Fundamental String (Membrane) Orbiting D5(M5)-branes

Rashmi R. Nayak

Centre for Theoretical Studies
Indian Institute of Technology Kharagpur,
Kharagpur- 721 302, India
E-mail: rashmi@cts.iitkgp.ernet.in

Pratap K. Swain

Department of Physics & Meteorology
Indian Institute of Technology Kharagpur,
Kharagpur, 721 302, India
E-mail: pratap@phy.iitkgp.ernet.in

Abstract: We study fundamental string (F-string) dynamics near $D5$-brane in some limits. We find that when the angular momentum of the probe is proportional to string length (l_s) and square root of string coupling constant ($\sqrt{g_s}$), the F-string lies in its metastable orbit at a finite distance from $D5$-branes. We further study the metastable orbits of a M2-brane in M5-brane background.

Keywords: D-branes.

Contents

1. Introduction and Summary

Type II string theories admit, in addition to the usual BPS branes, non-BPS branes [1] in its spectrum which are unstable due to the presence of the open string tachon. By now it has been well understood that the dynamics of this open string tachyon can be described by a Dirac-Born-Infeld (DBI) type of action which captures all the essential properties of the tachyon field [2], [3], [4], [5], [6], [7]. For a comprehensive review and complete list of references refer to [8]. The prefactor that appears in front of the Lagrangian density is the tachyon potential. Not so long back, it was noticed that the open string tachyon dynamics on the non-BPS brane has a geometric meaning in terms of the rolling of a BPS D-brane in the vicinity of a stack of NS5-branes, the so-called geometric tachyon [9], [10]. This is nothing but the radial distance between the probe D-brane and the NS5-brane, and the time dependent dynamics of this D-brane has shown to be astonishingly similar to that of the open string tachyon dynamics. As the D-brane moves in the vicinity of the gravitational potential produced by the NS5-brane [1], it pulls it towards its core and finally it gets absorbed into the NS5-brane [9]. For the energy of the incoming D-brane $E > \tau_p$, the D-brane escapes to infinity and for $E < \tau_p$, the D-brane will fall into the NS5-brane. [2]

However, more recently, in [16] it was argued that for a particular value of energy and angular momentum of the probe D-brane, the brane will lie in a metastable orbit and revolve around the NS5-brane by keeping a certain distance from NS5-brane at all times. In this limit of energy and angular momentum, the induced tachyon field on D-brane becomes a massless field due to the constant tachyon potential and the D-brane becomes a stable object. In this paper [16], this mechanism was well explained by constructing an classical equivalent radial action involving angular momentum. It was argued that the NS5-brane has no effecft on orbiting D-brane but the observer on the D-brane sees the change in tension due to NS5-brane.

[1] the Harmonic function for a stack of NS5-branes goes like $\frac{1}{r^2}$

[2] Further studies on rolling branes into NS5-brane have been considered, for example, in [11], [12], [13], [14], [15].

In an attempt to understand the origin of this geometric tachyon in [17] the time dependent dynamics of probe D-brane in the background of NS5-brane was explained in terms of proper acceleration. It was proposed that the tachyonic instability is due to the geodesic deviation caused by proper acceleration (which is formed due to the background dilaton field). This idea has been further extended to the system of F-string falling into the Dp-brane [18], even though there was no dilaton prefactor in the Nambo-Goto action for F-string in Dp-brane background. It was found that the tachyonic instability in (F-$D5$) system is due to the over all conformal factor of the induced metric on the F-string. Perhaps, this was in the expected line as (F-$D5$) system is S-dual to the ($D1$-NS5) and the later has a tachyonic instability due to the geometric tachyon. The resemblance of the (F-$D5$) system with that of open string dynamics on the non-BPS brane, it is interesting to study the F-string orbiting around the stack of Dp-branes. We shall study this system in the present paper.

Furthermore, in a related paper [19] the dynamics of the M2-brane was studied near the M5-brane background. As it is well known that M2-brane and M5-brane in 11-dimensions can be reduced to D-branes or other branes in string theory by compactifying some directions and applying T-dualities. Both M2-branes and M5-branes are stable and preserving some supersymmetries, but M2-brane in the vicinity of M5-brane breaks all supersymmetries. It was shown in [19] that the dynamics of M2-brane in the background of M5-brane behaves similarly as that of D-brane in the vicinity of NS5-brane after compactifying one of the transverse direction (x^{11}) at a periodic interval of $2\pi R_{11}$ with limit $1 << \frac{r}{R}$, where r is the radius of M5-brane along the transverse directions.

Motivated by the recent study of D-brane orbiting in NS5-brane background, we study further examples of such motion in various string and M-theory backgrounds. As specified earlier, the (F-Dp) system behaves in the same way as the open string tachyon condensation on non-BPS brane, we study the dynamics of F-string in the orbiting limit around a stack of $D5$-branes background. We find that when the angular momentum of the incoming F-string is related to the string coupling (g_s) and the string length (l_s), the F-string will lie in its metastable orbit keeping certain distance from the $D5$-brane. We further propose an action for the F-string written in terms of the radial coordinate only which gives the same equations of motion and conserved quantities but with a different tachyon-like potential compared to the one proposed in [18]. Then we study the dynamics of a Dp'-brane in the background of a stack of Dp-branes in the orbiting limit. [3] It so happens that only the ($D3 - D5$) system shows a similar behavior in orbiting limit with non-zero angular momentum.

Finally, we study the M2-brane dynamics in a stack of M5-branes background in orbiting limit. We show that there exists a solution where the angular momentum is written in terms of Planck length and the size of the eleven dimensional circle where the M2-brane will lie in the metastable orbit and revolve around the M5-brane by keeping a certain distance from it. Further we write the action of such a M2-brane, in terms of radial coordinate only,

[3]We have chosen the probe and the background in such a way that there is no Wess-Zumino term in the action of the Dp-brane and the dynamics is governed by the dilaton and by the metric.

which gives the same equations of motion and conserved quantities, but with an effective tension.

The rest of the paper is organized as follows. In section 2, we study the F-string orbiting around the $D5$-brane background. Section 3 is devoted to the study of orbiting limit of Dp'-brane in Dp-brane background. Finally, in section 4, we study the M2-brane in M5-brane background in orbiting limit.

2. F-string Orbiting a stack of D5-branes

(F-$D5$) is a non-supersymmetric system because they break different halves of supersymmetries. Therefore, F-string in the vicinity of $D5$-branes form a non-BPS system. The dynamics of F-string in $D5$-brane background has been discussed in [18]. We wish to find the condition where F-string is orbiting around $D5$-brane. The supergravity solutions of parallel coincident $D5$-branes are given by:

$$\begin{aligned}
ds^2 &= H(r)^{-\frac{1}{2}}\Big(-dt^2+\sum_{i=1}^{5}(dx^i)^2\Big)+H(r)^{\frac{1}{2}}\Big(dr^2+d{\Omega_3}^2\Big),\\
e^{2(\phi-\phi_0)} &= H(r)^{-1},\\
H(r) &= 1+\frac{Ng_s{l_s}^2}{r^2},\\
F_7 &= dH(r)^{-1}\wedge dt\wedge dx^1\wedge ...\wedge dx^5.
\end{aligned} \tag{2.1}$$

Here $H(r)$ is the harmonic function and $r=\sum_{a=6}^{9}(dx^a)^2$, ϕ is the dilaton, $d\Omega_3$ is the volume element of 3-sphere in transverse directions of $D5$-branes and F_7 is the 7-form Ramond-Ramond (R-R) field strength. The Nambo-Goto action of F-string is given by:

$$S_f=-\frac{1}{2\pi{l_s}^2}\int d^2\xi\sqrt{-\det\ \mathrm{G}_{\mu\nu}} \tag{2.2}$$

where, μ,ν runs from 0 to 1. and $d^2\xi$ represents the area element of the world sheet of F-string. $G_{\mu\nu}$ is the induced metric on the string given by

$$G_{\mu\nu}=\frac{\partial X^A}{\partial\xi^\mu}\frac{\partial X^B}{\partial\xi^\nu}G_{AB}, \tag{2.3}$$

where $A,B=0,1,...9$. We set, by reparametrization, $\xi^\mu=X^\mu$. The position of the F-string in the transverse space of the $D5$-brane gives rise to scalars on the world volume of the string. We further presume that the transverse directions are function of time (t) only. Under this, the action (2.2) for the string lying in (t,x^1) is given by:

$$S_f=-\frac{1}{2\pi{l_s}^2}\int dtdx^1\frac{1}{\sqrt{H}(X^m)}\sqrt{1-H(X^m)\dot{X}^m\dot{X}^m}. \tag{2.4}$$

The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2\pi {l_s}^2}\frac{1}{\sqrt{H}(X^m)}\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}, \tag{2.5}$$

and the nonzero components of stress-energy tensor $T_{\mu\nu}$ are given by:

$$T_{00} = \tau_f \frac{H(X^m)^{-\frac{1}{2}}}{\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}}, \tag{2.6}$$

$$T_{ij} = -\tau_f H(X^m)^{-\frac{1}{2}}\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}\,\delta_{ij}, \tag{2.7}$$

where $\tau_f = \frac{1}{2\pi {l_s}^2}$ is the fundamental string tension. The energy density (E) and angular momentum (L) are defined as

$$\begin{aligned} E &= P_n\dot{X}^n - \mathcal{L} \\ L &= X^m P^n - X^n P^m, \end{aligned} \tag{2.8}$$

where, P_n is the conjugate momentum defined by:

$$P_n = \frac{\delta\mathcal{L}}{\delta\dot{X}^n} = \tau_f \frac{H(X^m)^{\frac{1}{2}}\dot{X}^n}{\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}}. \tag{2.9}$$

Using P_n as given in (2.9) in equation (2.8), the angular momentum and energy density becomes

$$L = \tau_f \frac{H(X^m)^{\frac{1}{2}}}{\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}}\Bigg[X^m\dot{X}^n - X^n\dot{X}^m\Bigg], \tag{2.10}$$

$$E = \tau_f \frac{H(X^m)^{-\frac{1}{2}}}{\sqrt{1 - H(X^m)\dot{X}^m\dot{X}^m}}. \tag{2.11}$$

If F-string is always confined in the (X^6, X^7) plane, using polar coordinates as $X^6 = R\cos\theta$ and $X^7 = R\sin\theta$, the energy density and the angular momentum becomes

$$E = \tau_f \frac{H(R)^{-\frac{1}{2}}}{\sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}}, \tag{2.12}$$

and

$$L = \tau_f \frac{H(R)^{\frac{1}{2}}R^2\dot{\theta}}{\sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}}. \tag{2.13}$$

The equations for $\dot{\theta}$ and $\dot{R}$ derived from the above two equations become

$$\dot{\theta}^2 = \frac{l^2}{\epsilon^2 R^4 H(R)^2},$$

$$\dot{R}^2 = \frac{1}{\epsilon^2 H(R)^2}\left[\epsilon^2 H(R) - \left(1 + \frac{l^2}{R^2}\right)\right]. \tag{2.14}$$

Where we have defined $\epsilon = \frac{E}{\tau_f}$ and $l = \frac{L}{\tau_f}$. This radial equation can be compared with a particle of mass ($m = 2$) moving in a one dimensional effective potential (with zero energy) as:

$$V_{eff} = -\frac{1}{\epsilon^2 H(R)^2}\left[\epsilon^2 H(R) - \left(1 + \frac{l^2}{R^2}\right)\right]. \tag{2.15}$$

Note that the effective potential vanishes for $\epsilon = 1$ and $l = \sqrt{N g_s} l_s$. Similar type of calculation has been done for the D-brane orbiting around the NS5-branes in [16]. The orbiting angular momentum depends both on string length (l_s) and string coupling constant (g_s). In analogy with [16], we conclude that F-string maintains a particular desired orbit around the $D5$-brane as long as the above energy and angular momentum limits are satisfied, because for vanishing V_{eff}, there is no force that pulls the string towards the 5-brane or pushes it to infinity.

As in [16], we can write the action given in (2.4) in terms of polar coordinates as

$$S_f = -\tau_f \int dt dx^1 H(R)^{-\frac{1}{2}} \sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}. \tag{2.16}$$

The equations of motion for R and θ derived from this action are

$$\frac{d}{dt}\left(\frac{\dot{R} H(R)^{\frac{1}{2}}}{\sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}}\right) = \frac{R H(R)^{\frac{1}{2}}}{\sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}}\left[\dot{\theta}^2 - \omega^2(R)\right], \tag{2.17}$$

and

$$\frac{d}{dt}\left(\frac{R^2\dot{\theta} H(R)^{\frac{1}{2}}}{\sqrt{1 - H(R)(\dot{R}^2 + R^2\dot{\theta}^2)}}\right) = 0. \tag{2.18}$$

Where $\omega(R) = \frac{\sqrt{N g_s} l_s}{R^2 H(R)}$. One can check that $\dot{R} = 0$ with $\epsilon = 1$ and $l = \sqrt{N g_s} l_s$ becomes a solution to (2.17). Further the equation (2.18) guaranty that L is a constant of motion.

Now, we consider an equivalent radial action involving angular momentum as

$$S_f' = -\tau_f \int dt \sqrt{1 + \frac{l^2}{R^2}} \sqrt{\frac{1}{H(R)} - \dot{R}^2}. \tag{2.19}$$

Though this action looks same as given in [16], the expressions for H and l are not same. From the equivalent action, we get same radial equation of motion and energy density as that of original action (2.16). By comparing the action (2.19) with the open string tachyon effective action given in [2], we get the tachyon potential as $V(T) = \tau_f \frac{\sqrt{1+\frac{l^2}{R^2}}}{H(R(T))}$ which is different than that given in [18]. Note that for the orbiting condition, $l = \sqrt{N g_s} l_s$, the tachyon potential $\tilde{V}(T) = \tau_f$. The geometric tachyon does not roll, because the tachyon potential becomes flat. The geometric tachyon induced on the string becomes a massless scalar as described in [18]. Hence the decay process is suppressed and the F-string becomes stable.

3. Dp'-brane Orbiting Dp-branes

The supergravity solution of a stack of N coincident Dp-branes is given by the following form of metric, dilaton (ϕ), and R-R field $C_{(p+1)}$ as

$$\begin{aligned} ds^2 &= H_p{}^{-\frac{1}{2}}\sum_{a=0}^{p}(dx^a)^2 + +H_p{}^{\frac{1}{2}}\sum_{i=p+1}^{9}(dx^i)^2, \\ e^{2\phi} &= H_p{}^{\frac{3-p}{2}}, \\ H_p &= 1+\frac{Ng_s l_s{}^{7-p}}{r^{7-p}}, \\ C_{0\cdots p} &= H_p{}^{-1}. \end{aligned} \tag{3.1}$$

Where H_p is the harmonic function of N coincident Dp-branes in the transverse directions of Dp-brane. In the subsequent analysis, we shall assume that (1) the dimensionality of the probe is smaller than the background, (2) there is no magnetic flux on the probe, (3) we shall take a single probe brane at a time. Then the dynamics will be given by the induced metric and the dilaton prefactor only. The action of such a probe Dp' in a Dp brane background is given by DBI action as in [20]

$$S_{p'} = -\tau_{p'}V\int dt H_p{}^{\frac{p-p'-4}{4}}\sqrt{1-\dot{X}^m\dot{X}^m H_p}. \tag{3.2}$$

To get the action (3.2), we set the reparametrization invariance of the world volume coordinates of Dp'-brane (leveled as $\xi^a = x^a$), gives rise scalar fields (X^m) along the transverse directions of D-brane. We restrict the radial fluctuations along the transverse directions ($R=\sqrt{X^m X_m(\xi^a)}$) only and also the scalar fields are the function of time (t).

The nonzero components of stress-energy tensors $T_{\mu\nu}$ are given by

$$T_{00} = \tau_{p'}\frac{H_p{}^{\frac{p-p'-4}{4}}}{\sqrt{1-\dot{X}^m\dot{X}^m H_p}}, \tag{3.3}$$

$$T_{ij} = -\tau_{p'}H_p{}^{\frac{p-p'-4}{4}}\sqrt{1-\dot{X}^m\dot{X}^m H_p}\,\delta_{ij}, \tag{3.4}$$

where we have taken $\int d^p x = V = 1$. The conserved quantities are energy (E) and angular momentum (L) and they are defined as

$$\begin{aligned} E &= P_n\dot{X}^n - \mathcal{L}, \\ L &= X^m P^n - X^n P^m. \end{aligned} \tag{3.5}$$

Where the conjugate momentum P_n is given by:

$$P_n = \frac{\delta\mathcal{L}}{\delta\dot{X}^n} = \tau_{p'}\frac{{H_p}^{\frac{p-p'}{4}}\dot{X}^n}{\sqrt{1-\dot{X}^m\dot{X}^m H_p}}. \tag{3.6}$$

The angular momentum and the energy now becomes

$$L = \tau_{p'}\frac{{H_p}^{\frac{p-p'}{4}}}{\sqrt{1-\dot{X}^m\dot{X}^m H_p}}\left[X^m\dot{X}^n - X^n\dot{X}^m\right], \tag{3.7}$$

and

$$E = \tau_{p'}\frac{{H_p}^{\frac{p-p'-4}{4}}}{\sqrt{1-\dot{X}^m\dot{X}^m H_p}}. \tag{3.8}$$

Considering the motion of the Dp' lies in the plane (X^6, X^7) at all times and using polar coordinates $X^6 = R\cos\theta$ and $X^7 = R\sin\theta$, the angular momentum and energy becomes

$$E = \tau_{p'}\frac{{H_p}^{\frac{p-p'-4}{4}}}{\sqrt{1-(\dot{R}^2+R^2\dot{\theta}^2)H_p}}, \tag{3.9}$$

and

$$L = \tau_{p'}\frac{{H_p}^{\frac{p-p'}{4}}R^2\dot{\theta}}{\sqrt{1-(\dot{R}^2+R^2\dot{\theta}^2)H_p}}. \tag{3.10}$$

Solving the above two equtions, for $\dot{R}$ and $\dot{\theta}$, we get

$$\begin{aligned}
\dot{\theta}^2 &= \frac{l^2}{\epsilon^2 R^4 {H_p}^2},\\
\dot{R}^2 &= \frac{1}{\epsilon^2 {H_p}^2}\left[\epsilon^2 H_p - \left({H_p}^{\frac{p-p'-2}{2}} + \frac{l^2}{R^2}\right)\right].
\end{aligned} \tag{3.11}$$

Where once again we have defined $\epsilon = \frac{E}{\tau_{p'}}$ and $l = \frac{L}{\tau_{p'}}$. The radial equation of motion describes a particle of mass $(m = 2)$ moving in a one dimensional effective potential (with zero energy) as

$$V_{eff} = -\frac{1}{\epsilon^2 {H_p}^2}\left[\epsilon^2 H_p - \left({H_p}^{\frac{p-p'-2}{2}} + \frac{l^2}{R^2}\right)\right]. \tag{3.12}$$

Note that the effective potential vanishes for

$$\epsilon = 1,$$

$$p = p' + 2,$$

$$\text{and} \quad H_p = 1 + \frac{l^2}{R^2}. \tag{3.13}$$

The last two conditions together tell that V_{eff} vanishes only when $p = 5, p' = 3$ and $l = \sqrt{N g_s} l_s$. Thus the only consistent orbiting condition is found in case of $D3$-brane in $D5$-brane background. Rest of the analysis can be done by replacing $l = \sqrt{N g_s} l_s$ and $H = 1 + \frac{N g_s l_s^2}{R^2}$ in [16] and in section 2 of this paper. It will be interesting to find out further example of Dp-brane orbiting around stacks of D-brane bound states.

4. Membrane Orbiting M5-brane

In this section, we shall study the membranes orbit in a background generated by a periodic configuration of N coincident M5-branes along the x^{11} direction at an intervals $2\pi R_{11}$. In the limit of $1 << r/R_{11}$, the background of such array of M5-branes becomes [19], [21]

$$\begin{aligned}
ds^2 &= H^{-\frac{1}{3}} \eta_{\mu\nu} dx^\mu dx^\nu + H^{\frac{2}{3}} \delta_{ij} dx^i dx^j + H^{\frac{2}{3}} (dx^{11})^2, \\
H &= 1 + \frac{N l_p^3}{R_{11} r^2}, \\
F_4 &= \frac{2N\ell_p^3}{R_{11}} dv_{S^3} \wedge dx^{11}, \\
r^2 &= \sum_i (x^i)^2, \qquad x^{11} = R_{11}\phi,
\end{aligned} \tag{4.1}$$

where $\mu, \nu = 0, 1, \cdots, 5, \quad i, j = 6, \cdots, 9$, and $0 \leq \phi \leq 2\pi$, dv_{S^4} denotes the volume form of a unit S^4 and l_p is the Planck length in the 11-dimensional theory. We would like to study the dynamics of a M2-brane in the above background and study the homogeneous solutions only. The action for a single M2-brane is given by

$$S_{M2} = -T_2 \int d^3\xi \sqrt{-\mathrm{det} P[G]_{\mu\nu}} + T_2 \int P[A], \tag{4.2}$$

where $T_2 = \frac{1}{4\pi^2 {l_p}^3}$ is the tension of the M2-brane, $P[G]_{\mu\nu}$ and $P[A]$ are the pull back of the metric and the three form field onto the worldsheet given by

$$\begin{aligned}
P[G]_{\mu\nu} &= \frac{\partial X^M}{\partial \xi^\mu} \frac{\partial X^N}{\partial \xi^\nu} G_{MN}(X) \\
P[A] &= \frac{1}{6} \epsilon^{\mu\nu\rho} \frac{\partial X^M}{\partial \xi^\mu} \frac{\partial X^N}{\partial \xi^\nu} \frac{\partial X^p}{\partial \xi^\rho} A_{MNP}(X).
\end{aligned} \tag{4.3}$$

The indices M, N, P runs over the 11-dimensional spacetime. We shall restrict ourselves to the case when the transverse directions of the M5-brane depends only on time. So the dynamics will be governed only by the Nambu-Goto part of the action. The action of the M2-brane is given by

$$S_{M2} = -VT_2 \int dt \sqrt{H^{-1} - \dot{X^i} \dot{X^i} - \dot{X^{11}} \dot{X^{11}}} \ , \tag{4.4}$$

where V is the volume of the M2-brane. The action (4.4) of M2-brane in the background of stacks of M5-branes is given in terms of polar coordinates $X^6 = R\cos\theta, X^7 = R\sin\theta$ as

$$S_{M2} = -T_2 V \int dt \sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2} \tag{4.5}$$

The components of stress-energy tensor calculated from (4.5) are

$$T_{00} = E = \frac{T_2}{H}\frac{1}{\sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2}}, \tag{4.6}$$

$$T_{ij} = -T_2 R^2\sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2}, \tag{4.7}$$

$$T_{\phi\phi} = -T_2 {R_{11}}^2\sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2}. \tag{4.8}$$

The angular momenta are given by

$$L_\theta = \frac{T_2 R^2\dot{\theta}}{\sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2}}, \tag{4.9}$$

and

$$L_\phi = \frac{T_2 {R_{11}}^2\dot{\phi}}{\sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - {R_{11}}^2\dot{\phi}^2}}. \tag{4.10}$$

From (4.6) and (4.9) we can solve for $\dot{\theta}$ as

$$\dot{\theta} = \frac{L_\theta}{EHR^2}. \tag{4.11}$$

From (4.6) and (4.10) and also from (4.9) and (4.10), we get $\dot{\phi}$ as

$$\dot{\phi} = \frac{L_\phi}{EH{R_{11}}^2} = \frac{L_\phi R^2}{L_\theta {R_{11}}^2}\dot{\theta}. \tag{4.12}$$

Using (4.11) and (4.12) in (4.6), the equation for $\dot{R}$ reduces to

$$\dot{R}^2 = \frac{1}{H} - \frac{1}{E^2H^2}\left({T_2}^2 + \frac{{L_\theta}^2}{R^2} + \frac{{L_\phi}^2}{{R_{11}}^2}\right). \tag{4.13}$$

Defining ${T_e}^2 = {T_2}^2 + \frac{{L_\phi}^2}{{R_{11}}^2}$ as the effective tension, explained in [19], we can rewrite (4.13) as

$$\dot{R}^2 = \frac{1}{H} - \frac{1}{E^2H^2}\left({T_e}^2 + \frac{{L_\theta}^2}{R^2}\right). \tag{4.14}$$

The above equation can be thought of as a particle of mass ($m = 2$) (with zero energy) moving in an effective potential of the form:

$$V_{eff} = \frac{1}{E^2H^2}\left({T_e}^2 + \frac{{L_\theta}^2}{R^2}\right) - \frac{1}{H}. \tag{4.15}$$

Once again defining $\epsilon = \frac{E}{T_e}$ and $l = \frac{L_\theta}{T_e}$, the equation (4.15) becomes

$$V_{eff} = \frac{1}{\epsilon^2 H^2}\left(1 + \frac{l^2}{R^2}\right) - \frac{1}{H}. \tag{4.16}$$

In [19], it was discussed that for $E > T_e$, M2-brane escapes to infinity and for $E < T_e$, it falls into M5-branes. From eqn (4.16), it is clear that effective potential vanishes for particular values of energy density and angular momentum. Thus for $l = \sqrt{\frac{Nl_p{}^3}{R_{11}}}$ and $\epsilon = 1$, $V_{eff} = 0$. Hence, for this energy and angular momentum limits, the decay process of the M2-brane supressed and it becomes a stable object and revolves around the M5-branes. The radial equation of motion derived from the action (4.5) is given by:

$$\frac{d}{dt}\left(\frac{\dot{R}}{F(\dot{\theta},\dot{\phi})}\right) = \frac{R}{F(\dot{\theta},\dot{\phi})}\left(\dot{\theta}^2 - \frac{l^2}{H^2 R^4}\right), \tag{4.17}$$

where

$$F(\dot{\theta},\dot{\phi}) = \sqrt{H^{-1} - \dot{R}^2 - R^2\dot{\theta}^2 - R_{11}{}^2\dot{\phi}^2}. \tag{4.18}$$

In order to write the equation of motion only of radial component, we have to eliminate $\dot{\theta}$ and $\dot{\phi}$ in terms of $\dot{R}$ and R. Using (4.11) and (4.12) in (4.9) and simplifing we get

$$\begin{aligned} \dot{\theta}^2 &= \frac{L_\theta{}^2}{T_e{}^2 R^4}\left(\frac{H^{-1} - \dot{R}^2}{1 + \frac{l^2}{R^2}}\right) \\ &= \frac{l^2}{R^4 H^2} - \frac{\dot{R}^2 l^2}{R^4 H}. \end{aligned} \tag{4.19}$$

Again using (4.19) in (4.12) and simplifying we get

$$\dot{\phi}^2 = \frac{L_\phi{}^2}{T_e{}^2 R_{11}{}^4}\left(\frac{H^{-1} - \dot{R}^2}{1 + \frac{l^2}{R^2}}\right). \tag{4.20}$$

Now using (4.19) and (4.20) in (4.18), we get

$$\begin{aligned} F(\dot{\theta},\dot{\phi}) &= \frac{T_1}{T_e}\sqrt{\frac{H^{-1} - \dot{R}^2}{1 + \frac{l^2}{R^2}}} \\ &= \frac{F(0)}{\sqrt{1 + \frac{l^2}{R^2}}}, \end{aligned} \tag{4.21}$$

where

$$F(0) = \frac{T_1}{T_e}\sqrt{H^{-1} - \dot{R}^2}. \tag{4.22}$$

Finally using (4.19) and (4.21) in (4.17), we can rewrite the equation of motion in terms of radial coordinate as

$$\frac{d}{dt}\left(\frac{\dot{R}\sqrt{1 + \frac{l^2}{R^2}}}{F(0)}\right) = -\frac{\dot{R}^2 l^2}{R^3 H}\left(\frac{\sqrt{1 + \frac{l^2}{R^2}}}{F(0)}\right). \tag{4.23}$$

Similarly to the analysis given for F-string in $D5$-brane background, we can write a classically equivalent radial action of (4.5) which gives the same equation of motion as (4.23) and same energy density as (4.6). The classically equivalent radial action is given by

$$\tilde{S}_{M2} = -T_e V \int dt \sqrt{1 + \frac{l^2}{R^2}} \sqrt{H^{-1} - \dot{R}^2}. \tag{4.24}$$

Note that this action is written in terms of effective tension and orbital angular momentum. The effect of other non-zero angular momentum is absorbed in the definition of T_e.

However the exact analogy with [16] is unknown because here we don't have a tachyon potential. So it remains to be seen whether this action gives more information about the radial dynamics of the membrane.

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