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Holographic flows to IR Lifshitz spacetimes

Harvendra Singh

Theory Division, Saha Institute of Nuclear Physics
1/AF Bidhannagar, Kolkata 700064, India

E-mail: h.singh (AT) saha.ac.in

Abstract

Recently we studied 'vanishing' horizon limits of 'boosted' black D3-brane geometry [1]. The type IIB solutions obtained by taking these special double limits were found to describe nonrelativistic Lifshitz spacetimes at zero temperature. In the present work we study these limits for TsT black-hole solutions which include B-field. The new Galilean solutions describe a holographic RG flow from Schrödinger ($a = 2$) spacetime in UV to a Lifshitz universe ($a = 3$) in the IR.

1 Introduction

There are commonly two types of non-relativistic or Galilean string backgrounds, with broken Lorentzian symmetries, which are a subject of favorable attention currently [2]-[23]. The geometries which possess Schrödinger symmetries [2, 3] are given as written as

$$ds^2_{Sch} = \left(-\frac{\beta^2}{z^{2a}}(dx^+)^2 + \frac{-dx^+dx^- + dx_i^2}{z^2}\right) + \frac{dz^2}{z^2} \tag{1}$$

and the others with Lifshitz-like symmetries [4, 5] are written as

$$ds^2_{Lif} = \left(-\frac{\beta^2}{z^{2a}}dt^2 + \frac{dx_i^2}{z^2}\right) + \frac{dz^2}{z^2}. \tag{2}$$

In both these cases, x^i $(i = 1, ..., d)$ are flat spatial coordinates, $x^\pm$ are the light-cone coordinates, z is the holographic direction and parameter a is known as the dynamical exponent of the Galilean geometry. Some of these nonrelativistic geometries are claimed to be describing strongly coupled scaling phenomena near quantum critical points in dual nonrelativistic CFTs [2, 3]. The Schrödinger AdS spacetimes with dynamical exponent $a = 2$ can however be embedded in string theory as it has been shown in [6, 7]. The Schrödinger spacetimes with $a = 3$ can also be found in the massive type IIA string theory [16]. More recently, various Lifshitz spacetimes with light-like deformation were constructed as string solutions, as shown in [19]. These solutions require nontrivial dilaton field as well as metric deformations. Such solutions are further generalised in [20]. These recent Lifshitz solutions do avoid the early 'no-go' results of [24] because the field ansätze are some what less restrictive. This implies that a wider class of Lifshitz-like solutions can be found in string theory if we suitably excite other fields in the Lifshitz background. The hope is that some of these solutions could potentially describe interesting scaling phenomena in dual field theories where Lorentzian symmetry is explicitly broken and the system behaves quantum mechanically. Some new Lifshitz-like solutions with Janus-like configurations have been presented in [26].

1.1 A circle fibration over Lifshitz spacetime

In a recent work [1] we discussed a new type of nonrelativistic AdS_5 geometry which had one of the lightcone direction namely x^+ (time) being null while x^- being compact [1]

$$ds^2 = \left(-r^2dx^+dx^- + \frac{\beta^2}{4r^2}(dx^-)^2 + r^2(dx_1^2 + dx_2^2)\right) + \frac{dr^2}{r^2} \tag{3}$$

It was obtained by taking special vanishing horizon limits of 'boosted' black D3-brane geometry. For the thermodynamics these limits correspond to taking 'zero' temperature

[1] Most of our analysis in this work goes through even for the noncompact cases.

(or condensation) limits with vanishing chemical potential. The AdS_5 Galilean geometry may also be seen as a Lifshitz spacetime having a circle fibration

$$ds^2 \equiv ds^2_{Lif_4} + \frac{\beta^2}{4r^2}\Xi^2 \tag{4}$$

where 4-dimensional Lifshitz spacetime is

$$ds^2_{Lif_4} = -\frac{r^6}{\beta^2}(dx^+)^2 + r^2(dx_1^2 + dx_2^2) + \frac{dr^2}{r^2}, \qquad \text{and} \qquad \Xi = (dx^- - \frac{2r^4}{\beta^2}dx^+). \tag{5}$$

The size of the x^- fiber varies all over the Lifshitz base. Since x^- is a circle this geometry obviously cannot be trusted at large r where fiber size shrinks, nevertheless in some finite region inside the bulk we can trust this classical background. We shall discuss how to include a boundary configuration in these solutions.

In this paper we wish to extend our analysis to include backgrounds with NS-NS B-field. Specially we focus on the finite temperature T-s-T backgrounds obtained given in [7]. One useful feature of the TsT black-holes is that they are by construction asymptotically Schrödinger geometries with dynamical exponent $a = 2$. We would like to study what happens to these solutions under the 'vanishing' horizon double limits. The paper is organised as follows. In section-II we review the boosted black D3-brane solution and the vanishing horizon double limits where the black hole horizon shrinks to zero value while the 'boost' is simultaneously taken to be very large. The solutions thus obtained describe zero temperature non-relativistic Lifshitz geometry. These solutions are not well defined at the boundary. We discuss the issue of the boundary and propose a new gravity solution which includes a finite boundary configuration. In section III we repeat our analysis for TsT black hole solutions which have asymptotic Schrödinger symmetries. The section IV has the conclusions.

2 Simultaneous double limits of 'boosted' black 3-branes

2.1 Review

This section contains the review of our previous work [1]. We are particularly interested in studying the DLCQ of AdS_5 geometry as described in [7]. We start with the 'boosted' version of black D3-branes [7] where the near horizon solution is

$$\begin{aligned} ds^2_{D3} &= r^2\left(-\frac{1+f}{2}dx^+dx^- + \frac{1-f}{4}[\lambda^{-2}(dx^+)^2 + \lambda^2(dx^-)^2] + dx_1^2 + dx_2^2\right) + \frac{dr^2}{fr^2} + d\Omega_5^2\,, \\ F_5 &= 4(1+*)Vol(S^5) \end{aligned} \tag{6}$$

where $d\Omega_5^2$ is the line element of a unit size five-sphere S^5. The function $f(r) = 1 - r_0^4/r^4$, with $r = r_0$ being the horizon location and the boundary is at $r \to \infty$. The overall AdS_5 radius, L, has been set to unity. The black D3-branes (6) have large but finite momentum along compact direction x^-, $x^- \sim x^- + 2\pi r^-$. (We shall represent this circle as $\tilde{S}^1$ through out this paper as it will be present everywhere.) The boundary conformal field theory will have a DLCQ description in a given (discrete) momentum sector. However, due to x^- being compact, the geometry is well defined only in the interior region and not near the boundary. It should however be kept in mind that, since x^- is compact we simply cannot take $r_0 \to 0$, as it will make x^- a null direction.[2] Note that in (6) the size of x^- circle shrinks as we go near the boundary, but due to backreaction of the large lightcone momentum it stays finite within the bulk where we can trust this solution.

The boost parameter λ physically controls the size of x^-. As we see that $r_0^4\lambda^2$ effectively measures the size of this circle, therefore we consider a combined limit in which the size of horizon is allowed to shrink while boost is simultaneously taken to be large such that

$$r_0 \to 0, \qquad \lambda \to \infty, \qquad r_0^4\lambda^2 = \beta^2 = \text{fixed}. \tag{7}$$

In which case we find [1]

$$\begin{aligned} &(1+f) \to 2 + O(r_0^4), \qquad \frac{1-f}{\lambda^2} \to O(\frac{r_0^4}{\lambda^2}) \\ &(1-f)\lambda^2 \to \frac{\beta^2}{r^4} \end{aligned} \tag{8}$$

and the solution (6) simply reduces to

$$\begin{aligned} ds_{10}^2 &= r^2\left(-dx^+dx^- + \frac{\beta^2}{4r^4}(dx^-)^2 + dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2} + d\Omega_5^2 \\ F_5 &= 4(1+*)Vol(S^5) \end{aligned} \tag{9}$$

which itself is a complete solution of type IIB string theory at zero temperature. There is no curvature singularity. Actually the spacetime (9) is $AdS_5 \times S^5$ geometry but it is Galilean one and crucially the coordinate x^+ (time) is null. However we could always rewrite (9) as

$$ds_{10}^2 = ds_{Lif_4}^2 + \frac{\beta^2}{4r^2}\Xi^2 + d\Omega_5^2. \tag{10}$$

Thus the Galilean geometry (9) is indeed a Lifshitz four-universe along with a fibered product space $\tilde{S}^1 \times S^5$. It will represent a well defined system of Kaluza-Klein particles if we compactify the solution to four dimensions. In particular, the 9-dimensional type II solution schematically will be like

$$\begin{aligned} ds_9^2 &= \left(-\frac{r^6}{\beta^2}(dx^+)^2 + r^2(dx_1^2 + dx_2^2) + \frac{dr^2}{r^2}\right) + d\Omega_5^2 \\ e^{-2\phi} &= \frac{\beta}{2r}, \quad F_2^{(-)} \equiv dA^{(-)} = -\frac{8r^3}{\beta^2}dr \wedge dx^+, \end{aligned} \tag{11}$$

[2] Note, it generally is not a problem when x^- is noncompact, because in that case setting $r_0 = 0$ simply describes an extremal (BPS) limit which takes us to an ordinary AdS spacetime whose holographic dual is $N = 4$ super-Yang-Mills theory at large 't Hooft coupling.

there will be a scalar σ from internal metric component $e^{2\sigma} = G_{--}$ which couples to the KK gauge fields $A^{(-)}$. There will also be 3-form tensor field as a result of the reduction of the 4-form. That is we are effectively dealing with system of KK fields coupled to scalar field and a dilaton field in 4-dimensional Lifshitz universe while rest of the directions are all compact.

In summary, taking the double limits (7) of the 'boosted' black D3-branes allows us to exclusively 'zoom onto' the KK system in a Lifshitz universe such as (11). The Lifshitz geometry (9) is inherently nonrelativistic. There is an asymmetric scale (dilatation) invariance

$$r \to (1/\xi)r, \quad x^- \to \xi^{-1}x^-, \quad x^+ \to \xi^3 x^+, \quad x_{1,2} \to \xi x_{1,2} \tag{12}$$

where time scales with scaling dimension 3 and therefore the dynamical exponent is 3. There are also invariances under constant shifts (translations) as well as rotations in $x^1 - x^2$ plane, see [1]. However, (9) does not have any explicit invariance under the Galilean boosts.

$$x^+ \to x^+, \quad x^- \to x^- - \vec{v}.\vec{x} + \frac{v^2}{2}x^+, \quad \vec{x} \to \vec{x} - \vec{v}x^+. \tag{13}$$

Thus the solution (9) represents a geometry with broken Lorentzian symmetry in which the time has dynamical exponent $a = 3$ and which upon compactification to four dimensions simply gives us a Lifshitz universe along with scalar and gauge fields. The background (9) preserves at least 8 Poincaré supersymmetries.

2.2 Adding a boundary and resultant RG flow

The classical solution (9) obtained under the vanishing horizon limits is devoid of a useful description at the boundary at infinity because x^- tends to become null there while being a compact direction. However, there may be other possible ways to tackle this UV problem, in a rather adhoc manner we try to add a boundary configuration to the geometry (9). So we write down a completely different solution of type IIB, similar to (9),

$$\begin{aligned} &ds^2_{10} = r^2\left(-dx^+dx^- + \frac{1}{4}(1+\frac{\beta^2}{r^4})(dx^-)^2 + dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2} + d\Omega_5^2 \\ &F_5 = 4(1+*)Vol(S^5) \end{aligned} \tag{14}$$

which includes a distinct boundary configuration. It has a scale invariance with dynamical exponent $a = 1$ provided β also scales as

$$r \to \xi^{-1}r, \quad \beta \to \xi^2\beta, \quad x^- \to \xi^1 x^-, \quad x^+ \to \xi^1 x^+, \quad x_i \to \xi x_i. \tag{15}$$

Asymptotically the new geometry Eq.(14) becomes an anti-de Sitter spacetime

$$ds^2_{10} = r^2\left(-dx^+dx^- + \frac{1}{4}(dx^-)^2 + dx_1^2 + dx_2^2\right) + \frac{dr^2}{r^2} + d\Omega_5^2 \tag{16}$$

which has a flat boundary metric but in a 'shifted' basis. Thus the dual field theory will have a slightly changed DLCQ description, see appendix. Particularly the lightcone energy spectrum will be different. But the important thing to notice is that the size of the x^- circle now remains finite in the UV region including at the boundary. There may be some subtle issues involved in the CFT in doing this, but from gravity point of view we have got a classical background which is good for holographic study. Though, as discussed in the appendix, there appears to be no major difference in the two DLCQ descriptions so far as large lightcone momentum is involved.

In the deep IR region, as $r^4 \ll \beta^2$, the solution (14) becomes simply the Lifshitz solution (9) we started with. Thus the new type IIB background (14) describes a holographic flow of a DLCQ theory (with $a = 1$) in UV to a nonrelativistic Lifshitz theory (with $a = 3$) in IR.

3 Vanishing horizon limits of TsT background

Here we would like to focus on the 'TsT' black D3-brane solutions which have Schrödinger like asymptotic symmetry. These solutions have B field and can be obtained from (6) by applying a chain of T-dualities and a 'shift' [7]. The solution in the string frame can be written as

$$\begin{aligned} ds^2 &= r^2\Big[H^{-1}\left(-\frac{1+f}{2}dx^+dx^- + \frac{1-f}{4}[\lambda^{-2}(dx^+)^2 + \lambda^2(dx^-)^2] - \sigma^2 r^2 f(dx^+)^2\right) \\ &\qquad + dx_1^2 + dx_2^2\Big] + \frac{dr^2}{fr^2} + \frac{\eta^2}{H} + ds^2(\mathcal{B}_{KE}) \\ e^{2\phi} &= 1/H, \qquad F_5 = 4(1+*)Vol(SE) \\ B_{NS} &= \frac{\sigma}{2}r^2 H^{-1}[(1+f)dx^+ - (1-f)\lambda^2 dx^-] \wedge \eta \end{aligned} \tag{17}$$

where $H(r) = 1 + \sigma^2\lambda^2\frac{r_0^4}{r^2}$ and ϕ is the dilaton field. The σ is the 'shift' parameter incorporated in the T-s-T duality [7] and it is also related to the non-commutativity parameter in light-like noncommutative description of gauge theories [27]; also see [28]. The $Vol(SE)$ is the volume form over the Sasaki-Einstein metric

$$ds_{SE}^2 = ds^2(\mathcal{B}_{KE}) + \eta^2.$$

and $d\eta/2 = J$ is the Kähler 2-form over the base $\mathcal{B}_{KE}$ which is Kähler-Einstein. Unlike the 'boosted' black D3-branes of the last section the TsT black hole solutions (17) have desired Schrödinger asymptotics at infinity which is

$$\begin{aligned} ds^2 &= r^2[-dx^+dx^- - \sigma^2 r^2(dx^+)^2 + dx_1^2 + dx_2^2] + \frac{dr^2}{r^2} + \eta^2 + ds^2(\mathcal{B}_{KE}) \\ e^{2\phi} &= 1, \qquad F_5 = 4(1+*)Vol(SE) \\ B_{NS} &= \sigma r^2 dx^+ \wedge \eta, \end{aligned} \tag{18}$$

having a dynamical exponent $a = 2$, see [7] for further details.

We are now interested in applying the vanishing horizon double limits (7) on the TsT solutions (17). The resultant zero temperature solution is

$$
\begin{aligned}
ds^2 &= r^2\Big[H^{-1}\left(-dx^+dx^- + \frac{\beta^2}{4r^4}(dx^-)^2 - \sigma^2 r^2 (dx^+)^2\right) + dx_1^2 + dx_2^2\Big] + \frac{dr^2}{r^2} \\
&\quad + \frac{\eta^2}{H} + ds^2(\mathcal{B}_{KE}) \\
e^{-2\phi} &= H, \qquad F_5 = 4(1+*)Vol(SE) \\
B_{NS} &= \sigma r^2 H^{-1}[dx^+ - \frac{\beta^2}{2r^4}dx^-] \wedge \eta
\end{aligned} \tag{19}
$$

where $H(r) = 1 + \frac{\sigma^2\beta^2}{r^2}$. Note that the zero temperature solution (19) is still a Schrödinger solution asymptotically but it also has a nonrelativistic deformation in the IR region. The parameter β is a measure of this IR deformation. Interestingly the solution (19) has got an invariance in which parameter σ should also scale. The non-relativistic scale (dilatation) invariance is

$$
\begin{aligned}
&r \to (1/\xi)r, \qquad \sigma \to (1/\xi)\sigma, \qquad \beta \to \beta, \\
&x^- \to \xi^{-1}x^-, \qquad x^+ \to \xi^3 x^+, \qquad x_{1,2} \to \xi x_{1,2}.
\end{aligned} \tag{20}
$$

Hence the dynamical exponent for the solution (19) is $a = 3$. Let us also investigate what happens in the deep IR region where $r^2 \ll \sigma^2\beta^2$. In this region

$$
e^{2\phi} \approx \frac{r^2}{\sigma^2\beta^2}, \tag{21}
$$

so as the string coupling becomes weaker the solution (19) flows into a Lifshitz point

$$
\begin{aligned}
ds^2 &= r^2\Big[\frac{r^2}{\sigma^2\beta^2}\left(-dx^+dx^- + \frac{\beta^2}{4r^4}(dx^-)^2 - \sigma^2 r^2 (dx^+)^2\right) + dx_1^2 + dx_2^2\Big] \\
&\quad + \frac{dr^2}{r^2} + \frac{r^2}{\sigma^2\beta^2}\eta^2 + ds^2(\mathcal{B}_{KE}) \\
&\approx ds^2_{Lif_4} + \frac{1}{4\sigma^2}\Xi^2 + \frac{r^2}{\sigma^2\beta^2}\eta^2 + ds^2(\mathcal{B}_{KE}) \\
B_{NS} &= -\frac{1}{2\sigma}\Xi \wedge \eta \qquad \text{where} \qquad \Xi = dx^- - \frac{2r^4}{\beta^2}dx^+
\end{aligned} \tag{22}
$$

The scaling symmetry (20) survives in the deep IR region. Thus in deep IR region we have a Lifshitz four-universe (with $a = 3$) alongside a compact space $\tilde{S}^1 \times S^1 \times \mathcal{B}_{KE}$ as a fibration. The presence of the B-field in (22) is what distinguishes it from the Lifshitz background of (9) and (14). In conclusion, as we go from UV region to IR region the Galilean solution (19) basically flows from a Schrödinger spacetime (with $a = 2$) to a Lifshitz spacetime (with $a = 3$) in presence of B-field.

Although the Ξ radius becomes constant in the IR region, but we should be careful about of the size of the Hopf circle η in this region. In the deep IR region where

$$
L^2 \frac{r^2}{\sigma^2\beta^2} \le l_s^2,
$$

L being the AdS radius, we cannot trust the classical string solution (19) as the size of Hopf fibre η becomes sub-stringy there.

4 Conclusions

We reviewed the vanishing horizon double limit $r_0 \to 0$, $\lambda \to \infty$ of 'boosted' black D3-branes with a compact lightcone direction. The zero temperature Lifshitz solutions obtained as a result of taking the limits are not well defined in the UV, however by suitably adding appropriate boundary configuration we have been able to construct a new solution (14). The boundary configuration corresponds to $a = 1$ Lifshitz like configuration. The spectrum appears to be relativistic but with a shifted energy spectrum. The new solution describes a flow from $a = 1$ Lifshitz-like universe in UV to $a = 3$ Lifshitz universe in IR. We also study similar zero temperature limits for TsT black hole solutions which involve the B-field. Resulting zero temperature solutions describe a RG flow from Schrödinger spacetime ($a = 2$) in UV to the $a = 3$ Lifshitz universe in IR. However the latter class of Galilean solutions have an instability in the deep IR where the fibre direction over the Kähler base becomes sub-stringy. The main distinction between two type of flows is the presence of B-field.

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A Non-relativistic theories and DLCQ:

Let us consider a theory with the following $(d+2)$-dimensional flat spacetime metric

$$ds^2 = -dx^+dx^- + \frac{1}{4}(dx^-)^2 + d\vec{x}^2 \tag{23}$$

where $x^\pm = t \pm y$ are the lightcone coordinates. We shall denote the respective conjugate momenta by $p_\pm$, and the indices can be raised up by using the metric. Then the mass shell condition, $p_\mu p^\mu = 0$, for a 'massless' particle will become

$$4p_+p_- - \vec{p}^2 + (p_+)^2 = 0 \tag{24}$$

In these coordinates if we identify $-2p_- = M$, M being actual rest mass, and the lightcone energy as $E_+ \equiv -p_+$, then we can have

$$E_+ = \sqrt{\vec{p}^2 + M^2} - M. \tag{25}$$

It can be easily seen that if we set $E_+ = E - M$, Eq.(25) is precisely the standard relativistic mass shell relation for a free particle in Minkowski space with mass M and total energy E in $(d+1)$ dimensions. So the lightcone energies are just shifted from E to $E_+ = E - M$. The relation (25) remains valid even in the massless case ($p_- = 0$).

The momentum $(-p_-)$ is a continuous variable so far. We can discretize it by compactifying x^- direction on a circle, ($x^- \sim x^- + 2\pi r^-$), so that $-p_-$ will be quantized as $\frac{N}{r^-}$ for ($N \geq 0$). Thus the spectrum is separated into discrete mass sectors characterised by

N. This is known as discrete light cone quantization (DLCQ) of a relativistic theory, see for further discussion [7] and references therein. It is clear from the energy-mass relation (25) that a nonrelativistic limit in our DLCQ theory is achieved only when we focus on a given momentum sector with very large N, which means a large M. In the limit of very large M, $\vec{p}^2 \ll M^2$, we get from (25)

$$E_+ \simeq \frac{\vec{p}^2}{2M} + \cdots \tag{26}$$

where the $\cdots$ indicates suppressed relativistic corrections like the standard $M(\frac{\vec{v}^2}{c^2})^2$ etc. Thus in a large light-cone momentum sector the DLCQ theory is precisely a non-relativistic theory.

Notice the difference, the DLCQ of a relativistic theory in ordinary Minkowski metric $ds^2 = -dx^+dx^- + d\vec{x}^2$ gives a mass shell condition where lightcone energy is

$$E_+ = -p_+ = \frac{\vec{p}^2}{(-4p_-)} \tag{27}$$

which looks like the energy of a non-relativistic particle of mass $M \sim -2p_-$ from start. However, in reality Eq.(27) can be trusted as a nonrelativistic expression only in the large momentum sectors with $N \gg 0$, *i.e.* when $\vec{p}^2 \ll -p_-$. There is also a subtle issue here particularly involving the massless modes ($p_- = 0$), where the DLCQ needs to be treated differently, see [25].

We however see that the two expressions (25) and (27) coincide in the large momentum sectors which is of our immediate interest for the DLCQ in the gravity where we have included backreaction due to the large lightcone momentum. This analysis shows that the large lightcone momentum sectors are indeed non-relativistic.

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