Fermion localization on branes with generalized dynamics

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Abstract

In a recent paper published in this journal, Bazeia and collaborators [Phys. Lett. B **671**, (2009) 402] analyze braneworld models with nonstandard dynamics. It was found that the generalized braneworld scenario is classically stable and capable of localize gravity. In this present work, we complete the analyze of the above research, we will focus our attention on the matter energy density, energy of system and the Ricci scalar. Additionally, as a natural extension, we address the issue of fermion localization of fermions on a thick brane constructed from one scalar field with nonstandard kinetic terms coupled with gravity. The contribution of the nonstandard kinetic terms in the problem of fermion localization is analyzed. It is found that the simplest Yukawa coupling $\eta\bar{\Psi}\phi\Psi$ support the localization of fermions on the thick brane. It is shown that the zero mode for left-handed and right-handed fermions can be localized on the thick brane depending on the values for the coupling constant η and the brane model parameters. Furthermore, we present an argument that could jeopardize the results of previous work on massive modes.

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I. INTRODUCTION

In the past decade, the braneworld scenario has attracted a lot of interests for it gives an effective way to solve the hierarchy problem by introducing two 3-branes which are embedded in a five-dimensional anti-de Sitter (AdS_5) space-time [1]. As another attractive property, the Newtonian law of gravity with a correction is also given in this braneworld scenario [2]. In the Randall-Sundrum model [1], we can further add scalar fields [3] with usual dynamics and allow them to interact with gravity in the standard way. In this scenario, the smooth character of the solutions generate thick brane with a diversity of structures [4]-[7]. In the braneworld scenarios, an important issue is how gravity and different observable matter fields of the Standard Model of particle physics are localized on the brane. It has been shown that, in the Randall-Sundrum model in 5-dimensional space-time, graviton and spin 0 field can be localized on a brane with positive tension [2],[8]-[9]. And moreover spin 1/2 and 3/2 can be localized on a negative-tension brane [9]. The localization problem of spin-1/2 fermions on thick branes is interesting and important [8]-[26]. In order to achieve localization of fermions on a brane with positive tension, it seems that additional interactions except the gravitational interaction must be including in the bulk. On the other hand, the first recent observations [27]-[28] have led us with the intriguing fact that the Universe is presently undertaking accelerated expansion. These information directly contributed to establish some important advances in cosmology, one of them being the presence of dark energy. The presence of dark energy has opened some distinct routes of investigations. In recent years, there appeared some interesting models with noncanonical dynamics with focus on early time inflation or dark energy [29]-[32], as for instance, the so-called k-fields, first introduced in the context of inflation [32] and the k-essence models, suggested to solve the cosmic coincidence problem [31],[33]. The interaction between dark energy and fermion fields has already a precedent in the cosmology context [34]. We believe that the conditions for obtaining normalizable zero modes on brane model with generalized dynamics deserve to be more explored.

In this paper, we reinvestigate braneworld model with nonstandard dynamics. The model $\mathcal{L} = K(X) - V(\phi)$, where $K = X + \alpha |X| X$ (type I model in Ref. [29]), is considered, we will focus our attention on the matter energy, energy of system and the Ricci scalar. As the model is classically stable and capable of localize gravity, additionally we address the issue of fermion localization on a thick brane constructed from one scalar field with nonstandard kinetic terms coupled with gravity. We use the analytical expressions for small α and we investigate the contribution of this nonstandard kinetic terms in the problem of fermion localization. We find that the simplest Yukawa coupling $\eta\bar{\Psi}\phi\Psi$, where η is the coupling constant, allowed left-handed or right-handed fermions to posses a zero mode that localize on the thick brane under some conditions on the value for the coupling constant η and the brane model parameters. The organization of this paper is as follows: in Sec. II, we present a review of brane models with generalized dynamics (type I model)[29]. In Sec. III, we study the localization of spin-1/2 fermion for this model, we analyze the essential conditions for the localization with the simplest Yukawa coupling and we present an argument that could jeopardize the results of previous work on massive modes. Finally, our conclusions are presented in Sec. IV.

II. REVIEW OF SYSTEMS WITH GENERALIZED DYNAMICS

The action for this kind of system is described by [29]

$$S=\int d^5x\sqrt{|\,g|}\,\left[-\frac{1}{4}R+\mathcal{L}(\phi,X)\right], \tag{1}$$

where $g\equiv\mathrm{Det}(g_{ab})$ and $X=\frac{1}{2}\nabla^a\phi\nabla_a\phi$. The line element of the five-dimensional space-time can be written as

$$ds^2=g_{ab}dx^adx^b=\mathrm{e}^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu-dy^2, \tag{2}$$

where we are using the five-dimensional Newton constat $4\pi G^{(5)}=1$, $y=x^4$ is the extra dimension (the Latin indices run from 0 to 4), $\eta_{\mu\nu}$ is the Minkowski metric with signature $(+,-,-,-)$ and e^{2A} is the so-called warp factor (the Greek indices run from 0 to 3). We suppose that $A=A(y)$ and $\phi=\phi(y)$.

One can determine the static equations of motion for the above system

$$\left(\mathcal{L}_X+2X\mathcal{L}_{XX}\right)\phi''-\left(2X\mathcal{L}_{X\phi}-\mathcal{L}_\phi\right)=-4\mathcal{L}_XA'\phi', \tag{3}$$

$$A''+2A'^{\,2}=\frac{2}{3}\,\mathcal{L}\,, \tag{4}$$

$$A'^{\,2}=\frac{1}{3}\left(\mathcal{L}-2X\mathcal{L}_X\right), \tag{5}$$

where prime stands for derivate with respect to y. From eqs. (4) and (5), we obtain

$$A'' = \frac{4}{3}\, X \mathcal{L}_X. \tag{6}$$

The matter energy density is given by

$$\rho(y) = -\mathrm{e}^{2A(y)} \mathcal{L}\,, \tag{7}$$

and the scalar curvature is given by

$$R = -4\left(5A'^{\,2} + 2A''\right). \tag{8}$$

Let us consider the Lagrange density as

$$\mathcal{L} = K(X) - V(\phi), \tag{9}$$

for this case, from Eqs. (3), (5) and (6) the equations of motion can be expressed as

$$\left(K' + 2XK''\right)\phi'' - V_\phi = -4K'A'\phi', \tag{10}$$

$$A'' = \frac{4}{3}\, XK', \tag{11}$$

$$A'^{\,2} = \frac{1}{3}\left(K - V - 2XK'\right), \tag{12}$$

Now, we consider an explicit example for $K(X)$.

A. The model: $K(X) = X + \alpha|X|X$

This model is consider in [29], here α is a real, non-negative parameter and $X = -\frac{1}{2}\,\phi'^{\,2}$. For this case, the equations of motion becomes

$$\phi'' + 4A'\phi' - V_\phi = -\alpha\left(3\phi'' + 4\phi' A'\right)\phi'^{\,2}, \tag{13}$$

$$A'' = -\frac{2}{3}\,\phi'^{\,2}\left(1 + \alpha\phi'^{\,2}\right), \tag{14}$$

$$A'^{\,2} = \frac{1}{6}\left(1 + \frac{3}{2}\,\alpha\phi'^{\,2}\right)\phi'^{\,2} - \frac{1}{3}\,V\,, \tag{15}$$

we can rewrite (14) and (15) as

$$A'' + 2A'^{\,2} = -\frac{1}{3}\,\phi'^{\,2}\left(1 + \frac{\alpha}{2}\,\phi'^{\,2}\right) - \frac{2}{3}\,V\,, \tag{16}$$

this equation is the static equation of motion (4). Using the expression

$$A' = -\frac{1}{3}\,W(\phi)\,, \tag{17}$$

we can recast the Eq. (14) as

$$\phi' + \alpha\phi'^{\,3} = \frac{1}{2}\,W_\phi\,. \tag{18}$$

The real solution for (18) is given by

$$\phi' = \frac{m(W_\phi)}{6\alpha} - \frac{2}{m(W_\phi)}\,, \tag{19}$$

where

$$m(W_\phi) = \left(54\alpha^2 W_\phi + 6\sqrt{3}\left(16\alpha^3 + 27\alpha^4 W_\phi^2\right)^{1/2}\right)^{1/3}. \tag{20}$$

Substituting (17) in (15), we get

$$V(\phi) = \frac{1}{2}\,\phi'^{\,2} + \frac{3}{4}\,\alpha\phi'^{\,4} - \frac{1}{3}W(\phi)^2\,, \tag{21}$$

where ϕ' is given by (19).

On the other hand, we can define a functional as [4]

$$E\,[A,\phi] = \int dy\,(-\mathcal{L}_{\text{system}})\,, \tag{22}$$

where $\mathcal{L}_{\text{system}} = \sqrt{|g|}\,(-R/4 + \mathcal{L})$. For this case (22) becomes

$$E\,[A,\phi] = \int dy\,\mathrm{e}^{4A}\left\{\frac{1}{2}\,\phi'^{\,2} - 3A'^{\,2} + \frac{\alpha}{4}\,\phi'^{\,4} + V\right\}, \tag{23}$$

the functional $E\,[A,\phi]$ furnishes the static equations of motion (13) and (16). This functional is associated with the energy of system, as reported in [35].

In this point, it is instructive to analyze the matter energy, from (7) and (9) we obtain

$$E_\phi = \int dy\,\mathrm{e}^{2A}\left\{\frac{1}{2}\,\phi'^{\,2} + \frac{\alpha}{4}\,\phi'^{\,4} + V\right\}, \tag{24}$$

substituting (21) in (24), we get

$$E_\phi = \int dy\,\mathrm{e}^{2A}\left\{\phi'^{\,2} + \alpha\phi'^{\,4} - \frac{1}{3}\,W^2\right\}, \tag{25}$$

using (14) and (17) and integrating, we obtain

$$E_\phi = \frac{1}{2}\,\left(W\mathrm{e}^{2A}\right)_{-\infty}^{+\infty}\,, \tag{26}$$

this is the value of the matter energy for all α. Note that the matter energy depends on the asymptotic behavior of the warp factor (e^{2A}). In the same way, we can analyze the energy of system, from (21) and (23) and using (17) we get

$$E\,[A,\phi] = \int dy\,\mathrm{e}^{4A}\left\{\phi' + \alpha\phi'^{\,4} - 6A'^{\,2}\right\}, \tag{27}$$

substituting (14) and (17) in (27) and integrating, we obtain

$$E\,[A,\phi] = \frac{1}{2}\,\left(W\mathrm{e}^{4A}\right)_{-\infty}^{+\infty}\,, \tag{28}$$

one more time, the asymptotic behavior of the warp factor plays a leading role in the value of the functional $E\,[A,\phi]$.

We follow the same procedure of Ref. [29] and let us focus our study in the case of α very small. The solution of (18) becomes

$$\phi' = \frac{1}{2}\,W_\phi - \frac{\alpha}{8}\,W_\phi^3\,. \tag{29}$$

Substituting (29) in (21), we get

$$V(\phi) = \frac{1}{8}\,W_\phi^2 - \frac{\alpha}{64}\,W_\phi^4 - \frac{1}{3}\,W^2\,. \tag{30}$$

The solution for (29) becomes

$$\phi(y) = \phi_0(y) - \frac{\alpha}{4}\,W_\phi(\phi_0(y))W(\phi_0(y))\,, \tag{31}$$

where $\phi_0(y)$ is the solution when $\alpha = 0$. From (17) and (31), we obtain

$$A(y) = A_0(y) + \frac{\alpha}{12}\,W(\phi_0(y))^2, \tag{32}$$

where $A_0(y)$ represents $A(y)$ when $\alpha = 0$. The energy density given by (7) is

$$\rho = \mathrm{e}^{2A(y)}\left(\frac{1}{4}\,W_\phi^2 - \frac{1}{3}\,W^2 - \frac{\alpha}{16}\,W_\phi^4\right). \tag{33}$$

Substituting (31) and (32) in (33), we obtain

$$\rho = \rho_0 - \frac{\alpha}{48}\,\mathrm{e}^{2A_0(y)}\left(6W_{\phi\phi}W_\phi^2W - 10W^2W_\phi^2 + 3W_\phi^4 + \frac{8}{3}\,W^4\right)_{\phi=\phi_0}\,, \tag{34}$$

where

$$\rho_0 = \mathrm{e}^{2A_0(y)}\left(\frac{1}{4}\,W_\phi^2 - \frac{1}{3}\,W^2\right)_{\phi=\phi_0}\,, \tag{35}$$

note that the matter energy density (34) is a little bit different from that given in Ref. [29].

We consider that the superpotential $W(\phi)$ can be written as [6],[29]

$$W(\phi) = 3a\sin(b\phi)\,, \tag{36}$$

where a and b are real parameters. The classical solutions for (31) and (32) are given by

$$\phi(y) = \frac{1}{b}\arcsin\left[\tanh\left(\frac{3}{2}\,ab^2\,y\right)\right] - \frac{9a^2b\alpha}{4}\tanh\left(\frac{3}{2}\,ab^2\,y\right)\mathrm{sech}\left(\frac{3}{2}\,ab^2\,y\right), \tag{37}$$

and

$$A(y) = \frac{2}{3b^2}\ln\left[\mathrm{sech}\left(\frac{3}{2}\,ab^2\,y\right)\right] + \frac{3a^2\alpha}{4}\tanh^2\left(\frac{3}{2}\,ab^2\,y\right). \tag{38}$$

The profiles of the matter energy density is shown in Fig. 1 for some values of α, as in [29] the numerical study gives full support to the analytical expressions for α very small. The Fig. 1 clearly shows that the brane is localized at $y = 0$, because the region has a positive matter energy density. The contribution of the nonstandard kinetic term modifies the profile without altering the symmetrical form of matter energy density, as noted in Fig. 1. It known that the α-parameter, which indicates the strength of fourth-order kinetic term, used to modify the dynamics of the scalar field contributes to thicker the brane, as reported in [29]. The profiles of the matter energy density and the Ricci scalar are shown in Fig. 2 for $\alpha = 0.1$. Note that the presence of regions with positive Ricci scalar is connected with the localization of the brane and it reinforces the conclusion of the analyzes from the matter energy density. Also note that far from the brane R tends to a negative constant, characterizing the AdS_5 limit from the bulk, as reported in [23]. An similar behavior is obtained for $\alpha = 1$ and $\alpha = 10$. The profiles of the warp factor is shown in Fig. 3 for some values of α. The Fig. 3 shows that $\mathrm{e}^{2A} \to 0$ as $y \to \pm\infty$, therefore, the matter energy (26) and the energy of system (28) both are zero, this result is independent of α.

III. FERMION LOCALIZATION

The action for a Dirac spinor field coupled with the scalar fields by a general Yukawa coupling is

$$S = \int d^5x\sqrt{|\,g|}\,\left[i\bar{\Psi}\Gamma^M\nabla_M\Psi - \eta\bar{\Psi}F(\phi)\Psi\right]\,, \tag{39}$$

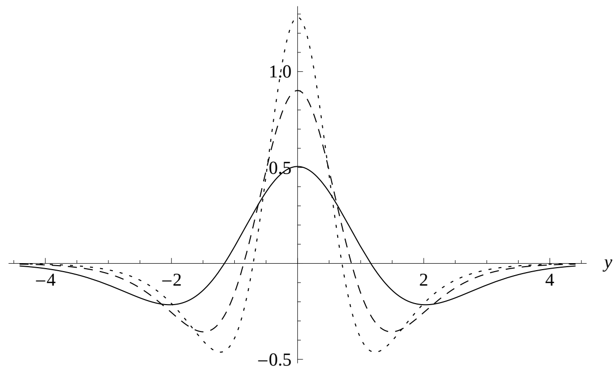

FIG. 1: The profiles of the energy density for $a = 1$, $b = \sqrt{2/3}$, $\alpha = 0.1$ (dot line), $\alpha = 1$ (dashed line) and $\alpha = 10$ (thin line).

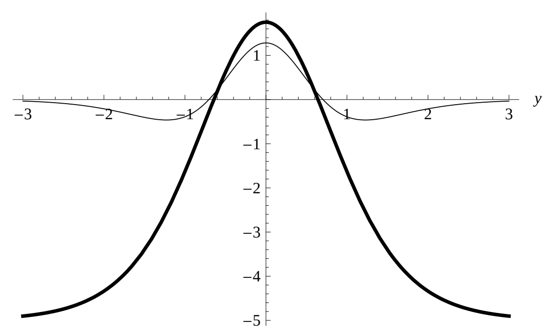

FIG. 2: The profiles of the matter energy density (thin line) and Ricci scalar (thick line) for $a = 1$, $b = \sqrt{2/3}$ and $\alpha = 0.1$.

where η is the positive coupling constant between fermions and the scalar field. Here we consider the field ϕ as a background field. The equation of motion is obtained as

$$i\,\Gamma^M \nabla_M \Psi - \eta F(\phi)\Psi = 0. \tag{40}$$

In this stage, it is useful to consider the current. The conservation law for J^M follows from the standard procedure and it becomes

$$\nabla_M J^M = \bar{\Psi}\left(\nabla_M \Gamma^M\right)\Psi\,, \tag{41}$$

where $J^M = \bar{\Psi}\Gamma^M\Psi$. Thus, if

$$\nabla_M \Gamma^M = 0\,, \tag{42}$$

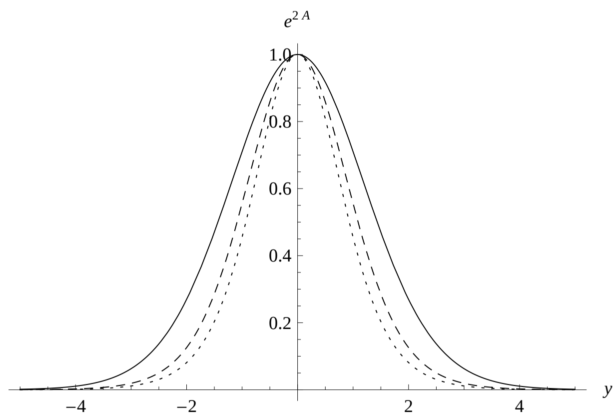

FIG. 3: The profiles of the warp factor for $a = 1$, $b = \sqrt{2/3}$, $\alpha = 0.1$ (dot line), $\alpha = 1$ (dashed line) and $\alpha = 10$ (thin line).

then four-current will be conserved. The condition (42) is the purely geometrical assertion that the curved-space gamma matrices are covariantly constant.

Using the same line element (2) and the representation for gamma matrices $\Gamma^M = \left(\mathrm{e}^{-A}\gamma^\mu, -i\gamma^5\right)$, the condition (42) is trivially satisfied and therefore the current is conserved.

The equation of motion (40) becomes

$$\left[i\gamma^\mu\partial_\mu + \gamma^5\mathrm{e}^A(\partial_y + 2\partial_y A) - \eta\,\mathrm{e}^A F(\phi)\right]\Psi = 0. \tag{43}$$

In this stage, we use the general chiral decomposition

$$\Psi(x,y) = \sum_n \psi_{L_n}(x)\alpha_{L_n}(y) + \sum_n \psi_{R_n}(x)\alpha_{R_n}(y), \tag{44}$$

with $\psi_{L_n}(x) = -\gamma^5\psi_{L_n}(x)$ and $\psi_{R_n}(x) = \gamma^5\psi_{R_n}(x)$. With this decomposition $\psi_{L_n}(x)$ and $\psi_{R_n}(x)$ are the left-handed and right-handed components of the four-dimensional spinor field, respectively. After applying (44) in (43), and demanding that $i\gamma^\mu\partial_\mu\psi_{L_n} = m_n\psi_{R_n}$ and $i\gamma^\mu\partial_\mu\psi_{R_n} = m_n\psi_{L_n}$, we obtain two equations for α_{L_n} and α_{R_n}

$$\left[\partial_y + 2\partial_y A + \eta F(\phi)\right]\alpha_{L_n} = m_n\mathrm{e}^{-A}\alpha_{R_n}\,, \tag{45}$$

$$\left[\partial_y + 2\partial_y A - \eta F(\phi)\right]\alpha_{R_n} = -m_n\mathrm{e}^{-A}\alpha_{L_n}\,. \tag{46}$$

Inserting the general chiral decomposition (44) into the action (39), using (45) and (46) and also requiring that the result take the form of the standard four-dimensional action for the massive chiral fermions

$$S=\sum_n\int d^4x\,\bar{\psi}_n\left(\gamma^\mu\partial_\mu-m_n\right)\psi_n, \tag{47}$$

where $\psi_n=\psi_{L_n}+\psi_{R_n}$ and $m_n\geq 0$, the functions α_{L_n} and α_{R_n} must obey the following orthonormality conditions

$$\int_{-\infty}^{\infty}dy\,\mathrm{e}^{3A}\alpha_{Lm}\alpha_{Rn}=\delta_{LR}\delta_{mn}. \tag{48}$$

Implementing the change of variables

$$z=\int_0^y\mathrm{e}^{-A(y')}dy', \tag{49}$$

$\alpha_{L_n}=\mathrm{e}^{-2A}L_n$ and $\alpha_{R_n}=\mathrm{e}^{-2A}R_n$, we get

$$-L_n''(z)+V_L(z)L_n=m_n^2L_n\,, \tag{50}$$

$$-R_n''(z)+V_L(z)R_n=m_n^2R_n\,, \tag{51}$$

where

$$V_L(z)\;=\;\eta^2\mathrm{e}^{2A}F^2(\phi)-\eta\partial_z\left(\mathrm{e}^AF(\phi)\right), \tag{52}$$

$$V_R(z)\;=\;\eta^2\mathrm{e}^{2A}F^2(\phi)+\eta\partial_z\left(\mathrm{e}^AF(\phi)\right). \tag{53}$$

Using the expressions $\partial_zA=\mathrm{e}^{A(y)}\partial_yA$ and $\partial_zF=\mathrm{e}^{A(y)}\partial_yF$, we can recast the potentials (52) and (53) as a function of y [24],[25]

$$V_L(z(y))\;=\;\eta\mathrm{e}^{2A}\left[\eta F^2-\partial_yF-F\partial_yA(y)\right] \tag{54}$$

$$V_R(z(y))\;=\;V_L(z(y))|_{\eta\to-\eta} \tag{55}$$

It is worthwhile to note that we can construct the Schrödinger potentials V_L and V_R from (54) and (55). This procedure is used just to have a qualitative analysis of the profile potential.

In this stage, it is instructive to state that with the change of variable (49) we get a geometry to be conformally flat

$$ds^2=\mathrm{e}^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu-dz^2\right). \tag{56}$$

Now we focus attention on the condition (42) for the line element (56). In this case we obtain

$$\nabla_M\Gamma^M=i(\partial_zA(z))\mathrm{e}^{-A(z)}\gamma^5. \tag{57}$$

Therefore, the current is not conserved for the line element (56). It is known that, in general, the reformulation of the theory in a new conformal frame leads to a different, physically inequivalent theory. This issue has already a precedent in cosmological models [36]. Recently, other inconsistencies are stated in [26].

Under this arguments, we use only the change of variable (49) to have a qualitative analysis of the potential profile, which is an fundamental ingredient for the fermion localization on the brane.

Now we focus attention on the calculation of the zero mode. Substituting $m_n = 0$ in (45) and (46) and using $\alpha_{L_n} = \mathrm{e}^{-2A} L_n$ and $\alpha_{R_n} = \mathrm{e}^{-2A} R_n$, respectively, we get

$$L_0 \propto \exp\left[-\eta \int_0^y dy' F(\phi)\right], \tag{58}$$

$$R_0 \propto \exp\left[\eta \int_0^y dy' F(\phi)\right]. \tag{59}$$

This fact is the same to the case of two-dimensional Dirac equation, in that context are called isolated solutions [37]. At this point is worthwhile to mention that the normalization of the zero mode and the existence of a minimum of the effective potential at the localization on the brane are essential conditions for the problem of fermion localization on the brane. Recently, this fact already has been reported in [25].

In order to check the normalization condition (48) for the left-handed fermion zero mode (58), the integral must be convergent, *i.e*

$$\int_{-\infty}^{\infty} dy \exp\left[-A(y) - 2\eta \int_0^y dy' F(\phi(y'))\right] < \infty. \tag{60}$$

This result clearly shows that the normalization of the zero mode is decided by the asymptotic behavior of $F(\phi(y))$. Furthermore, from (54) and (55) can be observed that the effective potential profile depends on the choice of $F(\phi(y))$. This fact implies that the existence of a minimum of the effective potential $V_L(z(y))$ or $V_R(z(y))$ at the localization on the brane is decided by $F(\phi(y))$. This point will be more clear when it is considered a specific Yukawa coupling. Therefore, the behavior of $F(\phi(y))$ plays a leading role for the fermion localization on the brane [25]. Having set up the two essential conditions for the problem of fermion localization on the brane, we are now in a position to choice some specific forms for Yukawa couplings.

A. Zero mode and fermion localization

From now on, we mainly consider the simplest case $F(\phi) = \phi$. First, we consider the normalizable problem of the solution. In this case, from Eqs. (37) and (38) the integrand in (60) can be expressed as

$$I = \exp\left[-\ln\left(\mathrm{sech}\left(a\,y\right)\right) - \frac{3a^2\alpha}{4}\tanh^2\left(a\,y\right) - \sqrt{6}\,\eta\bar{I}(y) - \frac{3\sqrt{6}}{2}\,a\eta\alpha\,\mathrm{sech}(ay)\right], \tag{61}$$

where $\bar{I} = \int dy' \arcsin\left[\tanh(ay')\right]$. We follow the same procedure of Ref. [16], we only need to consider the asymptotic behavior of the integrand and it becomes

$$I \to \exp\left[-\left(\frac{\pi\eta}{b} - a\right)|\,y| - \frac{3a\alpha\eta}{2b}\,\mathrm{e}^{-\frac{3}{2}ab^2|\,y|}\right]. \tag{62}$$

In this stage, it is instructive to note that the asymptotic behavior of the integrand dependent the signs of a and b. One can readily envisage that four different classes of solutions can be segregated:

- **Class A**. For $a > 0$ and $b > 0$, the behavior of (62) as $|\,y| \to \infty$ is given by

$$I \to \exp\left[-\left(\frac{\pi}{b}\,\eta - a\right)|\,y|\right] \to 0, \qquad \text{for} \quad \eta > ab/\pi\,. \tag{63}$$

  This result shows that the zero mode of the left-handed fermions is normalized only for $\eta > ab/\pi$. Note that the asymptotic behavior of the normalization condition for this case is independent of α. Now, under the change $\eta \to -\eta$ ($L_0 \to R_0$) we obtain that the right-handed fermions can not be a normalizable zero mode. The shape of the potentials for this case are shown in Fig. 4 for some values of α. The Fig. 4(a) shows that the potential of left-handed fermions, $(V_L)_A$, is indeed a volcano-like potential. The shapes of the energy density, $(V_L)_A$ potential and zero mode for this case are shown in Fig. 5 for $\alpha = 0.1$. An similar behavior is obtained for $\alpha = 1$ and $\alpha = 10$. The Fig. 5 clearly shows that the effective potential $(V_L)_A$ has a minimum at the localization of the brane, therefore, this clearly shows that the zero mode of the left-handed fermions is localized on the brane. On the other hand, the figure 4(a) shows a well structure that decreases as α grows. From this can be conclude that the ability to trap fermions decrease as the value of α grows. Figure 4(b) shows that the potential $(V_R)_A$ is always positive. This effective potential has a maximum that decreases as α grows. Therefore, the potential can not trap any bound fermions with right chirality.

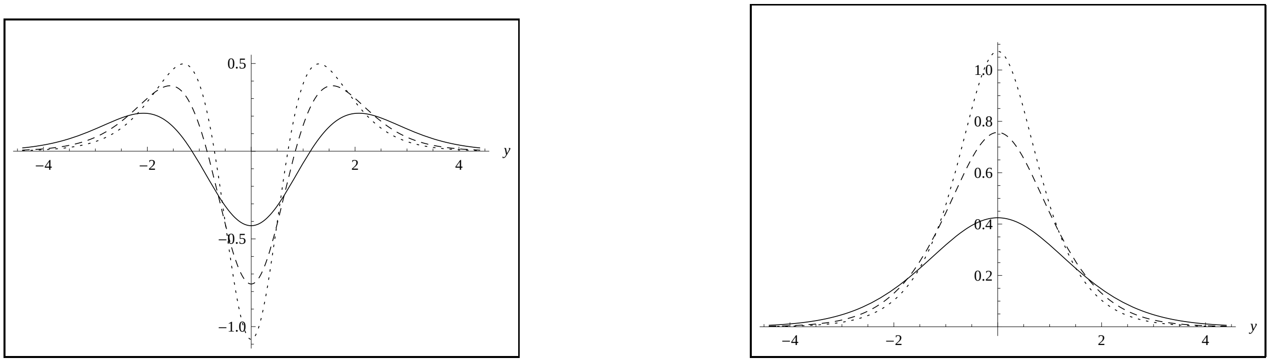

FIG. 4: Potential profile: (a) $(V_L(y))_A$ (left) and (b) $(V_R(y))_A$ (right) for $\eta = 1$, $a = 1$, $b = \sqrt{2/3}$, $\alpha = 0.1$ (dot line), $\alpha = 1$ (dashed line) and $\alpha = 10$ (thin line)

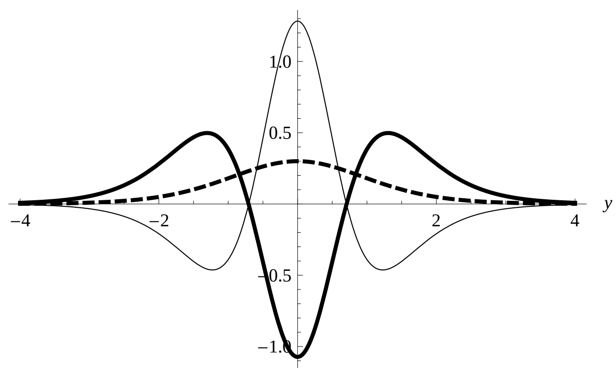

FIG. 5: The profiles of the energy density (thin line), $(V_L)_A$ (thick line) and zero mode (dashed thick line) for $\eta = 1$ and $\alpha = 0.1$.

- **Class B**. For $a > 0$ and $b < 0$, the behavior of (62) as $|y| \to \infty$ is

$$I \to \exp\left[\left(\frac{\pi}{|b|}\eta + a\right)|y|\right] \to \infty, \tag{64}$$

which leads to a non normalizable zero mode, therefore, the zero mode of the left-handed fermions can not be localized on the brane. Similar to previous case the asymptotic behavior of the normalization condition is independent of α. Otherwise, the change $\eta \to -\eta$ ($L_0 \to R_0$) allowed us to conclude that the right-handed fermions can be normalizable on the condition $\eta > ab/\pi$. It is instructive note that under the change $b \to -|b|$ in (37), we obtain $\phi \to -\phi$ (i.e $F(\phi) \to -F(\phi)$), therefore, the behavior of the potentials for the class **B** can be written out easily by replacing $(V_L)_B = (V_R)_A$ and $(V_R)_B = (V_L)_A$. From this, we conclude that the zero mode of the right-handed fermions is localized on the brane for $\eta > ab/\pi$.

- **Class C**. For $a<0$ and $b<0$, the behavior of (62) as $|\,y|\to\infty$ is

$$I\to\exp\left[-\left(|\,a|-\frac{\pi}{|\,b|}\,\eta\right)\,|\,y|-\frac{3\alpha\eta}{2}\,\frac{|\,a|}{|\,b|}\,\mathrm{e}^{\frac{3}{2}\,|\,a|b^2|\,y|}\right]. \tag{65}$$

At this point is worthwhile to mention that $\mathrm{e}^{\frac{3}{2}\,|\,a|b^2|\,y|}$ has a dominant asymptotic behavior that $\left(|\,a|-\frac{\pi}{|\,b|}\,\eta\right)\,|\,y|$ then $I\to 0$ independent the value of η. This result shows that the zero mode of the left-handed fermions is normalized for any value of η. This condition on η has not appeared in previous studies. Now, under the change $\eta\to-\eta$ ($L_0\to R_0$) we obtain that the right-handed fermions can not be a normalizable zero mode. One more time, note that under the change $b\to-|\,b|$ and $a\to-|\,a|$ in (37), we obtain $\phi\to\phi$ (i.e $F(\phi)\to F(\phi)$), therefore, the behavior of the potentials for the class **C** can be written out easily by replacing $(V_L)_C=(V_L)_A$ and $(V_R)_C=(V_R)_A$. From this, we conclude that the zero mode of the left-handed fermions is localized on the brane.

- **Class D**. For $a<0$ and $b>0$, the behavior of (62) as $|\,y|\to\infty$ is

$$I\to\exp\left[-\left(|\,a|+\frac{\pi}{b}\,\eta\right)\,|\,y|+\frac{3\alpha\eta}{2}\,\frac{|\,a|}{b}\,\mathrm{e}^{\frac{3}{2}\,|\,a|b^2|\,y|}\right]. \tag{66}$$

At this point is worthwhile to mention that $\mathrm{e}^{\frac{3}{2}\,|\,a|b^2|\,y|}$ has a dominant asymptotic behavior that $-\left(|\,a|+\frac{\pi}{b}\,\eta\right)\,|\,y|$ then $I\to\infty$ independent the value of η. This result shows that the zero mode of the left-handed fermions is not normalized. Now, under the change $\eta\to-\eta$ ($L_0\to R_0$) we obtain that the right-handed fermions is normalized for any value of η. One more time, note that under the change $a\to-|\,a|$ in (37), we obtain $\phi\to-\phi$ (i.e $F(\phi)\to-F(\phi)$), therefore, the behavior of the potentials for the class **D** can be written out easily by replacing $(V_L)_D=(V_R)_A$ and $(V_R)_D=(V_L)_A$. From this, we conclude that the zero mode of the right-handed fermions is localized on the brane.

In this four different classes of solutions the normalization condition of the zero mode is independent of α, but the ability to trap fermions is inversely proportional to α, because the well structure of the effective potential decreases as α grows. For α not necessarily small we do not have analytic expressions for the solution of this model, in this case the numerical study is essential. The numerical study done for a large range of values of α bear out our results.

IV. CONCLUSIONS

We have reinvestigated the braneworld model constructed from one scalar field with nonstandard kinetic terms coupled with gravity. We have considered the model $\mathcal{L} = K(X) - V(\phi)$, where $K = X + \alpha|X|X$ (type I model in Ref. [29]). This work completes the analyze of the research in Ref. [29]. We showed that the equations of motion can be deduced from the functional $E[A, \phi]$ (23), as done by Townsend in the case of standard dynamic. Also, we showed that the value of the matter energy and the energy of system depend of the asymptotic behavior of the warp factor. We found an expression for the matter energy density that differs slightly from [29], the numerical study gives full support to our matter energy density expression for α small.

We also have investigated the localization problem of fermions for the type model I. We have used the simplest Yukawa coupling $\bar{\Psi}\phi\Psi$ between the scalar and the spinor fields. In order to check the normalization condition for the zero mode, one can separate in four classes of solutions. For class A, we showed that the zero mode of left-handed fermions is normalizable under the condition $\eta > ab/\pi$ and it is independent of α. For this kind of solution, the effective potential of left-handed fermions $(V_L)_A$ is a volcano-like potential. $(V_L)_A$ has a minimum at the localization of the brane ($y = 0$), therefore, the zero mode of the left-handed is localized on the brane. On the other hand, the value of α adjust the minimum of $(V_L)_A$, if α increases the depth of the well structure decreases. Therefore, we can conclude that the ability to trap fermions of $(V_L)_A$ is inversely proportional to α. The right-handed fermions can not be localized on the brane, this fact is a consequence of the absence of a normalizable zero mode. For class B, we showed that the right-handed fermions can be localized on the brane for $\eta > ab/\pi$, on the contrary, the left-handed fermions can not be localized on the brane. For class C, the zero mode of the left-handed fermions can be localized on the brane for any value of η and the right-handed fermions does not. This condition on η has not appeared in previous studies. For class D, the zero mode of the right-handed fermions can be localized on the brane for any value of η and the left-handed fermions does not. For α not necessarily small, the numerical study done for a large range of values of α bear out our results.

Additionally, we showed that the change of variable $dz = \mathrm{e}^{-A(y)}dy$ leads to a non conserved current, because the curved-space gamma matrices are not covariantly constant. The effects of non-conserved current in the issue of massive modes is currently under consideration and will be the subject of another thorough study.

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