$D = 3$ $\mathcal{N} = 6$ superconformal symmetry of $AdS_4 \times \mathbb{CP}^3$ superstring

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Abstract

Invariance of the $AdS_4 \times \mathbb{CP}^3$ superstring under $D = 3$ $\mathcal{N} = 6$ superconformal symmetry is discussed in the sector described by the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset sigma-model action presented in the conformal basis for the $osp(4|6)/(so(1,3) \times u(3))$ Cartan forms. Transformation rules under $D = 3$ $\mathcal{N} = 6$ superconformal symmetry for the $(10|24)-$dimensional 'reduced' $AdS_4 \times \mathbb{CP}^3$ superspace coordinates are obtained and used to derive corresponding world-sheet currents.

1 Introduction

The first explicit example of the gauge/string duality [1], [2], [3] allowed to probe analytically previously unaccessible nonperturbative regime of $\mathcal{N} = 4$ super-Yang-Mills theory via the IIB superstring on $AdS_5 \times S^5$ background. It is in a sense the simplest instance of the AdS/CFT correspondence due to the maximal supersymmetry described by $PSU(2,2|4)$ supergroup both of the $AdS_5 \times S^5$ superbackground and $D = 4$ SCFT on the boundary of AdS_5 space. Another higher supersymmetric explicit example of the AdS/CFT correspondence that was put forward not long ago by Aharony, Bergman, Jafferis and Maldacena (ABJM) [4] provides description of the SCFT in the space-time of one lower dimension in terms of $M-$theory on $AdS_4 \times (S^7/\mathbb{Z}_k)$ background. In spite of the fact that lower dimensional theories basically have simpler dynamics compared to $4d$ ones the ABJM correspondence appears to be harder to verify since on both sides of the duality the isometry supergroup $OSp(4|6)$ isomorphic to $D = 3$ $\mathcal{N} = 6$ superconformal symmetry is lower than the maximally allowed one. Difficulties manifest itself already at the level of constructing the classical action for IIA superstring on $AdS_4 \times \mathbb{CP}^3$ superbackground that describes 't Hooft limit of the $D = 3$ SCFT proposed by ABJM [4]. Group-theoretic supercoset approach [5], [6], [7], [8] elaborated to describe IIB superstring on $AdS_5 \times S^5$ background when applied to the $AdS_4 \times \mathbb{CP}^3$ superstring gives only partial answer [9], [10] because $AdS_4 \times \mathbb{CP}^3$ background preserves only 3/4 space-time supersymmetries. The $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset action [9], [10] corresponds to fixing part of the gauge freedom related to $\kappa-$symmetry of the complete action [11] that is obtained via the double dimensional reduction [12] of the $D = 11$ supermembrane action on maximally supersymmetric $AdS_4 \times S^7$ background [13] due to the Hopf fibration realization of the 7-sphere $S^7 = \mathbb{CP}^3 \times U(1)$ [14], [15]. Although $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset sigma-model fails to describe all possible $AdS_4\times\mathbb{CP}^3$ superstring configurations [9], [11] it has clear group-theoretical structure and is classically integrable allowing one to utilize for its investigation many of the results obtained for the "elder brother" example of AdS_5/CFT_4 correspondence relying on the intregrable structures exhibited there [16], [17] (for the collection of recent reviews see, e.g., [18])[2].

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[2]The issue of seeking possible integrable structures for the $AdS_4 \times \mathbb{CP}^3$ superstring beyond the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model has been recently addressed in Ref.[19].

In Ref. [20] we have found explicit form of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model action in the conformal basis for $osp(4|6)$ Cartan forms based on the supercoset representative parametrized by Poincare coordinates for AdS_4 space with 24 fermionic coordinates split into two sets of 12 related to Poincare and conformal supersymmetries from the AdS boundary superspace perspective[3]. Such choice of the supercoset representative allows to formulate the stringy side of the duality in terms of the variables that contain those parametrizing $D = 3$ $\mathcal{N} = 6$ boundary superspace, where the ABJM theory could be formulated off-shell [25]-[27] aiming at getting new insights into the relation between both theories.

The goal of this paper is to establish transformation properties under $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset coordinates introduced in Ref. [20], as well as to derive Noether currents associated with the $D = 3$ $\mathcal{N} = 6$ superconformal invariance of the $OSp(4|6)/(SO(1,3) \times U(3))$ superstring action. Similar problem of deriving $D = 4$ $\mathcal{N} = 4$ superconformal transformations for the $AdS_5 \times S^5$ superspace coordinates relevant to the AdS_5/CFT_4 correspondence was addressed in [28], [29]. We start with reviewing the $AdS_4 \times \mathbb{CP}^3$ superstring action in conformal basis, then examine the action of left $D = 3$ $\mathcal{N} = 6$ superconformal transformations on the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset element and proceed to the derivation of $osp(4|6)$ Cartan forms transformation rules and Noether current densities for each of the individual transformations from the $D = 3$ $\mathcal{N} = 6$ superconformal symmetry.

2 $OSp(4|6)/(SO(1,3)\times U(3))$ superstring in conformal basis

The sigma-model action on the $(10|24)-$dimensional $OSp(4|6)/(SO(1,3)\times U(3))$ superspace was found in [9], [10] following the general prescription [5], [6], [7], [8] for constructing the sigma-model-type actions on supercoset spaces that admit 4-dimensional automorphism $\mathbb{Z}_4$ of the underlying isometry superalgebra. It relies on identifying $(10|24)-$dimensional supervielbein components with the Cartan forms associated with the $osp(4|6)/(so(1,3) \times u(3))$ supercoset 10 bosonic and 24 fermionic generators. Resulting action is invariant under global $OSp(4|6)$ supersymmetry, as well as gauge $SO(1,3)\times U(3)$ and $\kappa-$symmetries, describes the requisite number of the physical degrees of freedom, has correct bosonic limit and is classically integrable.

In [20] we have considered the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset element

$$\mathscr{G} = e^{x^m P_m + \theta^\mu_a Q^a_\mu + \bar\theta^{\mu a}\bar Q_{\mu a}} e^{\eta_{\mu a}S^{\mu a} + \bar\eta^a_\mu \bar S^\mu_a} e^{z^a V_a{}^4 + \bar z_a V_4{}^a} e^{\varphi D} \quad (1)$$

parametrized by $D = 3$ $\mathcal{N} = 6$ super-Poincare coordinates $(x^m, \theta^\mu_a, \bar\theta^{\mu a})$, AdS_4 radial direction coordinate φ related to the boundary-space dilatations, 3 complex coordinates $(z^a, \bar z_a)$ of the $\mathbb{CP}^3$ manifold, and 12 fermionic coordinates $(\eta_{\mu a},\ \bar\eta^a_\mu)$ corresponding to $D = 3$ $\mathcal{N} = 6$ conformal supersymmetry. Associated current 1-form in the conformal basis can be cast into the form

$$\begin{aligned} \mathscr{C}(d) = \mathscr{G}^{-1}d\mathscr{G} = \ & G^{mn}(d)M_{mn} + \hat\omega^m(d)P_m + \hat c^m(d)K_m + \Delta(d)D \\ + \ & \Omega_a{}^b(d)V_b{}^a + \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4 + \Omega_4{}^4(d)V_4{}^4 \\ + \ & \hat\omega^\mu_a(d)Q^a_\mu + \hat{\bar\omega}^{\mu a}(d)\bar Q_{\mu a} + \hat\chi_{\mu a}(d)S^{\mu a} + \hat{\bar\chi}^a_\mu(d)\bar S^\mu_a \end{aligned} \quad (2)$$

[3]Previously such conformal-type parametrizations were used to examine the string/brane models involved into the higher-dimensional counterparts of AdS/CFT correspondence [21], [22], [23], [24].

or manifesting the $\mathbb{Z}_4-$grading

$$\mathscr{C}(d) = \mathscr{C}_0(d) + \mathscr{C}_2(d) + \mathscr{C}_1(d) + \mathscr{C}_3(d), \tag{3}$$

where

$$\begin{aligned} \mathscr{C}_0(d) = & \; G_{mn}(d)M^{mn} + \tfrac{1}{2}(\hat{\omega}^m(d) - \hat{c}^m(d))(P_m - K_m) + \Omega_a{}^b(d)V_b{}^a + \Omega_4{}^4(d)V_4{}^4, \\ \mathscr{C}_2(d) = & \; \tfrac{1}{2}(\hat{\omega}^m(d) + \hat{c}^m(d))(P_m + K_m) + \Omega_a{}^4(d)V_4{}^a + \Omega_4{}^a(d)V_a{}^4, \\ \mathscr{C}_1(d) = & \; \hat{\omega}_a^\mu(d)Q_\mu^a + \hat{\chi}_{\mu a}(d)S^{\mu a}, \quad \mathscr{C}_3(d) = \hat{\bar{\omega}}^{\mu a}(d)\bar{Q}_{\mu a} + \hat{\bar{\chi}}_\mu^a(d)\bar{S}_a^\mu. \end{aligned} \tag{4}$$

Then the $\mathbb{Z}_4-$invariant $OSp(4|6)/(SO(1,3)\times U(3))$ superstring action in the conformal basis for Cartan forms (2) acquires the form

$$\begin{aligned} S = & \; -\tfrac{1}{2}\int d^2\xi\sqrt{-g}g^{ij}\left[\tfrac{1}{4}(\hat{\omega}_i^m + \hat{c}_i^m)(\hat{\omega}_{mj} + \hat{c}_{mj}) + \Delta_i\Delta_j + \tfrac{1}{2}(\Omega_{ia}{}^4\Omega_{j4}{}^a + \Omega_{ja}{}^4\Omega_{i4}{}^a)\right] \\ - & \; \tfrac{1}{2}\varepsilon^{ij}\int d^2\xi\left(\hat{\omega}_{ia}^\mu\varepsilon_{\mu\nu}\hat{\bar{\omega}}_j^{\nu a} + \hat{\chi}_{i\mu a}\varepsilon^{\mu\nu}\hat{\bar{\chi}}_{j\nu}^a\right). \end{aligned} \tag{5}$$

The first two summands in the kinetic part of the action (5) include Cartan forms associated with the generators P_m of the space-time translations on the $D = 3$ Minkowski boundary of the AdS_4 space

$$\hat{\omega}^m(d) = e^{-2\varphi}\omega^m(d), \quad \omega^m(d) = dx^m - id\theta_a^\mu\sigma_{\mu\nu}^m\bar{\theta}^{\nu a} + i\theta_a^\mu\sigma_{\mu\nu}^m d\bar{\theta}^{\nu a}, \tag{6}$$

conformal boost generators K_m

$$\begin{aligned} \hat{c}^m(d) = & \; e^{2\varphi}c^m(d), \quad c^m(d) = -id\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}\bar{\eta}_\nu^a + i\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}d\bar{\eta}_\nu^a \\ + & \; 2\left[\eta_{\mu a}\tilde{\sigma}^{m\mu\nu}(d\bar{\theta}_\nu^a + \tfrac{1}{4}\bar{\zeta}_\nu^a(d)) - (d\theta_{\mu a} + \tfrac{1}{4}\zeta_{\mu a}(d))\tilde{\sigma}^{m\mu\nu}\bar{\eta}_\nu^a\right](\bar{\eta}\eta), \;\; \bar{\eta}\eta \equiv \bar{\eta}_\rho^b\eta_b^\rho, \end{aligned} \tag{7}$$

where

$$\zeta_a^\mu(d) = -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\eta_{\nu a} = -\tilde{\omega}^{\mu\nu}(d)\eta_{\nu a}, \quad \bar{\zeta}^{\mu a}(d) = -\tilde{\sigma}^{m\mu\nu}\omega_m(d)\bar{\eta}_\nu^a = -\tilde{\omega}^{\mu\nu}(d)\bar{\eta}_\nu^a, \tag{8}$$

and dilatations

$$\Delta(d) = d\varphi + i(d\theta_a^\mu\bar{\eta}_\mu^a + d\bar{\theta}^{\mu a}\eta_{\mu a}) \tag{9}$$

with the corresponding generator D. Note that the generators $(D, P_m + K_m)$ can be identified as the $so(2,3)/so(1,3)$ coset generators[4] and the corresponding Cartan forms represent the AdS part of the $(10|24)-$supervielbein bosonic components.

Supervielbein components in the directions tangent to the $\mathbb{CP}^3$ manifold are identified with the $su(4)/u(3)$ Cartan forms $(\Omega_a{}^4(d), \Omega_4{}^a(d))$ that are the off-diagonal components of the traceless Hermitean matrix of $su(4)$ Cartan forms

$$\Omega_A{}^B(d) = \begin{pmatrix} \Omega_a{}^b & \Omega_a{}^4 \\ \Omega_4{}^b & \Omega_4{}^4 \end{pmatrix}, \quad \Omega_4{}^4 = -\Omega_a{}^a. \tag{10}$$

Using the isomorphism $SU(4) \sim SO(6)$ it is possible to accommodate $su(4)$ Cartan forms into the 6×6 matrix

$$\Omega_{\hat{a}}{}^{\hat{b}}(d) = \begin{pmatrix} \Omega_{\mathbf{b}a}{}^b - \delta_a^b\Omega_{\mathbf{b}c}{}^c & \varepsilon_{acb}\Omega_{\mathbf{b}4}{}^c \\ -\varepsilon^{acb}\Omega_{\mathbf{b}c}{}^4 & -\Omega_{\mathbf{b}b}{}^a + \delta_b^a\Omega_{\mathbf{b}c}{}^c \end{pmatrix} \tag{11}$$

[4](Anti)commutation relations of the $D = 3$ $\mathcal{N} = 6$ superalgebra can be found in Ref.[20].

antisymmetric w.r.t the metric

$$H_{\hat{a}\hat{b}} = \begin{pmatrix} 0 & \delta_a^b \\ \delta_b^a & 0 \end{pmatrix} \tag{12}$$

following the decomposition of the $D = 6$ vector representation as $\mathbf{3} \oplus \bar{\mathbf{3}}$ of $SU(3)$ [5]. For the considered choice of the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset element $su(4)$ Cartan forms are given by the sum of two contributions

$$\Omega_{\hat{a}}{}^{\hat{b}}(d) = \Omega_{\mathbf{b}\hat{a}}{}^{\hat{b}}(d) + \Omega_{\mathbf{f}\hat{a}}{}^{\hat{b}}(d) \tag{13}$$

coming from bosons and fermions. Bosonic contribution

$$\Omega_{\mathbf{b}\hat{a}}{}^{\hat{b}}(d) = iT_{\hat{a}}{}^{\hat{c}} d\bar{T}_{\hat{c}}{}^{\hat{b}} \tag{14}$$

is the $su(4)$ Catran form matrix associated with the $SU(4)/U(3)$ coset element

$$T_{\hat{a}}{}^{\hat{b}} = \begin{pmatrix} T_a{}^b & T_{ab} \\ T^{ab} & T^b{}_a \end{pmatrix} = \exp \begin{pmatrix} 0 & i\varepsilon_{acb} z^c \\ -\varepsilon^{acb} \bar{z}_c & 0 \end{pmatrix}. \tag{15}$$

Explicit expressions for the purely bosonic part of $\mathbb{CP}^3$ vielbein can be found in [20]. Fermionic contribution to (13)

$$\Omega_{\mathbf{f}\hat{a}}{}^{\hat{b}}(d) = (T\Psi(d)\bar{T})_{\hat{a}}{}^{\hat{b}} \tag{16}$$

is obtained by the $T-$transformation of the matrix

$$\Psi_{\hat{a}}{}^{\hat{b}}(d) = 2(d\theta^{\mu}_{\hat{a}} \eta^{\hat{b}}_{\mu} - d\theta^{\mu\hat{b}} \eta_{\mu\hat{a}} - \eta_{\mu\hat{a}} \tilde{\omega}^{\mu\nu}(d) \eta^{\hat{b}}_{\nu}), \tag{17}$$

where the fermionic coordinates have been written as the $D = 6$ vectors in $\mathbf{3} \oplus \bar{\mathbf{3}}$ basis

$$\theta^{\mu}_{\hat{a}} = \begin{pmatrix} \theta^{\mu}_a \\ \bar{\theta}^{\mu a} \end{pmatrix}, \quad \eta_{\mu\hat{a}} = \begin{pmatrix} \eta_{\mu a} \\ \bar{\eta}^a_{\mu} \end{pmatrix} \tag{18}$$

and $\theta^{\mu\hat{a}} = H^{\hat{a}\hat{b}} \theta^{\mu}_{\hat{b}}$, $\eta^{\hat{a}}_{\mu} = H^{\hat{a}\hat{b}} \eta_{\mu\hat{b}}$. The $T-$transformed $\mathbf{3} \oplus \bar{\mathbf{3}}$ vectors will be endowed with hats $\hat{\theta}^{\mu}_{\hat{a}} = T_{\hat{a}}{}^{\hat{b}} \theta^{\mu}_{\hat{b}}$, $\hat{\theta}^{\mu\hat{a}} = H^{\hat{a}\hat{b}} \hat{\theta}^{\mu}_{\hat{b}}$ etc. Using that $H_{\hat{a}\hat{c}} (\bar{T}^T)^{\hat{c}}{}_{\hat{d}} H^{\hat{d}\hat{b}} = T_{\hat{a}}{}^{\hat{b}}$ for the chosen realization of the matrix T the fermionic part of $su(4)$ Cartan form matrix can be cast into the form

$$\Omega_{\mathbf{f}\hat{a}}{}^{\hat{b}}(d) = 2(\hat{d\theta}^{\mu}_{\hat{a}} \hat{\eta}^{\hat{b}}_{\mu} - \hat{d\theta}^{\mu\hat{b}} \hat{\eta}_{\mu\hat{a}} - \hat{\eta}_{\mu\hat{a}} \tilde{\omega}^{\mu\nu}(d) \hat{\eta}^{\hat{b}}_{\nu}). \tag{19}$$

The Wess-Zumino (WZ) term of the action (5) in the $\mathbf{3} \oplus \bar{\mathbf{3}}$ basis can be written in the form

$$S_{WZ} = -\tfrac{i}{8} \varepsilon^{ij} \mathfrak{J}_{\hat{a}}{}^{\hat{b}} \int d^2\xi \left(\hat{\omega}^{\mu\hat{a}}_i \varepsilon_{\mu\nu} \hat{\omega}^{\nu}_{j\hat{b}} + \hat{\chi}^{\hat{a}}_{i\mu} \varepsilon^{\mu\nu} \hat{\chi}_{j\nu\hat{b}} \right), \tag{20}$$

[5] The metric $H_{\hat{a}\hat{b}}$ is the conventional unit $D = 6$ metric δ^{IJ} written in the $\mathbf{3} \oplus \bar{\mathbf{3}}$ basis. Both bases are connected by the transformation matrices

$$M^{I\hat{a}} = \frac{1}{2} (\tilde{\rho}^{Ia4}, \ \rho^I_{a4}), \quad M^{-1}{}_{\hat{a}I} = \begin{pmatrix} \rho^I_{4a} \\ \tilde{\rho}^{I4a} \end{pmatrix}: \quad MM^{-1} = I,$$

where ρ^I_{AB} and $\tilde{\rho}^{IAB}$ are $D = 6$ chiral $\gamma-$matrices, such that the components of a $D = 6$ vector O^I in these bases can be transformed into one another $O^I = M^{I\hat{a}} O_{\hat{a}}$ and $O_{\hat{a}} = M^{-1}{}_{\hat{a}I} O^I$. In particular, for the $D = 6$ metric we find that $\delta^{IJ} = -2M^{I\hat{a}} H_{\hat{a}\hat{b}} M^{J\hat{b}}$.

where $\mathfrak{J}_{\hat{a}}{}^{\hat{b}}$ is the $\mathbb{CP}^3$ Kahler form written in the $\mathbf{3}\oplus\bar{\mathbf{3}}$ basis[6]

$$\mathfrak{J}_{\hat{a}}{}^{\hat{b}} = 2i\left(\begin{array}{cc} \delta_a^b & 0 \\ 0 & -\delta_b^a \end{array}\right). \tag{21}$$

It contains the world-sheet projections of fermionic 1-forms

$$\hat{\omega}^{\mu}_{\hat{a}} = e^{-\varphi}T_{\hat{a}}{}^{\hat{b}}\omega^{\mu}_{\hat{b}}, \quad \omega^{\mu}_{\hat{b}}(d) = d\theta^{\mu}_{\hat{b}} + \zeta^{\mu}_{\hat{b}}(d) \tag{22}$$

related to Poincare supersymmetry generators $(Q^a_\mu, \bar{Q}_{\mu a})$, and

$$\hat{\chi}_{\mu\hat{a}} = e^{\varphi}T_{\hat{a}}{}^{\hat{b}}\chi_{\mu\hat{b}}, \quad \chi_{\mu\hat{a}}(d) = d\eta_{\mu\hat{a}} + 2i\eta^{\hat{b}}_{\mu}d\theta^{\nu}_{\hat{b}}\eta_{\nu\hat{a}} - i(\bar{\eta}\eta)(d\theta_{\mu\hat{a}} + \zeta_{\mu\hat{a}}(d)) \tag{23}$$

related to conformal supersymmetry generators $(S^{\mu a}, \bar{S}^{\mu}_{a})$.

3 $D=3$ $\mathcal{N}=6$ superconformal symmetry of the $OSp(4|6)/(SO(1,3)\times U(3))$ superstring: general properties and coordinate transformations

Global $OSp(4|6)$ transformations act on the $OSp(4|6)/(SO(1,3)\times U(3))$ coset representative from which the left-invariant Cartan forms (2) are constructed in the following way[7]

$$\mathscr{G}'H = G\mathscr{G}, \ G \in OSp(4|6) \tag{24}$$

with H being the compensating $SO(1,3)\times U(3)$ transformation or passing to infinitesimal transformations

$$\delta\mathscr{G} = g\mathscr{G} - \mathscr{G}h, \ g \in osp(4|6), \ h \in so(1,3)\oplus u(3). \tag{25}$$

Substituting above relations into (2) yields

$$\mathscr{C}(\delta) = \mathscr{G}^{-1}\delta\mathscr{G} = \mathscr{G}^{-1}g\mathscr{G} - h. \tag{26}$$

Consider the $osp(4|6)$ superalgebra valued transformation parameter in the conformal basis

$$\begin{array}{rl} g = & a^m P_m + b_m K^m + fD + \frac{1}{2}l^{mn}M_{mn} + y^a V_a{}^4 + \bar{y}_a V_4{}^a + w_a{}^b V_b{}^a + w_a{}^a V_b{}^b \\ + & \varepsilon^{\mu}_a Q^a_\mu + \bar{\varepsilon}^{\mu a}\bar{Q}_{\mu a} + \xi_{\mu a}S^{\mu a} + \bar{\xi}^a_\mu \bar{S}^\mu_a. \end{array} \tag{27}$$

It includes the parameters of $D=3$ Minkowski space-time translations a^m, conformal boosts b_m, dilatations f and Lorentz rotations l^{mn}, as well as anticommuting parameters of $D=3$ $\mathcal{N}=6$ Poincare supersymmetry $(\varepsilon^\mu_a, \bar{\varepsilon}^{\mu a})$ and conformal supersymmetry $(\xi_{\mu a}, \bar{\xi}^a_\mu)$

[6]In conventional $D=6$ vector basis it is given by the expression $J^{IJ} = \frac{i}{2}(\rho^I_{4a}\tilde{\rho}^{J4a} - \rho^J_{4a}\tilde{\rho}^{I4a})$. It takes simple diagonal form when contracted with the $6D$ rotation generators

$$J_A{}^B = J^{IJ}\rho^{IJ}{}_A{}^B = \left(\begin{array}{cc} -2i & 0 \\ 0 & 6i \end{array}\right).$$

The matrix $J_A{}^B$ can be shown to satisfy the following equation $J_A{}^C J_C{}^B - 4iJ_A{}^B + 12\delta^B_A = 0$.

[7]Since the supercoset string action is built out of the Cartan forms it is exactly invariant under the global symmetry in distinction with the original Green-Schwarz action [30] that is quasi-invariant because its WZ term cannot be presented as a 2-form in supercurrents. For detailed discussion on that point and the properties of WZ term on AdS backgrounds see, e.g., [31].

supplemented by $SU(4)$ $R-$symmetry parameters $(w_a{}^b, y^a, \bar{y}_a)$. Then the substitution of $OSp(4|6)/(SO(1,3)\times U(3))$ coset representative (1) into (26) yields

$$\begin{aligned}\mathscr{C}(\delta) = &\ (\hat{\omega}^m(\delta) - \hat{b}^m)P_m + (\hat{c}^m(\delta) + \hat{b}^m)K_m + \Delta(\delta)D + \tfrac{1}{2}(G^{mn}(\delta) + \hat{l}^{mn})M_{mn} \\ + &\ \Omega_4{}^a(\delta)V_a{}^4 + \Omega_a{}^4(\delta)V_4{}^a + (\Omega_a{}^b(\delta) + \hat{w}_a{}^b)V_b{}^a + (\Omega_a{}^a(\delta) + \hat{w}_a{}^a)V_b{}^b \\ + &\ \hat{\omega}^\mu_a(\delta)Q^a_\mu + \hat{\bar{\omega}}^{\mu a}(\delta)Q_{\mu a} + \hat{\chi}_{\mu a}(\delta)S^{\mu a} + \hat{\bar{\chi}}^a_\mu(\delta)\bar{S}^\mu_a.\end{aligned} \tag{28}$$

The quantities that cannot be accommodated into individual Cartan form variations like e.g., $\hat{\omega}^m(\delta) = i_\delta\hat{\omega}^m(d)$ represent parameters of the compensating transformations. In particular, vector parameter

$$\hat{b}^m = e^{2\varphi}A^{-1}b^m(\theta), \quad A = 1 - e^{4\varphi}(\bar{\eta}\eta)^2 \tag{29}$$

where

$$b^m(\theta) = b^m - i(\xi_a(\theta)\tilde{\sigma}^m\bar{\eta}^a + \bar{\xi}^a(\theta)\tilde{\sigma}^m\eta_a), \quad \xi_{\mu a}(\theta) = \xi_{\mu a} + b_{\mu\nu}\theta^\nu_a, \tag{30}$$

describes $SO(1,3)/SO(1,2)$ transformations, while the tensor one

$$\hat{l}^{mn} = l^{mn}(\theta) + ie^{2\varphi}(\eta_a\hat{\tilde{b}}\sigma^{mn}\bar{\eta}^a + \bar{\eta}^a\hat{\tilde{b}}\sigma^{mn}\eta_a) \tag{31}$$

with

$$l^{mn}(\theta) = l^{mn} + 2(b^m x^n - b^n x^m) + 2i(\bar{\theta}^a\sigma^{mn}\xi_a + \theta_a\sigma^{mn}\bar{\xi}^a) + i(\bar{\theta}^a\sigma^{mn}b\theta^a + \theta_a\sigma^{mn}b\bar{\theta}^a) \tag{32}$$

$SO(1,2)$ Lorentz rotations. Parameters of the compensating $U(3)$ rotations

$$\hat{w}_a{}^b = \widetilde{w}_a{}^b + \tfrac{i(1-\cos|z|)}{|z|\sin|z|}(\bar{z}_a\widetilde{y}^b - \widetilde{\bar{y}}_a z^b) + \tfrac{i(1-\cos|z|)^2}{2|z|^3\cos|z|\sin|z|}((\widetilde{y}\bar{z}) - (z\widetilde{\bar{y}}))\bar{z}_a z^b \tag{33}$$

have been presented in the form exhibiting explicit dependence on the entries

$$\begin{aligned}\widetilde{w}_a{}^b &= w_a{}^b(\theta) - e^{2\varphi}(2\eta_a\hat{\tilde{b}}\bar{\eta}^b - \delta^b_a\eta_c\hat{\tilde{b}}\bar{\eta}^c), \\ \widetilde{y}^a &= y^a(\theta) + e^{2\varphi}\varepsilon^{abc}\eta_b\hat{\tilde{b}}\eta_c, \quad \widetilde{\bar{y}}_a = \bar{y}_a(\theta) - e^{2\varphi}\varepsilon_{abc}\bar{\eta}^b\hat{\tilde{b}}\bar{\eta}^c,\end{aligned} \tag{34}$$

where

$$\begin{aligned}w_a{}^b(\theta) &= w_a{}^b - 2(\xi_{\mu a}\bar{\theta}^{\mu b} + \theta^\mu_a\bar{\xi}^b_\mu) + \delta^b_a(\xi_{\mu c}\bar{\theta}^{\mu c} - \bar{\xi}^c_\mu\theta^\mu_c) - 2\theta_a b\bar{\theta}^b + \delta^b_a\theta_c b\bar{\theta}^c, \\ y^a(\theta) &= y^a + 2\varepsilon^{abc}\xi_{\mu b}\theta^\mu_c + \varepsilon^{abc}\theta_b b\theta_c, \quad \bar{y}_a(\theta) = \bar{y}_a - 2\varepsilon_{abc}\bar{\xi}^b_\mu\bar{\theta}^{\mu c} - \varepsilon_{abc}\bar{\theta}^b b\bar{\theta}^c,\end{aligned} \tag{35}$$

of the $su(4)$ matrix

$$\widetilde{W}_{\hat{a}}{}^{\hat{b}} = \begin{pmatrix} \widetilde{w}_a{}^b - \delta^b_a\widetilde{w}_c{}^c & \varepsilon_{acb}\widetilde{y}^c \\ -\varepsilon^{acb}\widetilde{\bar{y}}_c & -\widetilde{w}_b{}^a + \delta^a_b\widetilde{w}_c{}^c \end{pmatrix} \tag{36}$$

that, as will be shown below, enters the transformation laws (41) of the $SU(4)/U(3)$ coset element T (15) under the $D=3$ $\mathcal{N}=6$ superconformal symmetry.

$D=3$ $\mathcal{N}=6$ superconformal transformations of the $OSp(4|6)/(SO(1,3)\times U(3))$ superspace coordinates that parametrize (1) include also the contributions proportional to the parameters of the compensating transformations (29), (31), (33). $D=3$ $\mathcal{N}=6$ Boundary superspace coordinates obey the following transformation rules[8]

$$\begin{aligned}\delta x^m = &\ a^m + l^m{}_n x^n + 2f x^m + b^m(x^2 + (\bar{\theta}\theta)^2) - 2x^m b_n x^n \\ - &\ i(\varepsilon_a\sigma^m\bar{\theta}^a + \bar{\varepsilon}^a\sigma^m\theta_a) - i(\xi_a\hat{\tilde{x}}\sigma^m\bar{\theta}^a + \bar{\xi}^a\hat{\tilde{x}}\sigma^m\theta_a) \\ + &\ e^{2\varphi}(\hat{b}^m + i\eta_a\hat{\tilde{b}}\sigma^m\bar{\theta}^a - i\theta_a\sigma^m\hat{\tilde{b}}\bar{\eta}^a),\end{aligned} \tag{37}$$

[8]Observe vanishing of the terms proportional to $SO(1,3)/SO(1,2)$ rotation parameters $\hat{b}$ in the boundary limit $\varphi \to -\infty$.

$$
\begin{aligned}
\delta\theta^\mu_a =& \ \varepsilon^\mu_a + \tfrac{1}{4}l^{mn}\theta^\nu_a\sigma_{mn\nu}{}^\mu + f\theta^\mu_a + iw_b{}^b\theta^\mu_a - iw_a{}^b\theta^\mu_b - i\varepsilon_{abc}y^b\bar\theta^{\mu c} + \hat{\tilde{x}}^{\mu\nu}b_{\nu\lambda}\theta^\lambda_a \\
+& \ \hat{\tilde{x}}^{\mu\nu}\xi_{\nu a} - 2i(\theta^\mu_b\bar\xi^b_\nu + \bar\theta^{\mu b}\xi_{\nu b})\theta^\nu_a + e^{2\varphi}\hat{\tilde{b}}^{\mu\nu}\eta_{\nu a}
\end{aligned}
\tag{38}
$$

and c.c., where $\hat{\tilde{x}}^{\mu\nu} = \tilde{x}^{\mu\nu} - i\varepsilon^{\mu\nu}(\bar\theta\theta)$, while that for the coordinate φ related to the AdS_4 space radial direction read

$$
\delta\varphi = f(\theta) = f - b_m x^m + i(\xi_{\mu a}\bar\theta^{\mu a} + \bar\xi^a_\mu\theta^\mu_a). \tag{39}
$$

Transformation properties of the $\mathbb{CP}^3$ complex coordinates

$$
\begin{aligned}
\delta z^a =& \ iz^b\widetilde{w}_b{}^a + i\widetilde{w}_b{}^b z^a + \tfrac{|z|\cos|z|}{\sin|z|}\widetilde{y}^a + \tfrac{1}{2|z|^2}\left(1 - \tfrac{|z|}{\cos|z|\sin|z|}\right)(\widetilde{y}\bar z)z^a \\
+& \ \tfrac{1}{2|z|^2}(1 + |z|(\tan|z| - \cot|z|))(z\widetilde{\bar y})z^a
\end{aligned}
\tag{40}
$$

can be summarized in the form of $SU(4)/U(3)$ coset representative (15) transformations

$$
\delta T_{\hat a}{}^{\hat b} = iT_{\hat a}{}^{\hat c}\widetilde{W}_{\hat c}{}^{\hat b} - i\widehat{W}_{\hat a}{}^{\hat c}T_{\hat c}{}^{\hat b}, \quad \delta\bar T_{\hat a}{}^{\hat b} = -i\widetilde{W}_{\hat a}{}^{\hat c}\bar T_{\hat c}{}^{\hat b} + i\bar T_{\hat a}{}^{\hat c}\widehat{W}_{\hat c}{}^{\hat b}, \tag{41}
$$

where the $SU(4)$ transformation parameter matrix $\widetilde{W}_{\hat a}{}^{\hat b}$ has been introduced above (36) and

$$
\widehat{W}_{\hat a}{}^{\hat b} = \begin{pmatrix} \hat w_a{}^b - \delta^b_a\hat w_c{}^c & 0 \\ 0 & -\hat w_b{}^a + \delta^a_b\hat w_c{}^c \end{pmatrix} \tag{42}
$$

represents $U(3)$ compensating transformation matrix. Finally Grassmann coordinates associated with the conformal supersymmetry generators transform as follows

$$
\begin{aligned}
\delta\eta_{\mu a} =& \ \xi_{\mu a}(\theta) - \tfrac{1}{4}l^{mn}(\theta)\sigma_{mn\mu}{}^\nu\eta_{\nu a} - f(\theta)\eta_{\mu a} + iw_b{}^b(\theta)\eta_{\mu a} - iw_a{}^b(\theta)\eta_{\mu b} - i\varepsilon_{abc}y^b(\theta)\bar\eta^c_\mu \\
+& \ 2ie^{2\varphi}\varepsilon_{\mu\lambda}\hat{\tilde{b}}^{\lambda\nu}\eta_{\nu a}(\bar\eta\eta) \\
=& \ -\tfrac{1}{4}l^{mn}\sigma_{mn\mu}{}^\nu\eta_{\nu a} - f\eta_{\mu a} + iw_b{}^b\eta_{\mu a} - iw_a{}^b\eta_{\mu b} - i\varepsilon_{abc}y^b\bar\eta^c_\mu + b_{\mu\nu}\theta^\nu_a - \eta_{\nu a}\hat{\tilde{x}}^{\nu\lambda}b_{\lambda\mu} \\
+& \ 2i\left[(\theta_a b\theta_b)\bar\eta^b_\mu + (\theta_a b\bar\theta^b)\eta_{\mu b}\right] + \xi_{\mu a} - 2i(\bar\xi^b_\mu\theta^\nu_b + \xi_{\mu b}\bar\theta^{\nu b})\eta_{\nu a} \\
-& \ 2i(\eta_{\mu b}\bar\xi^b_\nu + \bar\eta^b_\mu\xi_{\nu b})\theta^\nu_a + 2i\xi_{\nu a}(\theta^\nu_b\bar\eta^b_\mu + \bar\theta^{\nu b}\eta_{\mu b}) + 2ie^{2\varphi}\varepsilon_{\mu\lambda}\hat{\tilde{b}}^{\lambda\nu}\eta_{\nu a}(\bar\eta\eta)
\end{aligned}
\tag{43}
$$

and c.c.

$osp(4|6)$ Cartan forms associated with the numerator generators of $osp(4|6)/(so(1,3)\times u(3))$ are left-invariant under the above derived global transformations up to the compensating ones associated with the denominator generators that in its turn transform in a connection-type way. Bosonic 1-forms that are identified with the AdS_4 part of the supervielbein in general transform as

$$
\delta\hat\omega^m(d) + \delta\hat c^m(d) = \hat l^{mn}(\hat\omega_n(d) + \hat c_n(d)) + 4\hat b^m\Delta(d), \quad \delta\Delta(d) = \hat b_m(\hat\omega^m(d) + \hat c^m(d)), \tag{44}
$$

while e.g. $so(1,2)$ Cartan forms in the spinor realization $G^{\mu\nu}(d) = \varepsilon^{\mu\lambda}\sigma_{mn\lambda}{}^\nu G^{mn}(d)$ obey the following rule

$$
\delta G^{\mu\nu}(d) = \tfrac{1}{4}(G^{\mu\lambda}(d)\hat l_\lambda{}^\nu + G^{\nu\lambda}(d)\hat l_\lambda{}^\mu) + \hat b^\mu{}_\lambda(\hat{\tilde\omega}(d) - \hat{\tilde c}(d))^{\lambda\nu} + \hat b^\nu{}_\lambda(\hat{\tilde\omega}(d) - \hat{\tilde c}(d))^{\lambda\mu} - \tfrac{1}{2}d\hat l^{\mu\nu}. \tag{45}
$$

$su(4)$ Cartan forms are $OSp(4|6)$ left-invariant up to the $U(3)$ rotation

$$
\delta\Omega_{\hat a}{}^{\hat b}(d) = i(\Omega_{\hat a}{}^{\hat c}(d)\widehat{W}_{\hat c}{}^{\hat b} - \widehat{W}_{\hat a}{}^{\hat c}\Omega_{\hat c}{}^{\hat b}(d)) - d\widehat{W}_{\hat a}{}^{\hat b} \tag{46}
$$

with $su(4)/u(3)$ 1-forms identified with the $\mathbb{CP}^3$ part of the supervielbein transforming as

$$\delta\Omega_a{}^4(d) = i\hat{w}_b{}^b\Omega_a{}^4(d) - i\hat{w}_a{}^b\Omega_b{}^4(d), \quad \delta\Omega_4{}^a(d) = -i\hat{w}_b{}^b\Omega_4{}^a(d) + i\Omega_4{}^b(d)\hat{w}_b{}^a \tag{47}$$

and $u(3)$ 1-forms exhibiting connection-type transformation properties

$$\delta\Omega_a{}^b(d) = i(\Omega_a{}^c(d)\hat{w}_c{}^b - \hat{w}_a{}^c\Omega_c{}^b(d)) - d\hat{w}_a{}^b. \tag{48}$$

Cartan forms that are identified with the supervielbein fermionic components transform in the following way

$$\delta\hat{\omega}^\nu_{\hat{b}}(d) = \frac{1}{4}\hat{\omega}^\lambda_{\hat{b}}(d)\hat{l}^{mn}\sigma_{mn\lambda}{}^\nu + \hat{\bar{b}}^{\nu\lambda}\hat{\chi}_{\lambda\hat{b}}(d) - i\widehat{W}_{\hat{b}}{}^{\hat{c}}\hat{\omega}^\nu_{\hat{c}}(d) \tag{49}$$

and

$$\delta\hat{\chi}_{\nu\hat{b}}(d) = -\frac{1}{4}\hat{l}^{mn}\sigma_{mn\nu}{}^\lambda\hat{\chi}_{\lambda\hat{b}}(d) + \hat{b}_{\nu\lambda}\hat{\omega}^\lambda_{\hat{b}}(d) - i\widehat{W}_{\hat{b}}{}^{\hat{c}}\hat{\chi}_{\nu\hat{c}}(d). \tag{50}$$

For individual transformations from the $D = 3$ $\mathcal{N} = 6$ superconformal symmetry to be discussed below these expressions simplify by properly restricting the parameters of compensating transformations that will be indicated by the vertical line at the corresponding transformation parameters.

General structure of the Noether currents corresponding to $D = 3$ $\mathcal{N} = 6$ superconformal invariance of the superstring action can be formally presented in the following form

$$\mathcal{J}^i_\Sigma(\tau,\sigma) = \mathcal{J}_{AdS}{}^i_\Sigma + \mathcal{J}_{CP}{}^i_\Sigma + \mathcal{J}_{WZ}{}^i_\Sigma, \tag{51}$$

where Σ is a transformation parameter[9],

$$\mathcal{J}_{AdS}{}^i_\Sigma = -\sqrt{-g}g^{ij}\left(\tfrac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})\tfrac{\partial}{\partial\Sigma}(\hat{\omega}^m(\delta_\Sigma) + \hat{c}^m(\delta_\Sigma)) + \Delta_j\tfrac{\partial}{\partial\Sigma}\Delta(\delta_\Sigma)\right) \tag{52}$$

is the contribution of the AdS_4 and

$$\mathcal{J}_{CP}{}^i_\Sigma = -\tfrac{1}{2}\sqrt{-g}g^{ij}\left(\Omega_{ja}{}^4\tfrac{\partial}{\partial\Sigma}\Omega_4{}^a(\delta_\Sigma) + \Omega_{j4}{}^a\tfrac{\partial}{\partial\Sigma}\Omega_a{}^4(\delta_\Sigma)\right) \tag{53}$$

$\mathbb{CP}^3$ parts of the kinetic term of the action (5), as well as that of the Wess-Zumino term

$$\mathcal{J}_{WZ}{}^i_\Sigma = \tfrac{i}{4}\varepsilon^{ij}\mathfrak{I}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}^{\mu\hat{a}}_j\varepsilon_{\mu\nu}\tfrac{\partial}{\partial\Sigma}\hat{\omega}^\nu_{\hat{b}}(\delta_\Sigma) + \hat{\chi}^{\hat{a}}_\mu\varepsilon^{\mu\nu}\tfrac{\partial}{\partial\Sigma}\hat{\chi}_{\nu\hat{b}}(\delta_\Sigma)\right). \tag{54}$$

Below we specialize to discussion of the individual transformations from the $D = 3$ $\mathcal{N} = 6$ superconformal symmetry and present corresponding expressions for the Noether currents.

4 $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the $OSp(4|6)/(SO(1,3) \times U(3))$ superstring: Noether currents

4.1 Noether currents associated with $D = 3$ conformal symmetry

4.1.1 Space-time translations

$osp(4|6)$ Cartan forms are obviously invariant under the global translations of $D = 3$ Minkowski boundary coordinates. Their variation is hence governed by the corresponding

[9] We assume the right derivative for fermions.

current contributions related to the coordinate dependence of transformation parameters to be used below for the construction of the current density. In particular, Eq.(44) representing variation of the Cartan forms identified with the AdS part of the supervielbein, when restricted to the boundary space-time translations, reduces to

$$\delta_a\hat{\omega}^m(d) + \delta_a\hat{c}^m(d) \;\; = j^m{}_n da^n, \;\; \delta_a\Delta(d) = 0, \tag{55}$$

where the current contribution tensor equals

$$j^m{}_n = \frac{\partial(\hat{\omega}^m + \hat{c}^m)(\delta_a)}{\partial a^n} = e^{-2\varphi}A\delta^m_n. \tag{56}$$

$su(4)$ Cartan forms are also invariant under $3d$ translations

$$\delta_a\Omega_{\hat{a}}{}^{\hat{b}}(d) = J_{\hat{a}}{}^{\hat{b}}{}_m da^m \tag{57}$$

modulo the current contribution matrix

$$J_{\hat{a}}{}^{\hat{b}}{}_m = \frac{\partial}{\partial a^m}\Omega_{\hat{a}}{}^{\hat{b}}(\delta_a) = \left(\begin{array}{cc} j_a{}^b{}_m & j_{abm} \\ -\bar{j}^{ab}{}_m & -j_b{}^a{}_m \end{array}\right) = -2(\hat{\eta}_{\hat{a}}\sigma_m\hat{\eta}^{\hat{b}}). \tag{58}$$

As a result variation of the $\mathbb{CP}^3$ components of the supervielbein acquires the form

$$\delta_a\Omega_a{}^4(d) = -\frac{1}{2}\varepsilon_{abc}\bar{j}^{bc}{}_m da^m, \quad \delta_a\Omega_4{}^a(d) = -\frac{1}{2}\varepsilon^{abc}j_{bcm}da^m. \tag{59}$$

Variation of the fermionic supervielbein components associated with the Poincare supersymmetry under the localized boundary space-time translations reads

$$\delta_a\hat{\omega}^\mu_{\hat{a}}(d) = j^\mu_{\hat{a}\,m}da^m \tag{60}$$

with the current contribution

$$j^\mu_{\hat{a}\,m} = \frac{\partial\hat{\omega}^\mu_{\hat{a}}(\delta_a)}{\partial a^m} = -e^{-\varphi}\tilde{\sigma}^{\mu\nu}_m\hat{\eta}_{\nu\hat{a}}. \tag{61}$$

Then for the variation of remaining fermionic supervielbein components associated with the conformal supersymmetry we obtain

$$\delta_a\hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}m}da^m, \tag{62}$$

where

$$J_{\mu\hat{a}m} = \frac{\partial\hat{\chi}_{\mu\hat{a}}(\delta_a)}{\partial a^m} = -ie^{2\varphi}(\bar{\eta}\eta)\varepsilon_{\mu\nu}j^\nu_{\hat{a}\,m} \tag{63}$$

is the current contribution. So that the current density related to the superstring action (5) invariance under $D=3$ Minkowski space-time translations has the form

$$\mathcal{J}^i{}_m(\tau,\sigma) = \mathcal{J}^i_{AdSm} + \mathcal{J}^i_{CPm} + \mathcal{J}^i_{WZm}. \tag{64}$$

The AdS part of the current density

$$\mathcal{J}^i_{AdSm} = -\frac{1}{4}\sqrt{-g}g^{ij}(\hat{\omega}_{jn} + \hat{c}_{jn})j^n{}_m \tag{65}$$

is determined by the current contribution (56), the $\mathbb{CP}^3$ part

$$\mathcal{J}^i_{CPm} = \frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{ja}{}^4\varepsilon^{abc}j_{bcm} + \Omega_{j4}{}^a\varepsilon_{abc}\bar{j}^{bc}{}_m\right) \tag{66}$$

is contributed by Eq.(59), and the current contributions (61), (63) determine the WZ part of the current density

$$\mathcal{J}^i_{WZm} = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}^{\mu\hat{a}}_j\varepsilon_{\mu\nu}j^\nu_{\hat{b}\,m} + \hat{\chi}^{\hat{a}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{b}m}\right). \tag{67}$$

4.1.2 Conformal boosts

Transformation properties of the Cartan forms that enter the $AdS-$part of the $OSp(4|6)/(SO(1,3)\times U(3))$ superstring action (5) under the localized conformal boosts follow from the expressions (44) appropriately restricted

$$\begin{aligned} \delta_b\,\hat\omega^m(d)+\delta_b\hat c^m(d) &= j^{mn}db_n+(\hat l|_b)^{mn}(\hat\omega_n(d)+\hat c_n(d))+4(\hat b|_b)^m\Delta(d),\\ \delta_b\Delta(d) &= j^m db_m-(\hat b|_b)^m(\hat\omega_m(d)+\hat c_m(d)) \end{aligned} \tag{68}$$

modulo the current contributions

$$\begin{aligned} j^{mn} = \tfrac{\partial(\hat\omega^m+\hat c^m)(\delta_b)}{\partial b_n} &= e^{-2\varphi}A\left\{(x^2+(\bar\theta\theta)^2)\eta^{mn}-2x^mx^n+i\left[(\bar\theta^a\sigma^m\hat{\tilde x}\sigma^n\theta_a)+(\theta_a\sigma^m\hat{\tilde x}\sigma^n\bar\theta^a)\right]\right\}\\ &+ A\tfrac{\partial\hat b^m}{\partial b_n}+ie^{2\varphi}\left[(\bar\eta^a\hat{\tilde x}\sigma^n\tilde\sigma^m\eta_a)+(\eta_a\hat{\tilde x}\sigma^n\tilde\sigma^m\bar\eta^a)\right]+ie^{2\varphi}\left[(\bar\eta^a\tilde\sigma^m\Lambda_-\sigma^n\theta_a)+(\eta_a\tilde\sigma^m\Lambda_-\sigma^n\bar\theta^a)\right]\\ &+ 2e^{2\varphi}(\bar\eta\eta)\left[(\bar\eta^a\sigma^m\hat{\tilde x}\sigma^n\theta_a)+(\eta_a\sigma^m\hat{\tilde x}\sigma^n\bar\theta^a)\right], \end{aligned} \tag{69}$$

$$j^m=\frac{\partial\Delta(\delta_b)}{\partial b_m}=-x^m-i\left[(\bar\eta^a\hat{\tilde x}\sigma^m\theta_a)+(\eta_a\hat{\tilde x}\sigma^m\bar\theta^a)\right]. \tag{70}$$

Transformation properties of the $su(4)$ Cartan forms follow from Eq.(46) by specializing to the conformal boost parameter dependence

$$\delta_b\Omega_{\hat a}{}^{\hat b}(d)=\hat J_{\hat a}{}^{\hat bm}db_m+i\left(\Omega_{\hat a}{}^{\hat c}(d)(\widehat W|_b)_{\hat c}{}^{\hat b}-(\widehat W|_b)_{\hat a}{}^{\hat b}\Omega_{\hat b}{}^{\hat c}(d)\right)-d(\widehat W|_b)_{\hat a}{}^{\hat b}. \tag{71}$$

The current contribution matrix

$$\hat J_{\hat a}{}^{\hat bm}=\frac{\partial}{\partial b_m}\Omega_{\hat a}{}^{\hat b}(\delta_b)=\begin{pmatrix}\hat j_a{}^{bm} & \hat j_{ab}{}^m\\ -\hat{\hat j}{}^{abm} & -\hat j_b{}^{am}\end{pmatrix} \tag{72}$$

is obtained by $T-$transforming the matrix

$$\begin{aligned} J_{\hat a}{}^{\hat bm} = &\ \tfrac{\partial}{\partial b_m}(\widetilde W_{\hat a}{}^{\hat b}+\Psi_{\hat a}{}^{\hat b}(\delta_b))=-2\left[(\theta_{\hat a}\sigma^m\theta^{\hat b})+(x^2+(\bar\theta\theta)^2)(\eta_{\hat a}\tilde\sigma^m\eta^{\hat b})\right.\\ - &\ \left.2x^m(\eta_{\hat a}\tilde x\eta^{\hat b})-(\eta_{\hat a}\hat{\tilde x}\sigma^mZ^{\hat b})+(\eta^{\hat b}\hat{\tilde x}\sigma^mZ_{\hat a})\right], \end{aligned} \tag{73}$$

where $Z^\mu_a=\theta^\mu_a-i(\bar\theta\theta)\eta^\mu_a$, $\bar Z^{\mu a}=\bar\theta^{\mu a}-i(\bar\theta\theta)\bar\eta^{\mu a}$. The final form of the current contribution matrix (72) is

$$\begin{aligned} \hat J_{\hat a}{}^{\hat b} = &\ (TJ^m\bar T)_{\hat a}{}^{\hat b}=-2\left[(\hat\theta_{\hat a}\sigma^m\hat\theta^{\hat b})+(x^2+(\bar\theta\theta)^2)(\hat\eta_{\hat a}\tilde\sigma^m\hat\eta^{\hat b})\right.\\ - &\ \left.2x^m(\hat\eta_{\hat a}\tilde x\hat\eta^{\hat b})-(\hat\eta_{\hat a}\hat{\tilde x}\sigma^m\hat Z^{\hat b})+(\hat\eta^{\hat b}\hat{\tilde x}\sigma^m\hat Z_{\hat a})\right]. \end{aligned} \tag{74}$$

Thus variation of the supervielbein components tangent to the $\mathbb{CP}^3$ manifold is brought to the form

$$\begin{aligned} \delta_b\Omega_a{}^4(d) = &\ -\tfrac12\varepsilon_{abc}\hat{\hat j}{}^{bcm}db_m+i(\hat w|_b)_b{}^b\Omega_a{}^4(d)-i(\hat w|_b)_a{}^b\Omega_b{}^4(d),\\ \delta_b\Omega_4{}^a(d) = &\ -\tfrac12\varepsilon^{abc}\hat j_{bc}{}^m db_m-i(\hat w|_b)_b{}^b\Omega_4{}^a(d)+i\Omega_4{}^b(d)(\hat w|_b)_b{}^a. \end{aligned} \tag{75}$$

The expressions for variation of the fermionic 1-forms follows from (49) and (50). Namely, for Cartan forms related to Poincare supersymmetry one derives that

$$\delta_b\hat\omega^\mu_{\hat a}(d)=j^{\mu m}_{\hat a}db_m+\frac14\hat\omega^\nu_{\hat a}(d)(\hat l|_b)^{mn}\sigma_{mn\nu}{}^\mu+(\hat{\tilde b}|_b)^{\mu\nu}\chi_{\nu\hat a}(d)-i(\widehat W|_b)_{\hat a}{}^{\hat b}\hat\omega^\mu_{\hat b}(d), \tag{76}$$

where the current contributions read

$$j^{\mu m}_{\hat{a}} = \frac{\partial \hat{\omega}^{\mu}_{\hat{a}}(\delta_b)}{\partial b_m} = e^{-\varphi}\left[\hat{\tilde{x}}^{\mu\nu}\sigma^{m}_{\nu\lambda}\hat{Z}^{\lambda}_{\hat{a}} - (x^2+(\bar{\theta}\theta)^2)\tilde{\sigma}^{m\mu\nu}\hat{\eta}_{\nu\hat{a}} + 2x^m\tilde{x}^{\mu\nu}\hat{\eta}_{\nu\hat{a}} + i(\bar{\theta}\theta)\varepsilon^{\mu\nu}\hat{\eta}_{\rho\hat{a}}\hat{\tilde{x}}^{\rho\lambda}\sigma^{m}_{\lambda\nu}\right]. \tag{77}$$

Correspondingly for Cartan forms related to conformal supersymmetry we find

$$\delta_b\hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}}{}^{m}db_m - \frac{1}{4}(\hat{l}|_b)^{mn}\sigma_{mn\mu}{}^{\nu}\hat{\chi}_{\nu\hat{a}}(d) + (\hat{b}|_b)_{\mu\nu}\hat{\omega}^{\nu}_{\hat{a}}(d) - i(\widehat{W}|_b)_{\hat{a}}{}^{\hat{b}}\hat{\chi}_{\mu\hat{b}}(d) \tag{78}$$

with the current contributions

$$J_{\mu\hat{a}}{}^{m} = \frac{\partial\hat{\chi}_{\mu\hat{a}}(\delta_b)}{\partial b_m} = e^{\varphi}\left(\Lambda_{-\mu}{}^{\nu}\sigma^{m}_{\nu\lambda}\hat{\theta}^{\lambda}_{\hat{a}} - \hat{\eta}_{\nu\hat{a}}\hat{\tilde{x}}^{\nu\lambda}\sigma^{m}_{\lambda\rho}\Lambda_{-\mu}{}^{\rho} - ie^{\varphi}(\bar{\eta}\eta)\varepsilon_{\mu\nu}j^{\nu m}_{\hat{a}}\right), \tag{79}$$

where we have introduced the following $3d$ tensors $\Lambda_{\pm\mu}{}^{\nu} = \delta^{\nu}_{\mu} \pm 2i(\bar{\eta}^{a}_{\mu}\theta^{\nu}_{a} + \eta_{\mu a}\bar{\theta}^{\nu a})$ that will appear to be useful below. The substitution of Eqs.(68), (75), (76) and (78) into the superstring action variation under $D=3$ conformal boosts yields the current density

$$\mathcal{J}^{im}(\tau,\sigma) = \mathcal{J}^{i}_{AdS}{}^{m} + \mathcal{J}^{i}_{CP}{}^{m} + \mathcal{J}^{i}_{WZ}{}^{m}, \tag{80}$$

where

$$\begin{aligned} \mathcal{J}^{i}_{AdS}{}^{m} &= -\sqrt{-g}g^{ij}\left(\tfrac{1}{4}(\hat{\omega}_{jn}+\hat{c}_{jn})j^{nm} + \Delta_j j^m\right),\\ \mathcal{J}^{i}_{CP}{}^{m} &= \tfrac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{ja}{}^{4}\varepsilon^{abc}\hat{j}_{bc}{}^{m} + \Omega_{j4}{}^{a}\varepsilon_{abc}\hat{\bar{j}}^{bcm}\right),\\ \mathcal{J}^{i}_{WZ}{}^{m} &= \tfrac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}^{\mu\hat{a}}_{j}\varepsilon_{\mu\nu}j^{\nu m}_{\hat{b}} + \hat{\chi}^{\hat{a}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{b}}{}^{m}\right). \end{aligned} \tag{81}$$

4.1.3 Dilatations

$osp(4|6)/(so(1,3)\times u(3))$ Cartan forms identified with the $(10|24)-$supervielbein components are invariant under the global scale transformations due to the presence of appropriate exponents of the AdS_4 bulk coordinate φ. Hence their variation under the localized scale transformations is determined by the current contributions. In particular, for the components of the supervielbein tangent to the AdS_4 space we obtain that

$$\delta_f\,\hat{\omega}^m(d) + \delta_f\hat{c}^m(d) = j^m df, \quad \delta_f\Delta(d) = j df, \tag{82}$$

where

$$\begin{aligned} j^m &= \tfrac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_f)}{\partial f} = 2e^{-2\varphi}Ax^m + 2e^{2\varphi}(\bar{\eta}\eta)\left[(\bar{\eta}\sigma^m\theta) + (\eta\sigma^m\bar{\theta})\right],\\ j &= \tfrac{\partial\Delta(\delta_f)}{\partial f} = 1 + i(\theta^{\mu}_{a}\bar{\eta}^{a}_{\mu} + \bar{\theta}^{\mu a}\eta_{\mu a}). \end{aligned} \tag{83}$$

$su(4)$ Cartan forms are also scale-invariant

$$\delta_f\Omega_{\hat{a}}{}^{\hat{b}} = J_{\hat{a}}{}^{\hat{b}}df \tag{84}$$

modulo the current contributions

$$J_{\hat{a}}{}^{\hat{b}} = \frac{\partial}{\partial f}\Omega_{\hat{a}}{}^{\hat{b}}(\delta_f) = \begin{pmatrix} j_a{}^b & j_{ab} \\ -\bar{j}^{ab} & -j_b{}^a \end{pmatrix} = 2\left(\Theta^{\mu}_{\hat{a}}\hat{\bar{\eta}}^{\hat{b}}_{\mu} - \hat{\Theta}^{\mu\hat{b}}\hat{\eta}_{\mu\hat{a}}\right), \tag{85}$$

where $\Theta^{\mu}_{b} = \theta^{\mu}_{b} - \eta_{\nu b}\hat{\tilde{x}}^{\nu\mu}$ and $\bar{\Theta}^{\mu b} = \bar{\theta}^{\mu b} - \bar{\eta}^{b}_{\nu}\hat{\tilde{x}}^{\nu\mu}$. So that the variation of $\mathbb{CP}^3$ part of the supervielbein is governed by the appropriate components of the current contribution matrix (85)

$$\delta_f\Omega_a{}^4(d) = -\frac{1}{2}\varepsilon_{abc}\bar{j}^{bc}df, \quad \delta_f\Omega_4{}^a(d) = -\frac{1}{2}\varepsilon^{abc}j_{bc}df. \tag{86}$$

Fermionic supervielbein components related to Poincare supersymmetry obey the transformation rules

$$\delta_f\,\hat{\omega}^{\mu}_{\hat{a}}(d) = j^{\mu}_{\hat{a}}df \tag{87}$$

with the current contributions

$$j^{\mu}_{\hat{a}} = \frac{\partial\hat{\omega}^{\mu}_{\hat{a}}(\delta_f)}{\partial f} = e^{-\varphi}(\hat{\Theta}^{\mu}_{\hat{a}} - 2\tilde{x}^{\mu\nu}\hat{\eta}_{\nu\hat{a}}), \tag{88}$$

while those related to conformal supersymmetry transform as

$$\delta_f\hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}}df \tag{89}$$

with the corresponding current contributions given by

$$J_{\mu\hat{a}} = \frac{\partial\hat{\chi}_{\mu\hat{a}}(\delta_f)}{\partial f} = -e^{\varphi}\Lambda_{-\mu}{}^{\nu}\hat{\eta}_{\nu\hat{a}} - ie^{2\varphi}(\bar{\eta}\eta)\varepsilon_{\mu\nu}j^{\nu}_{\hat{a}}. \tag{90}$$

Above presented current contributions determine Noether current density related to the scale invariance of the superstring action (5)

$$\mathcal{J}^i(\tau,\sigma) = \mathcal{J}^i_{AdS} + \mathcal{J}^i_{CP} + \mathcal{J}^i_{WZ}. \tag{91}$$

Namely, the AdS part of the current density

$$\mathcal{J}^i_{AdS} = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})j^m + \Delta_j j\right) \tag{92}$$

is contributed by Eq.(83), the $\mathbb{CP}^3$ part

$$\mathcal{J}^i_{CP}{}^m = \frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{ja}{}^4\varepsilon^{abc}j_{bc} + \Omega_{j4}{}^a\varepsilon_{abc}\bar{j}^{bc}\right) \tag{93}$$

is determined by the current contributions that enter (86), and the WZ part

$$\mathcal{J}^i_{WZ} = \frac{i}{4}\varepsilon^{ij}\mathfrak{I}_{\hat{a}}{}^{\hat{b}}\left(\hat{\omega}^{\mu\hat{a}}_j\varepsilon_{\mu\nu}j^{\nu}_{\hat{b}} + \hat{\chi}^{\hat{a}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{b}}\right) \tag{94}$$

receives contributions from (88) and (90).

4.1.4 Lorentz rotations

Under the localized $SO(1,2)$ Lorentz rotations with parameters l^{mn} Cartan forms identified with the supervielbein components tangent to AdS_4 space exhibit the following transformation properties

$$\begin{aligned} \delta_l\hat{\omega}^m(d) + \delta_l\hat{c}^m(d) &= j^m{}_{kn}dl^{kn} + l^{mn}(\hat{\omega}_n(d) + \hat{c}_n(d)), \\ \delta_l\Delta(d) &= j_{mn}dl^{mn} \end{aligned} \tag{95}$$

with the current contributions

$$\begin{aligned} j^m{}_{kn} = {} & \tfrac{\partial(\hat{\omega}^m + \hat{c}^m)(\delta_l)}{\partial l^{kn}} = \tfrac{1}{2}e^{-2\varphi}A(\delta^m_k x_n - \delta^m_n x_k) + \tfrac{i}{2}e^{-2\varphi}A(\bar{\theta}\theta)\varepsilon^m{}_{kn} \\ & + \tfrac{1}{2}e^{2\varphi}(\bar{\eta}\eta)\varepsilon^m{}_{kn}\left[i + (\bar{\eta}\theta) + (\bar{\theta}\eta)\right] + \tfrac{1}{2}e^{2\varphi}(\bar{\eta}\eta)\left\{\delta^m_k\left[(\eta\sigma_n\bar{\theta}) + (\bar{\eta}\sigma_n\theta)\right] - (k\leftrightarrow n)\right\}, \\ j_{mn} = {} & \tfrac{\partial\Delta(\delta_l)}{\partial l^{mn}} = \tfrac{i}{4}\left[(\theta_a\sigma_{mn}\bar{\eta}^a) + (\bar{\theta}^a\sigma_{mn}\eta_a)\right]. \end{aligned} \tag{96}$$

The $su(4)$ Cartan forms are obviously $D = 3$ Lorentz invariant

$$\delta_l \Omega_{\hat{a}}{}^{\hat{b}}(d) = J_{\hat{a}}{}^{\hat{b}}{}_{mn} dl^{mn} \tag{97}$$

modulo the current contribution matrix

$$J_{\hat{a}}{}^{\hat{b}}{}_{mn} = \frac{\partial}{\partial l^{mn}} \Omega_{\hat{a}}{}^{\hat{b}}(\delta_l) = \begin{pmatrix} j_a{}^b{}_{mn} & j_{abmn} \\ -\bar{j}^{ab}{}_{mn} & -j_b{}^a{}_{mn} \end{pmatrix} = \frac{1}{2}\left[(\hat{\Theta}_{\hat{a}} \sigma_{mn} \hat{\eta}^{\hat{b}}) - (\hat{\Theta}^{\hat{b}} \sigma_{mn} \hat{\eta}_{\hat{a}}) \right]. \tag{98}$$

Hence variation of the supervielbein components tangent to $\mathbb{CP}^3$ manifold is extracted from (98)

$$\delta_l \Omega_a{}^4(d) = -\frac{1}{2} \varepsilon_{abc} \bar{j}^{bc}{}_{mn} dl^{mn}, \quad \delta_l \Omega_4{}^a(d) = -\frac{1}{2} \varepsilon^{abc} j_{bcmn} dl^{mn}. \tag{99}$$

Variation of the supervielbein fermionic components that are identified with the Cartan forms related to Poincare supersymmetry follows by restricting the general expressions (49)

$$\delta_l \hat{\omega}^{\mu}_{\hat{a}}(d) = j^{\mu}_{\hat{a}mn} dl^{mn} + \frac{1}{4} \hat{\omega}^{\nu}_{\hat{a}}(d) l^{mn} \sigma_{mn\nu}{}^{\mu}, \tag{100}$$

where the current contributions read

$$j^{\mu}_{\hat{a}mn} = \frac{\partial \hat{\omega}^{\mu}_{\hat{a}}(\delta_l)}{\partial l^{mn}} = \frac{1}{4} e^{-\varphi} \left(\hat{\Theta}^{\nu}_{\hat{a}} \sigma_{mn\nu}{}^{\mu} - \hat{\tilde{x}}^{\mu\lambda} \sigma_{mn\lambda}{}^{\nu} \hat{\eta}_{\nu\hat{a}} \right). \tag{101}$$

Transformation properties of the Cartan forms related to conformal supersymmetry identified with another half supervielbein fermionic components follow from (50)

$$\delta_l \hat{\chi}_{\mu\hat{a}}(d) = J_{\mu\hat{a}mn} dl^{mn} - \frac{1}{4} l^{mn} \sigma_{mn\mu}{}^{\nu} \hat{\chi}_{\nu\hat{a}}(d) \tag{102}$$

with the current contributions

$$J_{\mu\hat{a}mn} = \frac{\partial \hat{\chi}_{\mu\hat{a}}(\delta_l)}{\partial l^{mn}} = -\frac{1}{4} e^{\varphi} \Lambda_{-\mu}{}^{\nu} \sigma_{mn\nu}{}^{\lambda} \hat{\eta}_{\nu\hat{a}} - i e^{2\varphi} (\bar{\eta}\eta) \varepsilon_{\mu\nu} j^{\nu}_{\hat{a}mn}. \tag{103}$$

Then one is able to derive the current density related to global $SO(1,2)$ symmetry of the superstring action (5)

$$\mathcal{J}^i{}_{mn}(\tau, \sigma) = \mathcal{J}^i_{AdSmn} + \mathcal{J}^i_{CPmn} + \mathcal{J}^i_{WZmn}. \tag{104}$$

The contribution of the AdS part of the action

$$\mathcal{J}^i_{AdSmn} = -\sqrt{-g} g^{ij} \left(\frac{1}{4} (\hat{\omega}_{jk} + \hat{c}_{jk}) j^k{}_{mn} + \Delta_j j_{mn} \right), \tag{105}$$

is determined by Eq.(96), that of the $\mathbb{CP}^3$ part

$$\mathcal{J}^i_{CPmn} = \frac{1}{4} \sqrt{-g} g^{ij} \left(\Omega_{jc}{}^4 \varepsilon^{cde} j_{demn} + \Omega_{j4}{}^c \varepsilon_{cde} \bar{j}^{de}{}_{mn} \right) \tag{106}$$

by Eq.(99), and the WZ part of the current density

$$\mathcal{J}^i_{WZmn} = \frac{i}{4} \varepsilon^{ij} \mathfrak{I}_{\hat{a}}{}^{\hat{b}} \left(\hat{\omega}^{\mu\hat{a}}_j \varepsilon_{\mu\nu} j^{\nu}_{\hat{b}}{}_{mn} + \hat{\chi}^{\hat{a}}_{j\mu} \varepsilon^{\mu\nu} J_{\nu\hat{b}mn} \right) \tag{107}$$

is contributed by (101), (103).

4.2 Noether currents associated with $SU(4)$ $R-$symmetry

4.2.1 $U(3)$ Rotations

Global $U(3)$ rotations represent an obvious symmetry of the AdS_4 part of $(10|24)-$supervielbein thus the nontrivial part of its variation under the coordinate-dependent $U(3)$ rotations

$$\delta_w\hat{\omega}^m(d) + \delta_w\hat{c}^m(d) = j^m{}_a{}^b dw_b{}^a, \quad \delta_w\Delta = j_a{}^b dw_b{}^a \tag{108}$$

is concentrated in the current contributions

$$\begin{array}{rcl} j^m{}_a{}^b = & \frac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_w)}{\partial w_b{}^a} = 2e^{-2\varphi}A\left[\delta^b_a(\theta_c\sigma^m\bar{\theta}^c) - (\theta_a\sigma^m\bar{\theta}^b)\right] + 2e^{2\varphi}\delta^b_a\left\{(\eta_c\tilde{\sigma}^m\bar{\eta}^c)\right. \\ - & \left. i(\bar{\eta}\eta)\left[(\theta_c\sigma^m\bar{\eta}^c) + (\eta_c\sigma^m\bar{\theta}^c)\right]\right\} - 2e^{2\varphi}\left\{(\eta_a\sigma^m\bar{\eta}^b) - i(\bar{\eta}\eta)\left[(\theta_a\sigma^m\bar{\eta}^b) + (\eta_a\sigma^m\bar{\theta}^b)\right]\right\}, \\ j_a{}^b = & \frac{\partial\Delta(\delta_w)}{\partial w_b{}^a} = \delta^b_a(\bar{\theta}^{\mu c}\eta_{\mu c} + \bar{\eta}^c_\mu\theta^\mu_c) + \theta^\mu_a\bar{\eta}^b_\mu + \eta_{\mu a}\bar{\theta}^{\mu b}. \end{array} \tag{109}$$

Transformation properties of $su(4)$ Cartan forms under $U(3)$ rotations are derived from (46)

$$\delta_w\Omega_{\hat{c}}{}^{\hat{d}}(d) = \hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b dw_b{}^a + i(\Omega_{\hat{c}}{}^{\hat{e}}(d)(\widehat{W}|_w)_{\hat{e}}{}^{\hat{d}} - (\widehat{W}|_w)_{\hat{c}}{}^{\hat{e}}\Omega_{\hat{e}}{}^{\hat{d}}(d)) - d(\widehat{W}|_w)_{\hat{c}}{}^{\hat{d}}. \tag{110}$$

The current contribution matrix

$$\hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b = \frac{\partial}{\partial w_b{}^a}\Omega_{\hat{c}}{}^{\hat{d}}(\delta_w) = (TJ_a{}^b\bar{T})_{\hat{c}}{}^{\hat{d}} \tag{111}$$

is obtained by the $T-$transformation of the matrix

$$\begin{array}{rcl} J_{\hat{c}}{}^{\hat{d}}{}_a{}^b = & \frac{\partial}{\partial w_b{}^a}(W_{\hat{c}}{}^{\hat{d}} + \Psi_{\hat{c}}{}^{\hat{d}}(\delta_w)) = \delta_{\hat{c}}{}^b\delta_a{}^{\hat{d}} - \delta_{\hat{c}a}\delta^{b\hat{d}} + 4\eta_{\mu\hat{c}}(\theta^\mu_a\bar{\theta}^{\nu b} + \theta^\nu_a\bar{\theta}^{\mu b})\eta^{\hat{d}}_\nu \\ + & \delta^b_a\left[\frac{i}{2}\mathfrak{J}_{\hat{c}}{}^{\hat{d}} + (\mathfrak{J}\theta^\mu)_{\hat{c}}\eta^{\hat{d}}_\mu - (\mathfrak{J}\theta^\mu)^{\hat{d}}\eta_{\mu\hat{c}} - 4\eta_{\mu\hat{c}}(\theta^\mu_e\bar{\theta}^{\nu e} + \theta^\nu_e\bar{\theta}^{\mu e})\eta^{\hat{d}}_\nu\right] \\ + & 2i\left[\theta^\mu_a\left(\eta_{\mu\hat{c}}\delta^{b\hat{d}} - \eta^{\hat{d}}_\mu\delta_{\hat{c}}{}^b\right) + \bar{\theta}^{\mu b}\left(\eta^{\hat{d}}_\mu\delta_{\hat{c}a} - \eta_{\mu\hat{c}}\delta_a{}^{\hat{d}}\right)\right], \end{array} \tag{112}$$

where the following objects have been introduced

$$\delta_{\hat{c}}{}^b = \begin{pmatrix} \delta^b_c \\ 0 \end{pmatrix}, \quad \delta_{\hat{c}a} = \begin{pmatrix} 0 \\ \delta^c_a \end{pmatrix}, \quad \delta_a{}^{\hat{d}} = \begin{pmatrix} \delta^d_a & 0 \end{pmatrix}, \quad \delta^{b\hat{d}} = \begin{pmatrix} 0 & \delta^b_d \end{pmatrix}. \tag{113}$$

So that explicit form of the current contribution matrix is

$$\hat{J}_{\hat{c}}{}^{\hat{d}}{}_a{}^b = \mathcal{T}_{\hat{c}}^b\mathcal{T}^{\hat{d}}{}_a - \mathcal{T}_{\hat{c}a}\mathcal{T}^{\hat{d}b} + \frac{i}{2}\delta^b_a(\mathcal{T}\mathfrak{J}\bar{\mathcal{T}})_{\hat{c}}{}^{\hat{d}}. \tag{114}$$

The matrix $\mathcal{T}_{\hat{a}}{}^{\hat{b}}$ equal

$$\mathcal{T}_{\hat{a}}{}^{\hat{b}} = \begin{pmatrix} \mathcal{T}_a{}^b & \mathcal{T}_{ab} \\ \mathcal{T}^{ab} & \mathcal{T}^a{}_b \end{pmatrix} = T_{\hat{a}}{}^{\hat{b}} + 2i\bar{\eta}_{\nu\hat{a}}\theta^{\nu\hat{b}} \tag{115}$$

will also be used below. From (110) and (114) one derives the transformation properties of the $\mathbb{CP}^3$ part of the bosonic supervielbein

$$\begin{array}{rcl} \delta_w\Omega_c{}^4(d) = & -\frac{1}{2}\varepsilon_{cde}\hat{\bar{J}}^{de}{}_a{}^b dw_b{}^a + i(\hat{w}|_w)_d{}^d\Omega_c{}^4(d) - i(\hat{w}|_w)_c{}^d\Omega_d{}^4(d), \\ \delta_w\Omega_4{}^c(d) = & -\frac{1}{2}\varepsilon^{cde}\hat{J}_{cda}{}^b dw_b{}^a - i(\hat{w}|_w)_d{}^d\Omega_4{}^c(d) + i\Omega_4{}^d(d)(\hat{w}|_w)_d{}^c. \end{array} \tag{116}$$

Fermionic supervielbein components associated with the Poincare supersymmetry transform under $U(3)$ rotations as follows

$$\delta_w\hat{\omega}^\mu_{\hat{c}}(d) = j^\mu_{\hat{c}}{}_a{}^b dw_b{}^a - i(\widehat{W}|_w)_{\hat{c}}{}^{\hat{d}}\,\hat{\omega}^\mu_{\hat{d}}(d), \tag{117}$$

where

$$j^{\mu}_{\hat{c}\,a}{}^{b} = \frac{\partial \hat{\omega}^{\mu}_{\hat{c}}(\delta_w)}{\partial w_b{}^a} = e^{-\varphi}\left[\tfrac{1}{2}\delta^b_a(\mathcal{T}\mathfrak{J}\theta^{\mu})_{\hat{c}} - i\theta^{\mu}_a\mathcal{T}_{\hat{c}}{}^b + i\bar{\theta}^{\mu b}\mathcal{T}_{\hat{c}a}\right] \tag{118}$$

are the current contributions. Analogously fermionic supervielbein components related to conformal supersymmetry have the transformation properties

$$\delta_w\hat{\chi}_{\mu\hat{c}}(d) = J_{\mu\hat{c}a}{}^b dw_b{}^a - i(\widehat{W}|_w)_{\hat{c}}{}^{\hat{d}}\hat{\chi}_{\mu\hat{d}}(d) \tag{119}$$

with the current contributions

$$J_{\mu\hat{c}a}{}^b = \frac{\partial \hat{\chi}_{\mu\hat{c}}(\delta_w)}{\partial w_b{}^a} = e^{\varphi}\left[\tfrac{1}{2}\delta^b_a(\mathcal{T}\mathfrak{J}\eta_{\mu})_{\hat{c}} - i\eta_{\mu a}\mathcal{T}_{\hat{c}}{}^b + i\bar{\eta}^b_{\mu}\mathcal{T}_{\hat{c}a}\right]. \tag{120}$$

In summary the current density associated with $U(3)$ global invariance of the superstring action (5)

$$\mathcal{J}^i{}_a{}^b(\tau,\sigma) = \mathcal{J}^i_{AdSa}{}^b + \mathcal{J}^i_{CPa}{}^b + \mathcal{J}^i_{WZa}{}^b \tag{121}$$

consists of three summands

$$\mathcal{J}^i_{AdSa}{}^b = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})j^m{}_a{}^b + \Delta_j j_a{}^b\right), \tag{122}$$

$$\mathcal{J}^i_{CPa}{}^b = \frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{jc}{}^4\varepsilon^{cde}\hat{j}_{dea}{}^b + \Omega_{j4}{}^c\varepsilon_{cde}\hat{\bar{j}}^{de}{}_a{}^b\right), \tag{123}$$

and

$$\mathcal{J}^i_{WZa}{}^b = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{c}}{}^{\hat{d}}\left(\hat{\omega}^{\mu\hat{c}}_j\varepsilon_{\mu\nu}j^{\nu}_{\hat{d}a}{}^b + \hat{\chi}^{\hat{c}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{d}a}{}^b\right) \tag{124}$$

that are determined by the current contributions of the $osp(4|6)/(so(1,3)\times u(3))$ Cartan forms (109), (116), (118) and (120).

4.2.2 $SU(4)/U(3)$ transformations

As in the case of $U(3)$ rotations Cartan forms from the AdS_4 sector are invariant under the $SU(4)/U(3)$ transformations

$$\delta_y\hat{\omega}^m(d) + \delta_y\hat{c}^m(d) = j^m{}_a dy^a + \bar{j}^{ma}d\bar{y}_a, \quad \delta_y\Delta = j_a dy^a + \bar{j}^a d\bar{y}_a \tag{125}$$

modulo the current contributions

$$\begin{aligned} j^m{}_a &= \tfrac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_y)}{\partial y^a} = -\varepsilon_{abc}\left\{e^{-2\varphi}A(\bar{\theta}^b\sigma^m\bar{\theta}^c) + e^{\varphi}\left[(\bar{\eta}^b\tilde{\sigma}^m\bar{\eta}^c) - 2i(\bar{\eta}\eta)(\bar{\theta}^b\tilde{\sigma}^m\bar{\eta}^c)\right]\right\}, \\ \bar{j}^{ma} &= \tfrac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_y)}{\partial \bar{y}_a} = \varepsilon^{abc}\left\{e^{-2\varphi}A(\theta_b\sigma^m\theta_c) + e^{\varphi}\left[(\eta_b\tilde{\sigma}^m\eta_c) - 2i(\bar{\eta}\eta)(\theta_b\tilde{\sigma}^m\eta_c)\right]\right\}, \\ j_a &= \tfrac{\partial\Delta(\delta_y)}{\partial y^a} = \varepsilon_{abc}\bar{\theta}^{\mu b}\bar{\eta}^c_{\mu}, \; \bar{j}^a = \tfrac{\partial\Delta(\delta_y)}{\partial\bar{y}_a} = -\varepsilon^{abc}\theta^{\mu}_b\eta_{\mu c}. \end{aligned} \tag{126}$$

Transformation properties of $su(4)$ Cartan forms follow from the general formula (46) by specializing to $SU(4)/U(3)$ rotations

$$\delta_y\Omega_{\hat{b}}{}^{\hat{c}}(d) = J_{\hat{b}}{}^{\hat{c}}{}_a dy^a + \bar{J}_{\hat{b}}{}^{\hat{c}a}d\bar{y}_a + i\left(\Omega_{\hat{b}}{}^{\hat{d}}(d)(\widehat{W}|_y)_{\hat{d}}{}^{\hat{c}} - (\widehat{W}|_y)_{\hat{b}}{}^{\hat{d}}\Omega_{\hat{d}}{}^{\hat{c}}(d)\right) - d(\widehat{W}|_y)_{\hat{c}}{}^{\hat{d}}. \tag{127}$$

Corresponding current contribution matrices equal

$$J_{\hat{b}}{}^{\hat{c}}{}_a = \tfrac{\partial}{\partial y^a}\Omega_{\hat{b}}{}^{\hat{c}}(\delta_y) = \begin{pmatrix} j_b{}^c{}_a & j_{bca} \\ j^{bc}{}_a & -j_c{}^b{}_a \end{pmatrix} = -\varepsilon_{ade}\mathcal{T}_{\hat{b}}{}^d\mathcal{T}^{\hat{c}e} \tag{128}$$

and

$$\bar{J}_{\hat{b}}{}^{\hat{c}a} = \frac{\partial}{\partial \bar{y}_a}\Omega_{\hat{b}}{}^{\hat{c}}(\delta_y) = \begin{pmatrix} \bar{j}_b{}^{ca} & -\bar{j}_{bc}{}^{a} \\ -\bar{j}^{bc}{}_{a} & -\bar{j}_c{}^{ba} \end{pmatrix} = \varepsilon^{ade}\mathcal{T}_{\hat{b}d}\mathcal{T}^{\hat{c}}{}_{e}. \tag{129}$$

So that one extracts from the above expressions the transformation rules for the supervielbein bosonic components tangent to the $\mathbb{CP}^3$ manifold

$$\begin{aligned} \delta_y\Omega_b{}^4(d) &= \tfrac{1}{2}\varepsilon_{bcd}(j^{cd}{}_a dy^a - \bar{j}^{cda}d\bar{y}_a) + i(\hat{w}|_y)_c{}^c\Omega_b{}^4(d) - i(\hat{w}|_y)_b{}^c\Omega_c{}^4(d), \\ \delta_y\Omega_4{}^b(d) &= -\tfrac{1}{2}\varepsilon^{bcd}(j_{cda}dy^a - \bar{j}_{cd}{}^a d\bar{y}_a) - i(\hat{w}|_y)_c{}^c\Omega_4{}^b(d) + i\Omega_4{}^c(d)(\hat{w}|_y)_c{}^b. \end{aligned} \tag{130}$$

Cartan forms associated with Poincare supersymmetry have the following properties under the $SU(4)/U(3)$ transformations

$$\delta_y\hat{\omega}^{\mu}_{\hat{b}}(d) = j^{\mu}_{\hat{b}a}dy^a + j^{\mu a}_{\hat{b}}d\bar{y}_a - i(\widehat{W}|_y)_{\hat{b}}{}^{\hat{c}}\,\hat{\omega}^{\mu}_{\hat{c}}(d) \tag{131}$$

with the current contributions

$$\begin{aligned} j^{\mu}_{\hat{b}a} &= \frac{\partial\hat{\omega}^{\mu}_{\hat{b}}(\delta_y)}{\partial y^a} = ie^{-\varphi}\varepsilon_{acd}\mathcal{T}_{\hat{b}}{}^{c}\bar{\theta}^{\mu d}, \\ j^{\mu a}_{\hat{b}} &= \frac{\partial\hat{\omega}^{\mu}_{\hat{b}}(\delta_y)}{\partial \bar{y}_a} = -ie^{-\varphi}\varepsilon^{acd}\mathcal{T}_{\hat{b}c}\theta^{\mu}_{d}, \end{aligned} \tag{132}$$

while Cartan forms related to conformal supersymmetry transform as

$$\delta_y\hat{\chi}_{\mu\hat{b}}(d) = J_{\mu\hat{b}a}dy^a + J_{\mu\hat{b}}{}^{a}d\bar{y}_a - i(\widehat{W}|_y)_{\hat{b}}{}^{\hat{c}}\hat{\chi}_{\mu\hat{c}}(d), \tag{133}$$

where the current contributions can be brought to the form

$$\begin{aligned} J_{\mu\hat{b}a} &= \frac{\partial\hat{\chi}_{\mu\hat{b}}(\delta_y)}{\partial y^a} = ie^{\varphi}(\varepsilon_{acd}\mathcal{T}_{\hat{b}}{}^{c}\bar{\eta}^{d}_{\mu} + (\bar{\eta}\eta)\varepsilon_{\mu\nu}j^{\nu}_{\hat{b}}{}_{a}), \\ J_{\mu\hat{b}}{}^{a} &= \frac{\partial\hat{\chi}_{\mu\hat{b}}(\delta_y)}{\partial \bar{y}_a} = -ie^{\varphi}(\varepsilon^{acd}\mathcal{T}_{\hat{b}c}\eta_{\mu d} - (\bar{\eta}\eta)\varepsilon_{\mu\nu}j^{\nu a}_{\hat{b}}). \end{aligned} \tag{134}$$

Above derived current contributions of the supervielbein components define the current density associated with $SU(4)/U(3)$ global invariance of the superstring action

$$\mathcal{J}^{i}{}_{a}(\tau,\sigma) = \mathcal{J}^{i}_{AdSa} + \mathcal{J}^{i}_{CPa} + \mathcal{J}^{i}_{WZa}. \tag{135}$$

Each summand being contributed by Eqs.(126), (130), (132) and (134) respectively takes the form

$$\mathcal{J}^{i}_{AdSa} = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})j^{m}{}_{a} + \Delta_j j_a\right), \tag{136}$$

$$\mathcal{J}^{i}_{CPa} = \frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{jc}{}^{4}\varepsilon^{cde}j_{dea} - \Omega_{j4}{}^{c}\varepsilon_{cde}\bar{j}^{de}{}_{a}\right), \tag{137}$$

and

$$\mathcal{J}^{i}_{WZa} = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}{}^{\hat{c}}\left(\hat{\omega}^{\mu\hat{b}}_{j}\varepsilon_{\mu\nu}j^{\nu}_{\hat{c}\,a} + \hat{\chi}^{\hat{b}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{c}a}\right). \tag{138}$$

The expression for c.c. current density corresponding to $SU(4)/U(3)$ transformations with $\bar{y}_a$ parameters equals

$$\bar{\mathcal{J}}^{ia}(\tau,\sigma) = \bar{\mathcal{J}}^{i}_{AdS}{}^{a} + \bar{\mathcal{J}}^{i}_{CP}{}^{a} + \bar{\mathcal{J}}^{i}_{WZ}{}^{a}, \tag{139}$$

where

$$\bar{\mathcal{J}}^{i}_{AdS}{}^{a} = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm} + \hat{c}_{jm})\bar{j}^{ma} + \Delta_j\bar{j}^{a}\right), \tag{140}$$

$$\bar{\mathcal{J}}^{i}_{CP}{}^{a} = -\frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{jc}{}^{4}\varepsilon^{cde}\bar{j}_{de}{}^{a} - \Omega_{j4}{}^{c}\varepsilon_{cde}\bar{j}^{dea}\right), \tag{141}$$

and

$$\bar{\mathcal{J}}^{i}_{WZ}{}^{a} = \frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}{}^{\hat{c}}\left(\hat{\omega}^{\mu\hat{b}}_{j}\varepsilon_{\mu\nu}j^{\nu a}_{\hat{c}} + \hat{\chi}^{\hat{b}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{c}}{}^{a}\right). \tag{142}$$

4.3 $D=3$ $\mathcal{N}=6$ Poincare supersymmetry

The superstring action (5) is manifestly invariant under Poincare supersymmetry as $D=3$ $\mathcal{N}=6$ supercoordinates $(x^m, \theta^{\mu}_{a}, \bar\theta^{\mu a})$ that are the only non-trivially transforming ones enter through the supersymmetric Volkov-Akulov 1-forms [32]. Hence the non-invariance of the AdS_4 part of the supervielbein bosonic components is solely because of the current contributions

$$\delta_\varepsilon\hat\omega^m(d)+\delta_\varepsilon\hat c^m(d)=j^{ma}{}_{\mu}d\varepsilon^{\mu}_{a}-\bar j^{m}{}_{\mu a}d\bar\varepsilon^{\mu a},\quad \delta_\varepsilon\Delta(d)=j^a_\mu d\varepsilon^\mu_a-\bar j_{\mu a}d\bar\varepsilon^{\mu a}, \tag{143}$$

which explicit form is

$$\begin{aligned} j^{ma}{}_{\mu}=&\ \tfrac{\partial(\hat\omega^m+\hat c^m)(\delta_\varepsilon)}{\partial\varepsilon^\mu_a}=2\sigma^m_{\mu\nu}\left[ie^{-2\varphi}A\bar\theta^{\nu a}+e^{2\varphi}(\bar\eta\eta)\bar\eta^{\nu a}\right],\\ \bar j^{m}{}_{\mu a}=&\ -\tfrac{\partial(\hat\omega^m+\hat c^m)(\delta_\varepsilon)}{\partial\bar\varepsilon^{\mu a}}=-2\sigma^m_{\mu\nu}\left[ie^{-2\varphi}A\theta^\nu_a+e^{2\varphi}(\bar\eta\eta)\eta^\nu_a\right],\\ j^a_\mu=&\ \tfrac{\partial\Delta(\delta_\varepsilon)}{\partial\varepsilon^\mu_a}=-i\bar\eta^a_\mu,\ \bar j_{\mu a}=-\tfrac{\partial\Delta(\delta_\varepsilon)}{\partial\bar\varepsilon^{\mu a}}=i\eta_{\mu a}. \end{aligned} \tag{144}$$

$su(4)$ Cartan forms are also $D=3$ $\mathcal{N}=6$ super-Poincare invariant

$$\delta_\varepsilon\Omega_{\hat a}{}^{\hat b}(d)=J_{\hat a}{}^{\hat b a}{}_{\mu}d\varepsilon^\mu_a-\bar J_{\hat a}{}^{\hat b}{}_{\mu a}d\bar\varepsilon^{\mu a} \tag{145}$$

modulo the current contribution matrices

$$J_{\hat b}{}^{\hat c a}{}_{\mu}=\tfrac{\partial}{\partial\varepsilon^\mu_a}\Omega_{\hat b}{}^{\hat c}(\delta_\varepsilon)=\begin{pmatrix} j_b{}^{ca}{}_{\mu} & j_{bc}{}^{a}_{\mu}\\ j^{bca}{}_{\mu} & -j_c{}^{ba}{}_{\mu}\end{pmatrix}=2(\hat\eta_{\mu\hat b}\mathcal{T}^{\hat c a}-\mathcal{T}_{\hat b}{}^{a}\hat\eta^{\hat c}_{\mu}) \tag{146}$$

and

$$\bar J_{\hat b}{}^{\hat c}{}_{\mu a}=-\tfrac{\partial}{\partial\bar\varepsilon^{\mu a}}\Omega_{\hat b}{}^{\hat c}(\delta_\varepsilon)=\begin{pmatrix} \bar j_b{}^{c}{}_{\mu a} & -\bar j_{bc\mu a}\\ -\bar j^{bc}{}_{\mu a} & -\bar j_c{}^{b}{}_{\mu a}\end{pmatrix}=2(\hat\eta^{\hat c}_{\mu}\mathcal{T}_{\hat b a}-\hat\eta_{\mu\hat b}\mathcal{T}^{\hat c}{}_{a}) \tag{147}$$

so that the $\mathbb{CP}^3$ part of the bosonic supervielbein transforms as

$$\delta_\varepsilon\Omega_b{}^4(d)=\frac{1}{2}\varepsilon_{bcd}(j^{cda}{}_{\mu}d\varepsilon^\mu_a+\bar j^{cd}{}_{\mu a}d\bar\varepsilon^{\mu a}),\quad \delta_\varepsilon\Omega_4{}^b(d)=-\frac{1}{2}\varepsilon^{bcd}(j_{cd}{}^{a}_{\mu}d\varepsilon^\mu_a+\bar j_{cd\mu a}d\bar\varepsilon^{\mu a}). \tag{148}$$

Cartan forms associated with the Poincare supersymmetry are manifestly $D=3$ $\mathcal{N}=6$ super-Poincare invariant

$$\delta_\varepsilon\hat\omega^\nu_{\hat b}(d)=j^{\nu a}_{\hat b\,\mu}d\varepsilon^\mu_a+j^\nu_{\hat b\,\mu a}d\bar\varepsilon^{\mu a} \tag{149}$$

up to the current contributions

$$\begin{aligned} j^{\nu a}_{\hat b\,\mu}=&\ \tfrac{\partial\hat\omega^\nu_{\hat b}(\delta_\varepsilon)}{\partial\varepsilon^\mu_a}=e^{-\varphi}\left(\delta^\nu_\mu\mathcal{T}_{\hat b}{}^{a}+2i\hat\eta_{\mu\hat b}\bar\theta^{\nu a}\right),\\ j^\nu_{\hat b\,\mu a}=&\ \tfrac{\partial\hat\omega^\nu_{\hat b}(\delta_\varepsilon)}{\partial\bar\varepsilon^{\mu a}}=e^{-\varphi}\left(\delta^\nu_\mu\mathcal{T}_{\hat b a}+2i\hat\eta_{\mu\hat b}\theta^\nu_a\right), \end{aligned} \tag{150}$$

as well as Cartan forms associated with the conformal supersymmetry

$$\delta_\varepsilon\hat\chi_{\nu\hat b}(d)=J_{\nu\hat b\mu}{}^{a}d\varepsilon^\mu_a+J_{\nu\hat b\mu a}d\bar\varepsilon^{\mu a} \tag{151}$$

with the corresponding current contributions given by

$$\begin{aligned} J_{\nu\hat b\mu}{}^{a}=&\ \tfrac{\partial\hat\chi_{\nu\hat b}(\delta_\varepsilon)}{\partial\varepsilon^\mu_a}=ie^{\varphi}\left[2\hat\eta_{\mu\hat b}\bar\eta^a_\nu-e^{\varphi}(\bar\eta\eta)\varepsilon_{\nu\lambda}j^{\lambda a}_{\hat b\,\mu}\right],\\ J_{\nu\hat b\mu a}=&\ \tfrac{\partial\hat\chi_{\nu\hat b}(\delta_\varepsilon)}{\partial\bar\varepsilon^{\mu a}}=ie^{\varphi}\left[2\hat\eta_{\mu\hat b}\eta_{\nu a}-e^{\varphi}(\bar\eta\eta)\varepsilon_{\nu\lambda}j^{\lambda}_{\hat b\,\mu a}\right]. \end{aligned} \tag{152}$$

Putting all together contributions (144), (148), (150), (152) allows to determine the current density related to $D=3$ $\mathcal{N}=6$ supersymmetry invariance of the superstring action (5)

$$\mathcal{J}^{ia}_{\ \ \mu}(\tau,\sigma)=\mathcal{J}^{i\ \ a}_{AdS\mu}+\mathcal{J}^{i\ \ a}_{CP\mu}+\mathcal{J}^{i\ \ a}_{WZ\mu} \tag{153}$$

and the c.c. one

$$\bar{\mathcal{J}}^{i}_{\ \mu a}(\tau,\sigma)=\bar{\mathcal{J}}^{i}_{AdS\mu a}+\bar{\mathcal{J}}^{i}_{CP\mu a}+\bar{\mathcal{J}}^{i}_{WZ\mu a}. \tag{154}$$

The individual summands entering the current density (153) equal

$$\mathcal{J}^{i\ \ a}_{AdS\mu}=-\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm}+\hat{c}_{jm})j^{ma}_{\ \ \ \mu}+\Delta_j j^a_\mu\right), \tag{155}$$

$$\mathcal{J}^{i\ \ a}_{CP\mu}=\frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{jb}{}^4\varepsilon^{bcd}j_{cd\mu}^{\ \ \ \ a}-\Omega_{j4}{}^b\varepsilon_{bcd}j^{cda}_{\ \ \ \ \mu}\right) \tag{156}$$

and

$$\mathcal{J}^{i\ \ a}_{WZ\mu}=\frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}{}^{\hat{c}}\left(\hat{\omega}^{\mu\hat{b}}_j\varepsilon_{\mu\nu}j^{\nu a}_{\hat{c}\ \mu}+\hat{\chi}^{\hat{b}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{c}\mu}{}^a\right). \tag{157}$$

Analogously for the c.c. current density (154) one obtains

$$\bar{\mathcal{J}}^{i}_{AdS\mu a}=\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm}+\hat{c}_{jm})\bar{j}^{m}_{\ \ \mu a}+\Delta_j\bar{j}_{\mu a}\right), \tag{158}$$

$$\bar{\mathcal{J}}^{i}_{CP\mu a}=\frac{1}{4}\sqrt{-g}g^{ij}\left(\Omega_{jb}{}^4\varepsilon^{bcd}\bar{j}_{cd\mu a}-\Omega_{j4}{}^b\varepsilon_{bcd}\bar{j}^{cd}_{\ \ \ \mu a}\right) \tag{159}$$

and

$$\bar{\mathcal{J}}^{i}_{WZ\mu a}=\frac{i}{4}\varepsilon^{ij}\mathfrak{J}_{\hat{b}}{}^{\hat{c}}\left(\hat{\omega}^{\mu\hat{b}}_j\varepsilon_{\mu\nu}j^{\nu}_{\hat{c}\ \mu a}+\hat{\chi}^{\hat{b}}_{j\mu}\varepsilon^{\mu\nu}J_{\nu\hat{c}\mu a}\right). \tag{160}$$

4.4 $D=3$ $\mathcal{N}=6$ conformal supersymmetry

Transformation properties of the AdS part of supervielbein bosonic components can be read off from the general expressions (44) by appropriately restricting the parameters of $SO(1,3)$ compensating rotations

$$\begin{aligned}\delta_\xi\hat{\omega}^m(d)+\delta_\xi\hat{c}^m(d)=&\ \ j^{m\mu a}d\xi_{\mu a}-\bar{j}^{m\mu}_{\ \ \ a}d\bar{\xi}^a_\mu+(\hat{l}|_\xi)^{mn}(\hat{\omega}_n(d)+\hat{c}_n(d))+4(\hat{b}|_\xi)^m\Delta(d),\\ \delta_\xi\Delta(d)=&\ \ j^{\mu a}d\xi_{\mu a}-\bar{j}^\mu_a d\bar{\xi}^a_\mu-(\hat{b}|_\xi)^m(\hat{\omega}_m(d)+\hat{c}_m(d))\end{aligned} \tag{161}$$

and adding the current contribution terms

$$\begin{aligned}j^{m\mu a}=&\ \ \frac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_\xi)}{\partial\xi_{\mu a}}=2ie^{-2\varphi}A\bar{\theta}^{\lambda a}\sigma^m_{\lambda\nu}\hat{\dot{x}}^{\nu\mu}+2ie^{2\varphi}\bar{\eta}^a_\lambda\tilde{\sigma}^{m\lambda\nu}\Lambda_{-\nu}{}^\mu\\ +&\ \ 2e^{2\varphi}(\bar{\eta}\eta)\left(\bar{\eta}^{\lambda a}\sigma^m_{\lambda\nu}\hat{\dot{x}}^{\nu\mu}-\bar{\theta}^a_\lambda\tilde{\sigma}^{m\lambda\nu}\Lambda_{+\nu}{}^\mu\right),\\ \bar{j}^{m\mu}_{\ \ \ a}=&\ -\frac{\partial(\hat{\omega}^m+\hat{c}^m)(\delta_\xi)}{\partial\bar{\xi}^a_\mu}=-2ie^{-2\varphi}A\theta^\lambda_a\sigma^m_{\lambda\nu}\hat{\dot{x}}^{\nu\mu}-2ie^{2\varphi}\eta_{\lambda a}\tilde{\sigma}^{m\lambda\nu}\Lambda_{-\nu}{}^\mu\\ -&\ \ 2e^{2\varphi}(\bar{\eta}\eta)\left(\eta^\lambda_a\sigma^m_{\lambda\nu}\hat{\dot{x}}^{\nu\mu}-\theta_{\lambda a}\tilde{\sigma}^{m\lambda\nu}\Lambda_{+\nu}{}^\mu\right),\\ j^{\mu a}=&\ \ \frac{\partial\Delta(\delta_\xi)}{\partial\xi_{\mu a}}=-i\bar{\theta}^{\nu a}\Lambda_{-\nu}{}^\mu-i\bar{\eta}^a_\nu\hat{\dot{x}}^{\nu\mu},\quad \bar{j}^\mu_a=-\frac{\partial\Delta(\delta_\xi)}{\partial\bar{\xi}^a_\mu}=i\theta^\nu_a\Lambda_{-\nu}{}^\mu+i\eta_{\nu a}\hat{\dot{x}}^{\nu\mu}.\end{aligned} \tag{162}$$

Variation of the matrix of $su(4)$ Cartan forms under the coordinate-dependent conformal supersymmetry transformations is presented in the following form

$$\delta_\xi\Omega_{\hat{b}}{}^{\hat{c}}(d)=\hat{J}_{\hat{b}}{}^{\hat{c}\mu a}d\xi_{\mu a}-\hat{\bar{J}}_{\hat{b}}{}^{\hat{c}\mu}{}_a d\bar{\xi}^a_\mu+i\left(\Omega_{\hat{b}}{}^{\hat{d}}(d)(\widehat{W}|_\xi)_{\hat{d}}{}^{\hat{c}}-(\widehat{W}|_\xi)_{\hat{b}}{}^{\hat{d}}\Omega_{\hat{d}}{}^{\hat{c}}(d)\right)-d(\widehat{W}|_\xi)_{\hat{b}}{}^{\hat{c}}, \tag{163}$$

where the current contributions

$$\hat{J}_{\hat{b}}{}^{\hat{c}\mu a} = \frac{\partial}{\partial\xi_{\mu a}}\Omega_{\hat{b}}{}^{\hat{c}}(\delta_\xi) = (TJ^{\mu a}\bar{T})_{\hat{b}}{}^{\hat{c}} = \begin{pmatrix} \hat{j}_b{}^{c\mu a} & \hat{j}_{bc}{}^{\mu a} \\ \hat{j}^{bc\mu a} & -\hat{j}_c{}^{b\mu a} \end{pmatrix} \quad (164)$$

and

$$\hat{\bar{j}}_{\hat{b}}{}^{\hat{c}\mu}{}_a = -\frac{\partial}{\partial\bar{\xi}^a_\mu}\Omega_{\hat{b}}{}^{\hat{c}}(\delta_\xi) = (T\bar{J}^\mu_a\bar{T})_{\hat{b}}{}^{\hat{c}} = \begin{pmatrix} \hat{\bar{j}}_b{}^{c\mu}{}_a & -\hat{\bar{j}}_{bca}{}^{\mu} \\ -\hat{\bar{j}}^{bc\mu}{}_a & -\hat{\bar{j}}_c{}^{b\mu}{}_a \end{pmatrix} \quad (165)$$

are obtained by the $T-$transformation of the matrices

$$\begin{aligned} J_{\hat{b}}{}^{\hat{c}\mu a} &= \frac{\partial}{\partial\xi_{\mu a}}(W_{\hat{b}}{}^{\hat{c}} + \Psi_{\hat{b}}{}^{\hat{c}}(\delta_\xi)) = 2(\delta_{\hat{b}}{}^a\Theta^{\mu\hat{c}} - \delta^{a\hat{c}}\Theta^\mu_{\hat{b}}) - 4i\bar{\theta}^{\nu a}(\eta_{\nu\hat{b}}\Theta^{\mu\hat{c}} + \Theta^\mu_{\hat{b}}\eta^{\hat{c}}_\nu), \\ \bar{J}_{\hat{b}}{}^{\hat{c}\mu}{}_a &= -\frac{\partial}{\partial\bar{\xi}^a_\mu}(W_{\hat{b}}{}^{\hat{c}} + \Psi_{\hat{b}}{}^{\hat{c}}(\delta_\xi)) = 2(\delta_a{}^{\hat{c}}\Theta^\mu_{\hat{b}} - \delta_{\hat{b}}{}^a\Theta^{\mu\hat{c}}) + 4i\theta^\nu_a(\Theta^\mu_{\hat{b}}\eta^{\hat{c}}_\nu + \eta_{\nu\hat{b}}\Theta^{\mu\hat{c}}). \end{aligned} \quad (166)$$

So that the current contribution matrices (164) and (165) acquire the form

$$\hat{J}_{\hat{b}}{}^{\hat{c}\mu a} = 2(\mathcal{T}_{\hat{b}}{}^a\hat{\Theta}^{\mu\hat{c}} - \hat{\Theta}^\mu_{\hat{b}}\mathcal{T}^{\hat{c}a}) \quad (167)$$

and

$$\hat{\bar{J}}_{\hat{b}}{}^{\hat{c}\mu}{}_a = 2(\hat{\Theta}^\mu_{\hat{b}}\mathcal{T}^{\hat{c}}{}_a - \mathcal{T}_{\hat{b}a}\hat{\Theta}^{\mu\hat{c}}). \quad (168)$$

Then the variation under conformal supersymmetry transformations of the $\mathbb{CP}^3$ components of the supervielbein is brought to the form

$$\begin{aligned} \delta_\xi\Omega_b{}^4(d) =& \ \tfrac{1}{2}\varepsilon_{bcd}(\hat{j}^{cd\mu a}d\xi_{\mu a} + \hat{\bar{j}}^{cd\mu}{}_a d\bar{\xi}^a_\mu) + i(\hat{w}|_\xi)_c{}^c\Omega_b{}^4(d) - i(\hat{w}|_\xi)_b{}^c\Omega_c{}^4(d), \\ \delta_\xi\Omega_4{}^b(d) =& \ -\tfrac{1}{2}\varepsilon^{bcd}(\hat{j}_{cd}{}^{\mu a}d\xi_{\mu a} + \hat{\bar{j}}_{cda}{}^{\mu}d\bar{\xi}^a_\mu) - i(\hat{w}|_\xi)_c{}^c\Omega_4{}^b(d) + i\Omega_4{}^c(d)(\hat{w}|_\xi)_c{}^b. \end{aligned} \quad (169)$$

The variation of Cartan forms associated with the super-Poincare generators can be extracted from the general expression (49)

$$\delta_\xi\hat{\omega}^\nu_{\hat{b}}(d) = j^{\nu\mu a}_{\hat{b}}d\xi_{\mu a} + j^{\nu\mu}_{\hat{b}\,a}d\bar{\xi}^a_\mu + \frac{1}{4}\hat{\omega}^\lambda_{\hat{b}}(d)(\hat{l}|_\xi)^{mn}\sigma_{mn\lambda}{}^\nu + (\hat{\bar{b}}|_\xi)^{\nu\lambda}\hat{\chi}_{\lambda\hat{b}}(d) - i(\widehat{W}|_\xi)_{\hat{b}}{}^{\hat{c}}\hat{\omega}^\nu_{\hat{c}}(d) \quad (170)$$

with the current contributions given by

$$j^{\nu\mu a}_{\hat{b}} = \frac{\partial\hat{\omega}^\nu_{\hat{b}}(\delta_\xi)}{\partial\xi_{\mu a}} = e^{-\varphi}(\mathcal{T}_{\hat{b}}{}^a\hat{\tilde{x}}^{\nu\mu} + 2i\bar{\theta}^{\nu a}\hat{\Theta}^\mu_{\hat{b}}), \quad j^{\nu\mu}_{\hat{b}\,a} = \frac{\partial\hat{\omega}^\nu_{\hat{b}}(\delta_\xi)}{\partial\bar{\xi}^a_\mu} = e^{-\varphi}(\mathcal{T}_{\hat{b}a}\hat{\tilde{x}}^{\nu\mu} + 2i\theta^\nu_a\hat{\Theta}^\mu_{\hat{b}}). \quad (171)$$

Similarly the variation of Cartan forms associated with conformal supersymmetry generators reads

$$\delta_\xi\hat{\chi}_{\nu\hat{b}}(d) = J_{\nu\hat{b}}{}^{\mu a}d\xi_{\mu a} + J_{\nu\hat{b}a}{}^{\mu}d\bar{\xi}^a_\mu - \frac{1}{4}(\hat{l}|_\xi)^{mn}\sigma_{mn\nu}{}^\lambda\hat{\chi}_{\lambda\hat{b}}(d) + (\hat{b}|_\xi)_{\nu\lambda}\hat{\omega}^\lambda_{\hat{b}}(d) - i(\widehat{W}|_\xi)_{\hat{b}}{}^{\hat{c}}\hat{\chi}_{\nu\hat{c}}(d) \quad (172)$$

with the corresponding current contributions

$$\begin{aligned} J_{\nu\hat{b}}{}^{\mu a} =& \ \frac{\partial\hat{\chi}_{\nu\hat{b}}(\delta_\xi)}{\partial\xi_{\mu a}} = e^\varphi(\Lambda_{-\nu}{}^\mu\mathcal{T}^a_{\hat{b}} + 2i\bar{\eta}^a_\nu\hat{\Theta}^\mu_{\hat{b}} - ie^\varphi(\bar{\eta}\eta)\varepsilon_{\nu\lambda}j^{\lambda\mu a}_{\hat{b}}), \\ J_{\nu\hat{b}a}{}^{\mu} =& \ \frac{\partial\hat{\chi}_{\nu\hat{b}}(\delta_\xi)}{\partial\bar{\xi}^a_\mu} = e^\varphi(\Lambda_{-\nu}{}^\mu\mathcal{T}_{\hat{b}a} + 2i\eta_{\nu a}\hat{\Theta}^\mu_{\hat{b}} - ie^\varphi(\bar{\eta}\eta)\varepsilon_{\nu\lambda}j^{\lambda\mu}_{\hat{b}\,a}). \end{aligned} \quad (173)$$

Finally the current density associated with the superstring action (5) invariance under conformal supersymmetry takes the form

$$\mathcal{J}^{i\mu a}(\tau,\sigma) = \mathcal{J}^i_{AdS}{}^{\mu a} + \mathcal{J}^i_{CP}{}^{\mu a} + \mathcal{J}^i_{WZ}{}^{\mu a}. \quad (174)$$

Current contributions (162) enter the AdS part of the current density

$$\mathcal{J}^{i}_{AdS}{}^{\mu a}=-\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm}+\hat{c}_{jm})j^{m\mu a}+\Delta_j j^{\mu a}\right), \tag{175}$$

while those entering Eq.(169) determine the $\mathbb{CP}^3$ part of the superconformal current density

$$\mathcal{J}^{i}_{CP}{}^{\mu a}=\frac{1}{4}\sqrt{-g}g^{ij}(\Omega_{jb}{}^{4}\varepsilon^{bcd}\hat{j}_{cd}{}^{\mu a}-\Omega_{j4}{}^{b}\varepsilon_{bcd}\hat{j}^{cd\mu a}), \tag{176}$$

The form of WZ term contribution to the conformal supersymmetry current is determined by Eqs.(171) and (173)

$$\mathcal{J}^{i}_{WZ}{}^{\mu a}=\frac{i}{4}\varepsilon^{ij}\mathfrak{I}_{\hat{b}}{}^{\hat{c}}(\hat{\omega}^{\lambda\hat{b}}_{j}\varepsilon_{\lambda\nu}j^{\nu\mu a}_{\hat{c}}+\hat{\chi}^{\hat{b}}_{j\lambda}\varepsilon^{\lambda\nu}J_{\nu\hat{c}}{}^{\mu a}). \tag{177}$$

The expression for the current density related to c.c. transformation parameters $\bar{\xi}^{a}_{\mu}$ is

$$\bar{\mathcal{J}}^{i\mu}_{a}(\tau,\sigma)=\bar{\mathcal{J}}^{i}_{AdS}{}^{\mu}_{a}+\bar{\mathcal{J}}^{i}_{CP}{}^{\mu}_{a}+\bar{\mathcal{J}}^{i}_{WZ}{}^{\mu}_{a}. \tag{178}$$

Corresponding summands that enter c.c. current density equal

$$\bar{\mathcal{J}}^{i}_{AdS}{}^{\mu}_{a}=\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{jm}+\hat{c}_{jm})\bar{j}^{m\mu}{}_{a}+\Delta_j\bar{j}^{\mu}_{a}\right), \tag{179}$$

$$\bar{\mathcal{J}}^{i}_{CP}{}^{\mu}_{a}=\frac{1}{4}\sqrt{-g}g^{ij}(\Omega_{jb}{}^{4}\varepsilon^{bcd}\hat{\bar{j}}_{cd}{}^{\mu}_{a}-\Omega_{j4}{}^{b}\varepsilon_{bcd}\hat{\bar{j}}^{cd\mu}{}_{a}), \tag{180}$$

$$\bar{\mathcal{J}}_{WZ}{}^{i\mu}_{a}=\frac{i}{4}\varepsilon^{ij}\mathfrak{I}_{\hat{b}}{}^{\hat{c}}(\hat{\omega}^{\lambda\hat{b}}_{j}\varepsilon_{\lambda\nu}j^{\nu\mu}_{\hat{c}\,a}+\hat{\chi}^{\hat{b}}_{j\lambda}\varepsilon^{\lambda\nu}J_{\nu\hat{c}}{}^{\mu}_{a}). \tag{181}$$

5 Conclusion

The $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset sigma-model [9], [10] describes manifestly classically integrable part of the $AdS_4\times\mathbb{CP}^3$ superstring action [11]. By virtue of the isomorphism between the $osp(4|6)$ superalgebra and $D=3$ $\mathcal{N}=6$ superconformal algebra it can be presented in the conformal basis for $osp(4|6)/(so(1,3)\times u(3))$ Cartan forms [20] that are identified with the 'reduced' $(10|24)-$dimensional $D=10$ $\mathcal{N}=2A$ superspace vielbein obtained from the full one [11] by setting to zero 8 fermionic coordinates related to space-time supersymmetries broken by the $AdS_4\times\mathbb{CP}^3$ superbackground. The $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset sigma-model action is by construction invariant under global $OSp(4|6)$ supergroup transformations and hence is also invariant under $D=3$ $\mathcal{N}=6$ superconformal symmetry that is global symmetry of the ABJM gauge theory [33]. In this paper we have derived explicit expressions for the corresponding world-sheet current densities associated with each type of the transformations from $D=3$ $\mathcal{N}=6$ superconformal symmetry. Considering the $OSp(4|6)/(SO(1,3)\times U(3))$ supercoset element parametrized by $D=3$ $\mathcal{N}=6$ super-Poincare coordinates supplemented by those related to the generators of $3d$ dilatations and conformal supersymmetry we have also derived their full transformations under $D=3$ $\mathcal{N}=6$ superconformal symmetry. So that passing to the canonical formulation it should possible to calculate the algebra of associated supercharges.

Among the potential applications of the obtained results one could mention the semiclassical quantization around the solutions to the $OSp(4|6)/(SO(1,3)\times U(3))$ superstring equations of motion [34]. They are also the starting point to examine residual symmetry algebras surviving fixing the gauge symmetries of $OSp(4|6)/(SO(1,3)\times U(3))$ superstring action (see, e.g. [35]).

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