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## Redshift Drift in LTB Void Universes

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We study the redshift drift, i.e., the time derivative of the cosmological redshift in the Lemaître-Tolman-Bondi (LTB) solution in which the observer is assumed to be located at the symmetry center. This solution has often been studied as an anti-Copernican universe model to explain the acceleration of cosmic volume expansion without introducing the concept of dark energy. One of decisive differences between LTB universe models and Copernican universe models with dark energy is believed to be the redshift drift. The redshift drift is negative in all known LTB universe models, whereas it is positive in the redshift domain  $z \lesssim 2$  in Copernican models with dark energy. However, there have been no detailed studies on this subject. In the present paper, we prove that the redshift drift of an off-center source is always negative in the case of LTB void models. We also show that the redshift drift can be positive with an extremely large hump-type inhomogeneity. Our results suggest that we can determine whether we live near the center of a large void without dark energy by observing the redshift drift.

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## I. INTRODUCTION

The standard cosmological model is based on the so-called Copernican principle that we are not located in a special position in the universe. This model can naturally explain almost all observational data, and consequently seems to imply that the Copernican principle is a reality. However, we should not blindly rely on this principle without observational justifications.

The highly isotropic cosmic microwave background (CMB) implies the isotropy of our universe at our position. Then, if we assume that the Copernican principle is correct, we necessarily arrive at the conclusion that our universe is isotropic at every position, or equivalently, our universe is homogeneous and isotropic. By virtue of the homogeneity and isotropy, the standard cosmological model is determined by several parameters called the cosmological parameters. In order to determine the cosmological parameters, observational data are interpreted under the assumption of a homogeneous and isotropic universe on average. Here, we should note that it is not clear at all how large the systematic errors would be in the determination of the cosmological parameters, if the Copernican principle is abandoned. Thus, it is an unavoidable task in observational cosmology to investigate possible “anti-Copernican” universe models and test if such models can be observationally excluded.

Almost all anti-Copernican universe models are based on the Lemaître-Tolman-Bondi (LTB) solution which describes the dynamics of a spherically symmetric dust. In anti-Copernican universe models, an observer like us is usually assumed to be located in the neighborhood of the symmetry center [1–5]. In recent years, such models have attracted much attention, since some LTB universe models can recover the observed distance-redshift relation without introducing dark energy [6–22], and various ways to observationally test these models have been proposed by many authors [23–58].

As shown by previous studies, the LTB solution can explain various observational data besides the distance-redshift relation without the need to introduce dark energy. This is because this solution has functional degrees of freedom with respect to the comoving radial coordinate. In order to check the LTB universe models observationally, it is crucial to find observable quantities which can reveal differences between the LTB universe models and Copernican universe models with the dark energy. One such quantity is believed to be the

redshift drift, i.e., the time derivative of the cosmological redshift [46]. In the case of the  $\Lambda$ CDM model, which is the most likely Copernican model at present, the redshift drift is positive in the redshift domain  $z \lesssim 2$ , since the cosmological constant  $\Lambda$  causes repulsive gravity. By contrast, there is no exotic matter with the violation of the strong energy condition in the LTB solution. Thus, as long as there is no highly inhomogeneous structure, the redshift drift might be negative in LTB universe models. Although several authors have pointed out the importance of the redshift drift [41, 46, 59, 60], there has been no detailed study of its general behavior in LTB universe models. It is the purpose of this paper to investigate it.

This paper is organized as follows. In Sec. II, we briefly review the LTB solution. In Sec. III, we derive the equation for the redshift drift and show the behaviour of the redshift drift near an observer located at the symmetry center in LTB universe models. In Sec. IV, we define LTB void models and prove a theorem on the redshift drift in these models. In Sec. V, we show that the redshift drift can be positive even in an LTB universe model, if an extremely large hump-type mass density distribution exists. Sec. VI is devoted to the summary and discussion.

In this paper, we denote the speed of light and Newton's gravitational constant by  $c$  and  $G$ , respectively.

## II. THE LTB SOLUTION

As mentioned in the introduction, we consider a spherically symmetric inhomogeneous universe filled with dust. This universe is described by an exact solution of the Einstein equations, which is known as the Lemaître-Tolman-Bondi (LTB) solution. The metric of the LTB solution is given by

$$ds^2 = -c^2 dt^2 + \frac{(\partial_r R(t, r))^2}{1 - k(r)r^2} dr^2 + R^2(t, r) d\Omega^2, \quad (1)$$

where  $k(r)$  is an arbitrary function of the radial coordinate  $r$ . The matter is dust whose stress-energy tensor is given by

$$T^{\mu\nu} = \rho u^\mu u^\nu, \quad (2)$$

where  $\rho = \rho(t, r)$  is the mass density, and  $u^\mu$  is the four-velocity of the fluid element. The coordinate system in Eq. (1) is chosen in such a way that  $u^\mu = (c, 0, 0, 0)$ .

The circumferential radius  $R(t, r)$  is determined by one of the Einstein equations,

$$\left(\frac{\partial R}{\partial t}\right)^2 = \frac{2GM(r)}{R} - c^2kr^2, \quad (3)$$

where  $M(r)$  is an arbitrary function related to the mass density  $\rho$  by

$$\rho(t, r) = \frac{1}{4\pi R^2 \partial_r R} \frac{dM}{dr}. \quad (4)$$

$M(r)$  is known as the Misner-Sharp mass that is the quasi-local mass naturally introduced into the spherically symmetric spacetime[61]. Here, it should be noted that the Misner-Sharp mass is not necessarily a non-decreasing function of the comoving radial coordinate  $r$ , even if the mass density  $\rho$  is non-negative. In the case that a hypersurface labeled by  $t$  is a homogeneous and isotropic space with positive curvature,  $r$  can be chosen so that the circumferential radius is given by  $R = a(t) \sin r$ , where  $a(t)$  is a positive function of time (in this case,  $k(r) = r^{-2} \sin^2 r$ ). Thus,  $\partial_r R = a(t) \cos r$  is positive for  $0 \leq r < \pi/2$ , whereas it is negative  $\pi/2 < r \leq \pi$ . Since  $\rho$  is spatially constant by assumption, the derivative of the Misner-Sharp mass  $dM/dr$  is positive for  $0 \leq r < \pi/2$ , whereas it is negative for  $\pi/2 < r \leq \pi$ . However, in this paper, we assume that the Misner-Sharp mass is a monotonically increasing function of  $r$  in the domain of interest. This assumption is equivalent to the one that  $\partial_r R$  is positive if  $\rho$  is positive.

Following Ref. [62], we write the solution of Eq. (3) in the form,

$$R(t, r) = (6GM)^{1/3} [t - t_B(r)]^{2/3} S(x), \quad (5)$$

$$x = c^2kr^2 \left(\frac{t - t_B}{6GM}\right)^{2/3}, \quad (6)$$

where  $t_B(r)$  is an arbitrary function which determines the big bang time, and  $S(x)$  is a function defined implicitly as

$$S(x) = \begin{cases} \frac{\cosh \sqrt{-\eta} - 1}{6^{1/3}(\sinh \sqrt{-\eta} - \sqrt{-\eta})^{2/3}}; & x = \frac{-(\sinh \sqrt{-\eta} - \sqrt{-\eta})^{2/3}}{6^{2/3}} \quad \text{for } x \leq 0, \\ \frac{1 - \cos \sqrt{\eta}}{6^{1/3}(\sqrt{\eta} - \sin \sqrt{\eta})^{2/3}}; & x = \frac{(\sqrt{\eta} - \sin \sqrt{\eta})^{2/3}}{6^{2/3}} \quad \text{for } x > 0. \end{cases} \quad (7)$$

The function  $S(x)$  is analytic for  $x < (\pi/3)^{2/3}$ . Some characteristics of the function  $S(x)$  are given in Refs. [60] and [62].

As shown in the above, the LTB solution has three arbitrary functions,  $k(r)$ ,  $M(r)$  and  $t_B(r)$ . One of them is a gauge degree of freedom for the rescaling of  $r$ . In this paper, since

$M$  is assumed to be a monotonically increasing function of  $r$ , we can fix this freedom by setting

$$M = \frac{4}{3}\pi\rho_0 r^3, \quad (8)$$

where  $\rho_0$  is the mass density at the symmetry center at the present time  $t_0$ , i.e.,  $\rho_0 = \rho(t_0, 0)$ . By this choice, we have

$$R(t_0, r) = r + \mathcal{O}(r^2). \quad (9)$$

As in the case of the homogeneous and isotropic universe, the present Hubble parameter  $H_0$  is related to  $\rho_0$  as

$$H_0^2 + k(0)c^2 = \frac{8}{3}\pi G\rho_0. \quad (10)$$

### III. EQUATION FOR THE REDSHIFT DRIFT

In order to study the cosmological redshift and the redshift drift, we consider ingoing radial null geodesics. The cosmological redshift  $z$  of a light ray from a comoving source at  $r$  to the observer at the symmetry center  $r = 0$  is defined by

$$z(r) := \frac{k^t(\lambda(r))}{k^t(\lambda(0))} - 1, \quad (11)$$

where  $k^t$  is the time component of the null geodesic tangent, and  $\lambda$  is the affine parameter which can be regarded as a function of  $r$ . From the geodesic equations, we have the equation for the redshift  $z$  as

$$\frac{dz}{dr} = \frac{(1+z)\partial_t\partial_r R}{c\sqrt{1-kr^2}}. \quad (12)$$

The null condition leads to

$$\frac{dt}{dr} = -\frac{\partial_r R}{c\sqrt{1-kr^2}}. \quad (13)$$

We denote the trajectories of light rays observed by the central observer at  $t = t_0$  and  $t = t_0 + \delta t_0$ , respectively, by

$$\begin{cases} z = z_{\text{lc}}(r; t_0) \\ t = t_{\text{lc}}(r; t_0) \end{cases} \quad (14)$$

and

$$\begin{cases} z = z_{\text{lc}}(r; t_0 + \delta t_0) =: z_{\text{lc}}(r; t_0) + \delta z(r) \\ t = t_{\text{lc}}(r; t_0 + \delta t_0) =: t_{\text{lc}}(r; t_0) + \delta t(r) \end{cases}. \quad (15)$$

Here, by their definitions, we have  $t_{lc}(0; t_0) = t_0$ ,  $z_{lc}(0; t) = 0$ ,  $\delta z(0) = 0$  and  $\delta t(0) = \delta t_0$ . Substituting Eq. (15) into Eqs. (12) and (13), and regarding  $\delta z(r)$  and  $\delta t(r)$  as infinitesimal quantities, we obtain

$$\frac{d}{dr}\delta z = \frac{\partial_t \partial_r R}{c\sqrt{1-kr^2}}\delta z + \frac{(1+z)\partial_t^2 \partial_r R}{c\sqrt{1-kr^2}}\delta t, \quad (16)$$

$$\frac{d}{dr}\delta t = \frac{-\partial_t \partial_r R}{c\sqrt{1-kr^2}}\delta t, \quad (17)$$

where we have used the fact that (14) satisfies Eqs. (12) and (13), and the arguments of  $\partial_t \partial_r R$  and  $\partial_t^2 \partial_r R$  are  $t = t_{lc}(r, t_0)$  and  $r$ .

Hereafter, we consider the case where the cosmological redshift  $z$  is monotonically increasing with  $r$ . We say that such a model is *z-normal*. Then, we replace the independent variable  $r$  by  $z = z_{lc}(r; t_0)$ . By using

$$\frac{d}{dr} = \frac{dz}{dr} \frac{d}{dz} = \frac{(1+z)\partial_r \partial_r R}{c\sqrt{1-kr^2}} \frac{d}{dz}, \quad (18)$$

we have

$$\frac{d}{dz}\delta z = \frac{\delta z}{1+z} + \frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R}\delta t, \quad (19)$$

$$\frac{d}{dz}\delta t = -\frac{\delta t}{1+z}. \quad (20)$$

We can easily integrate Eq. (20) to obtain

$$\delta t = \frac{\delta t_0}{1+z}. \quad (21)$$

By using the above result, Eq. (19) is rewritten in the following form

$$\frac{d}{dz} \left( \frac{\delta z}{1+z} \right) = \frac{1}{(1+z)^2} \frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R} \delta t_0. \quad (22)$$

Here, let us study the redshift drift  $\delta z$  in the neighborhood of the symmetry center. By the regularity at  $r = 0$ , we have

$$k(r) = k_0 + \mathcal{O}(r), \quad (23)$$

$$t_B(r) = \mathcal{O}(r), \quad (24)$$

where, by using the freedom of the constant time translation, we have set  $t_B(0) = 0$ . From

Eq. (3), we have

$$\left(\frac{\partial_t R}{R}\right)^2 \Big|_{t=t_0, r=0} = -c^2 k_0 + \frac{8\pi G \rho_0}{3} = H_0^2, \quad (25)$$

$$\partial_t \partial_r R \Big|_{t=t_0, r=0} = H_0, \quad (26)$$

$$\partial_t^2 \partial_r R \Big|_{t=t_0, r=0} = -\frac{4\pi G \rho_0}{3} = -\frac{1}{2} \Omega_{m0} H_0^2, \quad (27)$$

where we have used Eq. (9) and  $\Omega_{m0} = 8\pi G \rho_0 / 3H_0^2$ . In the neighborhood of  $r = 0$ , we see from Eq.(22) that

$$\frac{d}{dz} \delta z \Big|_{t=t_0, r=0} = \frac{\partial_t^2 \partial_r R}{\partial_t \partial_r R} \Big|_{t=t_0, r=0} \delta t_0 + \mathcal{O}(z) = -\frac{1}{2} \Omega_{m0} \delta t_0 + \mathcal{O}(z). \quad (28)$$

Thus, we have

$$\frac{\delta z}{\delta t_0} = -\frac{1}{2} \Omega_{m0} z + \mathcal{O}(z^2). \quad (29)$$

The above equation shows that the redshift drift is non-positive near the center. This is the same behavior as that of the homogeneous and isotropic universe filled with dust.

#### IV. THE REDSHIFT DRIFT IN LTB VOID MODELS

We call an LTB universe model the LTB *void* model, if the following three conditions are satisfied.

1. the mass density is non-negative;
2. the mass density is increasing with  $r$  increasing in the domain  $r > 0$  on a spacelike hypersurface of constant  $t$ ;
3.  $\partial_r R$  is positive;
4.  $z$ -normality.

*Proposition 1* In LTB void models,  $\partial_t^2 \partial_r R$  is negative.

*Proof.* By Eq. (3), we obtain

$$\begin{aligned} \partial_t^2 \partial_r R(t, r) &= -\frac{G \partial_r M}{R^2} + \frac{2GM \partial_r R}{R^3} \\ &= 4\pi G \frac{\partial_r R}{R^3} \left( -\rho R^3 + 2 \int_0^r \rho(t, x) R^2(t, x) \partial_r R(t, x) dx \right), \end{aligned} \quad (30)$$

where we have used Eq. (4) in the second equality. Since  $\partial_r R$  is positive by the definition of LTB void models, we may replace the integration variable  $x$  by  $R = R(t, x)$  and obtain

$$\begin{aligned}\partial_t^2 \partial_r R(t, r) &= 4\pi G \frac{\partial_r R}{R^3} \left( -\rho R^3 + 2 \int_0^{R(t,r)} \rho R^2 dR \right) \\ &= -4\pi G \frac{\partial_r R}{R^3} \int_0^{R(t,r)} \left( \frac{d\rho}{dR} R^3 + \rho R^2 \right) dR.\end{aligned}\quad (31)$$

Since  $d\rho/dR = (\partial_r R)^{-1} \partial_r \rho$  is positive in the domain of  $R > 0$ , the integrand in the last equality of the above equation is positive. Q.E.D.

*Theorem* In LTB void models, the redshift drift of an off-center source observed at the symmetry center is negative.

*Proof.* Since the cosmological redshift  $z$  vanishes at  $r = 0$ ,  $z$  is non-negative by the assumption of  $z$ -normality. Further, the  $z$ -normality leads to  $\partial_t \partial_r R > 0$  through Eq. (12). Then, since  $\delta t_0 > 0$ , we see from Eq. (22) that Proposition 1 leads to the following inequality

$$\frac{d}{dz} \left( \frac{\delta z}{1+z} \right) < 0. \quad (32)$$

Since  $\delta z$  should vanish at  $z = 0$ , we have  $\delta z < 0$  for  $z > 0$  from the above inequality. Q.E.D.

## V. REDSHIFT DRIFT IN LTB UNIVERSE MODELS WITH A LARGE HUMP

In the preceding section, we showed that the redshift drift observed at the symmetry center is negative for  $r > 0$  in LTB void models. Conversely, if there is a domain in which the mass density is decreasing with increasing  $r$ , the redshift drift might be negative. In this section, we show that it is true with hump-type mass density distributions. We consider the following two LTB universe models,

- (i)  $k(r) = 0$  and  $t_B(r) = f(r; a, r_1, r_2)$   
with  $a = -1.7H_0^{-1}$ ,  $r_1 = 0.12cH_0^{-1}$  and  $r_2 = 0.9cH_0^{-1}$ ,
- (ii)  $t_B(r) = 0$  and  $k(r) = f(r; a, r_1, r_2)$   
with  $a = -100c^{-2}H_0^2$ ,  $r_1 = 0.1cH_0^{-1}$  and  $r_2 = 0.2cH_0^{-1}$ ,



where

$$f(r; a, r_1, r_2) = \begin{cases} 0 & \text{for } r < r_1, \\ \frac{a(r - r_1)^3 (r_1^2 - 5r_1r_2 + 10r_2^2 + 3r_1r - 15r_2r + 6r^2)}{(r_2 - r_1)^5} & \text{for } r_1 \leq r < r_2, \\ a & \text{for } r_2 \leq r. \end{cases} \quad (33)$$

In Figs.1 and 2, we show the redshift drifts of these models. Although we do not show the energy densities of these models, a large hump in the mass density distribution exists in each model as well as in  $t_B(r)$  or  $k(r)$ . Although there is a redshift domain with positive redshift drift in each example, the distance-redshift relations of these models do not agree with the observational data, and further, the inhomogeneities need to be very large.

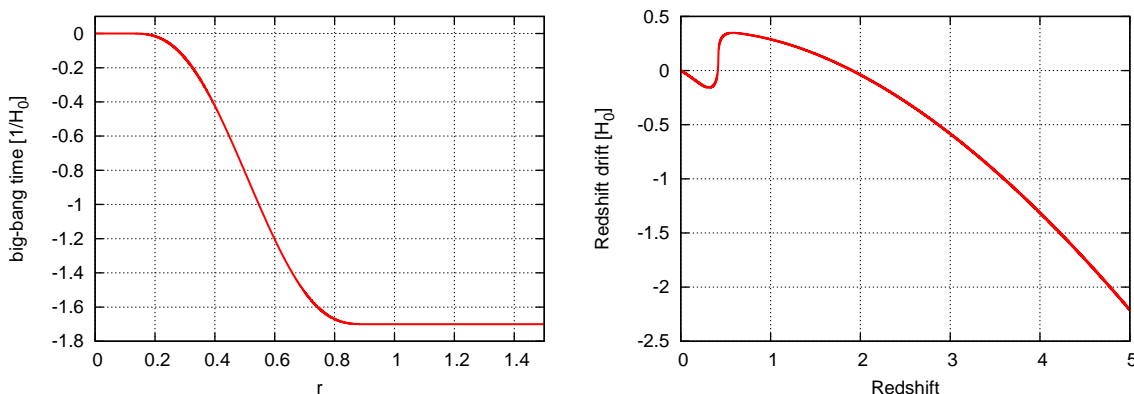


FIG. 1: The LTB universe model with  $k(r) = 0$ . The right panel depicts the redshift drift  $\delta z/\delta t_0$  of the LTB universe model with  $k(r) = 0$  as a function of the redshift  $z$ . The other arbitrary function  $t_B(r)$  is shown in the left panel.

## VI. SUMMARY AND DISCUSSION

In this paper, we studied the redshift drift in LTB universe models in which the observer is located at the symmetry center. We showed that, assuming that the mass density of the dust is positive, the redshift drift of an off-center source is negative if the mass density and the circumferential radius are increasing functions of the comoving radial coordinate. We also showed that if there is a very large hump structure around the symmetry center, the redshift drift can be positive. As a result, by observation of the redshift drift, we get a

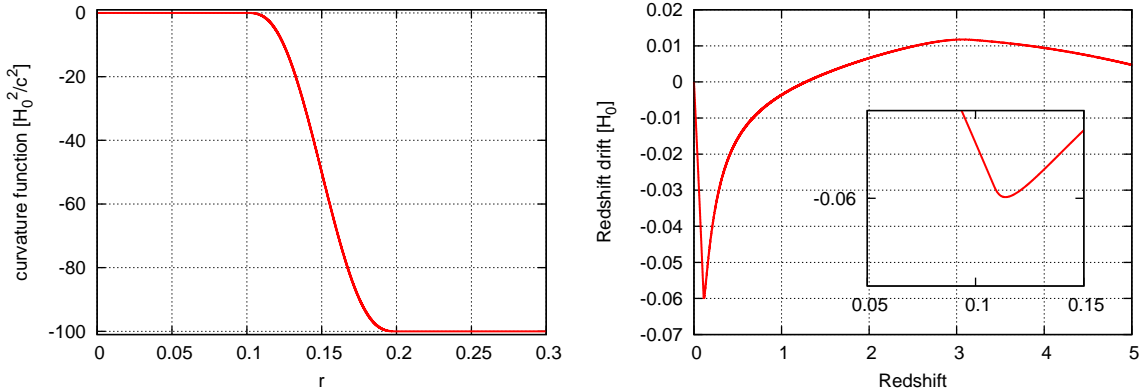


FIG. 2: The right panel depicts the redshift drift  $\delta z/\delta t_0$  of the LTB universe model with  $t_B(r) = 0$  as a function of the redshift  $z$ . The other arbitrary function  $k(r)$  is shown in the left panel.

strong constraint on void-type universe models: if the redshift drift turns out to be positive in some redshift domain, LTB void models can be rejected.

By projects such as the Cosmic Dynamics Experiment(CODEX) [63–65] in the European Extremely Large Telescope(E-ELT)[66] and PREcision Super Stable Observations (ESPRESSO) in the Very Large Telescope array(VLT) [64, 65, 67], it is possible to get highly accurate spectroscopic observational data. The observability of the redshift drift by CODEX has been analyzed in Refs. [64, 65, 68, 69], and the test of LTB universe models by this project has been investigated in Ref. [41]. Targets of these projects are QSOs whose redshifts are larger than two. On the other hand, observing the sign of the redshift drift in the redshift domain  $z \lesssim 2$  is very crucial to test LTB void universe models in contrast with Copernican universe models. This is because known examples of Copernican universe models with dark energy or modified gravity predict a positive redshift drift in the redshift domain  $z \lesssim 2$ , while it is negative for LTB void models. From, for example, the observation of compact binary stars by DECIGO [70–72] or BBO [73, 74], the redshift drift at  $z \simeq 1$  can be measured [70]. Observations by DECIGO or BBO over several years will make it possible to observe the sign of the redshift drift for  $z \lesssim 2$  and to test LTB void models [75].

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- [1] I. Zehavi, A. G. Riess, R. P. Kirshner, and A. Dekel, *Astrophys. J.* **503**, 483 (1998), arXiv:astro-ph/9802252, *A Local Hubble Bubble from SNe Ia?*
  - [2] K. Tomita, *Astrophys. J.* **529**, 38 (2000), arXiv:astro-ph/9906027, *Distances and lensing in cosmological void models.*
  - [3] K. Tomita, *Mon. Not. Roy. Astron. Soc.* **326**, 287 (2001), arXiv:astro-ph/0011484, *A Local Void and the Accelerating Universe.*
  - [4] K. Tomita, *Prog. Theor. Phys.* **106**, 929 (2001), arXiv:astro-ph/0104141, *Analyses of Type Ia Supernova Data in Cosmological Models with a Local Void.*
  - [5] M.-N. Celerier, *Astron. Astrophys.* **353**, 63 (2000), arXiv:astro-ph/9907206, *Do we really see a cosmological constant in the supernovae data ?*
  - [6] H. Alnes, M. Amarzguioui, and O. Gron, *Phys. Rev.* **D73**, 083519 (2006), arXiv:astro-ph/0512006, *An inhomogeneous alternative to dark energy?*
  - [7] H. Alnes and M. Amarzguioui, *Phys. Rev.* **D74**, 103520 (2006), arXiv:astro-ph/0607334, *CMB anisotropies seen by an off-center observer in a spherically symmetric inhomogeneous universe.*
  - [8] H. Alnes and M. Amarzguioui, *Phys. Rev.* **D75**, 023506 (2007), arXiv:astro-ph/0610331, *The supernova Hubble diagram for off-center observers in a spherically symmetric inhomogeneous universe.*
  - [9] S. Alexander, T. Biswas, A. Notari, and D. Vaid, *JCAP* **0909**, 025 (2009), arXiv:0712.0370, *Local Void vs Dark Energy: Confrontation with WMAP and Type Ia Supernovae.*
  - [10] K. Bolejko and J. S. B. Wyithe, *JCAP* **0902**, 020 (2009), arXiv:0807.2891, *Testing the Copernican Principle via Cosmological Observations.*

- [11] K. Enqvist, *Gen. Rel. Grav.* **40**, 451 (2008), arXiv:0709.2044, *Lemaitre-Tolman-Bondi model and accelerating expansion.*
- [12] K. Enqvist and T. Mattsson, *JCAP* **0702**, 019 (2007), arXiv:astro-ph/0609120, *The effect of inhomogeneous expansion on the supernova observations.*
- [13] S. February, J. Larena, M. Smith, and C. Clarkson, (2009), arXiv:0909.1479, *Rendering Dark Energy Void.*
- [14] J. Garcia-Bellido and T. Haugboelle, *JCAP* **0804**, 003 (2008), arXiv:0802.1523, *Confronting Lemaitre-Tolman-Bondi models with Observational Cosmology.*
- [15] J. Garcia-Bellido and T. Haugboelle, *JCAP* **0909**, 028 (2009), arXiv:0810.4939, *The radial BAO scale and Cosmic Shear, a new observable for Inhomogeneous Cosmologies.*
- [16] J. Garcia-Bellido and T. Haugboelle, *JCAP* **0809**, 016 (2008), arXiv:0807.1326, *Looking the void in the eyes - the kSZ effect in LTB models.*
- [17] D. Garfinkle, *Class. Quant. Grav.* **23**, 4811 (2006), arXiv:gr-qc/0605088, *Inhomogeneous spacetimes as a dark energy model.*
- [18] M. Kasai, *Prog. Theor. Phys.* **117**, 1067 (2007), arXiv:astro-ph/0703298, *Apparent Acceleration through Large-scale Inhomogeneities –Post-Friedmannian Effects of Inhomogeneities on the Luminosity Distance–.*
- [19] K. Tomita, *Astrophys. J.* **529**, 26 (2000), arXiv:astro-ph/9905278, *Bulk Flows and Cosmic Microwave Background Dipole Anisotropy in Cosmological Void Models.*
- [20] K. Tomita, *Prog. Theor. Phys.* **105**, 419 (2001), arXiv:astro-ph/0005031, *Anisotropy of the Hubble Constant in a Cosmological Model with a Local Void on Scales of 200 Mpc.*
- [21] J. P. Zibin, A. Moss, and D. Scott, *Phys. Rev. Lett.* **101**, 251303 (2008), arXiv:0809.3761, *Can we avoid dark energy?*
- [22] T. Clifton, P. G. Ferreira, and J. Zuntz, *JCAP* **0907**, 029 (2009), arXiv:0902.1313, *What the small angle CMB really tells us about the curvature of the Universe.*
- [23] T. Biswas and A. Notari, *JCAP* **0806**, 021 (2008), arXiv:astro-ph/0702555, *Swiss-Cheese Inhomogeneous Cosmology & the Dark Energy Problem.*
- [24] K. Bolejko, *PMC Phys.* **A2**, 1 (2008), arXiv:astro-ph/0512103, *Supernovae Ia observations in the Lemaitre–Tolman model.*
- [25] K. Bolejko, *Gen. Rel. Grav.* **41**, 1737 (2009), arXiv:0804.1846, *The Szekeres Swiss Cheese model and the CMB observations.*

- [26] K. Bolejko and P. Lasky, (2008), arXiv:0809.0334, *Pressure gradients, shell crossing singularities and acoustic oscillations - application to inhomogeneous cosmological models.*
- [27] N. Brouzakis, N. Tetradis, and E. Tzavara, JCAP **0702**, 013 (2007), arXiv:astro-ph/0612179, *The Effect of Large-Scale Inhomogeneities on the Luminosity Distance.*
- [28] N. Brouzakis, N. Tetradis, and E. Tzavara, JCAP **0804**, 008 (2008), arXiv:astro-ph/0703586, *Light Propagation and Large-Scale Inhomogeneities.*
- [29] S. Bhattacharya, P. S. Joshi, and K.-i. Nakao, (2009), arXiv:0911.2297, *Accelerated cosmic expansion in a scalar-field universe.*
- [30] C. Clarkson, B. Bassett, and T. H.-C. Lu, Phys. Rev. Lett. **101**, 011301 (2008), arXiv:0712.3457, *A general test of the Copernican Principle.*
- [31] R. R. Caldwell and A. Stebbins, Phys. Rev. Lett. **100**, 191302 (2008), arXiv:0711.3459, *A Test of the Copernican Principle.*
- [32] T. Clifton, P. G. Ferreira, and K. Land, Phys. Rev. Lett. **101**, 131302 (2008), arXiv:0807.1443, *Living in a Void: Testing the Copernican Principle with Distant Supernovae.*
- [33] M. P. Dabrowski and M. A. Hendry, Astrophys. J. **498**, 67 (1998), arXiv:astro-ph/9704123, *Non-Uniform Pressure Universes: The Hubble Diagram of Type Ia Supernovae and the Age of the Universe.*
- [34] J. Goodman, Phys. Rev. **D52**, 1821 (1995), arXiv:astro-ph/9506068, *Geocentrism reexamined.*
- [35] W. Godlowski, J. Stelmach, and M. Szydlowski, Class. Quant. Grav. **21**, 3953 (2004), arXiv:astro-ph/0403534, *Can the Stephani model be an alternative to FRW accelerating models?*
- [36] J. Jia and H.-b. Zhang, JCAP **0812**, 002 (2008), arXiv:0809.2597, *Can the Copernican principle be tested by cosmic neutrino background?*
- [37] P. D. Lasky and K. Bolejko, (2010), arXiv:1001.1159, *The effect of pressure gradients on luminosity distance - redshift relations.*
- [38] V. Marra, E. W. Kolb, S. Matarrese, and A. Riotto, Phys. Rev. **D76**, 123004 (2007), arXiv:0708.3622, *On cosmological observables in a swiss-cheese universe.*
- [39] J. W. Moffat, (2006), arXiv:astro-ph/0606124, *Inhomogeneous Cosmology, Inflation and Late-Time Accelerating Universe.*
- [40] J. F. Pascual-Sanchez, Mod. Phys. Lett. **A14**, 1539 (1999), arXiv:gr-qc/9905063, *Cosmic acceleration: Inhomogeneity versus vacuum energy.*

- [41] M. Quartin and L. Amendola, (2009), arXiv:0909.4954, *Distinguishing Between Void Models and Dark Energy with Cosmic Parallax and Redshift Drift.*
- [42] A. E. Romano, Phys. Rev. **D76**, 103525 (2007), *Redshift spherical shell energy in isotropic universes.*
- [43] J. Stelmach and I. Jakacka, Class. Quant. Grav. **23**, 6621 (2006), *Angular sizes in spherically symmetric Stephani cosmological models.*
- [44] M. Tanimoto, Y. Nambu, and K. Iwata, (2009), arXiv:0906.4857, *The Role of Anisotropy in the Void Models without Dark Energy.*
- [45] K. Tomita, (2009), arXiv:0912.4773, *Gauge-invariant treatment of the integrated Sachs-Wolfe effect on general spherically symmetric spacetimes.*
- [46] J.-P. Uzan, C. Clarkson, and G. F. R. Ellis, Phys. Rev. Lett. **100**, 191303 (2008), arXiv:0801.0068, *Time drift of cosmological redshifts as a test of the Copernican principle.*
- [47] C.-M. Yoo, K.-i. Nakao, and M. Sasaki, (2010), arXiv:1005.0048, *CMB observations in LTB universes: Part I: Matching peak positions in the CMB spectrum.*
- [48] C. A. Clarkson and R. Barrett, Class. Quant. Grav. **16**, 3781 (1999), arXiv:gr-qc/9906097, *Does the Isotropy of the CMB Imply a Homogeneous Universe? Some Generalised EGS Theorems.*
- [49] J. P. Zibin, (2008), arXiv:0804.1787, *Scalar Perturbations on Lemaitre-Tolman-Bondi Spacetimes.*
- [50] M. Regis and C. Clarkson, (2010), arXiv:1003.1043, *Do primordial Lithium abundances imply there's no Dark Energy?*
- [51] C. Clarkson and M. Regis, (2010), arXiv:1007.3443, *The Cosmic Microwave Background in an Inhomogeneous Universe.*
- [52] C. Clarkson and R. Maartens, Class. Quant. Grav. **27**, 124008 (2010), arXiv:1005.2165, *Inhomogeneity and the foundations of concordance cosmology.*
- [53] T. Biswas, A. Notari, and W. Valkenburg, (2010), arXiv:1007.3065, *Testing the Void against Cosmological data: fitting CMB, BAO, SN and  $H_0$ .*
- [54] H. Kodama, K. Saito, and A. Ishibashi, (2010), arXiv:1004.3089, *Analytic formulae for the off-center CMB anisotropy in a general spherically symmetric universe.*
- [55] S. Foreman, A. Moss, J. P. Zibin, and D. Scott, (2010), arXiv:1009.0273, *Spatial and temporal tuning in void models for acceleration.*

- [56] A. Moss, J. P. Zibin, and D. Scott, (2010), arXiv:1007.3725, *Precision Cosmology Defeats Void Models for Acceleration*.
- [57] V. Marra and M. Paakkonen, (2010), arXiv:1009.4193, *Observational constraints on the LLTB model*.
- [58] P. Zhang and A. Stebbins, (2010), arXiv:1009.3967, *Confirmation of the Copernican principle at Gpc radial scale and above from the kinetic Sunyaev Zel'dovich effect power spectrum*.
- [59] P. Dunsby, N. Goheer, B. Osano, and J.-P. Uzan, (2010), arXiv:1002.2397, *How close can an Inhomogeneous Universe mimic the Concordance Model?*
- [60] C.-M. Yoo, T. Kai, and K.-i. Nakao, Prog. Theor. Phys. **120**, 937 (2008), arXiv:0807.0932, *Solving Inverse Problem with Inhomogeneous Universe*.
- [61] C. W. Misner and D. H. Sharp, Phys. Rev. **136**, B571 (1964), *Relativistic equations for adiabatic, spherically symmetric gravitational collapse*.
- [62] M. Tanimoto and Y. Nambu, Class. Quant. Grav. **24**, 3843 (2007), arXiv:gr-qc/0703012, *Luminosity distance-redshift relation for the LTB solution near the center*.
- [63] L. Pasquini *et al.*, The Messenger **122**, 10 (2005), *CODEX: Measuring the Expansion of the Universe (and beyond)*.
- [64] S. Cristiani *et al.*, Nuovo Cim. **122B**, 1159 (2007), arXiv:0712.4152, *The CODEX-ESPRESSO experiment: cosmic dynamics, fundamental physics, planets and much more*.
- [65] J. Liske *et al.*, Mon. Not. Roy. Astron. Soc. **386**, 1192 (2008), arXiv:0802.1532, *Cosmic dynamics in the era of Extremely Large Telescopes*.
- [66] R. Gilmozzi and J. Spyromilio, The Messenger **127**, 11 (2007), *The European Extremely Large Telescope (E-ELT)*.
- [67] CODEX/ESPRESSO, V. D'Odorico, (2007), arXiv:0708.1258, *CODEX/ESPRESSO: the era of precision spectroscopy*.
- [68] P.-S. Corasaniti, D. Huterer, and A. Melchiorri, Phys. Rev. **D75**, 062001 (2007), arXiv:astro-ph/0701433, *Exploring the Dark Energy Redshift Desert with the Sandage-Loeb Test*.
- [69] A. Balbi and C. Quercellini, (2007), arXiv:0704.2350, *The time evolution of cosmological redshift as a test of dark energy*.
- [70] N. Seto, S. Kawamura, and T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001), arXiv:astro-ph/0108011, *Possibility of direct measurement of the acceleration of the universe*

*using 0.1-Hz band laser interferometer gravitational wave antenna in space.*

- [71] S. Kawamura *et al.*, *Class. Quant. Grav.* **23**, S125 (2006), *The Japanese space gravitational wave antenna DECIGO.*
- [72] S. Sato *et al.*, *J. Phys. Conf. Ser.* **154**, 012040 (2009), *DECIGO: The Japanese space gravitational wave antenna.*
- [73] C. Ungarelli, P. Corasaniti, R. A. Mercer, and A. Vecchio, *Class. Quant. Grav.* **22**, S955 (2005), arXiv:astro-ph/0504294, *Gravitational waves, inflation and the cosmic microwave background: towards testing the slow-roll paradigm.*
- [74] E. S. Phinney *et al.*, (NASA) (2003), *Big Bang Observer Mission Concept Study.*
- [75] K. Yagi, A. Nishizawa, and C. Yoo, in preparation .