Vacua of $N = 10$ three dimensional gauged supergravity

Auttakit Chatrabhuti[a,b] and **Parinya Karndumri**[c,d]
[a]Theoretical High-Energy Physics and Cosmology Group, Department of Physics, Faculty of Science, Chulalongkorn University, Bangkok 10330, Thailand
auttakit@sc.chula.ac.th
[b]Thailand Center of Excellence in Physics, CHE, Ministry of Education, Bangkok 10400, Thailand
[c]INFN, Sezione di Trieste, Italy
[d]International School for Advanced Studies (SISSA), via Bonomea 265, 34136 Trieste, Italy
karndumr@sissa.it

Abstract

We study scalar potentials and the corresponding vacua of $N = 10$ three dimensional gauged supergravity. The theory contains 32 scalar fields parametrizing the exceptional coset space $\frac{E_{6(-14)}}{SO(10)\times U(1)}$. The admissible gauge groups considered in this work involve both compact and non-compact gauge groups which are maximal subgroups of $SO(10)\times U(1)$ and $E_{6(-14)}$, respectively. These gauge groups are given by $SO(p)\times SO(10-p)\times U(1)$ for $p = 6, \dots 10$, $SO(5)\times SO(5)$, $SU(4,2)\times SU(2)$, $G_{2(-14)}\times SU(2,1)$ and $F_{4(-20)}$. We find many AdS$_3$ critical points with various unbroken gauge symmetries. The relevant background isometries associated to the maximally supersymmetric critical points at which all scalars vanish are also given. These correspond to the superconformal symmetries of the dual conformal field theories in two dimensions.

1 Introduction

Gauged supergravities play an important role in many aspects of string theory. Some of them arise as effective theories of string compactifications in the presence of fluxes of various p-form fields, see for example, [1] for a recent review. Furthermore, they are very useful in the AdS/CFT correspondence [2]. This is due to the fact that in gauged supergravity theories, supersymmetry allows scalar potentials which admit some critical points with negative cosmological constants, AdS critical points. These critical points are of particular interest in the context of the AdS/CFT correspondence because they correspond to conformal field theories on the boundary of AdS space.

In the original $\text{AdS}_5/\text{CFT}_4$ correspondence, critical points of $N = 8$ five dimensional gauged supergravity found in [3] describe various phases of $N = 4$ SYM. The correspondence is now extended to other dimensions as well. These include $\text{AdS}_4/\text{CFT}_3$ and $\text{AdS}_3/\text{CFT}_2$ correspondences. The former is of interest in the sense that it might give some insight to the condense matter systems, for example, superconductors. Gauged supergravities in four dimensions are useful to this study in much the same way as five dimensional gauged supergravities in $\text{AdS}_5/\text{CFT}_4$. Vacua of $N = 8$ four dimensional gauged supergravity have been classified in [4, 5] soon after its construction [6], and recently, some new vacua of this theory have been identified in [7, 8]. Although, a lot of works have been done in finding critical points of this theory, it is expected that many critical points remain to be found. On the other hand, $\text{AdS}_3/\text{CFT}_2$ correspondence is a good place to test and study many aspects of the AdS/CFT correspondence. This is because there are many known two dimensional conformal field theories, and things are more controllable in two dimensions. So, we hope to understand $\text{AdS}_3/\text{CFT}_2$ in much more detail than the higher dimensional analogues. In this case, three dimensional gauged supergravities are, of course, the natural framework. In comparison with the higher dimensional counterparts, $\text{AdS}_3/\text{CFT}_2$ is not only important for understanding the AdS/CFT correspondence but also for the study of black hole entropy, see [9] for a review and references therein.

Three dimensional Chern-Simons gauged supergravity, see, for example, [10, 11, 12, 13] and [14] for the construction, has a much richer structure than the analogous theories in higher dimensions due to the duality between vectors and scalars in three dimensions. The admissible gauge groups include compact, non-compact, non-semisimple and complex ones. Supersymmetry determines unique scalar target spaces for theories with $N > 8$, [15]. Some works have been done in studying critical points or vacua of gauged supergravities in three dimensions [16, 17, 18, 19, 20, 21]. The theories considered in these works have $N = 4, 8, 9, 16$ supersymmetry, respectively. In this paper, we study $N = 10$ theory whose 32 scalar fields parametrizing the coset $\frac{E_{6(-14)}}{SO(10)\times U(1)}$. The admissible gauge groups are subgroups of $E_{6(-14)}$. Some of compact and non-compact admissible gauge groups have been classified in [14]. These are gauge groups we will study in this work. The compact gauge groups are $SO(p)\times SO(10-p)\times U(1)$ for $p=6,\dots 10$ and $SO(5)\times SO(5)$. The non-compact gauge groups are $G_{2(-14)}\times SU(2,1)$, $SU(4,2)\times SU(2)$ and $F_{4(-20)}$. All of these gauge groups are maximal subgroups of $SO(10)\times U(1)$ and $E_{6(-14)}$, respectively.

We will study some critical points of the scalar potentials in all of the gaugings mentioned above by using the technique introduced in [4]. In this "subgroup method", we start by choosing a particular subgroup of the gauge group and study the potential on the restricted scalar manifold which is invariant under this subgroup. As a consequence of Schur's lemma, the critical points found on this invariant manifold are critical points of the potential on the whole scalar manifold, 32-dimensional $\frac{E_{6(-14)}}{SO(10)\times U(1)}$ manifold in this work. This method has been used to study critical points of scalar potentials of $N=16$ gauged supergravity in [20] and in other dimensions as well.

The paper is organized as follows. In section 2, we review some useful ingredients to construct $N=10$ gauged supergravity theory. We use the parametrization of the scalar coset manifold $\frac{E_{6(-14)}}{SO(10)\times U(1)}$ in much the same way as the $\frac{F_{4(-20)}}{SO(9)}$ coset in $N=9$ theory. All details of the gauge group generators and other needed information can be found in appendix A. Various vacua are given in section 3 including the background isometries of the maximally supersymmetric critical points at which all scalars vanish. The computations are carried out with the help of the computer program *Mathematica* [22]. We finally summarize our results and give some conclusions in section 4.

2 $N=10$ three dimensional gauged supergravity

In this section, we construct $N=10$ three dimensional gauged supergravity using the formulation given in [14]. The procedure is essentially the same as that given in [18], so we will give only the needed ingredients and refer the reader to [14] for the full detail of the construction.

We start by giving a description of symmetric spaces. In three dimensional gauged supergravity with $N>8$, scalar fields parametrize a unique coset space of the form G/H. The group G given by some non-compact real form of an exceptional group is the global symmetry of the theory with the maximal compact subgroup H. The subgroup H is further decomposed to $SO(N)\times H'$ in which $SO(N)$ is the R-symmetry. Note that the additional factor H' does not appear when $SO(N)$ is the maximal compact subgroup of G. This is the case for $N=9$ and $N=16$ theories in which G is given by $F_{4(-20)}$ and $E_{8(8)}$, respectively. The G generators $t^{\mathcal{M}}$ decompose into $\{X^{IJ},X^{\alpha}\}$ which are generators of $\{SO(N),H'\}$ and non-compact generators Y^A.

In general, the G algebra, $\mathfrak{g}$, is formed by the isometries of the target space that can be extended to an invariance of the Lagrangian. As shown in [14], under the map $\mathcal{V}$

$$\mathcal{V}:\mathfrak{g}\to\mathfrak{a},\qquad \mathcal{V}^{\mathcal{M}}_{\ \mathcal{A}}t^{\mathcal{A}}=\frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}t^{IJ}+\mathcal{V}^{\mathcal{M}}_{\ \alpha}t^{\alpha}+\mathcal{V}^{\mathcal{M}}_{\ A}t^{A}, \tag{1}$$

the algebra $\mathfrak{g}$ is mapped to an associative subalgebra of $\mathfrak{a}=\{t^{IJ},t^{\alpha},t^{A}\}$. The algebra $\mathfrak{a}$ is an extension of $SO(N)\times H'$ algebra, $\mathfrak{so}(N)\times\mathfrak{h}'$, with the commutation relations given by

$$\begin{aligned}
[t^{IJ},t^{KL}] &= -4\delta^{[I[K}t^{L]J]}, \qquad [t^{IJ},t^{A}]=-\frac{1}{2}f^{IJ,AB}t_{B}, \qquad [t^{\alpha},t^{\beta}]=f^{\alpha\beta}_{\ \ \gamma}t^{\gamma},\\
\left[t^{A},t^{B}\right] &= \frac{1}{4}f^{AB}_{IJ}t^{IJ}+\frac{1}{8}C_{\alpha\beta}h^{\beta AB}t^{\alpha}, \qquad [t^{\alpha},t^{A}]=h^{\alpha\ \ A}_{\ B}t^{B}
\end{aligned} \tag{2}$$

where $C_{\alpha\beta}$ and $h^{\alpha\ \ A}_{\ B}$ are an H' invariant tensor and anti-symmetric tensors defined in [14]. f^{IJ}_{ij} tensors are constructed from $N-1$ almost complex structures f^{P}, $2,\ldots N$. For symmetric target spaces, all the $\mathcal{V}$'s are given by the expansion

$$L^{-1}t^{\mathcal{M}}L=\frac{1}{2}\mathcal{V}^{\mathcal{M}IJ}X^{IJ}+\mathcal{V}^{\mathcal{M}}_{\ \alpha}X^{\alpha}+\mathcal{V}^{\mathcal{M}}_{\ A}Y^{A}, \tag{3}$$

and the map $\mathcal{V}$ is now an isomorphism, see [14] for further detail. We have introduced "flat" indices $A,B,\ldots$ for the scalar manifold. The target space metric g_{ij}, $i,j=1,2,\ldots d=\dim G/H$ is given by

$$g_{ij}=e^{A}_{i}e^{B}_{j}\delta_{AB} \tag{4}$$

where the vielbein e^{A}_{i} is encoded in the expansion

$$L^{-1}\partial_{i}L=\frac{1}{2}Q^{IJ}_{i}X^{IJ}+Q^{\alpha}_{i}X^{\alpha}+e^{A}_{i}Y^{A}\,. \tag{5}$$

Q^{IJ}_{i} and Q^{α}_{i} are composite connections for $SO(N)$ and H', respectively. R-symmetry indices $I,J,\ldots=1,\ldots,N$ and $\alpha,\beta,\ldots=1,\ldots,\dim H'$. Finally, the coset representative L transforms under G and H by multiplications from the left and right, respectively.

The scalar manifold of $N=10$ theory is a 32 dimensional symmetric space $\frac{E_{6(-14)}}{SO(10)\times U(1)}$. We will use the E_6 generators constructed in [24]. Notice that there is an additional factor $H'=U(1)$ in this theory in contrast to $N=9$ and $N=16$ theories studied in [18] and [20]. The 78 generators of E_6 are given in [23] for the first 52 generators and in [24] for the remaining 26. We can construct the non-compact form $E_{6(-14)}$ by making 32 generators non-compact using "Weyl unitarity". These are given by

$$Y^{A}=\left\{\begin{array}{ll} ic_{A+21} & \text{for } A=1,\ldots,8\\ ic_{A+28} & \text{for } A=9,\ldots,16\\ ic_{A+37} & \text{for } A=17,\ldots,32\end{array}\right.\,. \tag{6}$$

The 46 compact generators are the generators of $SO(10)\times U(1)$ and are given in appendix A. The next ingredient we need is the f^{IJ}_{ij} tensors which can be read off from the second commutator of (2) as we have described in [18].

We now come to various gaugings described by the gauge invariant embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$. From $\Theta_{\mathcal{M}\mathcal{N}}$, we can compute A_1 and A_2 tensors as well as the scalar potential via the so-called T-tensors using

$$\begin{aligned}
A_1^{IJ} &= -\frac{4}{N-2}T^{IM,JM}+\frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \\
A_{2j}^{IJ} &= \frac{2}{N}T^{IJ}{}_j+\frac{4}{N(N-2)}f^{M(Im}{}_j T^{J)M}{}_m+\frac{2}{N(N-1)(N-2)}\delta^{IJ}f^{KL}{}_j{}^m T^{KL}{}_m, \\
V &= -\frac{4}{N}g^2(A_1^{IJ}A_1^{IJ}-\frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ}) \qquad (7)
\end{aligned}$$

with T-tensors

$$T_{\mathcal{A}\mathcal{B}}=\mathcal{V}^{\mathcal{M}}_{\ \mathcal{A}}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}}_{\ \mathcal{B}}\,. \qquad (8)$$

The embedding tensors for the compact gaugings with gauge groups $SO(p)\times SO(10-p)\times U(1)$, $p=0,\ldots,4$ and $SO(5)\times SO(5)$ are given by [14]

$$\Theta_{IJ,KL}=\theta\delta^{KL}_{IJ}+\delta_{[I[K}\Xi_{L]J]}+\frac{1}{3}(5-p)\Theta_{U(1)} \qquad (9)$$

where

$$\Xi_{IJ}=\left\{\begin{array}{ll} 2\left(1-\frac{p}{10}\right)\delta_{IJ} & \text{for } I\leq p \\ -\frac{p}{5}\delta_{IJ} & \text{for } I>p \end{array}\right., \qquad \theta=\frac{p-5}{5}\,. \qquad (10)$$

For $p=5$, the gauge group is $SO(5)\times SO(5)$ which lies entirely in $SO(10)$. This is the case in which the $U(1)$ is not gauged. The generators for these gauge groups can be obtained by choosing appropriate generators of $SO(10)$, and the $U(1)$ generator is simply given by $2\tilde{c}_{70}$. We refer the reader to appendix A for further details.

Non-compact gaugings considered in this work are those given in [14]. The gauge groups are $SU(4,2)\times SU(2)$, $G_{2(-14)}\times SU(2,1)$ and $F_{4(-20)}$. We find the following embedding tensors

$$\begin{aligned}
G_{2(-14)}\times SU(2,1) &: \quad \Theta_{\mathcal{M}\mathcal{N}}=\eta^{G_2}_{\mathcal{M}\mathcal{N}}-\frac{2}{3}\eta^{SU(2,1)}_{\mathcal{M}\mathcal{N}} \qquad (11)\\
SU(4,2)\times SU(2) &: \quad \Theta_{\mathcal{M}\mathcal{N}}=\eta^{SU(4,2)}_{\mathcal{M}\mathcal{N}}-6\eta^{SU(2)}_{\mathcal{M}\mathcal{N}} \qquad (12)\\
F_{4(-20)} &: \quad \Theta_{\mathcal{M}\mathcal{N}}=\eta^{F_{4(-20)}}_{\mathcal{M}\mathcal{N}} \qquad (13)
\end{aligned}$$

where η^{G_0} is the Cartan Killing form of the gauge group G_0. The gauge generators of these three gaugings are given in appendix A.

We finally repeat the stationarity condition for the critical points of the scalar potential [14]

$$3A_1^{IK}A_{2j}^{KJ}+Ng^{kl}A_{2k}^{IK}A_{3lj}^{KJ}=0 \qquad (14)$$

where A^{KL}_{3lj} is defined by

$$A^{IJ}_{3ij} = \frac{1}{N^2}\Big[-2D_{(i}D_{j)}A^{IJ}_1 + g_{ij}A^{IJ}_1 + A^{K[I}_1 f^{J]K}_{ij} + 2T_{ij}\delta^{IJ} - 4D_{[i}T^{IJ}{}_{j]} - 2T_{k[i}f^{IJk}{}_{j]}\Big]. \tag{15}$$

For supersymmetric critical points, the unbroken supersymmetries are are encoded in the condition

$$A^{IK}_1 A^{KJ}_1 \epsilon^J = -\frac{V_0}{4g^2}\epsilon^I = \frac{1}{N}(A^{IJ}_1 A^{IJ}_1 - \frac{1}{2}Ng^{ij}A^{IJ}_{2i}A^{IJ}_{2i})\epsilon^I\,. \tag{16}$$

The notations and all definitions are the same as those in [14]. In the next section, we will give the scalar potential for each gauging along with the corresponding critical points.

3 Vacua of $N = 10$ gauged supergravity

In this section, we give some vacua of the $N = 10$ gauged theory with the gaugings described in the previous section. We will also discuss the isometry groups of the background with maximal supersymmetry at $L = \mathbf{I}$. This is a supersymmetric extension of the $SO(2,2) \sim SO(1,2) \times SO(1,2)$ isometry group of AdS$_3$. A similar study has been done in [20] and [18] for $N = 16$ and $N = 9$ theories, respectively. For the full list of superconformal groups in two dimensions, we refer the reader to [25]. As a general strategy, we give the trivial critical point in which all scalars are zero, $L = \mathbf{I}$, as the first critical point. It is also useful to compare the cosmological constants of other critical points with the trivial one. According to the AdS/CFT correspondence, the cosmological constant V_0 is related to the central charge in the dual CFT as $c \sim \frac{1}{\sqrt{-V_0}}$, so we will give the ratio of the central charges for each non trivial critical point with respect to the trivial critical point at $L = \mathbf{I}$. We first start with compact gaugings.

3.1 Vacua of compact gaugings

The compact gauging includes gauge groups $SO(p) \times SO(10 - p) \times U(1)$ for $p = 6, \ldots, 10$ and $SO(5) \times SO(5)$. We give the scalar potential in $SO(p) \times SO(10 - p) \times U(1)$ for $p = 7, \ldots, 10$ gaugings in the G_2 invariant scalar sector. For $SO(6) \times SO(4) \times U(1)$ gauging, we study the potential in $SO(4)_{\text{diag}}$ and $SO(3)_{\text{diag}}$ sectors. Finally, for $SO(5) \times SO(5)$ gauging, we study the potential in $SO(5)_{\text{diag}}$, $SO(4)_{\text{diag}}$ and $SO(3)_{\text{diag}}$ sectors. All notations are the same as in [16] and [18].

3.1.1 $SO(10) \times U(1)$ gauging

We will study the potential in the G_2 invariant scalar manifold. From 32 scalars, there are four singlets under $G_2 \subset SO(p)$, $p = 7, \ldots, 10$. These four scalars correspond to non-compact directions of $SU(2,1)$. We use the same parametrization as in [20] namely using three compact generators of the $SU(2)$ subgroup and one non-compact generator. With this parametrization, the coset representative takes the form

$$L = e^{a_1 c_{78}} e^{a_2 \tilde{c}_{53}} e^{a_3 c_{52}} e^{b_1 (Y_1 + Y_6)} e^{-a_3 c_{52}} e^{-a_2 \tilde{c}_{53}} e^{-a_1 c_{78}} \,. \tag{17}$$

This choice of L will also be used in the next three gauge groups. In this $SO(10) \times U(1)$ gauging, the potential is given by

$$V = \frac{1}{2} g^2 [-101 - 28 \cosh(2\sqrt{2} b_1) + \cosh(4\sqrt{2} b_1)]. \tag{18}$$

The potential does not depend on a_1, a_2 and a_3.

The first critical point is the trivial one in which all scalars are zero. We find

$$V_0 = -64 g^2, \qquad A_1 = -4 \mathbf{I}_{10} \,. \tag{19}$$

We use the notation $\mathbf{I}_n$ for an $n \times n$ identity matrix from now on. This is the critical point with (10,0) supersymmetry according to our convention. The corresponding background isometry is $Osp(10|2, \mathbb{R}) \times SO(2,1)$.

The second critical point is at $b_1 = \frac{\cosh^{-1} 2}{\sqrt{2}}$ with cosmological constant $V_0 = -100 g^2$. This is a non-supersymmetric point. The ratio of the central charges between this point and the maximally supersymmetric point is

$$\frac{c_{(0)}}{c_{(1)}} = \sqrt{\frac{V_0^{(1)}}{V_0^{(0)}}} = \frac{5}{4} \,. \tag{20}$$

Here and from now on, the notations $c_{(0)}$ and $c_{(i)}$ mean the central charges of the trivial and i^{th} non trivial critical points, respectively.

For $a_1 = a_3 = 0$, the coset representative (17) has a larger symmetry $SO(7)$. This $SO(7)$ is embedded in $SO(8)$ in such a way that it stabilizes one component of the $SO(8)$ spinor. In [20], this $SO(7)$ has been called $SO(7)^{\pm}$ according to a component of $\mathbf{8}_s$ or $\mathbf{8}_c$ is stabilized. Our critical point is parametrized only by b_1, so has $SO(7)$ symmetry. Notice that this point is very similar to the non-supersymmetric $SO(7) \times SO(7)$ critical point of the $SO(8) \times SO(8)$ gauged $N = 16$ theory given in [20] and the $SO(7)$ point in $SO(9)$ gauged $N = 9$ theory studied in [18]. The location and the value of the cosmological constant relative to the trivial point are similar for these points.

3.1.2 $SO(9) \times U(1)$ gauging

The potential in this gauging is much more complicated than the previous gauge group and depends on all four scalars. So, we use the local $H = SO(10) \times U(1)$ symmetry to remove $e^{-a_3 c_{52}} e^{-a_2 \tilde{c}_{53}} e^{-a_1 c_{78}}$ factor in the (17) to simplify the computation and reduce the calculation time. The potential is given in appendix C.

Although we do not have a systematic way of finding critical points of this complicated potential, we find some critical points, numerically.

The first critical point is the maximally supersymmetric (9,1) point

$$\begin{aligned} a_1 &= a_2 = a_3 = b_1 = 0, \qquad\qquad V_0 = -64g^2, \\ A_1 &= \mathrm{diag}\,(-4,-4,-4,-4,-4,-4,-4,-4,-4,4)\,. \end{aligned} \tag{21}$$

The background isometry is given by $Osp(9|2,\mathbb{R}) \times Osp(1|2,\mathbb{R})$.

The second critical point is given by

$$\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}\cosh^{-1}\frac{7}{3}, \qquad a_1 = \pi,\ a_2 = \frac{3\pi}{2},\ a_3 = \frac{\pi}{2},\ V_0 = -\frac{1024}{9}g^2, \\ A_1 &= \mathrm{diag}\left(-8,-8,-8,-8,-8,-8,-8,\frac{16}{3},-\frac{16}{3},-\frac{16}{3}\right). \end{aligned} \tag{22}$$

This G_2 critical point has (2,1) supersymmetry with

$$\frac{c_{(0)}}{c_{(1)}} = \frac{4}{3}. \tag{23}$$

This critical point should be compared with the (1,1) $G_2\times G_2$ point in the $SO(8)\times SO(8)$ gauged $N=16$ theory. The two point have similar locations and values of the cosmological constant relative to the trivial point.

The last critical point in this gauging is given by

$$\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}\cosh^{-1}2, \qquad a_1 = a_3 = \frac{\pi}{2}, \qquad a_2 = \text{arbitrary}, \qquad V_0 = -100g^2, \\ A_1 &= \mathrm{diag}\,(-7,-7,-7,-7,-7,-7,-7,-7,7,-5)\,. \end{aligned} \tag{24}$$

This is a (1,0) point with G_2 symmetry and

$$\frac{c_{(0)}}{c_{(2)}} = \frac{5}{4}. \tag{25}$$

3.1.3 $SO(8)\times SO(2)\times U(1)$ gauging

The potential in the G_2 sector is given by

$$\begin{aligned} V &= \frac{1}{4096}e^{-4\sqrt{2}b_1}g^2[3(-1+e^{\sqrt{2}b_1})^8\cos(4a_1)+4(-1+e^{\sqrt{2}b_1})^6\cos(2a_1)[27 \\ &\quad +170e^{\sqrt{2}b_1}+27e^{2\sqrt{2}b_1}+4(e^{\sqrt{2}b_1}-1)^2\cos^2a_1\cos(2a_3)] \\ &\quad +8(e^{\sqrt{2}b_1}-1)^6\cos^2a_1[2(13+86e^{\sqrt{2}b_1}+13e^{2\sqrt{2}b_1})\cos(2a_3) \\ &\quad +(e^{\sqrt{2}b_1}-1)^2\cos^2a_1\cos(4a_3)]-2e^{4\sqrt{2}b_1}[88549+21112\cosh(\sqrt{2}b_1) \\ &\quad +22148\cosh(2\sqrt{2}b_1)-56\cosh(3\sqrt{2}b_1)-681\cosh(4\sqrt{2}b_1)]]. \end{aligned} \tag{26}$$

The potential does not depend on a_2. We find the following critical points.

First of all, when $a_1 = a_2 = a_3 = b_1 = 0$, we find the maximally supersymmetric critical points. At this is point, we find

$$\begin{aligned} V_0 &= -64g^2, \\ A_1 &= \mathrm{diag}\,(-4,-4,-4,-4,-4,-4,-4,-4,4,4)\,. \end{aligned} \tag{27}$$

This point has (8,2) supersymmetry and $Osp(8|2,\mathbb{R}) \times Osp(2|2,\mathbb{R})$ as the background isometry group.

The next point is given by

$$b_1 = \cosh^{-1} 2, \qquad a_1 = a_3 = 0, \qquad V_0 = -100g^2\,. \tag{28}$$

This is an $SO(7)$ non-supersymmetric point with

$$\frac{c_{(0)}}{c_{(1)}} = \frac{5}{4}. \tag{29}$$

This point is very similar to the non-supersymmetric $SO(7)\times SO(7)$ point of the $SO(8)\times SO(8)$ gauged $N=16$ theory studied in [20].

The last critical point is given by

$$\begin{aligned} b_1 &= \frac{1}{\sqrt{2}}\cosh^{-1}\frac{7}{3}, \qquad a_1 = 0, \qquad a_3 = \frac{\pi}{2}, \qquad V_0 = -\frac{1024}{9}g^2, \\ A_1 &= \begin{pmatrix} -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{16}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_1 & x_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x_2 & x_3 \end{pmatrix} \end{aligned} \tag{30}$$

where

$$\begin{aligned} x_1 &= -\frac{4}{3}[-5+\cos(2a_2)] \qquad x_2 = \frac{4}{3}\sin(2a_2) \\ x_3 &= \frac{4}{3}[5+\cos(2a_2)]. \end{aligned} \tag{31}$$

We find that this is the (1,1) point with G_2 symmetry, and the diagonalized A_1 tensor is given by

$$A_1 = \mathrm{diag}\left(-8,-8,-8,-8,-8,-8,-8,8,-\frac{16}{3},\frac{16}{3}\right). \tag{32}$$

The ratio of the central charges is

$$\frac{c_{(0)}}{c_{(2)}} = \frac{4}{3}\,. \tag{33}$$

This point is similar to the $G_2 \times G_2$ point with (1,1) supersymmetry in $SO(8) \times SO(8)$ gauged $N = 16$ theory.

3.1.4 $SO(7) \times SO(3) \times U(1)$ gauging

In this gauging, we still work with the G_2 invariant scalar sector. The potential is given by

$$V = -\frac{1}{32} g^2 [1301 + 448\cosh(\sqrt{2}b_1) + 308\cosh(2\sqrt{2}b_1) - 9\cosh(4\sqrt{2}b_1)]. \tag{34}$$

This case is very similar to the $SO(10) \times U(1)$ gauging in the sense that the potential dose not depend on a_1, a_2 and a_3 and admits two critical points.

The first critical point is as usual at $L = \mathbf{I}$. This point is a (7,3) point with

$$\begin{aligned} V_0 &= -64g^2 \\ A_1 &= \mathrm{diag}(-4, -4, -4, -4, -4, -4, -4, 4, 4, 4). \end{aligned} \tag{35}$$

The background isometry is $Osp(7|2, \mathbb{R}) \times Osp(3|2, \mathbb{R})$.

The second critical point is given by

$$b_1 = \frac{1}{\sqrt{2}} \cosh^{-1}\frac{7}{3}, \qquad V_0 = -\frac{1024}{9} g^2\,. \tag{36}$$

The A_1 tensor is very complicated, so we give its explicit form in appendix B equation (104). Remarkably, the complicated matrix $M_3^{(1)}$ can be diagonalized to diag $\left(8, 8, \frac{16}{3}\right)$. This gives

$$A_1 = \mathrm{diag}\left(-8, -8, -8, -8, -8, -8, -8, 8, 8, \frac{16}{3}\right). \tag{37}$$

So, this critical point has (0,1) supersymmetry with

$$\frac{c_{(0)}}{c_{(1)}} = \frac{4}{3}\,. \tag{38}$$

Notice that this point has G_2 symmetry although it is characterized only by b_1. This is because the $SO(7)$ in the gauge group is not the same as $SO(7)^{\pm}$, and b_1 is not invariant under this $SO(7)$. The $SO(7)$ in the gauge group is embedded in $SO(8)$ as $\mathbf{8}_v \rightarrow \mathbf{7} + \mathbf{1}$. This point is similar to the (1,1) $G_2 \times G_2$ point in [20].

3.1.5 $SO(6) \times SO(4) \times U(1)$ gauging

We first study the potential in the $SO(4)_{\rm diag}$ scalar sector. There are four singlets in this sector corresponding the non-compact directions of $SO(2,2) \sim SO(2,1) \times SO(2,1)$. We parametrize the coset representative by

$$L = e^{a_1[V_1,V_2]}e^{b_1V_1}e^{-a_1[V_1,V_2]}e^{a_2[V_3,V_4]}e^{b_2V_1}e^{-a_2[V_3,V_4]} \tag{39}$$

where

$$\begin{aligned} V_1 &= j_1 + j_2 \\ V_2 &= j_3 - j_4 \\ V_3 &= j_3 + j_4 \\ V_4 &= j_1 - j_2 \end{aligned} \tag{40}$$

and

$$\begin{aligned} j_1 &= Y_1 + Y_5 - Y_9 + Y_{13} - Y_{17} - Y_{21} + Y_{30} + Y_{32} \\ j_2 &= Y_2 + Y_{10} - Y_{11} + Y_{18} + Y_{19} - Y_{28} + Y_{31} + Y_3 \\ j_3 &= Y_4 + Y_7 + Y_{12} - Y_{15} + Y_{20} + Y_{23} + Y_{26} - Y_{27} \\ j_4 &= Y_6 - Y_8 + Y_{14} + Y_{16} - Y_{22} + Y_{24} + Y_{25} - Y_{29}\,. \end{aligned} \tag{41}$$

We find the potential

$$\begin{aligned} V &= -2e^{-4\sqrt{2}(b_1+b_2)}[1 + 4e^{4\sqrt{2}b_1} + e^{8\sqrt{2}b_1} + 4e^{4\sqrt{2}b_2} + e^{8\sqrt{2}b_2} \\ &\quad +12e^{4\sqrt{2}(b_1+b_2)} + e^{8\sqrt{2}(b_1+b_2)} + 4e^{4\sqrt{2}(2b_1+b_2)} + 4e^{4\sqrt{2}(b_1+2b_2)}]g^2\,. \end{aligned} \tag{42}$$

Unfortunately, there is no non-trivial critical point in this potential. So, there is no critical point with $SO(4)_{\rm diag}$ symmetry.

The next sector we will consider is $SU(3)$ invariant sector. The $SU(3)$ is a subgroup of $SO(6) \sim SU(4)$. There are eight singlets in this sector. The coset representative is parametrized by

$$L = e^{a_1c_{36}}e^{a_2c_{51}}e^{a_3c_{52}}e^{a_4\tilde{c}_{53}}e^{a_5c_{77}}e^{a_6c_{78}}e^{b_1Y_1}e^{b_2Y_3} \tag{43}$$

in which the eight scalars correspond to non-compact directions of $SU(2,2)$. As usual, we have used local H symmetry to simplify the parametrization of L. The potential is given in appendix D. We find two critical points.

The trivial (6,4) critical point at $L = \mathbf{I}$ is given by

$$\begin{aligned} V_0 &= -64g^2, \\ A_1 &= \text{diag}\,(-4,-4,-4,-4,-4,-4,4,4,4,4)\,. \end{aligned} \tag{44}$$

The background isometry is $Osp(6|2,\mathbb{R}) \times Osp(4|2,\mathbb{R})$.

The non trivial critical point is given by

$$
\begin{aligned}
a_i &= \frac{\pi}{2}, \qquad i = 1, \ldots, 6, \\
b_1 &= b_2 = \cosh^{-1}\sqrt{3}, \qquad V_0 = -144g^2, \\
A_1 &= \mathrm{diag}\,(-10,-10,-10,-10,-10,-10,6,6,10,10)\,.
\end{aligned} \tag{45}
$$

This point preserves (0,2) supersymmetry and $SU(3)$ symmetry. The ratio of the central charges is

$$
\frac{c_{(0)}}{c_{(1)}} = \frac{3}{2}\,. \tag{46}
$$

3.1.6 $SO(5) \times SO(5)$ gauging

We start with the potential in the $SO(5)_{\mathrm{diag}}$ scalar sector. There are two singlets in this sector corresponding to the non-compact directions of $SL(2)$. We parametrize the coset representative by

$$
L = e^{a_1 V} e^{b_1 U} e^{-a_1 V} \tag{47}
$$

where the compact and non-compact generators of $SL(2)$ are given by

$$
V = \frac{1}{\sqrt{2}}\left(c_{11} - c_{17} + c_{32} - c_{48} + c_{75} + \frac{\sqrt{3}}{2}\tilde{c}_{70}\right), \tag{48}
$$

$$
U = Y_3 - Y_5 - Y_{12} + Y_{16} + Y_{17} - Y_{18} + Y_{27} + Y_{29}\,. \tag{49}
$$

The potential is given by

$$
V = -8g^2(5 + 3\cosh(4b_1)) \tag{50}
$$

which does not have any non-trivial critical points.

We then move to smaller unbroken gauge symmetry namely $SO(4)_{\mathrm{diag}}$. The parametrization of L is the same as in (39). The potential turns out to be the same as that of $SO(6) \times SO(4) \times U(1)$ gauging, and, of course, does not have any non trivial critical points.

To proceed further, we need to reduce the residual symmetry to a smaller group. The next sector we will consider is $SO(3)_{\mathrm{diag}}$. There are eight singlets in this sector. These are non-compact directions of $SO(4,2) \sim SU(2,2)$. We parametrize the coset representative in this sector by

$$
L = e^{a_1 c_{10}} e^{a_2 c_{14}} e^{a_3 c_{15}} e^{a_4 c_{19}} e^{a_5 c_{20}} e^{a_6 c_{21}} e^{b_1 Z_1} e^{b_2 Z_2} \tag{51}
$$

where

$$
Z_1 = Y_1 + Y_{11} - Y_{20} - Y_{29}, \qquad Z_2 = Y_2 + Y_{13} - Y_{24} + Y_{27}\,. \tag{52}
$$

The potential depends on all eight scalars. Its explicit form is given in appendix E.

The trivial (5,5) critical point at $L = \mathbf{I}$ is characterized by

$$V_0 = -64g^2, \qquad A_1 = \mathrm{diag}\,(-4,-4,-4,-4,-4,4,4,4,4,4)\,. \tag{53}$$

The corresponding background isometry group is $Osp(5|2,\mathbb{R}) \times Osp(5|2,\mathbb{R})$.

We find a non trivial critical point given by

$$\begin{aligned} a_i &= \frac{\pi}{2}, \;\; i = 1,\ldots,6, \qquad b_2 = 0, \\ b_1 &= \frac{\cosh^{-1} 5}{2} \qquad V_0 = -256g^2 \\ A_1 &= \mathrm{diag}\,(-8,-8,-8,16,16,-16,-16,16,16,16)\,. \end{aligned} \tag{54}$$

This critical point has (3,0) supersymmetry with the ratio of the central charges

$$\frac{c_{(0)}}{c_{(1)}} = 2\,. \tag{55}$$

3.2 Vacua of non-compact gaugings

We now consider non-compact gaugings with gauge groups $SU(4,2) \times SU(2)$, $G_{2(-14)} \times SU(2,1)$ and $F_{4(-20)}$. At $L = \mathbf{I}$, the gauge group is broken down to its maximal compact subgroup, and the bosonic part of the background isometry is formed by this subgroup and $SO(2,2)$. These gauge groups contain $SU(3)$ subgroup, so we study the potential in the $SU(3)$ scalar sector in all non-compact gaugings. For $G_{2(-14)} \times SU(2,1)$ and $F_{4(-20)}$ gaugings, the $SU(3) \subset G_2$ sector consists of eight scalars which is twice the number of scalars in the G_2 sector. The $SU(3)$ is embedded in G_2 as $\mathbf{7} \to \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$. The eight scalars correspond to non-compact directions of the $SO(4,2) \sim SU(2,2) \subset E_{6(-14)}$. For $SU(4,2) \times SU(2)$ gauging, the $SU(3)$ is embedded in $SU(4) \subset SU(4,2)$ as $\mathbf{4} \to \mathbf{3} + \mathbf{1}$. Similarly, the eight scalars are described by non-compact directions of $SU(2,2)$. This sector is essentially the same as that used in $SO(6) \times SO(4) \times U(1)$ gauging.

Fortunately, we do not need to deal with all eight scalars. In these three gaugings, four of the eight $SU(3)$ singlets lie along the gauge group, so only four directions orthogonal to the gauge group are relevant. This is because the singlets which are parts of the gauge group will drop out from the potential and correspond to flat directions of the potential. The relevant four singlets are contained in the $SU(2,1)$ sub group of $SU(2,2)$. We also study the potentials in other sectors specific to each gauging. The details of these sectors will be explained below.

3.2.1 $G_{2(-14)} \times SU(2,1)$ gauging

If we study the potential in the G_2 sector in this gauging, we will find the constant potential. This is because all scalars in the G_2 sector are parts of the gauge group and will drop out from the potential. We then start with $SU(3) \subset G_2$ sector. As discussed above, this sector contains four relevant scalars parametrized by

$$L = e^{a_1 c_{52}} e^{a_2 c_{78}} e^{a_3 \tilde{c}_{53}} e^{b_1 (Y_1 - Y_6)} e^{-a_3 \tilde{c}_{53}} e^{-a_2 c_{78}} e^{-a_1 c_{52}} \,. \tag{56}$$

The potential is given by

$$V = \frac{1}{18} g^2 [-101 - 28 \cosh(2\sqrt{2} b_1) + \cosh(4\sqrt{2} b_1)]. \tag{57}$$

There are two critical points. The first one is the trivial critical point given by $L = \mathbf{I}$ and

$$\begin{aligned} V_0 &= -\frac{64}{9} g^2, \\ A_1 &= \text{diag}\left(-\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}, \frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right). \end{aligned} \tag{58}$$

We find that this point has (7,3) supersymmetry. The symmetry of this point is given by the maximal compact subgroup $G_2 \times SU(2) \times U(1)$ of $G_{2(-14)} \times SU(2,1)$. The left handed supercharges transform as $\mathbf{7}$ under G_2 while the right handed supercharges transform as $\mathbf{3}$ under the $SU(2) \sim SO(3)$. So, the background isometry is given by $G(3) \times Osp(3|2, \mathbb{R})$.

The second critical point is characterized by

$$\begin{aligned} b_1 &= \frac{\cosh^{-1} 2}{\sqrt{2}}, \qquad V_0 = -\frac{100}{9} g^2, \\ A_1 &= \begin{pmatrix} -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{11}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{7}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_1 & y_4 & y_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_4 & y_2 & y_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y_5 & y_6 & y_3 \end{pmatrix} \end{aligned} \tag{59}$$

where

$$\begin{aligned} y_1 &= \frac{1}{6}[13 - \cos(2a_1) - 2\cos^2 a_1 \cos(2a_2)], \\ y_2 &= \frac{1}{6}[13 + \cos(2a_1) - 2\cos(2a_2)\sin^2 a_1] \\ y_3 &= \frac{1}{3}(6 + \cos(2a_2)), \qquad y_4 = \frac{1}{3}\cos^2 a_2 \sin(2a_1), \\ y_5 &= -\frac{1}{3}\cos a_1 \sin(2a_2), \qquad y_6 = \frac{1}{3}\sin a_1 \sin(2a_2)\,. \end{aligned} \tag{60}$$

We can diagonalize A_1 to

$$A_1 = \text{diag}\left(-\frac{11}{3}, -\frac{7}{3}, -\frac{7}{3}, -\frac{7}{3}, -\frac{7}{3}, -\frac{7}{3}, -\frac{7}{3}, \frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right) \tag{61}$$

from which we find that this is a (0,1) supersymmetric critical point. The ratio of the central charges relative to the $L = \mathbf{I}$ point is

$$\frac{c_{(0)}}{c_{(1)}} = \frac{5}{4}\,. \tag{62}$$

This $SU(3)$ point is closely related to the (0,1) $SU(3)$ point in $G_{2(-14)} \times SL(2)$ gauged $N = 9$ theory in [18].

We now study the potential in different sector, $SU(2)_{\text{diag}}$ sector. From the $SU(3)$ sector discussed above, the next symmetry to consider could be the $SU(2) \subset SU(3)$. In general, we expect more scalars than those appearing in the $SU(3)$ sector. This will make the calculation takes much longer time. We then consider $SU(2)_{\text{diag}}$ sector in which $SU(2)_{\text{diag}} \subset SU(2) \times SU(2)$. The first and second $SU(2)$'s are subgroups of $SU(3) \subset G_{2(-14)}$ and $SU(2,1)$, respectively. There are four singlets in this sector corresponding to the non-compact directions of $SO(4,1) \sim Sp(1,1)$. We choose to parametrize the coset representative by applying three $SO(3) \subset SO(4) \sim SO(3) \times SO(3)$ rotations as follow

$$L = e^{a_1 c_8} e^{a_2 c_{17}} e^{a_3 c_{20}} e^{b_1 (Y_2 - Y_{16} + Y_{19} + Y_{29})} e^{-a_3 c_{20}} e^{-a_2 c_{17}} e^{-a_1 c_8}\,. \tag{63}$$

The potential is

$$V = \frac{1}{72} g^2 [-269 - 192\cosh(2b_1) - 52\cosh(4b_1) + \cosh(8b_1)]\,. \tag{64}$$

There is one non trivial critical points given by

$$b_1 = \cosh^{-1}\sqrt{2}, \qquad V_0 = -16g^2\,. \tag{65}$$

This is a supersymmetric point with the associated A_1 tensor given in appendix B equation (106). After diagonalization, we find

$$A_1 = \text{diag}\left(-4, -4, -4, -4, -\frac{10}{3}, -2, -2, 2, 2, 2, 2\right) \tag{66}$$

which gives (2,3) supersymmetry. The ratio of the central charges is

$$\frac{c_{(0)}}{c_{(2)}} = \frac{3}{2}\,. \tag{67}$$

This critical point has $SU(2)_{\text{diag}} \times U(1)$ symmetry.

3.2.2 $F_{4(-20)}$ gauging

In this gauging with simple gauge group, we study the potential in the G_2 and $SU(3)$ scalar sectors. We start with the G_2 sector. Two of the four scalars are parts of the gauge group, so we only need to parametrize the coset representative with the other two scalars. These two scalars correspond to the non-compact directions of $SL(2)$. The L is then parametrized by

$$L = e^{a_1 c_{52}} e^{b_1(Y_{25}+Y_{30})} e^{-a_1 c_{52}}\,. \tag{68}$$

The potential is

$$V = \frac{g^2}{8}[-101 - 28\cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1)]. \tag{69}$$

There are two critical points. The first one is trivial and given by

$$\begin{aligned} L &= \mathbf{I}, \qquad V_0 = -16g^2, \\ A_1 &= \mathrm{diag}\,(-2,-2,-2,-2,-2,-2,-2,-2,-2,2)\,. \end{aligned} \tag{70}$$

This is the maximally supersymmetric point with (9,1) supersymmetry. The gauge symmetry is broken down to its maximal compact subgroup $SO(9)$, and the background isometry is $Osp(9|2,\mathbb{R}) \times Osp(1|2,\mathbb{R})$.

The second critical point is given by

$$\begin{aligned} b_1 &= \frac{\cosh^{-1}2}{\sqrt{2}}, \qquad V_0 = -25g^2, \\ A_1 &= \begin{pmatrix} -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_1 & w_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_3 & w_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} \end{pmatrix} \end{aligned} \tag{71}$$

where

$$w_1 = -3 - \frac{1}{2}\cos(2a_1), \;\; w_2 = \frac{1}{2}[-6+\cos(2a_1)], \;\; w_3 = \cos a_1 \sin a_1\,. \tag{72}$$

The A_1 tensor can be diagonalized to

$$A_1 = \mathrm{diag}\left(\frac{11}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{7}{2}, -\frac{5}{2}\right). \tag{73}$$

This critical point is a (1,0) point with

$$\frac{c_{(0)}}{c_{(1)}} = \frac{5}{4} \tag{74}$$

and preserves $SO(7) \subset SO(9) \subset F_{4(-20)}$ symmetry.

In the $SU(3)$ sector, there are eight singlets, but four of them are parts of the $F_{4(-20)}$. So, there are four singlets orthogonal to the gauge group. These are non-compact directions of $SU(2,1)$, and L can be parametrized by

$$L = e^{a_1 c_{34}} e^{a_2 c_{49}} e^{a_3 c_{52}} e^{b_1 Y_{21}} e^{-a_3 c_{52}} e^{-a_2 c_{49}} e^{-a_1 c_{34}} . \tag{75}$$

The potential is given by

$$V = \frac{g^2}{8}[-101 - 28\cosh(2\sqrt{2}b_1) + \cosh(4\sqrt{2}b_1)] \tag{76}$$

which is the same as the potential in the G_2 sector. The non-trivial critical point is at the same position and cosmological constant, $b_1 = \cosh^{-1} 2$, $V_0 = -25g^2$. The residual symmetry is $SO(7)$ as in the previous critical point. Although the A_1 tensor in this case is more complicated, it is the same as (73) after diagonalization. The explicit form of A_1 is given in appendix B equation (108).

3.2.3 $SU(4,2) \times SU(2)$ gauging

This gauging is the most difficult one to find a suitable scalar sector in order to reveal non trivial critical points and still have a manageable number of scalars. We start with the $SO(4)_{\text{diag}}$ scalar sector. The $SO(4)_{\text{diag}}$ is formed by taking the subgroup $SU(2) \times SU(2) \times SU(2) \times SU(2)$ of $SU(4,2) \times SU(2)$. The first two $SU(2)$'s are subgroups of $SU(4) \subset SU(4,2)$, the third $SU(2)$ is the $SU(2) \subset SU(4,2)$. Our $SO(4)_{\text{diag}}$ is the diagonal subgroup of $(SU(2) \times SU(2)) \times (SU(2) \times SU(2)) \sim SO(4) \times SO(4)$. There are two singlets in this sector. These are non-compact directions of $SL(2)$, and L can be parametrized by

$$\begin{aligned} L &= e^{a_1 c_{15}} e^{b_1 \tilde{Y}} e^{-a_1 c_{15}}, \\ \tilde{Y} &= Y_1 + Y_2 - Y_6 - Y_7 - Y_9 + Y_{10} - Y_{14} + Y_{15} \\ &\quad + Y_{17} - Y_{18} - Y_{22} + Y_{23} - Y_{27} + Y_{28} - Y_{29} - Y_{32} \end{aligned} \tag{77}$$

which, unfortunately, gives a constant potential $V = -16g^2$. So, we move to a smaller residual symmetry to obtain a non trivial structure of the potential.

We now study the potential in the scalar sector parametrizing the $SU(3)$ invariant manifold. This $SU(3)$ is a subgroup of $SU(4) \subset SU(4,2)$. The eight singlet scalars in this sector are the non-compact directions of $SO(4,2) \sim SU(2,2)$. The four directions which are orthogonal to the gauge group are non-compact directions of $SU(2,1) \subset SU(2,2)$. The coset representative is given by

$$\begin{aligned} L &= e^{a_1(c_{51}+c_{78})} e^{a_2(c_{36}+\tilde{c}_{53})} e^{a_3(c_{77}-c_{52})} e^{b_1(Y_1 - Y_{23})} \\ &\quad e^{-a_3(c_{77}-c_{52})} e^{-a_2(c_{36}+\tilde{c}_{53})} e^{-a_1(c_{51}+c_{78})} . \end{aligned} \tag{78}$$

We find the potential

$$V = -2g^2(5 + 3\cosh(2b_1)) \tag{79}$$

which, again, does not admit any non trivial critical points.

The next sector we will study is $SU(2)_{\text{diag}}$. This symmetry is a diagonal subgroup of $SU(2) \times SU(2)$ in which the first $SU(2)$ is a subgroup of $SU(4) \subset SU(4,2)$, and the second $SU(2)$ is the $SU(2)$ factor in the gauge group. There are four scalars in this sector. These scalars are non-compact directions of $SU(2,1)$, and L can be parametrized by

$$L = e^{a_1 c_{10}} e^{a_2 c_{14}} e^{a_3 c_{15}} e^{b_1 Y} e^{-a_3 c_{15}} e^{-a_2 c_{14}} e^{-a_1 c_{10}} \tag{80}$$

where

$$Y = Y_7 - Y_6 - Y_{12} - Y_{16} + Y_{17} + Y_{18} + Y_{30} + Y_{31}\,. \tag{81}$$

The corresponding potential is

$$V = \frac{g^2}{8}[-101 - 28\cosh(4\sqrt{2}b_1) + \cosh(8\sqrt{2}b_1)]\,. \tag{82}$$

We now discuss its trivial critical point at $L = \mathbf{I}$. This point is characterized by

$$V_0 = -16g^2, \qquad A_1 = \text{diag}\,(-2,-2,-2,-2,-2,-2,2,2,2,2,2)\,. \tag{83}$$

The critical point has (6,4) supersymmetry. The gauge group is broken down to its maximal compact subgroup $SU(4) \times SU(2) \times U(1) \times SU(2)$. The left handed supercharges transform as $\mathbf{6}$ under $SU(4) \sim SO(6)$ while the right handed supercharges transform as $\mathbf{4}$ under $SU(2) \times SU(2) \sim SO(4)$. So, the background isometry is given by $Osp(6|2,\mathbb{R}) \times Osp(4|2,\mathbb{R})$.

The non trivial critical point with $SU(2)_{\text{diag}} \times SU(2) \times SU(2) \times U(1)$ symmetry is given by

$$b_1 = \frac{1}{\sqrt{2}} \cosh^{-1}\sqrt{\frac{3}{2}}, \qquad V_0 = -25g^2\,. \tag{84}$$

The associated A_1 tensor is given in appendix B equation (110) which can be diagonalized to

$$A_1 = \text{diag}\left(\frac{11}{2},\frac{11}{2},\frac{11}{2},\frac{11}{2},-\frac{7}{2},-\frac{7}{2},-\frac{5}{2},-\frac{5}{2},-\frac{5}{2},-\frac{5}{2}\right). \tag{85}$$

So, this is a (4,0) point with

$$\frac{c_{(0)}}{c_{(1)}} = \frac{5}{4}\,. \tag{86}$$

4 Conclusions

In this paper, we have studied critical points of $N = 10$ three dimensional gauged supergravity with both compact and non-compact gauge groups. Remarkably, all critical points found in this paper are AdS critical points. This is in contrast to the results of [20] in which some Minkowski and dS vacua have been found. In $SO(10) \times U(1)$ gauging, there is one non trivial critical point which completely breaks all the supersymmetries and breaks the gauge group down to $SO(7)$. In $SO(9) \times U(1)$ gauging, we have found two non trivial critical points with (2,1) and (1,1) supersymmetries. Both of them break the gauge group down to G_2. In $SO(8) \times SO(2) \times U(1)$ gauging, there are two non trivial critical points. One of them completely breaks all of the supersymmetries and preserves $SO(7)$ subgroup of the gauge group. Another critical point preserves G_2 symmetry and (1,1) supersymmetry. In $SO(7) \times SO(3) \times U(1)$ gauging, there is one non trivial critical point with (0,1) supersymmetry and G_2 symmetry. In $SO(6) \times SO(4) \times U(1)$ gauging, there is one non trivial $SU(3)$ critical point with (0,2) supersymmetry. In $SO(5) \times SO(5)$ gauging, we have found one nontrivial critical point breaking the gauge group down to $SO(3)_{\text{diag}}$ and preserving (3,0) supersymmetry.

Our results in non-compact gaugings are as follows. In $G_{2(-14)} \times SU(2,1)$ gauging, we have found two non trivial critical points. One of them has (0,1) supersymmetry and preserves $SU(3)$ symmetry of the gauge group. The second critical point breaks the gauge group down to $SU(2)_{\text{diag}} \times U(1)$ and preserves (2,3) supersymmetry. In $F_{4(-20)}$ gauging, we have found one $SO(7)$ critical point with (1,0) supersymmetry. Finally, in $SU(4,2) \times SU(2)$ gauging, there is one non trivial (4,0) critical point with $SU(2)_{\text{diag}} \times SU(2) \times SU(2) \times U(1) \sim SU(2)_{\text{diag}} \times SO(4) \times U(1)$ symmetry.

The gauge groups considered in this work are only maximal subgroups of $E_{6(-14)}$. It is interesting to study gaugings with other gauge groups which are not maximal subgroups of $E_{6(-14)}$ along with their scalar potentials and the corresponding critical points. In particular, non-semisimple gaugings are very interesting in the sense that they are related to semisimple Yang-Mills gaugings which arise from dimensional reductions of higher dimensional theories [26]. Furthermore, studies of RG flows between critical points identified in this work are of particular interest in studying deformations of the two dimensional CFT's. We hope to give further results on these issues in future works.

Acknowledgement We gratefully thank the Extreme Condition Physics Research Laboratory at Department of Physics, Faculty of Science, Chulalongkorn University for computing facilities.

A Essential formulae

In this appendix, we give all necessary formulae in order to obtain the scalar potential. We use the 52 generators of the F_4 subgroup of E_6 from [23]. The remaining 26 generators are given in [24]. The generators are normalized by

$$\mathrm{Tr}(c_i c_j) = -6\delta_{ij}. \tag{87}$$

With this normalization, we find that

$$\begin{aligned}
\mathcal{V}^{\alpha IJ} &= -\frac{1}{6}\mathrm{Tr}(L^{-1}T^{\alpha}_{G}LX^{IJ}) && (88)\\
\mathcal{V}^{\alpha A} &= \frac{1}{6}\mathrm{Tr}(L^{-1}T^{\alpha}_{G}LY^{A}) && (89)\\
\mathcal{V}^{IJ}_{U(1)} &= -\frac{1}{6}\mathrm{Tr}(L^{-1}XLX^{IJ}) && (90)\\
\mathcal{V}^{A}_{U(1)} &= \frac{1}{6}\mathrm{Tr}(L^{-1}XLY^{A}) && (91)
\end{aligned}$$

where we have introduced the symbol T^{α}_{G} for gauge group generators as in [18]. T^{α}_{G} will be replaced by some appropriate generators of the gauge group being considered in each gauging.

The following mapping provides the relation between c_i and X^{IJ}, generators of $SO(10)$,

$$\begin{aligned}
X^{12} &= c_1,\ X^{13} = -c_2,\ X^{23} = c_3,\ X^{34} = c_6,\ X^{14} = c_4,\ X^{24} = -c_5,\\
X^{15} &= c_7,\ X^{25} = -c_8,\ X^{35} = c_9,\ X^{45} = -c_{10},\ X^{56} = -c_{15},\ X^{16} = c_{11},\\
X^{26} &= -c_{12},\ X^{46} = -c_{14},\ X^{36} = c_{13},\ X^{17} = c_{16},\ X^{27} = -c_{17},\ X^{47} = -c_{19},\\
X^{37} &= c_{18},\ X^{67} = -c_{21},\ X^{57} = -c_{20},\ X^{78} = -c_{36},\ X^{18} = c_{30},\ X^{28} = -c_{31},\\
X^{48} &= -c_{33},\ X^{38} = c_{32},\ X^{68} = -c_{35},\ X^{58} = -c_{34},\ X^{29} = -c_{46},\ X^{19} = c_{45},\\
X^{49} &= -c_{48},\ X^{39} = c_{47},\ X^{69} = -c_{50},\ X^{59} = -c_{49}, X^{89} = -c_{52},\ X^{79} = -c_{51},\\
X^{1,10} &= -c_{71},\ X^{2,10} = c_{72},\ X^{3,10} = -c_{73},\ X^{4,10} = c_{74},\ X^{5,10} = c_{75},\\
X^{6,10} &= c_{76},\ X^{7,10} = c_{77},\ X^{8,10} = c_{78},\ X^{9,10} = \tilde{c}_{53}\,. && (92)
\end{aligned}$$

The $\tilde{c}_{53}$ and $\tilde{c}_{70}$ are defined by [24]

$$\tilde{c}_{53} = \frac{1}{2}c_{53} + \frac{\sqrt{3}}{2}c_{70} \qquad \text{and} \qquad \tilde{c}_{70} = -\frac{\sqrt{3}}{2}c_{53} + \frac{1}{2}c_{70}\,. \tag{93}$$

All the f^{IJ}'s components can be obtained from the structure constants of the $[X^{IJ}, Y^A]$ given in [23] and [24].

Generators of the $SO(p)\times SO(10-p)$ compact gauge group are given by

$$\begin{aligned}
T^{IJ}_1 &= X^{IJ}, && I,J = 1,\dots p,\\
T^{IJ}_2 &= X^{IJ}, && I,J = p+1,\dots 10\,. && (94)
\end{aligned}$$

The $U(1)$ subgroup is generated by $X = 2\tilde{c}_{70}$.

In the non-compact $G_{2(-14)} \times SU(2,1)$ gauging, the generators of $G_{2(-14)}$ can be obtained from combinations of $SO(7)$ generators [27]

$$\begin{aligned}
T_1 &= \frac{1}{\sqrt{2}}(X^{36} + X^{41}), \;\; T_2 = \frac{1}{\sqrt{2}}(X^{31} - X^{46}), \\
T_3 &= \frac{1}{\sqrt{2}}(X^{43} - X^{16}), \;\; T_4 = \frac{1}{\sqrt{2}}(X^{73} - X^{24}), \\
T_5 &= -\frac{1}{\sqrt{2}}(X^{23} + X^{47}), \;\; T_6 = -\frac{1}{\sqrt{2}}(X^{26} + X^{71}), \\
T_7 &= \frac{1}{\sqrt{2}}(X^{76} - X^{21}), \;\; T_8 = \frac{1}{\sqrt{6}}(X^{16} + X^{43} - 2X^{72}), \\
T_9 &= -\frac{1}{\sqrt{6}}(X^{41} - X^{36} + 2X^{25}), \;\; T_{10} = -\frac{1}{\sqrt{6}}(X^{31} + X^{46} - 2X^{57}), \\
T_{11} &= \frac{1}{\sqrt{6}}(X^{73} + X^{24} + 2X^{15}), \;\; T_{12} = -\frac{1}{\sqrt{6}}(X^{74} - X^{23} + 2X^{65}), \\
T_{13} &= \frac{1}{\sqrt{6}}(X^{26} - X^{71} + 2X^{35}), \;\; T_{14} = \frac{1}{\sqrt{6}}(X^{21} + X^{76} - 2X^{45}).
\end{aligned} \tag{95}$$

These generators are essentially the same as those used in [18], but we repeat them here for conveniences. The $SU(2,1)$ generators are given by

$$\begin{aligned}
J_1 &= -c_{52}, \qquad J_2 = -\tilde{c}_{53}, \qquad J_3 = -c_{78}, \qquad J_4 = \tilde{c}_{70}, \\
J_5 &= \frac{1}{\sqrt{2}}(Y_1 + Y_6), \qquad J_6 = \frac{1}{\sqrt{2}}(Y_9 + Y_{14}), \\
J_7 &= \frac{1}{\sqrt{2}}(Y_{21} + Y_{24}), \qquad J_8 = \frac{1}{\sqrt{2}}(Y_{25} + Y_{30})\,.
\end{aligned} \tag{96}$$

We have normalized these generators according to the embedding tensor given in section 2.

In $SU(4,2) \times SU(2)$ gauging, the relevant generators are given by

- $SU(4,2)$:

$$\begin{aligned}
Q_i &= c_i, \qquad i=1,\ldots,15,\\
Q_{16} &= \frac{1}{\sqrt{2}}(c_{52}+c_{77}),\ \ Q_{17}=\frac{1}{\sqrt{2}}(c_{51}-c_{78}),\ \ Q_{18}=\frac{1}{\sqrt{2}}(\tilde{c}_{53}-c_{36}),\\
Q_{19} &= \tilde{c}_{70},\ \ Q_{20}=\frac{1}{\sqrt{2}}(Y_1+Y_{23}),\ \ Q_{21}=\frac{1}{\sqrt{2}}(Y_2-Y_{22}),\\
Q_{22} &= \frac{1}{\sqrt{2}}(Y_3+Y_{24}),\ \ Q_{23}=\frac{1}{\sqrt{2}}(Y_4-Y_{21}),\ \ Q_{24}=\frac{1}{\sqrt{2}}(Y_5+Y_{20}),\\
Q_{25} &= \frac{1}{\sqrt{2}}(Y_6+Y_{18}),\ \ Q_{26}=\frac{1}{\sqrt{2}}(Y_7-Y_{17}),\ \ Q_{27}=\frac{1}{\sqrt{2}}(Y_8-Y_{19}),\\
Q_{28} &= \frac{1}{\sqrt{2}}(Y_9+Y_{27}),\ \ Q_{29}=\frac{1}{\sqrt{2}}(Y_{10}-Y_{29}),\ \ Q_{30}=\frac{1}{\sqrt{2}}(Y_{11}-Y_{25}),\\
Q_{31} &= \frac{1}{\sqrt{2}}(Y_{12}+Y_{30}),\ \ Q_{32}=\frac{1}{\sqrt{2}}(Y_{13}+Y_{26}),\ \ Q_{33}=\frac{1}{\sqrt{2}}(Y_{14}-Y_{28}),\\
Q_{34} &= \frac{1}{\sqrt{2}}(Y_{15}-Y_{32}),\ \ Q_{35}=\frac{1}{\sqrt{2}}(Y_{16}+Y_{31})\,. \qquad (97)
\end{aligned}$$

- $SU(2)$:

$$K_1=\frac{1}{2}(c_{51}+c_{78}),\ \ K_2=-\frac{1}{2}(c_{52}-c_{77}),\ \ K_3=\frac{1}{2}(c_{36}+\tilde{c}_{53}). \qquad (98)$$

To find the above generators, we first look at the generators of the compact subgroup $SU(4)\times SU(2)\times U(1)$ of the $SU(4,2)$. Using the fact that $SU(4)\sim SO(6)$ and $SU(2)\times SU(2)\sim SO(4)$, we can identify $SU(4)\times SU(2)\times SU(2)$ with $SO(6)\times SO(4)\subset SO(10)$. The $U(1)$ generator is simply $\tilde{c}_{70}$.

The final non-compact gauge group is $F_{4(-20)}$. Its generators can be easily identified by $c_1,\ldots,c_{52}$ in the construction of the E_6 given in [24].

We can now compute the T-tensors using

$$\begin{aligned}
T^{IJ,KL} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{KL,\beta}\delta^{SO(p)}_{\alpha\beta}-\mathcal{V}^{IJ,\alpha}\mathcal{V}^{KL,\beta}\delta^{SO(10-p)}_{\alpha\beta}+\frac{1}{3}(5-p)\mathcal{V}^{IJ}_{U(1)}\mathcal{V}^{KL}_{U(1)}, \qquad (99)\\
T^{IJ,A} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{A,\beta}\delta^{SO(p)}_{\alpha\beta}-\mathcal{V}^{IJ,\alpha}\mathcal{V}^{A,\beta}\delta^{SO(10-p)}_{\alpha\beta}+\frac{1}{3}(5-p)\mathcal{V}^{IJ}_{U(1)}\mathcal{V}^{A}_{U(1)} \qquad (100)
\end{aligned}$$

for compact gaugings and

$$\begin{aligned}
T^{IJ,KL} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{KL,\beta}\eta^{G_1}_{\alpha\beta}-K\mathcal{V}^{IJ,\alpha}\mathcal{V}^{KL,\beta}\eta^{G_2}_{\alpha\beta}, \qquad (101)\\
T^{IJ,A} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{A,\beta}\eta^{G_1}_{\alpha\beta}-K\mathcal{V}^{IJ,\alpha}\mathcal{V}^{A,\beta}\eta^{G_2}_{\alpha\beta} \qquad (102)
\end{aligned}$$

for non-compact gaugings with K being $\frac{2}{3}$ and 6 for $G_1\times G_2$ being $G_{2(-14)}\times SU(2,1)$ and $SU(4,2)\times SU(2)$, respectively. As in [18], we use summation convention over gauge indices α, β with the notation δ^{G_0} and η^{G_0} meaning that the summation is restricted to the G_0 generators. For $F_{4(-20)}$ gauging, we have the simpler expressions for the T-tensors namely

$$\begin{aligned}
T^{IJ,KL} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{KL,\beta}\eta^{F_{4(-20)}}_{\alpha\beta} \\
T^{IJ,A} &= \mathcal{V}^{IJ,\alpha}\mathcal{V}^{A,\beta}\eta^{F_{4(-20)}}_{\alpha\beta}\,.
\end{aligned} \tag{103}$$

B Explicit forms of the A_1 tensors

In this section, we give the explicit forms of the A_1 tensors mentioned in the main text. We collect them here due to their lengthly and complicated forms.

- $SO(7)\times SO(3)\times U(1)$ gauging
  G_2 sector:

$$A_1 = \begin{pmatrix} -8\mathbf{I}_7 & 0 \\ 0 & M_3^{(1)} \end{pmatrix} \qquad M_3^{(1)} \begin{pmatrix} m_1 & m_4 & m_5 \\ m_4 & m_2 & m_6 \\ m_5 & m_6 & m_3 \end{pmatrix}. \tag{104}$$

The elements of the matrix $M^{(1)}$ are given by

$$\begin{aligned}
m_1 &= \frac{1}{3}[21-\cos(2a_3)-\cos(2a_1)(1+3\cos(2a_3))+4\cos(2a_2)\sin^2(a_1)\sin^2 a_3 \\
&\quad +4\sin(2a_1)\sin a_2\sin(2a_3)] \\
m_2 &= -\frac{2}{3}[-11+\cos(2a_2)-2\cos^2 a_2\cos(2a_3)] \\
m_3 &= \frac{1}{3}[21-\cos(2a_3)+\cos(2a_1)(1+3\cos(2a_3))+4\cos^2 a_1\cos(2a_2)\sin^2 a_3 \\
&\quad -4\sin(2a_1)\sin a_2\sin(2a_3)] \\
m_4 &= \frac{8}{3}\cos a_2\sin a_3[\cos a_1\cos a_3-\sin a_1\sin a_2\sin a_3] \\
m_5 &= \frac{1}{3}[[-2\cos^2 a_2+(-3+\cos(2a_2))\cos(2a_3)]\sin(2a_1) \\
&\quad -4\cos(2a_1)\sin a_2\sin(2a_3)] \\
m_6 &= \frac{8}{3}\cos a_2\sin a_3(\cos a_3\sin a_1+\cos a_1\sin a_2\sin a_3).
\end{aligned} \tag{105}$$

- $G_{2(-14)}\times SU(2,1)$ gauging

$SU(2)_{\rm diag}$ sector:

$$A_1=\begin{pmatrix} -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 & m_{52} & 0 & m_{72} & 0 & 0 & 0 \\ 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{52} & 0 & 0 & m_{55} & 0 & m_{75} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4 & 0 & 0 & 0 & 0 \\ 0 & m_{72} & 0 & 0 & m_{75} & 0 & m_{77} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \tag{106}$$

where

$$\begin{aligned} m_{22} &= \frac{1}{12}[-30+\cos[2(a_1-a_2)]+\cos[2(a_1+a_2)]-2\cos(2a_3) \\ &\quad +\cos(2a_1)(2+6\cos(2a_3))+\cos(2a_2)(2-4\cos^2 a_1\cos(2a_3)) \\ &\quad +8\sin(2a_1)\sin a_2\sin(2a_3)] \\ m_{52} &= \frac{1}{6}[(-2\cos^2 a_2+(-3+\cos(2a_2))\cos(2a_3))\sin(2a_1) \\ &\quad +4\cos(2a_1)\sin a_2\sin(2a_3)] \\ m_{72} &= \frac{4}{3}\cos a_2\sin a_3(\cos a_3\sin a_1-\cos a_1\sin a_2\sin a_3) \\ m_{55} &= \frac{1}{12}[-\cos[2(a_1-a_2)]-\cos[2(a_1+a_2)]-2\cos(2a_1)(1+3\cos(2a_3)) \\ &\quad +\cos(2a_2)(2-4\cos(2a_3)\sin^2 a_1)-2(15+\cos(2a_3) \\ &\quad +4\sin(2a_1)\sin a_2\sin(2a_3))] \\ m_{75} &= \frac{4}{3}\cos a_2\sin a_3(\cos a_1\cos a_3+\sin a_1\sin a_2\sin a_3) \\ m_{77} &= \frac{1}{3}[-7-\cos(2a_2)+2\cos^2 a_2\cos(2a_3)]. \end{aligned} \tag{107}$$

- $F_{4(-20)}$ gauging
  $SU(3)$ sector:

$$A_1=\begin{pmatrix} -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & 0 & 0 & a_{85} & a_{95} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{7}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{85} & 0 & 0 & a_{88} & a_{98} & 0 \\ 0 & 0 & 0 & 0 & a_{95} & 0 & 0 & a_{98} & a_{99} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} \end{pmatrix} \tag{108}$$

where

$$
\begin{aligned}
a_{55} &= \frac{1}{16}[-50-2\cos(2a_1)+3\cos[2(a_1-a_3)]-8\cos^2 a_1\cos(2a_2)\cos^2 a_3 \\
&\quad -2\cos(2a_3)+3\cos[2(a_1+a_3)]+8\sin(2a_1)\sin a_2\sin(2a_3)] \\
a_{85} &= \frac{1}{8}[[2\cos^2 a_2+(-3+\cos(2a_2))\cos(2a_3)]\sin(2a_1) \\
&\quad +4\cos(2a_1)\sin a_2\sin(2a_3)] \\
a_{95} &= \frac{1}{2}\cos a_2[2\cos a_1\cos^2 a_3\sin a_2+\sin a_1\sin(2a_3)] \\
a_{88} &= \frac{1}{16}[\cos[2(a_1-a_2)]+\cos[2(a_1+a_2)]+\cos(2a_1)(2-6\cos(2a_3)) \\
&\quad -2\cos(2a_2)[1+2\cos(2a_3)\sin^2 a_1]-2(25+\cos(2a_3) \\
&\quad +4\sin(2a_1)\sin a_2\sin(2a_3))] \\
a_{99} &= \frac{1}{4}[-13+\cos(2a_2)+2\cos^2 a_2\cos(2a_3)] \\
a_{98} &= \frac{1}{2}\cos a_2[-2\cos^2 a_3\sin a_1\sin a_2+\cos a_1\sin(2a_3)]. \qquad (109)
\end{aligned}
$$

- $SU(4,2)\times SU(2)$ gauging
  $SU(2)_{\mathrm{diag}}$ sector:

$$
A_1 = \begin{pmatrix}
-\frac{5}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{5}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{5}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u_1 & u_4 & u_5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u_4 & u_2 & u_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u_5 & u_6 & u_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2}
\end{pmatrix} \qquad (110)
$$

where

$$\begin{aligned}
u_1 &= \frac{1}{16}[-50 - \cos[2(a_1 - a_2)] - \cos[2(a_1 + a_2)] + 2\cos(2a_3) \\
&\quad -2\cos(2a_1)(1 + 3\cos(2a_3)) + \cos(2a_2)(-2 + 4\cos^2 a_1 \cos(2a_3)) \\
&\quad -8\sin(2a_1)\sin a_2 \sin(2a_3)] \\
u_2 &= \frac{1}{16}[-50 + 2\cos(2a_1) + \cos[2(a_1 - a_2)] - 2\cos(2a_2) \\
&\quad +\cos[2(a_1 + a_2)] + \cos(2a_3)(2 + 6\cos(2a_1) + 4\cos(2a_2)\sin^2 a_1) \\
&\quad +8\sin(2a_1)\sin a_2 \sin(2a_3)] \\
u_3 &= \frac{1}{4}[-13 + \cos(2a_2) - 2\cos^2 a_2 \cos(2a_3)] \\
u_4 &= \frac{1}{8}[(2\cos^2 a_2 - (-3 + \cos(2a_2))\cos(2a_3))\sin(2a_1) \\
&\quad -4\cos(2a_1)\sin a_2 \sin(2a_3)] \\
u_5 &= \cos a_2 \sin a_3(-\cos a_3 \sin a_1 + \cos a_1 \sin a_2 \sin a_3) \\
u_6 &= -\cos a_2] \sin a_3(\cos a_1 \cos a_3 + \sin a_1 \sin a_2 \sin a_3) \qquad (111)
\end{aligned}$$

C Scalar potential for $SO(9)\times U(1)$ gauging in G_2 sector

$$
\begin{aligned}
V \;=\; & -\frac{1}{327680}g^2 e^{-4\sqrt{2}b_1}\Big[-2(4(-1+e^{\sqrt{2}b_1})^3(1+e^{\sqrt{2}b_1})\cos[2a_1](1+3\cos[2a_3]) \\
& +4(-1+e^{2\sqrt{2}b_1})(29+6e^{\sqrt{2}b_1}+29e^{2\sqrt{2}b_1}-(-1+e^{\sqrt{2}b_1})^2\cos[2a_3] \\
& +4(-1+e^{\sqrt{2}b_1})^2(\cos[a_1]^2\cos[2a_2]\sin[a_3]^2-\sin[2a_1]\sin[a_2]\sin[2a_3])))^2 \\
& +20((-1+e^{\sqrt{2}b_1})^4(4\cos[a_2]^2\cos[2a_3]+2\cos[2a_1](-2\cos[a_2]^2 \\
& +(-3+\cos[2a_2])\cos[2a_3])+8\sin[2a_1]\sin[a_2]\sin[2a_3]))^2 \\
& -2621440e^{4\sqrt{2}b_1}\cos[a_1]^2\cos[a_2]^2(\cos[a_3]\sin[a_1]+\cos[a_1]\sin[a_2]\sin[a_3])^2\times \\
& \sinh[\frac{b_1}{\sqrt{2}}]^6-384e^{\sqrt{2}b_1}(-1+e^{\sqrt{2}b_1})^6(4\cos[2a_3]\sin[2a_1]\sin[a_2] \\
& +(3\cos[2a_1]-2\cos[a_1]^2\cos[2a_2]-1)\sin[2a_3])^2 \\
& -96(-1+e^{2\sqrt{2}b_1})^2(2(4(3+2e^{\sqrt{2}b_1}+3e^{2\sqrt{2}b_1})+4(-1+e^{\sqrt{2}b_1})^2\cos[a_1]^2\cos[a_3]^2 \\
& +(-1+e^{\sqrt{2}b_1})^2((3+\cos[2a_2]-2\cos[2a_1]\sin[a_2]^2)\sin[a_3]^2 \\
& -2\sin[2a_1]\sin[a_2]\sin[2a_3])))^2-4(-1+e^{2\sqrt{2}b_1})^2(2(29-2e^{\sqrt{2}b_1}(-3+\cos[2a_2]) \\
& +\cos[2a_2]+e^{2\sqrt{2}b_1}(29+\cos[2a_2])+(e^{\sqrt{2}b_1}-1)^2\cos[2a_1](2\cos[a_2]^2 \\
& -(\cos[2a_2]-3)\cos[2a_3])-2(e^{\sqrt{2}b_1}-1)^2(\cos[a_2]^2\cos[2a_3] \\
& +2\sin[2a_1]\sin[a_2]\sin[2a_3])))^2-16e^{\sqrt{2}b_1}(-1+e^{\sqrt{2}b_1})^6(12\cos[2a_1]\sin[2a_3] \\
& +16\cos[2a_3]\sin[2a_1]\sin[a_2]-4(1+2\cos[a_1]^2\cos[2a_2])\sin[2a_3])^2 \\
& -(-4(-1+e^{\sqrt{2}b_1})^3(1+e^{\sqrt{2}b_1})\cos[2a_1](1+3\cos[2a_3]) \\
& -4(-1+e^{2\sqrt{2}b_1})(29+6e^{\sqrt{2}b_1}+29e^{2\sqrt{2}b_1}-(-1+e^{\sqrt{2}b_1})^2\cos[2a_3] \\
& +4(-1+e^{\sqrt{2}b_1})^2(\cos[a_1]^2\cos[2a_2]\sin[a_3]^2-\sin[2a_1]\sin[a_2]\sin[2a_3])))^2\Big]
\end{aligned}
\tag{112}
$$

D Scalar potential for $SO(6)\times SO(4)\times U(1)$ gauging in $SU(3)$ sector

$$
\begin{aligned}
V \;=\; & -4g^2\Bigg[\tfrac{1}{64}(-11+\cosh[2b_1]-24\cosh[b_1]\cosh[b_2]+2\cosh[b_1]^2\cosh[2b_2])^2 \\
& -5(\tfrac{1}{10}(-3+\cosh[b_1]\cosh[b_2])^2(\sinh[b_1]^2+\cosh[b_1]^2\sinh[b_2]^2) \\
& +\tfrac{1}{129600}(\tfrac{1}{2}(\tfrac{1}{48}(-48\csc[a_4]\sin[2a_4](-\cosh[\tfrac{b_2}{2}](\sin[a_1]\sin[a_3]\sin[a_5] \\
& +\cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4]+\cos[a_3]\sin[a_2]\sin[a_5]))\sinh[\tfrac{b_1}{2}] \\
& -\cosh[\tfrac{b_1}{2}](\cos[a_6](\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_4] \\
& +(\cos[a_5]\sin[a_1]\sin[a_3]+\cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2] \\
& -\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\cos[a_1]\times \\
& \cos[a_2]\cos[a_6]+(\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6]) \\
& +\cos[a_5](-\cos[a_3]\sin[a_1]+\cos[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]) \\
& +48\csc[a_4]\sin[2a_4](\cosh[\tfrac{b_2}{2}](\sin[a_1]\sin[a_3]\sin[a_5] \\
& +\cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4]+\cos[a_3]\sin[a_2]\sin[a_5]))\sinh[\tfrac{b_1}{2}] \\
& +\cosh[\tfrac{b_1}{2}](\cos[a_6](\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_4] \\
& +(\cos[a_5]\sin[a_1]\sin[a_3]+\cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2] \\
& -\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\cos[a_1]\times \\
& \cos[a_2]\cos[a_6]+(\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6]) \\
& +\cos[a_5](-\cos[a_3]\sin[a_1]+\cos[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]) \\
& -96\cosh[\tfrac{b_2}{2}]\sinh[b_1]\cos[a_4]\cos[a_5]\cosh[\tfrac{b_2}{2}](\cos[a_3]\sin[a_1] \\
& -\cos[a_1]\sin[a_2]\sin[a_3])(\cos[a_6](\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\times \\
& \sin[a_4]+(\cos[a_5]\sin[a_1]\sin[a_3]+\cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2]-\cos[a_2]\times \\
& \sin[a_4]\sin[a_5]))\sin[a_6])-192\cosh[\tfrac{b_1}{2}]^2\cosh[\tfrac{b_2}{2}]\cos[a_4](\cos[a_1]\cos[a_2]\times \\
& \cos[a_6]+(\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])\times \\
& (\cos[a_6](-\cos[a_3]\sin[a_1]+\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_4] \\
& -(\cos[a_5]\sin[a_1]\sin[a_3]+\cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2] \\
& -192\sinh[\tfrac{b_1}{2}]^2\sinh[\tfrac{b_2}{2}]\cos[a_4]\cos[a_5]\cosh[\tfrac{b_2}{2}](\cos[a_3]\sin[a_1] \\
& -\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_2}{2}]-\cos[a_1]\sin[a_2]\sin[a_3])(\sin[a_1]\times \\
& \sin[a_3]\sin[a_5]+\cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4]+\cos[a_3]\sin[a_2]\sin[a_5])) \\
& +96\sinh[b_1]\sinh[\tfrac{b_2}{2}]\cos[a_4](\sin[a_1]\sin[a_3]\sin[a_5]+\cos[a_1]\times \\
& (\cos[a_2]\cos[a_5]\sin[a_4]+\cos[a_3]\sin[a_2]\sin[a_5]))(\cos[a_1]\cos[a_2]\cos[a_6] \\
& +(\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])\sinh[\tfrac{b_2}{2}] \\
& +192\cos[a_4](\cosh[\tfrac{b_2}{2}](\cos[a_2]\cos[a_5]\sin[a_1]\sin[a_4]+(\cos[a_3]\sin[a_1]\sin[a_2] \\
& -\cos[a_1]\sin[a_3])\sin[a_5])\sinh[\tfrac{b_1}{2}]+\cosh[\tfrac{b_1}{2}](-\cos[a_6](\cos[a_1]\cos[a_3] \\
& +\sin[a_1]\sin[a_2]\sin[a_3])\sin[a_4]+(\cos[a_5](\cos[a_3]\sin[a_1]\sin[a_2] \\
& -\cos[a_1]\sin[a_3])-\cos[a_2]\sin[a_1]\sin[a_4]\sin[a_5])\sin[a_6])\sinh[\tfrac{b_2}{2}])\times \\
& (\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\cos[a_2]\cos[a_6]\sin[a_1]-(\cos[a_1]\cos[a_3] \\
& +\sin[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])+\cos[a_5](\cos[a_1]\cos[a_3]
\end{aligned}
$$

$$+\sin[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]) + 192\cos[a_4](\cos[a_5]\cosh[\tfrac{b_2}{2}]\times$$
$$(\cos[a_1]\cos[a_3] + \sin[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_1}{2}] + \cosh[\tfrac{b_1}{2}]\times$$
$$(\cos[a_2]\cos[a_6]\sin[a_1] - (\cos[a_1]\cos[a_3] + \sin[a_1]\sin[a_2]\times$$
$$\sin[a_3])\sin[a_5]\sin[a_6])\sinh[\tfrac{b_2}{2}])(-\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\cos[a_6](\cos[a_1]\cos[a_3]$$
$$+\sin[a_1]\sin[a_2]\sin[a_3])\sin[a_4] + (-\cos[a_3]\cos[a_5]\sin[a_1]\sin[a_2]$$
$$+\cos[a_1]\cos[a_5]\sin[a_3] + \cos[a_2]\sin[a_1]\sin[a_4]\sin[a_5])\sin[a_6])$$
$$+(\cos[a_2]\cos[a_5]\sin[a_1]\sin[a_4] + (\cos[a_3]\sin[a_1]\sin[a_2]$$
$$-\cos[a_1]\sin[a_3])\sin[a_5])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}])$$
$$+96(\sinh[\tfrac{b_1}{2}]\cos[a_2]\cos[a_5]\cosh[\tfrac{b_2}{2}]\sin[a_3] - \cosh[\tfrac{b_1}{2}](\cos[a_6\sin[a_2]$$
$$+\cos[a_2]\sin[a_3]\sin[a_5]\sin[a_6])\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\sin[2a_4]\sin[a_2]\times$$
$$\sin[a_5]\sin[a_6] + \cos[a_2](-\sin[2a_4]\cos[a_6]\sin[a_3]$$
$$+2\cos[a_4]\cos[a_3]\cos[a_5]\sin[a_6])) + (-\sin[2a_4]\cos[a_5]\sin[a_2]$$
$$+2\cos[a_4]\cos[a_2]\cos[a_3]\sin[a_5])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]))$$
$$+\tfrac{1}{4}(4\csc[a_4]\sin[2a_4](\cosh[\tfrac{b_2}{2}](\cos[a_1]\cos[a_2]\cos[a_6]$$
$$+(\cos[a_3]\sin[a_1] - \cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])\sinh[\tfrac{b_1}{2}]$$
$$-\cos[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_2}{2}])(-\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\sin[a_1]\sin[a_3]\sin[a_5]$$
$$+\cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4] + \cos[a_3]\sin[a_2]\sin[a_5]))$$
$$-(\cos[a_6](-\cos[a_3]\sin[a_1] + \cos[a_1]\sin[a_2]\sin[a_3])\sin[a_4]$$
$$-(\cos[a_5]\sin[a_1]\sin[a_3] + \cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2]$$
$$-\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}])$$
$$+\cos[a_5]\cosh[\tfrac{b_1}{2}](\cos[a_3]\sin[a_1] - 4\csc[a_4]\sin[2a_4](\cosh[\tfrac{b_2}{2}](\cos[a_1]\cos[a_2]\times$$
$$\cos[a_6] + (\cos[a_3]\sin[a_1] - \cos[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])\sinh[\tfrac{b_1}{2}]$$
$$+\cos[a_5]\cosh[\tfrac{b_1}{2}](\cos[a_3]\sin[a_1] - \cos[a_1]\sin[a_2]\sin[a_3])\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\times$$
$$\cosh[\tfrac{b_2}{2}](\sin[a_1]\sin[a_3]\sin[a_5] + \cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4]$$
$$+\cos[a_3]\sin[a_2]\sin[a_5])) + (\cos[a_6](-\cos[a_3]\sin[a_1] + \cos[a_1]\sin[a_2]\times$$
$$\sin[a_3])\sin[a_4] - (\cos[a_5]\sin[a_1]\sin[a_3] + \cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2]$$
$$-\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}])$$
$$+(8\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}]\sin[2a_4]\cos[a_2]\cos[a_5]\sin[a_1]$$
$$+8\sin[2a_4]\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]\cos[a_1]\cos[a_3]\cos[a_6]$$
$$+8\sin[2a_4]\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]\cos[a_6]\sin[a_1]\sin[a_2]\sin[a_3]$$
$$+16\cos[a_4]\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}]\cos[a_3]\sin[a_1]\sin[a_2]\sin[a_5]$$
$$-16\cos[a_4]\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]\cos[a_3]\cos[a_5]\sin[a_1]\sin[a_2]\sin[a_6]$$
$$+8\sin[2a_4]\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]\cos[a_2]\sin[a_1]\sin[a_5]\sin[a_6]$$
$$-16\cos[a_4]\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}]\cos[a_1]\sin[a_3]\sin[a_5]$$
$$+16\cos[a_4]\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]\cos[a_1]\cos[a_5]\sin[a_3]\sin[a_6])(\cosh[\tfrac{b_2}{2}]\times$$
$$(-\cos[a_2]\cos[a_6]\sin[a_1] + (\cos[a_1]\cos[a_3] + \sin[a_1]\sin[a_2]\sin[a_3])\times$$
$$\sin[a_5]\sin[a_6])\sinh[\tfrac{b_1}{2}] + \cos[a_5]\cosh[\tfrac{b_1}{2}](\cos[a_1]\cos[a_3] + \sin[a_1]\times$$
$$\sin[a_2]\sin[a_3])\sinh[\tfrac{b_2}{2}]) + 16\cos[a_4](\cosh[\tfrac{b_2}{2}](\cos[a_6](\cos[a_1]\cos[a_3]$$
$$+\sin[a_1]\sin[a_2]\sin[a_3])\sin[a_4] + (-\cos[a_3]\cos[a_5]\sin[a_1]\sin[a_2]$$
$$+\cos[a_1]\cos[a_5]\sin[a_3] + \cos[a_2]\sin[a_1]\sin[a_4]\sin[a_5])\sin[a_6])\sinh[\tfrac{b_1}{2}]$$
$$+\cosh[\tfrac{b_1}{2}](\cos[a_2]\cos[a_5]\sin[a_1]\sin[a_4] + (\cos[a_3]\sin[a_1]\sin[a_2]$$

$$
\begin{aligned}
&-\cos[a_1]\sin[a_3])\sin[a_5])\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\cos[a_5]\cosh[\tfrac{b_2}{2}]\times\\
&(\cos[a_1]\cos[a_3]+\sin[a_1]\sin[a_2]\sin[a_3])-\sinh[\tfrac{b_1}{2}](\cos[a_2]\cos[a_6]\sin[a_1]\\
&-(\cos[a_1]\cos[a_3]+\sin[a_1]\sin[a_2]\sin[a_3])\sin[a_5]\sin[a_6])\sinh[\tfrac{b_2}{2}])\\
&-16\cos[a_4](\cosh[\tfrac{b_2}{2}](\cos[a_6](-\cos[a_3]\sin[a_1]+\cos[a_1]\sin[a_2]\sin[a_3])\times\\
&\sin[a_4]-(\cos[a_5]\sin[a_1]\sin[a_3]+\cos[a_1](\cos[a_3]\cos[a_5]\sin[a_2]\\
&-\cos[a_2]\sin[a_4]\sin[a_5]))\sin[a_6])\sinh[\tfrac{b_1}{2}]+\cosh[\tfrac{b_1}{2}](\sin[a_1]\sin[a_3]\sin[a_5]\\
&+\cos[a_1](\cos[a_2]\cos[a_5]\sin[a_4]+\cos[a_3]\sin[a_2]\sin[a_5]))\sinh[\tfrac{b_2}{2}])\times\\
&(\cos[a_5]\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\cos[a_3]\sin[a_1]-\cos[a_1]\sin[a_2]\sin[a_3])\\
&+(\cos[a_3]\sin[a_1]\sin[a_5]\sin[a_6]+\cos[a_1](\cos[a_2]\cos[a_6]\\
&-\sin[a_2]\sin[a_3]\sin[a_5]\sin[a_6]))\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}])+4(\cosh[\tfrac{b_2}{2}](\cos[a_6]\sin[a_2]\\
&+\cos[a_2]\sin[a_3]\sin[a_5]\sin[a_6])\sinh[\tfrac{b_1}{2}]+\cos[a_2]\cos[a_5]\cosh[\tfrac{b_1}{2}]\sin[a_3]\times\\
&\sinh[\tfrac{b_2}{2}])(\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](-2\sin[2a_4]\cos[a_5]\sin[a_2]\\
&+4\cos[a_4]\cos[a_2]\cos[a_3]\sin[a_5])-\sinh[\tfrac{b_1}{2}](2\sin[2a_4]\sin[a_2]\sin[a_5]\sin[a_6]\\
&+\cos[a_2](-2\sin[2a_4]\cos[a_6]\sin[a_3]+4\cos[a_4]\cos[a_3]\cos[a_5]\sin[a_6]))\times\\
&\sinh[\tfrac{b_2}{2}])-8(\sinh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}](\sin[2a_4]\sin[a_2]\sin[a_5]\sin[a_6]\\
&+\cos[a_2](-\sin[2a_4]\cos[a_6]\sin[a_3]+2\cos[a_4]\cos[a_3]\cos[a_5]\sin[a_6]))\\
&-\cosh[\tfrac{b_1}{2}](-\sin[2a_4]\cos[a_5]\sin[a_2]+2\cos[a_4]\cos[a_2]\cos[a_3]\sin[a_5])\times\\
&\sinh[\tfrac{b_2}{2}])(\cos[a_2]\cos[a_5]\cosh[\tfrac{b_1}{2}]\cosh[\tfrac{b_2}{2}]\sin[a_3]+(\cos[a_6]\sin[a_2]\\
&+\cos[a_2]\sin[a_3]\sin[a_5]\sin[a_6])\sinh[\tfrac{b_1}{2}]\sinh[\tfrac{b_2}{2}]))))^2)\Bigg]
\end{aligned}
\tag{113}
$$

E Scalar potential for $SO(5) \times SO(5)$ gauging in $SO(3)_{\mathbf{diag}}$ sector

$$
\begin{aligned}
V \;=\; & -4g^2\Big[4(1+\cosh[2b_1]\cosh[2b_2])^2 - \tfrac{1}{2}(-1+\cosh[2b_1]\cosh[2b_2])^2\times \\
& (1+\cosh[2b_1]\cosh[2b_2])(\cos[a_2]^2(\cos[2a_3]+\cos[2a_4])\cos[a_5]^2 \\
& -2\sin[a_2]^2\sin[a_5]^2+\sin[2a_2]\sin[a_3]\sin[a_4]\sin[2a_5]) \\
& +\tfrac{1}{64}(-1+\cosh[2b_1]\cosh[2b_2])^4(\cos[a_2]^2(\cos[2a_3] \\
& +\cos[2a_4])\cos[a_5]^2-2\sin[a_2]^2\sin[a_5]^2+\sin[2a_2]\sin[a_3]\times \\
& \sin[a_4]\sin[2a_5])^2-5(\tfrac{33}{100}(\sinh[2b_1]^2+\cosh[2b_1]^2\sinh[2b_2]^2) \\
& -\tfrac{1}{6400}(-1+\cosh[2b_1]\cosh[2b_2])(-41+23\cosh[4b_1] \\
& +2\cosh[2b_1]^2(8\cosh[4b_2]+\cosh[8b_2]))(\cos[2(a_2-a_4)] \\
& +\cos[2(a_2+a_4)]+8\cos[a_2]^2\cos[2a_3]\cos[a_5]^2 \\
& +\cos[2a_4](2+4\cos[a_2]^2\cos[2a_5])-16\sin[a_2]^2\sin[a_5]^2 \\
& +8\sin[2a_2]\sin[a_3]\sin[a_4]\sin[2a_5]) \\
& +\tfrac{1}{12800}(-1+\cosh[2b_1]\cosh[2b_2])^3(1+\cosh[2b_1]\cosh[2b_2])\times \\
& (\cos[2(a_2-a_4)]+\cos[2(a_2+a_4)]+8\cos[a_2]^2\cos[2a_3]\times \\
& \cos[a_5]^2+\cos[2a_4](2+4\cos[a_2]^2\cos[2a_5])-16\sin[a_2]^2\sin[a_5]^2 \\
& +8\sin[2a_2]\sin[a_3]\times\sin[a_4]\sin[2a_5])^2 \\
& +\tfrac{1}{800}(\cos[a_2]^2(5+\cos[2a_3]+\cos[2a_4]+(3+\cos[2a_3] \\
& +\cos[2a_4])\cos[2a_5])+8\cos[2a_5]\sin[a_2]^2-3(-3+\cos[2a_2])\times \\
& \sin[a_5]^2+4\cosh[2b_1]\cosh[2b_2](\cos[a_2]^2(-\cos[a_4]^2 \\
& +\cos[a_5]^2\sin[a_3]^2)+(\cos[a_4]^2+\sin[a_2]^2\sin[a_4]^2)\sin[a_5]^2) \\
& -2(-1+\cosh[2b_1]\cosh[2b_2])\sin[2a_2]\sin[a_3]\sin[a_4]\times \\
& \sin[2a_5])\sinh[2b_1]^2+\tfrac{1}{800}\sinh[2b_1]^2(\cos[a_1]^2\cos[a_2]^2\cos[a_4]^2\times \\
& (-1+\cosh[2b_1]\cosh[2b_2])+\cos[a_2]^2\cos[a_4]^2(-1+\cosh[2b_1]\times \\
& \cosh[2b_2])\sin[a_1]^2-\cos[a_4]^2(3+\cosh[2b_1]\cosh[2b_2])\sin[a_5]^2 \\
& -(3+\cosh[2b_1]\cosh[2b_2])(\cos[a_2]\cos[a_5]\sin[a_3] \\
& -\sin[a_2]\sin[a_4]\sin[a_5])^2-2(\cos[2a_5]\sin[a_2]^2 \\
& +\cos[a_2]^2(\cos[2a_3]\cos[a_5]^2-\cos[2a_4]\sin[a_5]^2) \\
& +\sin[2a_2]\sin[a_3]\sin[a_4]\sin[2a_5]))^2 \\
& +\tfrac{1}{3200}(-1+\cosh[2b_1]^2\cosh[4b_2])(28+\cos[2(a_2-a_3)] \\
& +\cos[2(a_2+a_3)]+8\cos[a_2]^2\cos[2a_4]\cos[a_5]^2+4\cos[2a_5] \\
& +\cos[2a_3](2+4\cos[a_2]^2\cos[2a_5])+8\cos[2a_2]\sin[a_5]^2 \\
& +16\cosh[2b_1]\cosh[2b_2](\cos[a_2]^2(-\cos[a_4]^2+\cos[a_5]^2\sin[a_3]^2) \\
& +(\cos[a_4]^2+\sin[a_2]^2\sin[a_4]^2)\sin[a_5]^2)-8(-1+\cosh[2b_1]\times \\
& \cosh[2b_2])\sin[2a_2]\sin[a_3]\sin[a_4]\sin[2a_5])+\tfrac{1}{204800}(-1+\cosh[2b_1]^2\times \\
& \cosh[4b_2])(28+\cos[2(a_2-a_3)]+\cos[2(a_2+a_3)]+8\cos[a_2]^2\times \\
& \cos[2a_4]\cos[a_5]^2+4\cos[2a_5]+\cos[2a_3](2+4\cos[a_2]^2\cos[2a_5]) \\
& +8\cos[2a_2]\sin[a_5]^2+16\cosh[2b_1]\cosh[2b_2](\cos[a_2]^2(-\cos[a_4]^2 \\
& +\cos[a_5]^2\sin[a_3]^2)+(\cos[a_4]^2+\sin[a_2]^2\sin[a_4]^2)\sin[a_5]^2) \\
& -8(-1+\cosh[2b_1]\cosh[2b_2])\sin[2a_2]\sin[a_3]\sin[a_4]\sin[2a_5])^2
\end{aligned}
$$

$$+\tfrac{1}{400}(\sinh[2b_1]^2+\cosh[2b_1]^2\sinh[2b_2]^2)(4\cos[a_2]^2\cos[a_4]^2$$
$$+\cos[a_4]^2(3+\cosh[2b_1]\cosh[2b_2])\sin[a_2]^2+(3+\cosh[2b_1]\times$$
$$\cosh[2b_2])\sin[a_4]^2-(-1+\cosh[2b_1]\cosh[2b_2])(\cos[a_3]\times$$
$$\cos[a_5]\sin[a_1]-\cos[a_1](\cos[a_5]\sin[a_2]\sin[a_3]+\cos[a_2]\sin[a_4]\times$$
$$\sin[a_5]))^2-(-1+\cosh[2b_1]\cosh[2b_2])(\cos[a_1]\cos[a_3]\cos[a_5]$$
$$+\sin[a_1](\cos[a_5]\sin[a_2]\sin[a_3]+\cos[a_2]\sin[a_4]\sin[a_5]))^2)^2$$
$$+\tfrac{1}{20}\cos[a_2]^2\cos[a_4]^2(-1+\cosh[2b_1]\cosh[2b_2])^3(\cos[a_3]^2\times$$
$$\sin[a_2]^2+(\cos[a_2]\cos[a_5]\sin[a_4]-\sin[a_2]\sin[a_3]\sin[a_5])^2)$$
$$+\tfrac{3}{400}(-1+\cosh[2b_1]\cosh[2b_2])^2(-\cos[a_5]^2\sin[2a_2]\sin[a_3]\times$$
$$\sin[a_4](\cosh[b_2]\sinh[b_1]\cos[a_6]-\cosh[b_1]\sinh[b_2]\sin[a_6])$$
$$+(\sin[2a_2]\sin[a_3]\sin[a_4]\sin[a_5]^2+\sin[a_2]^2\sin[2a_5])\times$$
$$(\cosh[b_2]\sinh[b_1]\cos[a_6]-\cosh[b_1]\sinh[b_2]\sin[a_6])$$
$$-\cos[a_3]\sin[2a_2]\sin[a_4]\sin[a_5](\cosh[b_1]\sinh[b_2]\cos[a_6]$$
$$+\cosh[b_2]\sinh[b_1]\sin[a_6])+\cos[a_2]^2\cos[a_5]\times$$
$$(\cos[a_6](\cosh[b_1]\sinh[b_2]\sin[2a_3]+\cosh[b_2]\sinh[b_1](\cos[2a_3]$$
$$+\cos[2a_4])\sin[a_5])+(\cosh[b_2]\sinh[b_1]\sin[2a_3]$$
$$-\cosh[b_1]\sinh[b_2](\cos[2a_3]+\cos[2a_4])\sin[a_5])\sin[a_6]))^2$$
$$+\tfrac{3}{6400}(-1+\cosh[2b_1]\cosh[2b_2])^2(\cos[a_3]\sin[2a_2]\sin[a_4]\times$$
$$\sin[a_5](4\cosh[b_2]\sinh[b_1]\cos[a_6]-4\cosh[b_1]\sinh[b_2]\sin[a_6])$$
$$-\cos[a_5]^2\sin[2a_2]\sin[a_3]\sin[a_4](4\cosh[b_1]\sinh[b_2]\cos[a_6]$$
$$+4\cosh[b_2]\sinh[b_1]\sin[a_6])+(\sin[2a_2]\sin[a_3]\sin[a_4]\sin[a_5]^2$$
$$+\sin[a_2]^2\sin[2a_5])(4\cosh[b_1]\sinh[b_2]\cos[a_6]+4\cosh[b_2]\times$$
$$\sinh[b_1]\sin[a_6])+\cos[a_2]^2\cos[a_5](\cos[a_6](-4\cosh[b_2]\times$$
$$\sinh[b_1]\sin[2a_3]+4\cosh[b_1]\sinh[b_2](\cos[2a_3]+\cos[2a_4])\times$$
$$\sin[a_5])+(4\cosh[b_1]\sinh[b_2]\sin[2a_3]+4\cosh[b_2]\sinh[b_1]\times$$
$$(\cos[2a_3]+\cos[2a_4])\sin[a_5])\sin[a_6]))^2$$
$$+\tfrac{3}{400}(-1+\cosh[2b_1]\cosh[2b_2])^2(\cosh[b_2](2\cos[a_2]\cos[a_3]\times$$
$$\cos[a_6](\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])$$
$$+(\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]-2\cos[a_5](\cos[a_3]^2$$
$$+\sin[a_2]^2\sin[a_3]^2-\cos[a_2]^2\sin[a_4]^2)\sin[a_5])\sin[a_6])\sinh[b_1]$$
$$+\cosh[b_1](\cos[a_6](\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]$$
$$-2\cos[a_5](\cos[a_3]^2+\sin[a_2]^2\sin[a_3]^2-\cos[a_2]^2\sin[a_4]^2)\times$$
$$\sin[a_5])+2\cos[a_2]\cos[a_3](-\cos[a_2]\cos[a_5]\sin[a_3]$$
$$+\sin[a_2]\sin[a_4]\sin[a_5])\sin[a_6])\sinh[b_2])^2$$
$$+\tfrac{1}{50}(-1+\cosh[2b_1]\cosh[2b_2])^2(\cos[a_4]^2\cos[a_5]\sin[a_5]\times$$
$$(-\cos[a_6]\cosh[b_2]\sinh[b_1]+\cosh[b_1]\sin[a_6]\sinh[b_2])$$
$$-(\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])(-\cosh[b_2]\times$$
$$(\cos[a_5]\cos[a_6]\sin[a_2]\sin[a_4]+\cos[a_2]\times$$
$$(\cos[a_6]\sin[a_3]\sin[a_5]-\cos[a_3]\sin[a_6]))\sinh[b_1]$$
$$+\cosh[b_1](\cos[a_5]\sin[a_2]\sin[a_4]\sin[a_6]+\cos[a_2]\times$$

$$
\begin{aligned}
&(\cos[a_3]\cos[a_6]+\sin[a_3]\sin[a_5]\sin[a_6]))\sinh[b_2]))^2\\
&+\tfrac{1}{100}(-1+\cosh[2b_1]\cosh[2b_2])^2((\cos[a_2]^2\cos[a_3]\cos[a_4]^2\times\\
&\cos[a_5](\cosh[b_2](\cos[a_3]\cos[a_6]\sin[a_5]+\sin[a_3]\sin[a_6])\times\\
&\sinh[b_1]+\cosh[b_1](\cos[a_6]\sin[a_3]-\cos[a_3]\sin[a_5]\sin[a_6])\times\\
&\sinh[b_2])-(\cos[a_2]\cos[a_5]\sin[a_3]\sin[a_4]-\sin[a_2]\sin[a_5])\times\\
&(\cosh[b_2](\cos[a_5]\cos[a_6]\sin[a_2]+\cos[a_2]\sin[a_4]\times\\
&(\cos[a_6]\sin[a_3]\sin[a_5]-\cos[a_3]\sin[a_6]))\sinh[b_1]\\
&-\cosh[b_1](\cos[a_5]\sin[a_2]\sin[a_6]+\cos[a_2]\sin[a_4]\times\\
&(\cos[a_3]\cos[a_6]+\sin[a_3]\sin[a_5]\sin[a_6]))\sinh[b_2])))^2\\
&+\tfrac{3}{400}(-1+\cosh[2b_1]\cosh[2b_2])^2(\cosh[b_2](\cos[a_6]\times\\
&(-\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]+2\cos[a_5](\cos[a_3]^2\\
&+\sin[a_2]^2\sin[a_3]^2-\cos[a_2]^2\sin[a_4]^2)\sin[a_5])+2\cos[a_2]\times\\
&\cos[a_3](\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])\times\\
&\sin[a_6])\sinh[b_1]+\cosh[b_1](2\cos[a_2]\cos[a_3]\cos[a_6]\times\\
&(\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])\\
&+(\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]-2\cos[a_5](\cos[a_3]^2\\
&+\sin[a_2]^2\sin[a_3]^2-\cos[a_2]^2\sin[a_4]^2)\sin[a_5])\sin[a_6])\sinh[b_2])^2\\
&+\tfrac{3}{400}(-1+\cosh[2b_1]\cosh[2b_2])^2(\cos[a_4]^2\cos[a_5]\sin[a_5]\times\\
&(\cosh[b_1]\sinh[b_2]\cos[a_6]+\cosh[b_2]\sinh[b_1]\sin[a_6])\\
&-(\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])(\cos[a_5]\times\\
&\sin[a_2]\sin[a_4](\cosh[b_1]\sinh[b_2]\cos[a_6]+\cosh[b_2]\times\\
&\sinh[b_1]\sin[a_6])+\cos[a_2](\cos[a_3](\cosh[b_2]\sinh[b_1]\times\\
&\cos[a_6]-\cosh[b_1]\sinh[b_2]\sin[a_6])+\sin[a_3]\sin[a_5]\times\\
&(\cosh[b_1]\sinh[b_2]\cos[a_6]+\cosh[b_2]\sinh[b_1]\sin[a_6])))))^2\\
&+\tfrac{1}{80}(-1+\cosh[2b_1]\cosh[2b_2])^2(-\cos[a_4]^2\cos[a_5]\sin[a_5]\times\\
&(\cosh[b_2]\sin[a_6]\sinh[b_1]+\cos[a_6]\cosh[b_1]\sinh[b_2])\\
&+(\cos[a_2]\cos[a_5]\sin[a_3]-\sin[a_2]\sin[a_4]\sin[a_5])(\cosh[b_2]\times\\
&(\cos[a_5]\sin[a_2]\sin[a_4]\sin[a_6]+\cos[a_2](\cos[a_3]\cos[a_6]\\
&+\sin[a_3]\sin[a_5]\sin[a_6]))\sinh[b_1]+\cosh[b_1](\cos[a_5]\cos[a_6]\sin[a_2]\sin[a_4]\\
&+\cos[a_2](\cos[a_6]\sin[a_3]\sin[a_5]-\cos[a_3]\sin[a_6]))\sinh[b_2]))^2\\
&+\tfrac{1}{100}(-1+\cosh[2b_1]\cosh[2b_2])^2((\cos[a_2]^2\cos[a_3]\cos[a_4]^2\cos[a_5]\times\\
&(\cosh[b_2](-\cos[a_6]\sin[a_3]+\cos[a_3]\sin[a_5]\sin[a_6])\sinh[b_1]\\
&+\cosh[b_1](\cos[a_3]\cos[a_6]\sin[a_5]+\sin[a_3]\sin[a_6])\sinh[b_2])\\
&-(\cos[a_2]\cos[a_5]\sin[a_3]\sin[a_4]-\sin[a_2]\sin[a_5])(\cosh[b_2]\times\\
&(\cos[a_5]\sin[a_2]\sin[a_6]+\cos[a_2]\sin[a_4](\cos[a_3]\cos[a_6]\\
&+\sin[a_3]\sin[a_5]\sin[a_6]))\sinh[b_1]+\cosh[b_1](\cos[a_5]\cos[a_6]\sin[a_2]\\
&+\cos[a_2]\sin[a_4](\cos[a_6]\sin[a_3]\sin[a_5]-\cos[a_3]\sin[a_6]))\sinh[b_2])))^2\\
&+\tfrac{1}{1600}(-1+\cosh[2b_1]\cosh[2b_2])^2((\cosh[b_2](\cos[a_6](-2\cos[2a_5]\times\\
&\sin[2a_2]\sin[a_3]\sin[a_4]+(1-\cos[2a_2]+\cos[a_2]^2(\cos[2a_3]\\
&+\cos[2a_4]))\sin[2a_5])+2(\cos[a_2]^2\cos[a_5]\sin[2a_3]
\end{aligned}
$$

$$
\begin{aligned}
&-\cos[a_3]\sin[2a_2]\sin[a_4]\sin[a_5])\sin[a_6])\sinh[b_1]+\cosh[b_1]\times\\
&(2\cos[a_6](\cos[a_2]^2\cos[a_5]\sin[2a_3]-\cos[a_3]\sin[2a_2]\sin[a_4]\times\\
&\sin[a_5])+(2\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]-(\cos[a_2]^2\times\\
&(\cos[2a_3]+\cos[2a_4])+2\sin[a_2]^2)\sin[2a_5])\sin[a_6])\sinh[b_2]))^2\\
&+\frac{1}{1600}(-1+\cosh[2b_1]\cosh[2b_2])^2((\cosh[b_2](2\cos[a_6]\times\\
&(-\cos[a_2]^2\cos[a_5]\sin[2a_3]+\cos[a_3]\sin[2a_2]\sin[a_4]\sin[a_5])\\
&+(-2\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]+(\cos[a_2]^2(\cos[2a_3]\\
&+\cos[2a_4])+2\sin[a_2]^2)\sin[2a_5])\sin[a_6])\sinh[b_1]+\cosh[b_1]\times\\
&(\cos[a_6](-2\cos[2a_5]\sin[2a_2]\sin[a_3]\sin[a_4]+(1-\cos[2a_2]\\
&+\cos[a_2]^2(\cos[2a_3]+\cos[2a_4]))\sin[2a_5])+2(\cos[a_2]^2\times\\
&\cos[a_5]\sin[2a_3]-\cos[a_3]\sin[2a_2]\sin[a_4]\sin[a_5])\sin[a_6])\sinh[b_2]))^2)\Bigg]
\end{aligned}
\tag{114}
$$

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