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On Black Attractors in 8D and Heterotic/Type IIA Duality

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ABSTRACT: Motivated by the study of black attractors in *8D* supergravity with *16* supersymmetries, we use the field theory approach and *8D* supersymmetry with non trivial central charges to shed light on the exact duality between heterotic string on T^2 and type IIA on real connected and compact surfaces Σ_2. We investigate the two constraints that should be obeyed by Σ_2 and give their solutions in terms of intersecting 2-cycles as well their classification using Dynkin diagrams of affine Kac-Moody algebras. It is shown as well that the moduli space of these dual theories is given by $SO\left(1,1\right) \times \frac{SO(2,r+2)}{SO(2) \times SO(r+2)}$ where r stands for the rank of the gauge symmetry G_r of the *10D* heterotic string on T^2. The remarkable cases $r = -2, -1, 0$ as well as other features are also investigated.

KEYWORDS: supergravity in *8D*, superstring compactifications, heterotic / type IIA duality, extremal black attractors..

Contents

1. Introduction

In the last few years, there has been a growing interest in the study the attractor mechanism of black brane configurations in higher dimensional supergravity theories with typical moduli space given by the coset group manifold G/H [1]-[9]. This mechanism was first discovered in the context of supersymmetric black holes in 4D $\mathcal{N} = 2$ supergravity coupled to vector supermultiplets [10]-[13]; and was extended for black branes living in diverse dimensions; for reviews see [14]-[19] and refs therein. The attractor mechanism tells that the values $\phi^I(r_H)$ of the moduli at the horizon geometry of the black attractor are independent of the asymptotic values $\phi^I(r \to \infty)$; and, up on solving the attractor equations, they are completely determined by the electric Q_Λ and magnetic P^Λ charges respectively given by the fluxes $\int_{S^{6-p}} \tilde{\mathcal{F}}_{6-p|\Lambda}$ and $\int_{S^{p+2}} \mathcal{F}^\Lambda_{p+2}$ of the various massless abelian gauge fields of the supergravity theory including p-branes.

A remarkable class of the supergravity theories that have attracted much interest in the recent literature concerns the ultra-violet (UV) finite ones arising from superstrings and M-theory compactifications having interpretation in terms of D- and M- branes wrapping cycles of the internal space. There, the Maxwell type gauge fields are associated with semi-realistic string models with multitude of $U(1)$ gauge factors that survive in the typical gauge symmetry breaking

$$G_r \longrightarrow \prod_i G_{N_i} \quad , \tag{1.1}$$

where G_r can be thought of as the $E_8 \times E_8$ or $SO(32)$ gauge symmetries of the *10D* heterotic string, or a subsymmetry of them. The G_{N_i}s are rank N_i subgroups of G_r with the property $\sum_i N_i \leq r \leq 16$ and associated 2-form gauge field strengths $\mathcal{F}_2$ having non trivial fluxes along the $U^{N_i}(1)$ directions and playing a central role in the study of black attractors in these supergravity theories. In this issue, several results have been obtained on attractor mechanism of extremal (vanishing temperature for non-zero entropy) BPS and non BPS black branes living in *4D*, *5D*, *6D*, *7D* and *8D* supergravities with various numbers of conserved supersymmetries [8, 9] and refs therein.

On the other hand, in completing missing results on intersecting attractors in non chiral $\mathcal{N} = 1$ supergravity in eight dimensions [20] arising from *10D* heterotic string on 2-torus and involving $(2 + 2 + n)$ 1-form gauge fields together with $2(2 + n)$ real scalars parameterizing the moduli space

$$\frac{SO(2,2+n)}{SO(2)\times SO(2+n)} \times SO(1,1) \quad , \quad 0 \leq n \leq 16 \quad , \tag{1.2}$$

we have noticed an interesting property concerning their type IIA interpretation using D- branes. This property, which is investigated in this paper, concerns the two following things:

(**1**) the geometric engineering of the *exact* duality between the *10D* heterotic superstring on T^2 with some gauge symmetry G_r; and the type IIA superstring compactified on real compact surfaces Σ_2 with D- branes wrapping cycles of Σ_2.

(**2**) The heterotic/ type IIA interpretation of the *singular limit* describing the coupling of the *8D* $\mathcal{N} = 1$ supergravity multiplet to *one* Maxwell gauge supermultiplet. The moduli space of this special supersymmetric gauge theory is given by the particular real 3-dimensional coset manifold,

$$\frac{SO(2,1)}{SO(2)} \times SO(1,1) \quad , \tag{1.3}$$

which does not appear in the moduli space family (1.2); although it could be recovered by going beyond the lower bound of the positive integer n and setting $n = -1$ in eq(1.2).

Moreover, because of the heterotic/type IIA duality in *8D*, we also show that the real compact surfaces Σ_2, used in the compactification scenario

$$\mathcal{M}^{1,9} \longrightarrow \mathcal{M}^{1,7} \times \Sigma_2 \quad , \tag{1.4}$$

should obey two basic consistency constraints. The first one, which is familiar in string compactification, deals with the number of conserved supersymmetries after space time dimension reduction. The second one however concerns the connected 2-cycle homology of Σ_2 that requires a vanishing self intersection; i.e : $\Sigma_2 \cdot \Sigma_2 = 0$. This constraint turns out to capture two remarkable information:

(**a**) it allows to explain the stringy origin of the non chiral $8D$ $\mathcal{N} = 1$ supergravity models with moduli spaces $\frac{SO(2,1)}{SO(2)} \times SO(1,1)$ and $SO(1,1)$ which miss in the listing (1.2) and respectively recovered by roughly setting $n = -1, -2$.

(**b**) it captures data on superconformal invariance in $8D$ that underly known results on $4D$ $\mathcal{N} = 2$ conformal models. Recall that $4D$ supersymmetric conformal models were first obtained in [21], see also [22] for an extension, from $10D$ type II superstrings on Calabi-Yau threefolds with affine ADE singularities; they may be re-derived from non chiral $8D$ $\mathcal{N} = 1$ supergravity by a further compactification on K3 complex surfaces that reduce the 16 conserved supersymmetries down to 8 [26, 27, 28].

To fix the ideas, let us briefly describe the two constraints that should be obeyed by the real compact surface Σ_2 used in type IIA compactification:

- it should preserve only 16 supercharges amongst the initial 32 existing supersymmetries living in $10D$ type IIA superstring. A far as duality is concerned, this implies, amongst others, that heterotic string on T^2 and type IIA string on Σ_2 should have the same underlying non chiral $8D$ $\mathcal{N} = 1$ supersymmetric algebra, the same types of central charges and the same extremal black attractors.

- the 2-cycle homology of Σ_2 should lead to the same fields content one obtains in heterotic string on T^2 with some gauge invariance Gr. This means that, on both sides, we should have the same number of supersymmetric representations (supergravity and Yang-Mills multiplets). In particular, we should have the same number of $8D$ gauge fields $\left\{\mathcal{A}_\mu^\Lambda\right\}$ and the same number of $8D$ scalars $\left\{\phi^I\right\}$ that parameterize the moduli space (1.2) of these theories.

The presentation is as follows: In section 2, we give general features as well as constraints on non chiral $8D$, $\mathcal{N} = 1$ supergravity arising from superstrings compactification. In section 3, we revisit the duality between heterotic string on T^2 and type IIA on Σ_2 and show that exact duality between the massless spectrum of the two string theories requires, amongst others, the vanishing of the self intersection of the compact surface; i.e: $\Sigma_2 \cdot \Sigma_2 = 0$. In section 4, we give three examples of non chiral $8D$ $\mathcal{N} = 1$ supergravity models; the first one aims to illustrate the general result of the paper; and the two others correspond to two extreme situations including the case where the 2-torus degenerate. In section 5, we give our conclusion.

2. Non chiral $8D$ $\mathcal{N} = 1$ supergravity

We begin by recalling that in eight space time dimensions; one has two kinds of non chiral supergravity theories: the maximal one arising from M-theory on 3-torus; it has *32* supersymmetries and a moduli space,

$$M_{\text{M-th}/\text{T}^3}^{8D,\mathcal{N}=2} = \frac{SL(3,R)}{SO(3)} \times \frac{SL(2,R)}{SO(2)} \quad . \tag{2.1}$$

The non chiral $8D$ $\mathcal{N} = 1$ supergravity with a moduli space (1.2); it may be viewed as describing the physics at Plank energy following from two dual superstring models [26]; either from the *10D* heterotic superstring compactified on T^2 with some G_r invariance; or from *10D* type IIA superstring on some compact surface Σ_2 together with D- branes wrapping cycles of Σ_2. Recall that in *10D* space time, the bosonic massless spectrum of these two theories is as follows

$$10\text{D heterotic} \quad : \quad \mathcal{G}_{MN}, \mathcal{B}_{MN}, \Phi \quad ; \quad \{\mathcal{A}_M^a\} \tag{2.2}$$

$$10\text{D type IIA} \quad : \quad \mathcal{G}_{MN}, \mathcal{B}_{MN}, \Phi \quad ; \quad \mathcal{A}_M, \mathcal{C}_{MNP} \tag{2.3}$$

where $\mathcal{G}_{MN}$, $\mathcal{B}_{MN}$, Φ are respectively the graviton, the NS-NS antisymmetric field and the dilaton; and where the gauge fields $\mathcal{A}_M^a$ are given by the trace of $(T^a \mathcal{A}_M)$ with $\{T^a\}$ generating the G_r invariance contained in $E_8 \times E_8$ or $SO(32)$ gauge symmetry. The other gauge fields $\mathcal{A}_M, \mathcal{C}_{MNP}$ are associated with the D0- and D2- branes of type IIA superstring.

2.1 Compactification to $8D$

Under compactification, the resulting massless spectrum following from heterotic superstring on T^2 is familiar from the view of standard quantum field theory and is easily derived. However, the field content following from type IIA string on Σ_2, and which should be the same as in the case of heterotic on T^2, needs a careful treatment as it implies D-branes wrapping cycles of the compact manifold. From group theoretical view, these extended objects are manifested through a generalized supersymmetric algebra going beyond the usual Haag-Lopozanski-Sohnius one. The latter has central charges Z carrying no Lorentz space time charges and predicts only black holes. The generalized superalgebra involves however operators $Z_{\mu_1 \dots \mu_p}$ with non trivial $SO(1,7)$ charges having interpretation in terms of p-branes; see eqs(2.15) for details.

2.1.1 Deriving the homology of Σ_2

Focussing on non chiral *8D* supergravity with *16* supercharges, one may use the duality in *6D* space time between heterotic string on $T^4 = T^2 \times T^2$; and the type IIA string on $K3$ in order to study the geometric structure of the real compact and connected Σ_2 surfaces that are needed for the brane interpretation of the extremal BPS and non BPS black attractors in *8D*. By using the adiabatic argument between the non chiral supergravities with *16* supersymmetries in *6D* and *8D* arising from superstring compactifications; and roughly thinking about the 4-dimensional compact manifold (complex surface) K3 as given by the local description $T^2 \times S^2$; one ends with the following possibility

$$\Sigma_2 = S^2. \tag{2.4}$$

This is a simple solution that agrees with the well known result that the compactification on the 2-sphere preserves half of the initial supersymmetries namely *16*. But eq(2.4) let also understand that generally speaking the compact surface Σ_2 may be also thought of as given by the intersection of several copies of 2-spheres as given below

$$\Sigma_2 = \bigcup_{a=1}^{N} u_a \left[S_a^2 \right] \quad , \quad u_a \in \mathbb{Z}_+ \quad , \tag{2.5}$$

where u_as are positive integers that will be interpreted later on as Dynkin weights of affine Kac-Moody algebras. As we will see, the duality between heterotic on T^2 and type IIA on Σ_2 puts a strong constraint on the intersection matrix between the 2-spheres S_a^2 of the reducible 2-cycle (2.5). It happens that Σ_2 should have a vanishing self intersection

$$\Sigma_2 \cdot \Sigma_2 \quad = 0 = \quad \sum_{a,b} u_a \mathcal{I}_{ab} u_b \quad , \tag{2.6}$$

with intersection matrix $\mathcal{I}_{ab} = \left[S_a^2 \right] \cdot \left[S_b^2 \right]$.

Below, we first focus on the case (2.4) and determine the gauge fields as well as the moduli space associated with type IIA superstring on S^2. The investigation of the general possibility (2.5) will be done in next section. The reason for splitting this study into two cases: $N = 1$ and $N > 1$, comes from the fact that the first case corresponds to a singular limit which deserves to be treated separately.

2.1.2 Type IIA string on S^2

Starting from the spectrum (2.3) and computing the fields content that follow from the compactification of the *10D* IIA supergravity multiplet on S^2, we get various 8D fields and too particularly the following real 3- dimensional moduli space

$$M_{8D\text{-type IIA}}^{\mathcal{N}=1} = SO\left(1,1\right) \times \frac{SO(2,1)}{SO(2)} \quad . \tag{2.7}$$

Here the factor $SO\left(1,1\right)$ is associated with the *8D* dilaton σ while the 2- dimensional coset group $SO\left(2,1\right)/SO\left(2\right)$ has to do with S^2. The two real moduli (φ_1, φ_2) parameterizing this non compact manifold correspond to the Kahler parameter of the 2-sphere and the value of the B$_{NS}$ field on S^2.

$$\varphi_1 = \int_{S^2} J_{\text{Kahler}} \quad , \quad \varphi_2 = \int_{S^2} B_{NS} \quad . \tag{2.8}$$

Actually this is a remarkable result; since heterotic/type IIA duality predicts at least *four* scalars; that is two Maxwell multiplets. To shed light on this property, we study here below the moduli space of the heterotic string on a regular T^2.

2.1.3 Heterotic string on T^2

Starting from (2.2) and doing the same thing by using the dual description, given by the heterotic superstring on T^2 with abelian $U^r\left(1\right)$ gauge subsymmetry, we find that the moduli space of the resulting non chiral *8D*, $\mathcal{N} = 1$ supergravity is generally given by

$$SO\left(1,1\right) \times \frac{SO(2,r+2)}{SO(2) \times SO(r+2)} \quad , \quad 0 \le r \le 16 \tag{2.9}$$

Here $SO\left(1,1\right)$ is as in eq(2.7) and the integer r stands for the number of Maxwell gauge multiplets in the non chiral *8D* supergravity that results from the compactification of the *10D* SYM sector of the heterotic string; in other words it is the rank of the gauge symmetry group

$$G_r \subseteq E_8 \times E_8 \quad or \quad SO\left(32\right) \quad . \tag{2.10}$$

Notice that for the case $r = 0$; that is in the case of pure *10D* $\mathcal{N}{=}1$ supergravity on T^2 and SYM sector ignored, the above moduli space reduces to

$$M_{8D\text{-het}}^{\mathcal{N}=1} = SO\left(1,1\right) \times \tfrac{SO(2,2)}{SO(2)\times SO(2)} \quad , \tag{2.11}$$

where the factor $\frac{SO(2,2)}{SO(2)\times SO(2)}$ is a 4-dimensional non compact manifold parameterized by *4* real moduli ϕ_{ij} with the following interpretation:

- *3* of the four real scalars namely those given by the symmetric term $\phi_{(ij)}$ are given by the value of the metric $\mathcal{G}_{ij}$ on T^2; they describe the Kahler and the complex structure of the 2-torus. For later use, we denote by R_1 and R_2 the radii of the two 1-cycles of T^2 and set

$$k \sim R_1 R_2 \quad , \quad \tau \sim \tfrac{R_1}{R_2} e^{i\vartheta} \quad . \tag{2.12}$$

  respectively describing the Kahler and the shape (complex structure) parameters of the 2-torus.

- the fourth parameter $\phi_{[ij]} = \varepsilon_{[ij]} b$ is given by the value of the B_{NS} field on T^2, i.e: $b = \int_{T^2} B_{NS}$. This modulus combine with $k \sim R_1 R_2$ to give the complex Kahler modulus $t = k + ib$.

Under compactification of the *10D* supergravity multiplet (2.2) on T^2; we also have *4* gauge fields $\mathcal{G}_\mu^i$ and $\mathcal{B}_\mu^i$ that respectively follow from the metric $\mathcal{G}_{MN}$ and the NS-NS $\mathcal{B}_{MN}$ fields; two of these gauge fields; say $\mathcal{G}_\mu^i$, are involved in the non chiral *8D* $\mathcal{N} = 1$ supergravity multiplet while the two $\mathcal{B}_\mu^i$s combine with ϕ^{ij} to make two *8D* $\mathcal{N} = 1$ Maxwell multiplets.

$$\begin{array}{lll}
8D \text{ gravity} & : & \mathcal{G}_{\mu\nu}, \mathcal{B}_{\mu\nu}, \sigma, \mathcal{G}_\mu^i \quad , \\
8D \text{ Maxwell} & : & \mathcal{B}_\mu^i, \phi^{ij} \quad\quad\quad\quad\quad ,
\end{array} \tag{2.13}$$

with internal indices taking the values $i, j = 1, 2$.

2.2 Singular limits

By comparing the moduli spaces $M_{8D\text{-type IIA}}^{\mathcal{N}=1}$ (2.7) and $M_{8D\text{-het}}^{\mathcal{N}=1}$ (2.11), it follows that the non chiral *8D* $\mathcal{N} = 1$ supergravities following from the type IIA superstring on S^2 and the (gravity sector of the) heterotic superstring on T^2 are not exactly identical. As such, one concludes that they are not exactly dual

$$\text{type IIA on } S^2 \quad \nleftrightarrow \quad \text{heterotic on } T^2 \quad ; \tag{2.14}$$

but still have:

- the same gravity supermultiplet with the bosonic fields $\mathcal{G}_{\mu\nu}$, $\mathcal{B}_{\mu\nu}$, σ, $\mathcal{G}^i_\mu$,

- the same non chiral *8D* $\mathcal{N} = 1$ superalgebra generated by the fermionic generators Q_α and $\bar{Q}_{\dot\alpha}$.

Recall that, in absence of central charges, the supercharges Q_α and $\bar{Q}_{\dot\alpha}$ obey the typical relation $Q_\alpha \bar{Q}_{\dot\beta} + \bar{Q}_{\dot\beta} Q_\alpha \sim \sigma^\mu_{\alpha\dot\beta} P_\mu$; but in general they satisfy the following extended anticommutation relations,

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \delta_{\alpha\beta} Z_0 + \sigma^{\mu\nu\rho\sigma}_{(\alpha\beta)} \, Z_{\mu\nu\rho\sigma} \quad , \\
\{Q_\alpha, \bar{Q}_{\dot\beta}\} &= \sigma^\mu_{\alpha\dot\beta} Z^0_\mu + \sigma^{\mu\nu\rho}_{\alpha\dot\beta} \, Z^0_{\mu\nu\rho} \quad , \\
\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} &= \delta_{\dot\alpha\dot\beta} \bar{Z}_0 + \sigma^{\mu\nu\rho\sigma}_{(\alpha\beta)} \, \tilde{Z}_{\mu\nu\rho\sigma} \quad ,
\end{aligned} \tag{2.15}$$

where the bosonic operators $Z_{\mu_1\ldots\mu_p}$ are "central charges" carrying non trivial $SO(1,7)$ charges and having interpretation in terms of electric and magnetic charges of p-branes. The $\sigma^{\mu_1\ldots\mu_p}$'s are *antisymmetric* products of the 8×8 Pauli- Dirac Γ^μ- matrices in *8D*.

Notice also that to get the exact duality

$$\text{type IIA on S}^2 \quad \longleftrightarrow \quad \text{heterotic on } T^2 \tag{2.16}$$

one has to put *2* constraint relations on the *4* real fields ϕ_{ij} (2 complex ones t and τ) in order to equate the moduli spaces (2.7) and (2.11). This may be achieved either by fixing the complex structure τ of the 2-torus or its complexified Kahler parameter. In the last case; this corresponds to a singular limit where one (or both) of the two circles of T^2 shrinks to zero,

$$t = k + ib \quad \rightarrow \quad 0 \quad , \tag{2.17}$$

and happens whenever one of the two radii is reduced to zero; i.e $R_i \rightarrow 0$ in eq(2.14). The singular case where both $t \rightarrow 0$ and $\tau \rightarrow 0$ is associated, on the side of type IIA compactification side, with the shrinking of S^2 down to zero. This special situation corresponds precisely to dealing with pure non chiral *8D* $\mathcal{N} = 1$ supergravity with moduli space $SO(1,1)$.

3. Heterotic/type IIA duality in *8D* revisited

We start by recalling the degrees of freedom of non chiral *8D* $\mathcal{N} = 1$ supergravity by focussing on those used in the study of black attractors namely the abelian gauge fields and the scalars. Then we derive the constraint relation (2.6) on the compact surfaces Σ_2 that lead to the exact heterotic/type IIA duality in *8D*.
Restricting the investigation to bosons, it is interesting to notice that the fields of the non chiral *8D* $\mathcal{N} = 1$ supergravity organize into two basic supermultiplets:

- *the gravity multiplet*: it contains, in addition to the gravitino and graviphotino cap-
turing *40+8* on shell degrees of freedom, the usual graviton $\mathcal{G}_{\mu\nu}$, one antisymmetric
$\mathcal{B}_{\mu\nu}$ field, *2* graviphotons denoted as $\mathcal{G}_{\mu}^{i} = \left(\mathcal{G}_{\mu}^{1}, \mathcal{G}_{\mu}^{2}\right)$ and the dilaton σ. The on shell
degrees of freedom of these fields are respectively as follows:

$$20 + 15 + 2 \times 6 + 1 = 48 \tag{3.1}$$

- N *Maxwell multiplets*: they contain N gauge fields $\mathcal{A}_{\mu}^{a} = \left(\mathcal{A}_{\mu}^{1}, ..., \mathcal{A}_{\mu}^{N}\right)$ and $2N$ real
scalar fields $\phi^{ia} = \left(\phi^{i1}, ..., \phi^{iN}\right)$ and $i = 1, 2$. The number of gauge supermultiplets
depends on the gauge symmetry of the theory. In the case of non chiral *8D* $\mathcal{N} = 1$
supergravity arising from heterotic string on regular 2-torus, this number should be
as

$$N \leq 18. \tag{3.2}$$

The gauge fields $\mathcal{G}_{\mu}^{i}$ and $\mathcal{A}_{\mu}^{a}$ as well as the real scalar moduli ϕ^{ia} have geometric and stringy
meaning in the framework of superstring compactifications; but with different interpreta-
tions depending on whether they are following from heterotic string on T^2 or from type
IIA string on Σ_2.

3.1 Heterotic string on T^2

In the case of heterotic string on T^2 with a $U^r(1)$ abelian gauge subsymmetry, the $2N$ real
scalars ϕ^{ia} of the non chiral *8D* $\mathcal{N} = 1$ supergravity decomposes like

$$2N = 4 + 2r \quad , \quad N \geq 0 \tag{3.3}$$

and are interpreted as follows:

- *the real 4 scalars*: they split as *3+1* having respectively a geometric and stringy
origin; the three scalars describe the Kahler and complex structure of the 2-torus;
the fourth is given by the B$_{NS}$- field on T^2. These are precisely the complex fields
moduli t and τ encountered previously.

- *2r real scalars* ϕ^{ia}: they come from the $U^r(1)$ gauge fields of the SYM$_{10}$ sector of
the heterotic string along the 8-th and 9-th compact directions as given below

$$\phi^{ia} = \begin{pmatrix} \mathcal{A}_8^a \\ \mathcal{A}_9^a \end{pmatrix} \quad , \quad a = 1, ..., r \quad . \tag{3.4}$$

Notice that $U^r(1)$ is just the Cartan subgroup of the gauge symmetry $G_r \subseteq E_8 \times E_8$
or $SO(32)$. Notice also that the T^2 directions fill the space coordinates $\left(x^8, x^9\right)$.

3.2 Type IIA superstring on Σ_2

Here we use a field theoretical method and requires the *exact* heterotic/type IIA duality to
derive the 2-cycle homology of the compact surfaces Σ_2 as well as the constraint relation
(2.6).

3.2.1 Field theory method

In non chiral *8D* $\mathcal{N} = 1$ supergravity with a generic moduli space $\frac{SO(2,N)}{SO(2) \times SO(N)} \times SO(1,1)$, the 1-form gauge fields and the scalars are as follows:

- *1-form gauge fields*: there are $(2 + N)$ gauge fields that may be written collectively like

$$\mathcal{A}_{\mu}^{\Lambda} = \left(\mathcal{G}_{\mu}^{i}, \mathcal{C}_{\mu}^{a} \right) \quad , \tag{3.5}$$

where for later use, it is useful to think about the two graviphotons $\mathcal{G}_{\mu}^{i}$ as follows

$$\mathcal{G}_{\mu}^{i} = \left(\mathcal{A}_{\mu}^{0}, \mathcal{C}_{\mu}^{0} \right) \quad . \tag{3.6}$$

These $(2 + N)$ gauge fields transform differently under the $SO(2) \times SO(N)$ symmetry of the moduli space. We have:

$$\mathcal{G}_{\mu}^{i} \quad \sim \quad (2,1) \quad , \quad \mathcal{C}_{\mu}^{a} \quad \sim \quad (1,N) \quad . \tag{3.7}$$

In the type IIA picture on Σ_2, the above gauge fields are associated with two kinds of D0- branes as described below:

- the gauge field $\mathcal{A}_{\mu}^{0}$ of eq(3.6); it is associated with the standard D0- brane that descend from *10D* type IIA.

- the gauge fields $\mathcal{C}_{\mu}^{0}$ of eq(3.6) and the N gauge fields $\mathcal{C}_{\mu}^{a}$ of eq(3.7); they are associated with those D0-branes following from D2-branes wrapping 2-cycles in Σ_2.

From the second property as well as the fact that half of the *32* supersymmetries should be preserved, we learn that the compact surface Σ_2 should have $(1 + N)$ irreducible 2-spheres as shown below.

$$\Sigma_2^{(N+1)} = \bigcup_{a=0}^{N} u_a \left[S_a^2 \right] \quad , \quad u_a \in \mathbb{Z}_+ \quad . \tag{3.8}$$

where the positive integers u_a are as mentioned before.

Using this 2-cycle homology, one clearly sees that the gauge fields are given by

$$\mathcal{C}_{\mu}^{a} = \int_{S_a^2} \mathcal{C}_{\mu z \bar{z}} \, dz \wedge d\bar{z} \quad , \quad a = 0, 1, ..., N \quad , \tag{3.9}$$

where z_a, $\bar{z}_a$ are complex coordinates that parameterize the 2-sphere S_a^2 and $\mathcal{C}_{\mu z \bar{z}}$ is the gauge 3- form associated with D2- brane wrapping S_a^2.

However to determine the intersection matrix $\mathcal{I}_{ab}$ between the irreducible 2- spheres; we need an extra information that turns out to be exactly given by matching the number of the real scalar fields in the supergravity. This feature is discussed below.

- *matching the scalar fields*: Besides the dilaton σ, there are $2N$ real scalars ϕ^{ia} transforming under the $SO(2) \times SO(N)$ symmetry of the moduli space as follows,

$$\phi^{ia} \quad \sim \quad (2,N) \quad . \tag{3.10}$$

Since each 2-sphere contributes with two real scalars φ_1 and φ_2; one associated with the Kahler parameters and the other given by the value of the B_{NS} field S_a^2, we have the following.

A complex scalar given by

$$\Phi^0 = \int_{S_0^2} (J_{\text{Kahler}} + iB_{NS}) \quad , \tag{3.11}$$

and N similar others as follows:

$$\Phi^a = \int_{S_a^2} (J_{\text{Kahler}} + iB_{NS}) \quad , \quad a = 1, ..., N \quad , \tag{3.12}$$

where $J_{\text{Kahler}} + iB_{NS}$ stands for the complexified Kahler form and where we have set $\Phi = \varphi_1 + i\varphi_2$.

From this construction, we learn that there are two real (one complex) undesired scalars since N Maxwell supermultiplets need only $2N$ real scalars as in eq(3.10). This means that the (complex) scalars Φ_a should obey one complex (two real) condition. A natural constraint equation corresponds to express Φ_0 in terms of the others as given by the following linear relation,

$$\Phi_0 \quad = \quad \sum_{b=1}^{r} \lambda_b \Phi_b \quad , \tag{3.13}$$

that lead to the appropriate number of scalars. To get the explicit expression of the coefficients λ_b, we use a well known result [29, 30] giving a correspondence between (irreducible) 2-spheres S_a^2 and (simple) roots $\boldsymbol{\alpha}_a$ of affine Kac-Moody algebras,

$$\begin{aligned}
\left[S_a^2 \right] & \longleftrightarrow & \boldsymbol{\alpha}_a & \quad , \\
\boldsymbol{\alpha}_0 & = & \delta - \sum_{b=1}^{r} u_b \boldsymbol{\alpha}_b & .
\end{aligned} \tag{3.14}$$

In the second relation of the above eqs, which should be put in 1:1 with eq(3.13), the $\boldsymbol{\alpha}_0$ is the simple root that is associated with affine node in the Dynkin diagram of affine Kac-Moody algebras, $\boldsymbol{\alpha}_1, ..., \boldsymbol{\alpha}_r$ are the usual simple roots of finite dimensional Lie algebras and u_b the Dynkin weights. δ is an imaginary root; we will ignore it here.

Using this correspondence, we end with an intersection matrix $\mathcal{I}_{ab} = \left[S_a^2 \right] \cdot \left[S_b^2 \right]$ given by minus the generalized Cartan matrix $\mathcal{K}_{ab}$ of affine Kac-Moody algebras. More precisely, we have

$$\mathcal{I}_{ab} = -\mathcal{K}_{ab} \tag{3.15}$$

with

$$\mathcal{K}_{ab} = \frac{2 (\boldsymbol{\alpha}_a, \boldsymbol{\alpha}_b)}{(\boldsymbol{\alpha}_a, \boldsymbol{\alpha}_a)} \tag{3.16}$$

With this intersection matrix and the identities $\sum_a u_a \mathcal{I}_{ab} = 0$ and $\sum_b \mathcal{I}_{ab} u_b = 0$, we end with the following self intersection property $\Sigma_2 \cdot \Sigma_2 = 0$.

3.2.2 Result

The exact duality between heterotic string on T^2 with some gauge symmetry G_r and type IIA string on Σ_2 requires:

- real compact and connected surfaces type

$$\Sigma_2 \;=\; \bigcup_{a=1}^{N} u_a \left[S_a^2 \right]$$

  with the properties:

  - have vanishing self intersection $\Sigma_2 \cdot \Sigma_2 = 0$
  - preserve *16* of the *32* supersymmetries

- surfaces classified by the affine Kac-Moody extension of the G_r since:

  - the intersection matrix $\mathcal{I}_{ab} = \left[S_a^2 \right] \cdot \left[S_b^2 \right]$ between 2-spheres of Σ_2 is given by minus the generalized Cartan matrix $K_{ij}\left(\hat{g}\right)$ of affine Kac-Moody algebras $\hat{g}$

$$\mathcal{I}_{ab} = -K_{ab}\left(\hat{g}\right) \quad . \tag{3.17}$$

  Since affine Kac-Moody algebras $\hat{g}$ are classified; it follows that the Σ_2's are given by the Dynkin diagrams of the affine Kac- Moody- Lie algebras. In the case symmetric generalized Cartan matrices $K_{ab} = K_{ba}$, these surfaces are given by the Dynkin diagrams of the following simply laced Kac-Moody algebras

$$\widehat{su}\left(m\right), \quad \widehat{so}\left(2m\right), \quad \hat{E}_6, \quad \hat{E}_7, \quad \hat{E}_8, \tag{3.18}$$

  - A graphic representation of examples of $\Sigma_2\left(\hat{g}_r\right)$ are depicted in figure 1 where the 2-spheres are given by the nodes of the quivers and the intersection by lines joining the nodes.
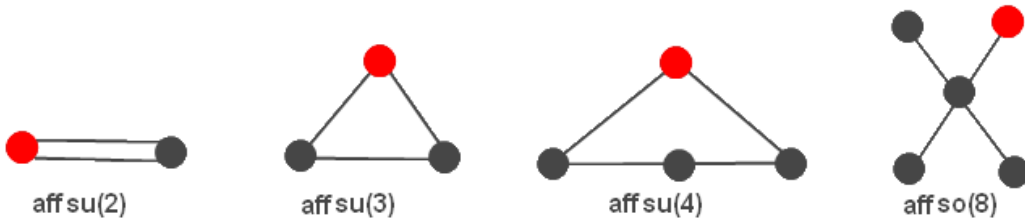

Figure 1: Four examples of compact real surfaces given by intersecting 2-spheres. These quivers are in one in to one correspondence with the affine Dynkin diagrams; the red node gives the affine extension.

These surfaces $\Sigma_2\left(\hat{g}_r\right)$ capture also the nature of the non abelian gauge symmetry that may live in the non chiral *8D* $\mathcal{N} = 1$ supergravity theory. Since affine Lie algebras $\hat{g}_r$ are built out of ordinary Lie algebras g_r; one can recover the non abelian gauge invariance by switching off the vevs (moduli).

Below we discuss three examples of dual models namely those involving compact and connected surfaces given by the Dynkin diagrams of $\widehat{su}\left(3\right)$, $\widehat{so}\left(32\right)$ and $\widehat{su}\left(2\right)$.

4. Heterotic/type IIA dual models

In this section, we study three models using duality between type IIA string a on compact real surface $\Sigma_2\,(\hat{g}_N)$ and heterotic string on T^2 with $U^{N-1}\,(1)$ gauge symmetry. These are:

- the $\widehat{su}\,(3)$ model, which we use to illustrate the general idea,

- the $\widehat{so}\,(32)$ model since the group $SO\,(32)$ is one of the two largest gauge symmetries of the *10D* heterotic string theory,

- the $\widehat{su}\,(2)$ model as it corresponds to a singular limit for heterotic string on T^2.

4.1 $\widehat{su}\,(3)$ model

In this supergravity model, the compact surface $\Sigma_{2,\widehat{su}_3}$ is associated with $\widehat{su}\,(3)$ affine Kac-Moody algebra having three simple roots α_0, α_1, α_2 with $\alpha_0 = -\alpha_1 - \alpha_2$. Using the correspondence (3.14), we then have

$$\Sigma_{2,\widehat{su}_3} = \quad [S_0^2] \bigcup [S_1^2] \bigcup [S_2^2] \tag{4.1}$$

with intersection matrix like

$$\mathcal{I}_{ab} = \begin{pmatrix} -2 & +1 & +1 \\ +1 & -2 & +1 \\ +1 & +1 & -2 \end{pmatrix} \tag{4.2}$$

The 1-form gauge fields as well as the moduli of this model are as follows

- *4* Maxwell gauge fields partitioned as follows:

  - the $\mathcal{A}_\mu^0$ graviphoton associated with the D0-brane (3.6);
  - *3* Maxwell gauge fields $\mathcal{C}_\mu^a$ given by

  $$\mathcal{C}_\mu^a = \int_{S_a^2} \mathcal{C}_{\mu z \bar{z}} \, dz \wedge d\bar{z} \quad , \quad a = 0, 1, 2. \tag{4.3}$$

  These *3* fields correspond to D2-brane wrapping the three 2-spheres S_a^2.

- 4 real scalars:

  A priori there are *3* complex (6 real) moduli coming from the values of the complexified Kahler form on the 2-spheres of $\Sigma_{2,\widehat{su}_3}$

  $$\Phi_a = \int_{S_a^2} (J_{\text{Kahler}} + iB_{NS}) \quad . \tag{4.4}$$

  But these field are not independent; they obey the condition

  $$\Phi_0 + \Phi_1 + \Phi_2 = 0 \tag{4.5}$$

  associated with Lie algebraic relation $\alpha_0 + \alpha_1 + \alpha_2 = 0$.

As such one is left with 4 real scalars parameterizing the moduli space $\frac{SO(2,2)}{SO(2)\times SO(2)}$.
This model is dual to the *10D* heterotic string on T^2 where the fluxes of the gauges fields in the SYM_8 sector and the VEVs of the corresponding scalars completely vanish.

$$\int_{S^2}\mathcal{F}_2^{(G_r)}=0 \quad , \quad \int_{S^6}\tilde{\mathcal{F}}_6^{(G_r)}=0 \quad , \quad <\Phi>=0 \quad , \tag{4.6}$$

with the field strength 2-forms $\mathcal{F}_2^{(G_r)}=\sum\tau_I\mathcal{F}_2^I$, their Hodge duals $\tilde{\mathcal{F}}_6^{(G_r)}=\sum\tau_I\tilde{\mathcal{F}}_6^I$ and the adjoint matter $\Phi=\sum\tau_I\Phi^I$ are valued in the Lie algebra $G_r\subset E_8\times E_8$ or $SO(32)$ gauge symmetry with generators τ_I.

4.2 $\widehat{so}(32)$ model

In this *8D* supergravity model, the compact real surface Σ_2, whose quiver is depicted in figure 2, is given by the intersection of seventeen 2-spheres

$$\Sigma_2=\bigcup_{a=0}^{16}u_a\left[S_a^2\right] \tag{4.7}$$

with Dynkin weights $u_a=(1,1,2,2...2,2,1,1)$ and intersection matrix $\mathcal{I}_{ab}$ given by minus the generalized Cartan matrix $-K_{ab}^{\widehat{so}(32)}$ of the affine Kac-Moody $\widehat{so}(32)$.
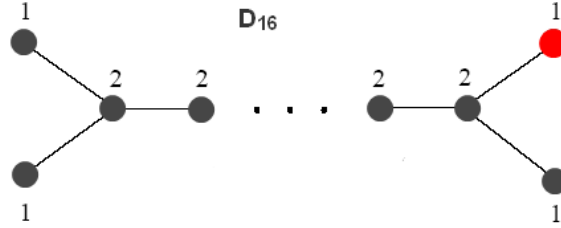

Figure 2: Dynkin diagram of affine $SO(32)$

     This model involves *18* Maxwell gauge fields; *2* of them (graviphotons) belong to the gravity multiplet and the *16* others are valued in the Cartan subalgebra of the $SO(32)$ gauge symmetry. There are also *32* scalar fields parameterizing the moduli space $\frac{SO(2,16)}{SO(2)\times SO(16)}$.

4.3 $\widehat{su}(2)$ model

This is a simple and special model that follows from type IIA superstring on a compact real surface given by the intersection of two 2-spheres

$$\Sigma_2=\left[S_0^2\right]\bigcup\left[S_1^2\right] \quad , \tag{4.8}$$

with intersection matrix as

$$\mathcal{I}_{ab}^{\widehat{su}_2}=\begin{pmatrix}-2 & +2\\ +2 & -2\end{pmatrix} \quad . \tag{4.9}$$

The resulting non chiral *8D* $\mathcal{N}=1$ supergravity has, amongst others, *3* gauge fields and one complex scalar in addition to the dilaton σ. These fields are as follows:

- *3 Maxwell gauge fields:* they are given by:

  - the $\mathcal{A}_\mu^0$ graviphoton associated with the D0-brane (3.6);
  - the $\mathcal{C}_\mu^0$ graviphoton associated with the D2-brane wrapping S_0^2 and reading as

  $$\mathcal{C}_\mu^0 = \int_{S_0^2} \mathcal{C}_{\mu z \bar{z}} \; dz \wedge d\bar{z} \quad , \tag{4.10}$$

  with $\mathcal{C}_{MNP}$ the RR gauge 3-form of the *10D* type IIA spectrum.

  - the Maxwell gauge fields $\mathcal{C}_\mu^1$ associated with the D2-brane wrapping S_1^2; it is given by

  $$\mathcal{C}_\mu^1 = \int_{S_1^2} \mathcal{C}_{\mu z \bar{z}} \; dz \wedge d\bar{z} \quad , \tag{4.11}$$

- one free complex scalar Φ (*2* real scalars) solving the condition the constraint relation $\Phi_0 + \Phi_1 = 0$ with Φ_0 and Φ_1 given by the complexified Kahler parameters

$$\begin{aligned} \Phi_0 &= \int_{S_0^2} \left(J_{\text{Kahler}} + iB_{NS} \right) \quad ,. \\ \Phi_1 &= \int_{S_1^2} \left(J_{\text{Kahler}} + iB_{NS} \right) \quad , \end{aligned} \tag{4.12}$$

Recall that the constraint relation $\Phi_0 + \Phi_1 = 0$ is associated with the $\widehat{su}(2)$ affine Kac-Moody algebraic relation between the simple roots namely $\alpha_0 + \alpha_1 = 0$.

This $\widehat{su}(2)$ involves *2* real moduli rather than the *4* scalars that we have in the *10D* heterotic string on T^2 whose moduli space is given by $\frac{SO(2,2)}{SO(2) \times SO(2)}$ as shown on eq(2.11). It corresponds therefore to a singular configuration where the complexified Kahler structure

$$t + ib \quad = \quad \int_{a_1 \times a_2} \left(J_{\text{Kahler}} + iB_{NS} \right) \quad , \tag{4.13}$$

and the values of the the ten dimensional B_{MN} field along the two 1-cycles a_i, generating the periods of the 2-torus T^2,

$$B_{\mu i} \quad = \quad \int_{a_i} B_{\mu \nu} dy^\nu \quad , \tag{4.14}$$

vanish identically. But the torus still has a complex structure described by a complex moduli τ that should be put in one to one correspondence with the Φ of (4.12).

5. Conclusion

Motivated by the study of black attractors in non chiral *8D* $\mathcal{N} = 1$ supergravity with gauge 2-form $\mathcal{B}_{\mu \nu}$, $(2 + N)$ Maxwell gauge fields $\mathcal{A}_\mu^\Lambda = \left(\mathcal{A}_\mu^i, \mathcal{A}_\mu^a \right)$ and $(1 + 2N)$ real scalars $\left(\sigma, \phi^{ia} \right)$ parameterizing the moduli space

$$SO(1,1) \times \frac{SO(2,N)}{SO(2) \times SO(N)}, \tag{5.1}$$

and using an explicit field theoretic method, we have studied in this paper two main things: First, we have investigated the exact duality between the heterotic string on a *regular* T^2,

with $U^r(1)$ abelian symmetry contained in some rank r gauge group G_r, and the type IIA string on a connected compact real surface Σ_2. Because of the fact that Σ_2 should preserve *16* supersymmetries; that is the half of the *32* existing supercharges in the *10D* type IIA superstring and because of heterotic/type IIA duality in *8D*, we have shown that connected and compact Σ_2 should have a vanishing self intersection,

$$\Sigma_2 \cdot \Sigma_2 = 0, \tag{5.2}$$

that is having the same homology as the 2-torus used in the heterotic side. Moreover using known results on geometric engineering of QFTs embedded in string theory, we have shown that 2-cycle homology of the surface Σ_2 is given by intersecting 2-spheres S_a^2 with intersection matrices $\mathcal{I}_{ab}$ classified by the generalized Cartan matrices $\mathcal{K}_{ab}(\hat{g})$ of the affine Kac-Moody Lie algebras $\hat{g}$.

Second, we have studied some special dual models; in particular the non chiral *8D* $\mathcal{N} = 1$ supergravity theory having *3* Maxwell gauge fields and *1+2* real scalar parameterizing the moduli space

$$SO(1,1) \times \frac{SO(2,1)}{SO(2)}. \tag{5.3}$$

This real 3- dimensional manifold does not belong to the well known family $SO(1,1) \times \frac{SO(2,2+r)}{SO(2) \times SO(2,r)}$ with $0 \leq r \leq 16$. We have shown that, from heterotic side, this supergravity model follows from compactification on a *singular* torus T^2 with vanishing complexified Kahler structure. In the type IIA picture, the model follows from the compactification on a real surface given by two intersecting 2-spheres $[S_0^2] \bigcup [S_1^2]$ with branch cuts and matrix intersection

$$[S_0^2] \cdot [S_0^2] = -2 \quad , \quad [S_1^2] \cdot [S_1^2] = -2 \quad , \quad [S_0^2] \cdot [S_1^2] = 2 \tag{5.4}$$

given by minus the generalized Cartan matrix of the twisted affine Kac-Moody algebra $\widehat{su}(2)$. Accordingly, one of the *3* gauge fields is associated with the D0 brane while the 2 others follow from the D2 brane wrapping the 2-spheres of Σ_2. We end this study by noting that it would be interesting to explore the issue regarding the properties of type IIA compactification on non connected surfaces. Progress in this direction will reported in a future occasion.

6. ACKNOWLEDGMENTS

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