty}(-1)^{k+l}c_{2k+1,2l+2}(g)J_{2k}(t)J_{2l+1}(t')\right],$$

which gives $f(g) = \frac{\hat\sigma^{(1)}(0)}{\pi}$. Even if an explicit expression for $f(g)$ was not found, weak coupling [12, 6] and strong coupling [13] expansions were easily obtained.

Going further at the order $(\ln s)^0$, one has the linear integral equation [10]

$$\hat\sigma^{(0)}(k) = F^{(0)}(k) - \frac{g^2k}{\sinh\frac{k}{2}}\int_0^{+\infty} dte^{-\frac{t}{2}}\hat K(\sqrt{2}gk,\sqrt{2}gt)\hat\sigma^{(0)}(t), \tag{2.4}$$

which has the same kernel as BES equation, but different forcing term:

$$\begin{aligned} F^{(0)}(k) &= 4g^2\frac{\pi k}{\sinh\frac{k}{2}}\int_0^{+\infty}\frac{dt}{e^t-1}[\hat K(\sqrt{2}gk,\sqrt{2}gt) - \hat K(\sqrt{2}gk,0)] + \\ &+ \frac{\pi L}{\sinh\frac{k}{2}}[1-J_0(\sqrt{2}gk)] + 4g^2\gamma_E\frac{\pi k}{\sinh\frac{k}{2}}\hat K(\sqrt{2}gk,0) + \\ &+ g^2(L-2)\frac{\pi k}{\sinh\frac{k}{2}}\int_0^{+\infty} dte^{-\frac{t}{2}}\hat K(\sqrt{2}gk,\sqrt{2}gt)\frac{1-e^{\frac{t}{2}}}{\sinh\frac{t}{2}}. \end{aligned}$$

The solution of (2.4) gives $f_{sl}(g,L) = \frac{\hat{\sigma}^{(1)}(0)}{\pi}$: one easily gets weak coupling perturbative expansion [14]:

$$f_{sl}(g,L) = (\gamma_E - (L-2)\ln 2) f(g) + 8(2L-7)\zeta(3)\left(\frac{g}{\sqrt{2}}\right)^4 -$$
$$-\frac{8}{3}\left(\pi^2\zeta(3)(L-4) + 3(21L-62)\zeta(5)\right)\left(\frac{g}{\sqrt{2}}\right)^6 +$$
$$+\frac{8}{15}\left(\pi^4\zeta(3)(3L-13) + 75(46L-127)\zeta(7) + 5(11L-32)\pi^2\zeta(5)\right)\left(\frac{g}{\sqrt{2}}\right)^8 + \ldots$$

The strong coupling asymptotic series [15, 14] requires a slightly bigger effort:

$$\begin{aligned} f_{sl}(g,L) &= 2\sqrt{2}\, g\Big[\ln\frac{2\sqrt{2}}{g} - 1 - \frac{3\ln 2}{2\sqrt{2}\pi\, g}\ln\frac{2\sqrt{2}}{g} + \frac{6\ln 2 - \pi + (2-L)\pi}{2\sqrt{2}\pi g} - \\ &- \frac{\mathrm{K}}{8\pi^2 g^2}\ln\frac{2\sqrt{2}}{g} + \frac{4\mathrm{K} - 9(\ln 2)^2}{16\pi^2\, g^2} + O\big(\frac{\ln g}{g^3}\big)\Big]. \end{aligned} \tag{2.5}$$

Importantly, result (2.5) agrees with string theory computations [16].

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