There are no Static Solutions in Source-Free Non-commutative $U_\star(1)$ Gauge Theory

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The vanishing of self-stress for static systems excludes finite energy time-independent solutions of source-free $U_\star(1)$ theory in (3+1) dimensions. This implies that static solutions in case $\theta^{0i} \neq 0$ for non-commutative electromagnetic fields are forbidden.

Keywords: Non-commutative Geometry, Non-commutative QED

Introduction

Now, there has been a great deal of interest in the study of noncommutative geometry because it is a very rich framework for modifying the theoretical physics and also the fact that we can discover the standard results just by requiring the vanishing of the deformed parameter which means also the vanishing of noncommutativity.

Usual noncommutative theories are based on an antisymmetric quantity of rank two. For this, it does not exhibit the Lorentz symmetry and it does not make much sense to look for noncommutative theories invariant by general coordinates[1]. But, many people believe that this problem can be solved by Hopf algebra. This hope has led to further study of noncommutative dynamics. These studies revealed some peculiar features of noncommutative quantum models. Much attention has been paid also to quantum field theories on noncommutative space time, in particular noncommutative Yang - Mills theory as well as noncommutative QED.

The aim of this paper is to study another aspect of the noncommutativity framework adapted to the source-free static solutions of noncommutative Maxwell equations. As well known, the Maxwell four laws describe the evolution in time and space of the electric and magnetic fields E and B. Recently, many people attended to a new approach leading to noncommutative electrodynamics and many details were extracted[2, 3, 4, 5, 6, 7]. In this paper I will study of values of E and B and I will show that there are no static solutions of source-free noncommutative $U_\star(1)$ in case $\theta^{0i} \neq 0$. In this case, we will remove all $\star$'s related to metric tensor, because we do not search the momentum of metric. In [8] S. Deser presents the static solutions in source-free Yang - Mills theory are forbidden. His work is nonabelian electrodynamics in commutative space and I think this idea can not be generalized to noncommutative electrodynamics comprehensive.

Physics Notes

I now turn to the case of noncommutative $U_\star(1)$ theory in d-dimensions in case $\theta^{0i} \neq 0$ and $\theta^{ij} = 0$, that is

$$\begin{aligned} &[\hat{x}^0,\ \hat{x}^j] = \imath\theta^{0j} \\ &[\hat{x}^i,\ \hat{x}^j] = 0 \end{aligned} \tag{1}$$

with action

$$\mathbf{S} = \int dx\ \frac{-1}{4}(F_{\mu\nu} \star F_{\alpha\beta})g^{\mu\alpha}g^{\nu\beta} \tag{2}$$

where $F^{\mu\nu}$ denotes the strength of the noncommutative $U_\star(1)$ gauge field A_μ that is $F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha - \imath e[A^\alpha,\ A^\beta]_\star$ and I remove all $\star$'s related to metric tensor, because I do not search the momentum of metric. Also, I work in Minkowski space time so the $g = \eta$ is x^μ-independent. In standard $U_\star(1)$ theory the canonical energy - momentum tensor is neither symmetric nor gauge invariant. But by applying the Noether or Belinfante procedure one finds a symmetric and gauge invariant tensor[9]. With this tools for noncommutative $U_\star(1)$ theory we get to[2]

$$4\mathbf{T}_\star^{\mu\nu} = -2\{F^{\mu\alpha},\ F_\alpha{}^\nu\}_\star - g^{\mu\nu}(F^{\alpha\beta} \star F_{\alpha\beta}) \tag{3}$$

with $D_\mu \star \mathbf{T}_\star^{\mu\nu} = \partial_\mu \mathbf{T}_\star^{\mu\nu} - \imath e[A_\mu,\ \mathbf{T}_\star^{\mu\nu}]_\star = 0$ I show here that for the case $d = 4$ and $\theta^{0i} \neq 0$ (of course, $\theta^{ij} = 0$) there are no static solutions to self interacting models of $U_\star(1)$ type. I know the one of $\star$'s can be removed so the stress tensor for a $U_\star(1)$ field has components

$$\int dx\ \mathbf{T}^\mu_{\star\mu} = \int dx\ \mathbf{T}^\mu_\mu = \int dx\ \frac{1}{4}(4-d)F^{\alpha\beta}F_{\alpha\beta} \tag{4}$$

then $\int dx\ \mathbf{T}^0_0 = \int dx\ \frac{1}{2}(F^2_{0i} + \frac{1}{2}F^2_{ij})$ where $F^2 = F^{\mu\nu}F_{\nu\mu}$ now, compactness of gauge group indeed to F^2_{0i} and F^2_{ij} be positive. I assume that in boundaries of space, all fields and related them must will be vanish because all of $F^{\mu\nu}$ to fall of faster than $|\ \vec{r}\ |^{-\frac{1}{2}(d-1)}$ so

$$\int d^{d-1}x\ x_j \star D_\mu \star \mathbf{T}_\star^{\mu j} = \int d^{d-1}x\ x_j \star (\partial_\mu \mathbf{T}_\star^{\mu j} - \imath e[A_\mu,\ \mathbf{T}_\star^{\mu j}]_\star) = 0 \tag{5}$$

but all fields are static then

$$\int d^{d-1}x\ (-\mathbf{T}_\star^{jj} - \imath e x_j \star [A_\mu,\ \mathbf{T}_\star^{\mu j}]_\star) = 0 \tag{6}$$

in case $\theta^{i0} \neq 0$ the second term will be vanish because $\int\ x_j \star [A_\mu,\ \mathbf{T}_\star^{\mu j}]_\star = \int\ A_\mu \star [x_j,\ \mathbf{T}_\star^{\mu j}]_\star = \int\ \imath\theta^{\alpha\beta} A_\mu \star \partial_\alpha \mathbf{T}_\star^{\mu j} \delta_j^\beta$ then $\alpha = 0$ but all fields and related them must be static then the second term will be vanish. The vanishing of second term implies that $\int d^{d-1}x\ \mathbf{T}^{ii} = 0$ or

$$\int dx\ \mathbf{T}^i_i = \int dx\ \frac{1}{2}(\frac{(5-d)}{2}F^2_{0i} + (d-3)\frac{1}{2}F^2_{ij}) = 0 \tag{7}$$

for $d = 4$ first and second term must all vanish and for $d > 5$ we learn nothing further above from eq-(7). But, the time independent solutions of gauge fields are chosen so the electric field reduces to $F_{0i} = -D_i \star A_0$ where $D_i = \partial_i - \imath e[A_i,\ ]_\star$ it follows that

$D^i \star F_{0i} = 0$ so I have $\int d^{d-1}x A_0 \star D^i \star F_{0i} = -\int d^{d-1}x D^i \star A_0 \star F_{0i} = -\int d^{d-1}x F_{0i}^2 = 0$ and consequently that for any values of 'd' I have $F_{0i} = 0$ that the result is the absence of electric fields. With this result eq-(7) yields $F_{ij} = 0$ expect for $d = 5$. This means that the static solutions in case $\theta^{0i} \neq 0$ in (3+1) dimensions for source-free $U_\star(1)$ theory is absence.

For case $\theta^{ij} \neq 0$ the part of F^{0i} is still zero, this means that electric fields are still absence in (3+1) dimensions but the part of F^{ij} can be no zero in arbitrary dimensions because $F_{ij}^2 \sim \int d^{d-1}x \; x_i \star [A_\mu, \; T^\mu_j]_\star$.

Discussion

In this work by using the action of quantum electrodynamics in noncommutative space time and $\theta^{0i} \neq 0$ we saw that the momentum conjugate of $g^{\mu\nu}$ does not exhibit so the metric term did not participate with star product in Lagrangian density and also for the typical energy-momentum tensor, from general way, we saw that the vanishing of self-stress for static systems excludes finite energy time-independent solutions of source-free $U_\star(1)$ theory in (3+1) dimensions. This implies that static solutions in case $\theta^{0i} \neq 0$ for non-commutative electromagnetic fields are forbidden. For case $\theta^{ij} \neq 0$ we say that just the electric fields is zero and the magnetic fields may be no zero!.

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