

Head-on collisions of unequal mass black holes in $D = 5$ dimensions

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We study head-on collisions of unequal mass black hole binaries in $D = 5$ spacetime dimensions, with mass ratios between 1:1 and 1:4. Information about gravitational radiation is extracted by using the Kodama-Ishibashi gauge-invariant formalism and details of the apparent horizon of the final black hole. For the first time, we present waveforms, total integrated energy and momentum for this process. Our results show surprisingly good agreement, within 5 % or less, with those extrapolated from linearized, point-particle calculations. Our results also show that consistency with the area theorem bound requires that the same process in a large number of spacetime dimensions must display new features.

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I. INTRODUCTION

Black holes (BHs) have been at the center stage of fundamental physics in the last decades: supermassive, astrophysical BHs lurk at the center of most galaxies while large numbers of stellar-mass BHs are thought to populate each galaxy [1, 2]; highly dynamical BH binaries are strong sources of gravitational waves and, perhaps, power jets and other extreme phenomena [3, 4]. In high-energy physics, BHs are a central piece of the gauge-gravity duality [5], and are the generic outcome of particle collisions at center-of-mass energies above the Planck scale [6]. In this regime the particular nature of the particles' structure should become irrelevant, as indicated by Thorne's "hoop" conjecture [7], and "no-hair theorem"-type of arguments. These can be invoked to argue that, in general, trans-Planckian collisions of particles are well described by collisions of highly boosted BHs. In this context, scenarios such as TeV-gravity are especially interesting, as they lower the Planck scale to the level at which BHs would be produced in cosmic rays and particle accelerators [8–17]. Thus, high-energy BH collisions could be used to look for signatures of extra dimensions and BH production in ground-based experiments in the forthcoming years. At the fundamental level, BHs might hold the key for a theory of quantum gravity, and might help understand important issues such as cosmic censorship, information loss and the maximum possible luminosities in any physical process [18, 19].

The above arguments illustrate the necessity to understand accurately dynamical BH spacetimes, and their potential across a wide variety of fields. Since the full system of Einstein equations needs to be carefully understood, this is by all means a monumental task, and typically requires numerical methods. With these fundamental issues as motivation, long-term efforts to understand dynamical BHs in generic spacetimes have been initiated [20–25], ranging from the inspiralling of BH binaries [26, 27], high-energy collisions of BHs in four [18, 19, 28] and low energy collisions in higher spacetime dimensions [21, 22], stability studies in higher dimensions [29–31] and BH evolutions in non asymptotically flat spacetimes [32].

Our group has recently studied head-on collisions of equal-mass black holes in higher dimensions, in particular $D = 5$, for the first time [22]; here, we wish to extend that study to the case of unequal mass BH binaries. This is an interesting extension for several reasons, perhaps the most important of which is the non-trivial comparison with point-particle (PP) calculations in the linearized regime. We will compare radiated energy, momentum and multipolar dependence of our full non-linear results with results from linearized Einstein equations. It turns out that the agreement is remarkable, providing an outstanding consistency check on our codes and results. A thorough analysis of the linearized Einstein equations is done in an accompanying paper [33].

This paper is organized as follows: In Sec. II we summarize our numerical method and setup and present the numerical results. We finish by giving some conclusions and final remarks in Sec. III.

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II. NUMERICAL RESULTS

The numerical simulations have been performed with the LEAN code, originally introduced in Refs. [34, 35] and adapted to higher dimensional space-times by Zilhão et al [21]. The LEAN code is based on the Cactus computational toolkit [36] and uses the Carpet mesh refinement package [37, 38] and Thornburg's apparent horizon finder AHFinderDirect [39, 40].

By dimensional reduction, the D dimensional vacuum Einstein equations yield an effectively $3 + 1$ dimensional system of gravity coupled to a scalar field [21]. We have evolved this system using the BSSN [41, 42] formulation along the lines presented in Ref. [21], together with the moving puncture approach [43, 44]. The initial data consists in the time-symmetric Brill-Lindquist initial data in the form presented in Witek et al. [22] (Eq. (2.15) therein). Gravitational waves have been extracted using the Kodama-Ishibashi (KI) formalism [45, 46]. For details of the wave extraction implementation we refer the reader to Ref. [22]. We have evolved BH binaries, colliding head-on from rest with mass ratios $q \equiv M_1/M_2 = r_{S,1}^{D-3}/r_{S,2}^{D-3} = 1, 1/2, 1/3, 1/4$, where M_i is the mass of the i -th BH. The mass parameter $r_{S,1}^{D-3}/r_{S,2}^{D-3}$ of the smaller BH is given in Table I and we adapt the value of the second BH accordingly. The initial coordinate separation of the two BHs is set to $d/r_S = 6.37$ which translates to a proper initial separation of $L/r_S = 6.33$. Further details of the setup of the simulations are summarized in Table I. Unless denoted otherwise, our discussion will always refer to the highest resolution runs with $h_f/r_S = 1/84$, $h_f/r_S = 1/102.9$, $h_f/r_S = 1/118.8$ and $h_f/r_S = 1/132.8$ for models HD5a, HD5b, HD5c and HD5d_f in Table I, respectively. The energy flux is computed according to Eq. (2.56) in Ref. [22] (see Eq. (21) in Ref. [47] for the corresponding expression in Fourier space). The momentum flux can be obtained from

$$\frac{dP^i}{dt} = \int_{S_\infty} d\Omega \frac{d^2E}{dt d\Omega} n^i, \quad (1)$$

with n^i a unit radial vector on the sphere at infinity S_∞ . This results in an infinite series coupling different multipoles. Using only the first two terms in the series, we find, for instance, that in $D = 5$ the momentum flux in the collision direction is given by

$$\frac{dP}{dt} = \frac{1}{4\pi} \Phi_{,t}^{l=3} (5\Phi_{,t}^{l=2} + 21\Phi_{,t}^{l=4}). \quad (2)$$

Here, $\Phi_{,t}^l$ is the l -pole component of the KI gauge-invariant wavefunction [22, 45, 46]. From the momentum radiated, the recoil velocity of the system can be obtained as

$$v_{\text{recoil}} = \left| \int_{-\infty}^{\infty} dt \frac{dP}{dt} \right|. \quad (3)$$

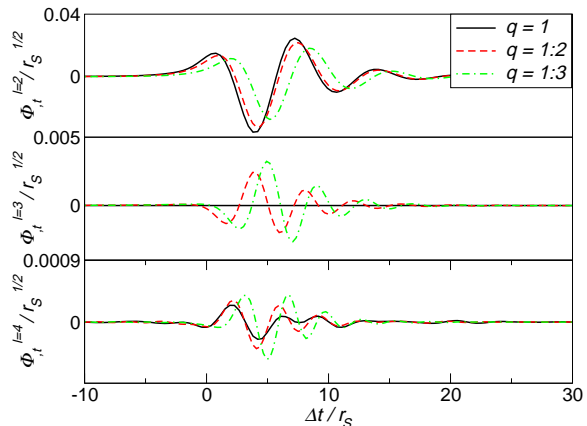


FIG. 1. (Color online) From top to bottom: $l = 2, l = 3$ and $l = 4$ modes of the KI waveform for the different mass ratios; $q = 1$ (black solid lines), $q = 1/2$ (red dashed lines) and $q = 1/3$ (green dash-dotted lines). The curves have been shifted in time such that the formation of the common apparent horizon corresponds to $\Delta t/r_S = 0$ and taking into account the time that it takes the waves to propagate to the extraction radius.

A. Waveforms

In Fig. 1 we show the $l = 2, 3, 4$ waveforms for different mass ratios, zoomed in around the time of the merger. The waveforms have been shifted in time such that $\Delta t/r_S = (t - r_{ex} - t_{CAH})/r_S = 0$ corresponds to the time t_{CAH} at which the common apparent horizon forms and taking into account the propagation time of the waves to the extraction radius $r_{ex}/r_S = 60, 49, 42.4, 37.9$. The waveform is similar to previous four-dimensional results (see, e.g. Ref. [48], a more detailed study is in preparation [49]). Although not shown in Fig. 1 we observe a small, spurious signal starting around $(t - r_{ex})/r_S \approx 0$, which is an artifact of the initial data.

The actual physical part of the waveform is dominated by the merger signal at $\Delta t/r_S \approx 0$ followed by the quasinormal ringdown. We estimate that the different ringdown modes are given by

$$\begin{aligned} \omega_{l=2} r_S &= 0.955 \pm 0.005 - i(0.255 \pm 0.005), \\ \omega_{l=3} r_S &= 1.60 \pm 0.01 - i(0.31 \pm 0.01), \\ \omega_{l=4} r_S &= 2.25 \pm 0.03 - i(0.35 \pm 0.05). \end{aligned}$$

These results agree well, and within uncertainties, with estimates from linearized theory [47, 50–52], providing a strong consistency check on our results. Finally, we consider numerical convergence of our waveforms. This study is summarized in Fig. 2 for the $l = 2$ mode of the KI wavefunction, and for the most challenging mass ratio, $q = 1/4$, model HD5d in Table I. We have evolved this setup at three different resolutions, namely $h_c/r_S = 1/113.8$, $h_m/r_S = 1/123.3$ and $h_f/r_S = 1/132.8$,

Run	q	$r_{S,1}^{D-3}/r_S^{D-3}$	r_S^{D-3}	z_1/r_S	z_2/r_S	Grid Setup
HD5a	1	0.5	1	3.185	-3.185	$\{(256, 128, 64, 32, 16, 8) \times (2, 1, 0.5), h = 1/84\}$
HD5b	1/2	0.33	1.5	4.247	-2.123	$\{(209, 104.5, 52.3, 26.1, 13.1, 6.5) \times (1.6, 0.8, 0.4), h = 1/102.9\}$
HD5c	1/3	0.25	2	4.777	-1.592	$\{(181.0, 90.5, 45.3, 22.6, 11.3) \times (2.8, 1.4, 0.7, 0.4), h = 1/118.8\}$
HD5d _c	1/4	0.2	2.5	5.096	-1.274	$\{(161.9, 80.9, 40.5, 20.2, 10.1) \times (2.5, 1.3, 0.6, 0.3), h = 1/113.8\}$
HD5d _m	1/4	0.2	2.5	5.096	-1.274	$\{(161.9, 80.9, 40.5, 20.2, 10.1) \times (2.5, 1.3, 0.6, 0.3), h = 1/123.3\}$
HD5d _f	1/4	0.2	2.5	5.096	-1.274	$\{(161.9, 80.9, 40.5, 20.2, 10.1) \times (2.5, 1.3, 0.6, 0.3), h = 1/132.8\}$

TABLE I. Grid structure and initial parameters of the head-on collisions from rest in $D = 5$. The grid setup is given in terms of the “radii” of the individual refinement levels, as well as the resolution near the punctures h , in units of r_S (see Sec. II E in [34] for details). We give the Schwarzschild radius $r_S^{D-3} = r_{S,1}^{D-3} + r_{S,2}^{D-3}$ of the final BH, the mass of the smaller BH $r_{S,1}^{D-3}/r_S^{D-3}$ and the quantity $q \equiv M_1/M_2 = r_{S,1}^{D-3}/r_{S,2}^{D-3}$ denotes the mass ratio. z_i/r_S is the initial position of i -th BH.

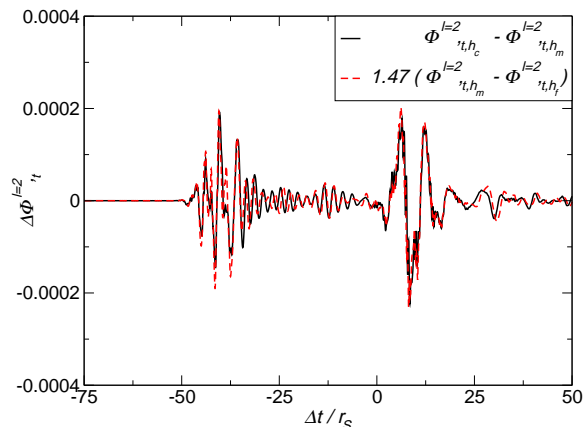


FIG. 2. (Color online) Convergence analysis of the $l = 2$ mode for model HD5d in Table I. We show the differences between the coarse and medium resolution waveform (black solid line) and the medium and high resolution waveform (red dashed line). The latter has been amplified by the factor $Q = 1.47$, indicating fourth order convergence. The curves have been shifted in time such that the formation of the common apparent horizon corresponds to $\Delta t/r_S = 0$ and taking into account the time that it takes the waves to propagate to the extraction radius.

which we will refer to as “coarse”, “medium” and “high” resolution in the following. We show the difference between the coarse and medium as well as between the medium and high resolution waveforms. The latter has been amplified by the factor $Q = 1.47$, which indicates fourth order convergence. We obtain the same order of accuracy for the higher modes. The discretization error in the waveforms is estimated to be $\approx 1.5\%$.

B. Radiated energy

Table II lists some of the most important physical quantities which characterize the head-on collision of BHs

q	$E^{\text{rad}}/M(\%)$	$E_{l=2}^{\text{rad}}(\%)$	$E_{l=3}^{\text{rad}}(\%)$	$E_{l=4}^{\text{rad}}(\%)$	$v_{\text{recoil}}(\text{km/s})$
1/1	0.089(0.090)	99.9	0.0	0.1	0.00
1/2	0.073(0.067)	97.7	2.2	0.1	11.37
1/3	0.054(0.051)	94.8	4.8	0.4	12.64
1/4	0.040(0.035)	92.4	7.0	0.6	11.38

TABLE II. Summary of our results concerning unequal mass head-on collisions of BHs in $D = 5$. We show the total radiated energy E/M as measured from the energy flux at r_{ex} , the quantity in parenthesis refers to the estimate obtained using properties of the apparent horizon (see Ref. [22] for details). The next three columns show the fraction of energy E^l excited in the l -th mode as compared to the total radiated energy. The last column refers to the recoil velocity v_{recoil} in km/s .

in $D = 5$. In particular, we show the radiated energy in units of total mass M , and the recoil velocity of the final BH in km/s . The maximum amount of energy is emitted in the equal mass case ($E^{\text{rad}}/M = 0.089\%$ as found previously in Ref. [22]), and it decreases for smaller mass ratios. We estimate the error in the radiated energy to be about 5%. These results were obtained by integrating the energy flux as given by the KI master waveform. We have also estimated the radiated energy using properties of the apparent horizon [22]. We estimate the discretization error to be about $\approx 10\%$ when using this method. The apparent horizon estimate for the total radiated energy is shown in parenthesis in Table II, and is consistent with the flux computation within numerical uncertainties. Table II also shows the fraction of energy emitted in different multipoles. Higher multipoles are clearly enhanced as the mass ratio decreases, in agreement with what we expect in the extreme case of a PP falling into a BH. In fact, we can make this statement more precise. Post-Newtonian arguments, which extend to generic D -dimensions, allow one to expect the functional dependence for the total radiated energy [53], $E^{\text{rad}}/M \propto \eta^2$, where $\eta = q/(1+q)^2$ is the dimensionless reduced mass. For clarity, we show the ratio $E^{\text{rad}}/(M\eta^2)$ in the top panel of Fig. 3, which can be seen to depend very weakly on η^2 . We can refine this argument by fitting our numerical results to an improved expression of

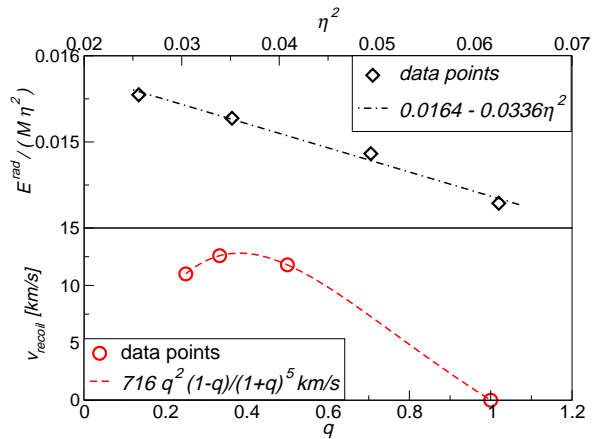


FIG. 3. (Color online) Top: Total integrated energy for different mass ratios, as function of $\eta^2 = [q/(1+q)]^2$. The black diamonds denote the numerical data and the black dashed-dotted line is the corresponding fitting function, Eq. (4). Bottom: Recoil velocity v_{recoil} in km/s as function of the mass ratio q . The red circles denote the numerical data and the red dashed line is the corresponding fitting function, Eq. (8).

the form $E^{\text{rad}}/M\eta^2 = A_0 + A_1\eta^2$. We find

$$\frac{E^{\text{rad}}}{M\eta^2} = 0.0164 - 0.0336\eta^2. \quad (4)$$

Moreover, the following expressions for the multipolar content provide a good fit to our numerical data,

$$\frac{E_{l=2}^{\text{rad}}}{E^{\text{rad}}} = 0.79 + 0.83\eta, \quad \frac{E_{l=3}^{\text{rad}}}{E^{\text{rad}}} = 0.19 - 0.77\eta. \quad (5)$$

Linearized, PP calculations presented in an accompanying paper [33] show that in the limit of zero mass ratio one obtains

$$\frac{E_{\text{PP}}^{\text{rad}}}{M\eta^2} = 0.0165, \quad (6)$$

which agrees with the extrapolation of our numerical results within less than 1%. The multipole content in the PP limit are

$$\frac{E_{l=2}^{\text{rad}}}{E_{\text{PP}}^{\text{rad}}} = 0.784, \quad \frac{E_{l=3}^{\text{rad}}}{E_{\text{PP}}^{\text{rad}}} = 0.167, \quad (7)$$

still in very good agreement with the extrapolation of our full numerical results to the zero mass ratio limit. In fact, bearing in mind that we are extrapolating from mass ratios of 1/4 down to the zero mass ratio limit, the agreement is impressive. Finally, all these results are consistent with the fact that higher multipoles contribute more to the radiation than in $D = 4$, where for instance the $l = 3$ mode contributed roughly 10% of the total energy

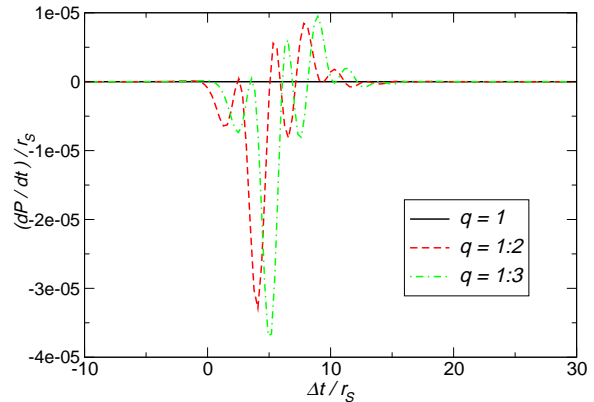


FIG. 4. (Color online) Momentum flux for different mass ratios. The curves have been shifted in time such that the formation of the common apparent horizon corresponds to $\Delta t/r_s = 0$ and taking into account the time that it takes the waves to propagate to the extraction radius.

in the PP limit [54]. Linearized, point-particle calculations show that the trend is consistent and continues in higher- D , which might mean that accurate wave extraction will become extremely difficult, as higher resolution is necessary to resolve higher- l modes [33].

C. Radiated momentum

For unequal-mass collisions, the asymmetric emission of radiation along the collision axis causes a net momentum to be carried by gravitational waves. As such, the final BH will “recoil”, according to Eq. (3). Momentum fluxes and recoil velocity for different mass-ratios are shown in Fig. 4 and the bottom panel of Fig. 3, respectively. We estimate the errors in the recoil velocity to be $\approx 5\%$. The general functional form for the dependence of momentum on the mass parameters of the individual holes has been worked out by M. Lemos in generic spacetime dimensions [53] and is the same as in four dimensions

$$v_{\text{recoil}} = C \frac{q^2(1-q)}{(1+q)^5}. \quad (8)$$

By fitting this function to our numerical data, we obtain $C = 716 \text{ km/s}$. Observe that v_{recoil} reaches a maximum value at $q = 2 - \varphi \simeq 0.38$, where φ is the golden ratio. The quality of the fit can be seen in the bottom panel of Fig. 3, where we overplot the numerical data points with the fitting function, Eq. (8). This exercise is interesting because we can again extrapolate our results to the PP limit. In an accompanying paper, Berti et al. [33] find

$$v_{\text{recoil}} = 779q^2 \text{ km/s}, \quad (9)$$

in reasonably good agreement (better than 10%) with our extrapolation. We note that momentum emission is given by a non-trivial interference between different multipoles, so this is a non-trivial agreement.

III. CONCLUSIONS

For the first time, we have evolved unequal-mass BH binaries in higher dimensions, by solving Einstein equations in the full nonlinear regime. We have focused on head-on collisions in $D = 5$ spacetime dimensions.

The gravitational waveforms are similar to the $D = 4$ counterparts [48, 49], and we were able to estimate the ringdown frequencies of the lowest multipoles. We find good agreement with published values for the quasi-normal frequencies, extracted in a linearized formalism. When extrapolated to the zero-mass-ratio limit, our results agree with linearized calculations [33] at the % level or better for the energy and momentum radiated, as well as for the multipolar dependence. This outstanding agreement is one of the main results of this work.

Our findings, supported by linearized analysis, indicate that the higher multipoles become more important for larger D . This will certainly make wave extraction at sufficiently large D a more demanding task, since higher resolutions are necessary to resolve these modes. The momentum structure is similar to the four-dimensional case, it would be interesting to understand if other aspects, such as the anti-kick, can still be interpreted in like-manner [55]. Finally, it would be very interesting to perform an exhaustive set of simulations in higher D : our results, together with linearized analysis [33], suggest a qualitative change in radiation emission for $D \leq 12 - 13$.

In fact, this change is required by the fact that Hawking's area theorem forces the total amount of gravitational radiation to decrease with D , at sufficiently large D [22, 33]. Understanding the mechanism at play requires extension of our results to arbitrary spacetime dimensions.

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