Lorentz-preserving fields in Lorentz-violating theories

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November 5, 2010

Abstract

We identify a fairly general class of field configurations (of spins $0, \frac{1}{2}$ and 1) which preserve Lorentz invariance in effective field theories of Lorentz violation characterized by a constant timelike vector. These fields concomitantly satisfy the equations of motion yielding cubic dispersion relations similar to those found earlier. They appear to have prospective applications in inflationary scenarios.

Invariance under Lorentz transformation is known till date to be a global symmetry of the standard theory of elementary particles when gravitation is ignored. However, questions have been raised regarding the validity of this symmetry at small length scales owing to probable quantum gravity effects. The natural mass scale of quantum gravity is the Planck mass M_{Pl}. Departures, suppressed by the Planck mass, from the standard special relativistic dispersion relation of free particles of mass m at large energies have been accepted as a signature of Lorentz invariance violation and has been the principal *objet de l'attention* of experimental and theoretical probes of Lorentz violation. These hypothesised corrections due to Lorentz non-invariance must have their origin in new terms in the action of the system. Myers and Pospelov [1] have studied this issue within the framework of effective field theory involving fields of spins 0, 1/2 and 1, by incorporating into the action dimension five operators containing a *constant* timelike 4-vector $\mathbf{n}$ which ostensibly breaks Lorentz invariance. Choosing a Lorentz frame where $n^\mu = (1, \vec{0})$, corrections of $O(p^3)$ to the dispersion relation of each of the three fields have been obtained in [1] in the limit of relatively high energies E ($M_{Pl} >> E >> m$).

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For a complex scalar field this is given by

$$\omega^2 \simeq |\vec{p}|^2 + \frac{\kappa}{M_{Pl}}|\vec{p}|^3. \tag{1}$$

For the Maxwell field, the dispersion relation obtained takes the form (for circularly polarized photons)

$$(\omega^2 - |\vec{p}|^2 \pm \frac{2\xi}{M_{Pl}}|\vec{p}|^3)(\epsilon_x \pm i\epsilon_y) = 0. \tag{2}$$

In the case of a Dirac spinor one gets,

$$\omega^2 - |\vec{p}|^2 - \frac{2|\vec{p}|^3}{M_{PL}}(\eta_1 + \eta_2\gamma_5) = 0. \tag{3}$$

Many experiments aimed at constraining the parameters $\kappa, \xi, \eta_1, \eta_2$ quantifying Lorentz violation have been proposed in the past few years. Lorentz violating effects scale with energy making astrophysical observations a perfect arena for detecting them. The simplest astrophysical observations that provide interesting constraints on lack of Lorentz symmetry at Planck scale measure the differences in arrival times of photons emitted simultaneously from distant sources of radiation like γ-ray bursts, active galactic nuclei and pulsars [2], [3], [4]. The authors of [5] found the strongest limits on ξ of $\xi < 47$ by observing a strong flare in the TeV band of active galactic nuclei Markarian 501. The lowest order corrections in the photon dispersion relation (2) also imply the birefringence of vacuum (different group velocities for different helicities of photons). In 2008, Maccione *et al.* [6] used polarimetric observations of hard x-ray from the Crab nebula to impose a bound on Lorentz violation in quantum electrodynamics of $|\xi| < 9 \times 10^{-10}$ at 95% confidence level.

Complementary constraints have also been obtained from the threshold reactions of photon decay, fermion pair emission, synchrotron radiation, vacuum Cerenkov radiation and helicity changing decays. In [7], the authors analyzed synchrotron radiation from the Crab nebula to deduce $\eta > -7 \times 10^{-8}$. Observational details and their phenomenological consequences have been exhaustively discussed in [8], [9], [10].

It is clear that the deformed dispersion relation has been the object of extensive observational scrutiny of departure from Lorentz invariance. Does it unequivocally imply Lorentz violation? We explore here the possibility that special field configurations exist for which the apparently Lorentz symmetry violating theory [1] may still be Lorentz *invariant* analogous to what happens in magnetic monopole theory, as shown by Zwanziger in [11]. In Zwanziger's work, a local, manifestly anisotropic Lagrangian density has been shown to preserve Lorentz invariance when the fields obey certain constraints. In this paper we consider the Nöther current corresponding to Lorentz transformation for the higher derivative theory of [1]. Requiring that this Nöther current is conserved leads to special field configurations which conserve Lorentz symmetry, despite the presence of the constant vector **n**. Identical configurations also appear when we demand that the *action* changes at most by a constant when the fields transform under an infinitesimal Lorentz transformation while the 4-vector $\mathbf{n}$ stays fixed.

Spin 0 *fields:* The Lagrangian density for a complex scalar field ϕ put forth in [1] is,

$$\mathcal{L}_{MP_\phi} = |\partial\phi|^2 - m^2|\phi|^2 + \frac{i\kappa}{M_{Pl}}\phi^*\partial_n^3\phi, \tag{4}$$

with κ being a real, dimensionless parameter. The Nöther current corresponding to Lorentz transformations has the spacetime divergence given by

$$\partial_\mu \mathcal{J}^\mu_{\alpha\beta} = -\frac{\partial\mathcal{L}_{MP_\phi}}{\partial n^\lambda}(\delta n^\lambda)_{\alpha\beta} = n_{[\alpha}\frac{\partial\mathcal{L}_{MP_\phi}}{\partial n^{\beta]}} \ . \tag{5}$$

If Lorentz transformations are symmetries of the Lagrangian (4), we must have $\partial_\mu\mathcal{J}^\mu_{\alpha\beta} = 0 = n_{[\alpha}\frac{\partial\mathcal{L}_{MP}}{\partial n^{\beta]}}$. Requiring this yields the condition

$$n_{[\alpha}\partial_{\beta]}\partial_n^2\Phi = 0.$$

A possible non-trivial solution is,

$$\partial_n^2\phi = f(\mathbf{x}.\mathbf{n}) = f(z) \tag{6}$$

where $z \equiv \mathbf{x}.\mathbf{n}$. This condition involves the derivative only along the direction $\mathbf{n}$ in spacetime. It is convenient in flat spacetime to resolve the coordinate 4-vector along $\mathbf{n}:\ x^\mu = (z/n^2)n^\mu + x^\mu_\perp$ where $\mathbf{n}\cdot\mathbf{x}_\perp = 0$. It is straightforward to show that

$$\partial_n\phi(x) = n^2\partial_z\phi(z, x_\perp) \ . \tag{7}$$

This implies in its turn

$$\phi(x) = \phi_\parallel(z) + \phi_\perp(\mathbf{x}_\perp) \tag{8}$$

where, $\phi_\parallel$ and $\phi_\perp$ are arbitrary functions of their arguments. If $\mathbf{n}$ is timelike, we can choose coordinates such that x^0 lies along n. Then our condition (8) implies that when the full scalar field is a linear combination of a time-dependent, spatially homogeneous piece and a static spatially inhomogeneous piece, *the theory will possess Lorentz symmetry.*

Maxwell (spin 1) field : In this case, the usual kinetic term of the free Maxwell field and a dimension five, $\mathbf{n}$ dependent operator constitute the modified Lagrangian density proposed in [1],

$$S = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\xi}{M_{Pl}}n^\mu F_{\mu\nu}n^\alpha\partial_\alpha n_\rho\tilde{F}^{\rho\nu}\right] \tag{9}$$

where ξ is a dimensionless parameter constraining Lorentz violation. For convenience, we define $n^\mu F_{\mu\nu} \equiv F_{n\nu}, n_\rho\tilde{F}^{\rho\nu} = \tilde{F}^{n\nu}$. Directly studying the variation of the action or the divergence of the Nöther current of Lorentz transformation in parallel with the argument given in case of the scalar field, the condition for the theory to be Lorentz invariant is,

$$n_{[\alpha}F_{\beta]\nu}\partial_n\tilde{F}^{n\nu} + F_{n\nu}n_{[\alpha}\partial_{\beta]}\tilde{F}^{n\nu} + F^{n\nu}\partial_n n_{[\alpha}\tilde{F}_{\beta]\nu} = 0 \ . \tag{10}$$

The last term is always zero because $F^{n\nu}\partial_n n_{[\alpha}\tilde{F}_{\beta]\nu} = F^{n\nu}\partial_n n_{[\alpha}\epsilon_{\beta]\nu\lambda\sigma}F^{\lambda\sigma}$ contains a fifth rank completely antisymmetric tensor ($\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}F_{\lambda\sigma}$) in four spacetime dimensions.

We transform to the Lorentz frame defined by $\mathbf{n} = (1, \vec{0})$, to get a better physical picture of the problem in terms of electric and magnetic field 3-vectors identified as $E_i = F_{0i} = \frac{1}{2}\epsilon_{lmi}\tilde{F}_{lm}, B_i = \tilde{F}_{0i} = -\frac{1}{2}\epsilon_{ijk}F_{jk}$ where we have used $\epsilon^{0ijk} = \epsilon_{ijk}$. The condition for Lorentz invariance becomes,

$$\epsilon_{ijm}\partial_0 B_j B_m - E_j\partial_i B_j = 0 \tag{11}$$

$$\dot{\vec{B}} \times \vec{B} - \vec{\nabla}\vec{B}\cdot\vec{E} = 0 \tag{12}$$

It is easy to see that if the fields are harmonic functions of spacetime as

$$\vec{E} = Re(\vec{\mathcal{E}}_0 exp(-iwt + i\vec{k}\cdot\vec{x})) \tag{13}$$

$$\vec{B} = Re(\vec{\mathcal{B}}_0 exp(-iwt + i\vec{k}\cdot\vec{x})) \tag{14}$$

they satisfy (12) when the relation $\vec{E}\cdot\vec{B} = 0$ (deduced from Bianchi identity) is incorporated. This ensures that it is possible to have Lorentz symmetric electromagnetic fields in the modified electrodynamics of [1].

Spin $\frac{1}{2}$ field : In [1], the action describing a Dirac spinor has been modified to,

$$\begin{aligned} S &= \int d^4x\ \overline{\psi}[(i\gamma.\partial - m)\psi + \frac{\gamma.\mathbf{n}}{M_{Pl}}(\eta_1 + \eta_2\gamma_5)\partial^2_{\mathbf{n}}\psi] \\ &\equiv S_D + S_{V_D}\ , \end{aligned} \tag{15}$$

where S_D is the standard Dirac action of a spinor field ψ and S_{V_D} accounts for Lorentz violation. The dimensionless parameters η_1, η_2 give the measure of Lorentz violation.

The only source of Lorentz violation is, by assumption, the appearance of the constant 4-vector $\mathbf{n}$ in S_V. Thus, there are no constant vectors in the theory independent of $\mathbf{n}$. It is straightforward to show that, under an infinitesimal Lorentz transformation, the action S changes by,

$$\begin{aligned} \delta_{\alpha\beta}S_V = \frac{1}{M_{Pl}}\int d^4x\ \overline{\psi}\{&\mathbf{n}_{[\alpha}\gamma_{\beta]}(\eta_1 + \eta_2\gamma_5)\partial^2_{\mathbf{n}}\psi \\ &+ \gamma.\mathbf{n}(\eta_1 + \eta_2\gamma_5)n_{[\alpha}\partial_{\beta]}\partial_{\mathbf{n}}\psi \end{aligned} \tag{16}$$

If we set

$$\partial_n\psi = \chi(z) \tag{17}$$

where $z = \mathbf{x}.\mathbf{n}$, then the second term in (16) vanishes. After a partial integration (dropping the surface term), the first term reduces to,

$$
\begin{aligned}
\delta_{\alpha\beta} S_{V_D} &= \frac{1}{M_{Pl}} - \int d^4x \left[\eta_1 n_{[\alpha} \overline{\chi} \gamma_{\beta]} \chi + \eta_2 n_{[\alpha} \overline{\chi} \gamma_{\beta]} \gamma_5 \chi\right] \\
&= \frac{1}{M_{Pl}} - \int d^4x \left[\eta_1 n_{[\alpha} J_{\beta]}(z) + \eta_2 n_{[\alpha} J^5_{\beta]}(z)\right] \ ,
\end{aligned} \tag{18}
$$

where $J_\alpha(z) \equiv \overline{\chi}\gamma_\alpha\chi$, $J^5_\alpha(z) \equiv \overline{\chi}\gamma_{\beta]}\gamma_5\chi$ etc.

Now, one can decompose the currents $\mathbf{J}$ and $\mathbf{J}^5$ as $\mathbf{J} = \left(\frac{\mathbf{n}.\mathbf{J}}{n^2}\right)\mathbf{n} + \mathbf{J}_\perp$ and $\mathbf{J}^5 = \left(\frac{\mathbf{n}.\mathbf{J}^5}{n^2}\right)\mathbf{n} + \mathbf{J}^5_\perp$ where $\mathbf{n}.\mathbf{J}_\perp = 0$, $\mathbf{n}.\mathbf{J}^5_\perp = 0$. Inserting this decomposition into (18), it is clear that

$$
\delta_{\alpha\beta} S_V = -\int d^4x \left[\eta_1 n_{[\alpha} J_{\perp,\beta]}(z) + \eta_2 n_{[\alpha} J^5_{\perp,\beta]}(z)\right] \ , \tag{19}
$$

so that Lorentz violation now depends on the current 4-vectors $\mathbf{J}_\perp$ and $\mathbf{J}^5_\perp$.

It should be noted, however, that *these current 4-vectors are orthogonal to* $\mathbf{n}$ *and are constants in the direction they point!* If, for example, $\mathbf{n}$ is timelike, the currents $\mathbf{J}_\perp$ and $\mathbf{J}^5_\perp$ must be spacelike and yet must be *spatially homogeneous*, being functions of z. This makes them constant 4-vectors *independent* of $\mathbf{n}$. Since, by assumption there are no constant 4-vectors in the problem apart from $\mathbf{n}$, these currents must vanish. As illustrated for the scalar field, our condition ((17)) implies that,

$$
\psi(\mathbf{x}) = \psi_\parallel(z) + \psi_\perp(\mathbf{x}_\perp). \tag{20}
$$

In the preferred frame $\mathbf{n} = (1, \vec{0})$, $\psi_\parallel(z)$ is a spatially homogeneous spinor whereas the spinor $\psi_\perp(x_\perp)$ is time independent.

Now that we have found non-trivial and quite general field configurations that make the modified [1] scalar, vector and spinor theories Lorentz invariant, the next step will entail calculating the dispersion relations obeyed by these special fields.

Scalar field : The scalar field $\phi(x)$ assumed to be given by (8) leads to the equation of motion $(\Box + m^2)\phi = \frac{i\kappa}{M_{Pl}}\partial^3_n\phi$ to be written as

$$
(\nabla^2_\perp + m^2)\phi_\perp = -[\ddot{\phi}_\parallel + m^2\phi_\parallel] + \frac{i\kappa}{M}\dddot{\phi}_\parallel \tag{21}
$$

It is obvious that to make sense of (21) we must set both sides to a constant which we choose to vanish for convenience. In the inertial frame defined earlier, by taking the simple ansätz $\phi_\perp \sim \exp -i\mathbf{k}_\perp \cdot \mathbf{x}_\perp$ and $\phi_\parallel \sim \exp iEz$, it is easy to see that the following equations emerge

$$
\begin{aligned}
E^2 &= m^2 + \frac{\kappa}{M} E^3 \\
k^2_\perp &= m^2
\end{aligned} \tag{22}
$$

One can now eliminate m^2 from these equations and use $\mathbf{k}_\perp = (0, \vec{k})$ to get the dispersion relation,

$$E^2 \simeq |\vec{k}|^2 + \frac{\kappa}{M_{Pl}} \, |\vec{k}|^3 \tag{23}$$

which is same as the dispersion relation (1) computed in [1].

Vector field : The equations of motion obtained by the variation of the action (9) is (derived in [12]),

$$\partial_\mu F^{\mu\nu} + \frac{\xi}{M_{Pl}} \left(n_\rho \epsilon^{\rho\sigma\mu\nu} \partial_\mu \partial_n F_{n\sigma} - \partial_n^2 \tilde{F}^{n\nu} \right) = 0 \tag{24}$$

The above equation and the Bianchi identity $\partial_{[\mu} F_{\nu\rho]} = 0$ in the chosen reference frame are equivalent to the following equations:

$$\vec{\nabla} \cdot \vec{E} = 0 = \vec{\nabla} \cdot \vec{B} \tag{25}$$

$$\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \tag{26}$$

$$-\dot{\vec{E}} + \vec{\nabla} \times \vec{B} + \frac{\xi}{M_{Pl}} (\ddot{\vec{B}} - \vec{\nabla} \times \dot{\vec{E}}) = 0 \tag{27}$$

These are the modified free Maxwell equations. If we take the curl of both sides of (27), simplify using the Bianchi identity (25), substitute the LI solution for the magnetic field and assume $\mathbf{k} = (\omega, 0, 0, k^3)$, the dispersion relations at high energy $\omega \simeq |\vec{k}|$ are:

$$\omega^2 - |\vec{k}|^2 \simeq \pm \frac{2\xi}{M_{Pl}} |\vec{k_\perp}|^3. \tag{28}$$

The plus and minus signs appear for right and left circularly polarised electromagnetic waves respectively ($B_{R,L} = B_1 \pm i B_2$).

Spinor field : Likewise, the equation of motion of a spinor described by (15) is $(\Box + m^2)\psi = \frac{2i}{M_{Pl}} (\eta_1 + \eta_2 \gamma_5) \partial_{\mathbf{n}}^3 \psi$. In the chosen Lorentz frame, if we take the spacetime dependance of the fields in (20) to be $\psi_\parallel \sim exp(-iwt)$, $\psi_\perp \sim exp(-i\mathbf{k}_\perp \cdot \mathbf{x}_\perp) = exp(i\vec{k} \cdot \vec{x})$ then the dispersion relation turns out to be,

$$\omega^2 \simeq |\vec{k}|^2 + \frac{2}{M_{Pl}} \, (\eta_1 + \eta_2 \gamma_5) |\vec{k}|^3 \tag{29}$$

when $E \simeq |\vec{k}| >> m$. In this limit of negligible mass, the spinor $\psi(\mathbf{n})$ can be chirality operator eigenstates and the dispersion relation can be rewritten in terms of $\eta_{R,L} \equiv \eta_1 \pm \eta_2$ as $\omega^2 \simeq |\vec{k}|^2 + \frac{2}{M_{Pl}} \, \eta_{R,L} |\vec{k}|^3$.

The similarity of the dispersion relations obtained here for the Lorentz invariant scalar, vector and spinor fields to the modified dispersion relations (1), (2), (3) obtained in [1] indicates that observation of a modified dispersion relation is not sufficient to guarantee violation of Lorentz symmetry *for all field configurations* in the proposed effective field theories. These special Lorentz-preserving configurations enable the use of the entire gamut of standard techniques when applied to analyse the effective field theories in question. Furthermore, the configurations themselves have aspects of intrinsic interest when one considers prospective application to cosmology as in inflationary scenarios. The fact that there is a natural decomposition in Lorentz-preserving (scalar) fields between spatially homogeneous and inhomogeneous parts implies that while the former, in a Friedmann-Robertson-Walker background spacetime, can play the role of the inflaton field, the latter, acting as a perturbation on the former, may provide natural seeds for the growth of inhomogeneities in a Lorentz symmetric manner.

The energy momentum tensor has the form $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\mathcal{L}_{\mathcal{MP}_\phi}$. For fields of the form (8), the energy density is given by

$$T_{00} = \frac{1}{2}\dot{\phi}_{\parallel}^{\,2} + \frac{i\kappa}{M_{Pl}}\phi_{\parallel}\dddot{\phi}_{\parallel} + \frac{i\kappa}{M_{Pl}}\phi_{\perp}\dddot{\phi}_{\parallel} + \frac{1}{2}(\nabla\phi_{\perp})^2 \tag{30}$$

The last two terms in the equation above can be interpreted as perturbations over the homogeneous energy density of the field $\phi_{\parallel}$. The Lorentz-preserving perturbations due to the field $\phi_{\perp}(\mathbf{x}_{\perp})$ ought to lead to growth of Lorentz invariant inhomogeneities in spacetime. We hope to report on this in future.

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