CALT-68-2810, IPMU10-0201, SCIPP 10/20

Spatially Modulated Phase in Holographic Quark-Gluon Plasma

Hirosi Ooguri[1,2] and Chang-Soon Park[3]

[1] *California Institute of Technology, 452-48, Pasadena, CA 91125, USA*
[2] *Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8586, Japan*
[3] *Santa Cruz Institute for Particle Physics and Department of Physics, University of California, Santa Cruz, CA 95064, USA*

We present a string theory construction of a gravity dual of a spatially modulated phase. In our earlier work, we showed that the Chern-Simons term in the 5-dimensional Maxwell theory destabilizes the Reissner-Nordström black holes in anti-de Sitter space if the Chern-Simons coupling is sufficiently high. In this paper, we show that a similar instability is realized on the worldvolume of 8-branes in the Sakai-Sugimoto model in the quark-gluon plasma phase. We also construct and analyze a non-linear solution describing the end-point of the transition. Our result suggests a new spatially modulated phase in quark-gluon plasma when the baryon density is above $0.8 N_f$ fm^{-3} at temperature 150 MeV.

PACS numbers: 11.25.Tq, 12.38.Mh, 04.60.Cf

INTRODUCTION

The 5-dimensional Maxwell theory with the Chern-Simons term is tachyonic in the presence of a constant electric field [1]. The tachyonic modes with non-zero spatial momenta can destabilize the Reissner-Nordström black holes in 5-dimensional anti-de Sitter space (AdS_5) if the Chern-Simons coupling is larger than a certain critical value. If it happens, the holographically dual quantum field theory in (3+1) dimensions encounters a spatially modulated phase transition. In [2], we constructed and analyzed non-linear solutions in the bulk which would describe the end-point of the phase transition.

However, an explicit realization of such an instability in superstring theory has been missing. For example, the Chern-Simons coupling of the minimal gauged supergravity in 5 dimensions is $\alpha = 1/2\sqrt{3} = 0.2887...$ which is slightly lower than the critical value $\alpha_{\rm crit} = 0.2896...$ for the instability of the extremal Reissner-Nordström black hole [1]. Similarly, the three-charge extremal black hole in the type IIB superstring theory on $AdS_5 \times S^5$ is found to be barely stable.

In this paper, we show that such an instability is realized in the quark-gluon plasma phase of the Sakai-Sugimoto model for QCD with N_f flavors of massless quarks [3]. On the worldvolume of the 8-branes, there is a $U(N_f)$ gauge field, and its diagonal $U(1)$ part is dual to the quark number ($= N_c$ times the baryon number). The baryons are identified with instanton solutions on the worldvolume in this model [4]. Worldvolume solutions representing QCD states with finite baryon density and temperature have been studied [3, 5, 6]. See also [7–9] for related papers.

Most of the solutions with finite baryon density are singular at the sources of baryons charges, and it is not clear whether the supergravity approximation is applicable. One of the exceptional cases is the quark-gluon plasma phase, where there is a smooth solution representing a finite baryon density configuration.

In the Sakai-Sugimoto model, the gluon degrees of freedom are realized on N_c D4 branes compactified on a circle S^1_c with supersymmetry breaking boundary condition [10]. At finite temperature, we compactify the Euclidean time on another circle S^1_T, and the D4 brane world volume has the topology of $S^1_c \times S^1_T \times \mathbb{R}^3$. In the confinement phase, S^1_c is contractible in the bulk, and the topology of the bulk geometry is then $S^1_T \times \mathbb{R}^3 \times S^4$ times a disk bounded by S^1_c. Each 8-brane wraps the thermal $S^1_T \times S^4$ and is extended in $\mathbb{R}^3$. In this phase, the 8-brane starts as a D8 brane at a point on S^1_c, meanders in the bulk, and ends as a $\overline{\rm D8}$ brane at another point on S^1_c.

In the deconfinement phase, the thermal S^1_T becomes contractible in the bulk geometry [10]. Depending on the relative locations of the 8-branes, the chiral symmetry restoration takes place at or above the deconfinement temperature [3, 5, 6]. Above the chiral symmetry restoration temperature, D8 and $\overline{\rm D8}$ branes become separated, and each of them has the topology of a disk bounded by S^1_T times S^4. This describes a holographic dual of the quark-gluon plasma in this model. In this phase, it is possible to construct a solution with finite baryon density that is smooth everywhere on the worldvolume, as we will discuss below. In this paper, we will focus on this case.

The dynamics of the 8-brane worldvolume is described by the Dirac-Born-Infeld (DBI) action with the Chern-Simons term. We show that there is a critical baryon density above which the brane configuration becomes unstable by tachyonic modes carrying non-zero momenta. To understand the nature of the phase transition, we construct a solution to the full non-linear equations. Though the solution carries non-zero momentum, its energy is lower than that of the original configuration which is spatially homogeneous. This suggests a spatially modulated phase with a baryon density wave.

A holographic dual of a baryon density wave was discussed in a phenomenological model in [11]. The instability of the Sakai-Sugimoto model has been studied earlier, for example in [12], but not in the chiral symmetric phase. To our knowledge, it has not been shown whether the Chern-Simons coupling on the worldvolume theory on the 8-branes is large enough to trigger the spatially modulated phase transition. In this paper, we will show that the Chern-Simons coupling is 3 times the critical value required for the instability.

INSTABILITY OF HOMOGENEOUS SOLUTION

The bulk geometry above the deconfining temperature is given by [10],

$$\begin{aligned} ds^2 =& \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(-f(U)dX_0{}^2 + d\vec{X}^2 + dX_4{}^2\right) \\ &+ \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) , \qquad (1) \\ e^{\phi} =& g_s \left(\frac{U}{R}\right)^{\frac{3}{4}} , \quad F_4 = dC_3 = \frac{3N_c}{4\pi}\epsilon_4 , \end{aligned}$$

where $f(U) = 1 - U_T^3/U^3$, $R^3 = \pi g_s N_c l_s^3$, ϵ_4 is the volume form of a unit S^4, and $d\Omega_4^2$ is a metric for a unit four-sphere. The temperature T is $\frac{3}{4\pi}U_T^{1/2}R^{-3/2}$. It sets the periodicity of the X_0 direction (along the thermal circle S^1_T) in the Euclidean solution, while the period of the compact X_4 direction (along the S^1_c) is arbitrary. In the chiral symmetry restoration phase, each 8-brane is located at a constant X_4 [3, 5, 6]. The induced metric on the branes is

$$\begin{aligned} ds^2 =& \left(\frac{U}{R}\right)^{\frac{3}{2}} \left(-f(U)dX_0^2 + d\vec{X}^2\right) \\ &+ \left(\frac{R}{U}\right)^{\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right) . \end{aligned} \qquad (2)$$

The D8 and $\overline{\text{D8}}$ branes are separated in the chiral symmetric phase. For now, let us focus on the dynamics on the D8 branes. The DBI action on the D8 brane is given by

$$S = -T_{D8}\int d^9\sigma e^{-\phi}\sqrt{-\det(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta})} + S_{CS} , \qquad (3)$$

where $T_{D8} = (2\pi)^{-8} l_s^{-9}$. The Chern-Simons action is given by

$$S_{CS} = \frac{1}{48\pi^3}\int_{D8} F_4 \wedge \omega_5(A) , \qquad (4)$$

where $F_4 = dC_3$ is the RR 4-form field which satisfies $\frac{1}{2\pi}\int_{S^4} F_4 = N_c$ in our convention and $\omega_5(A) = A\wedge F\wedge F$ is the Chern-Simons 5-form.

For our purpose, it is sufficient to turn on the $U(1)$ part of the gauge field on the worldvolume. To the quadratic order, the $U(1)$ part does not couple to the $SU(N_f)$ part of the gauge field or fluctuations of the 8-brane in the transverse direction. Couplings to the bulk degrees of freedom are suppressed by $1/N_c$. To simplify our equations, we rescale the gauge field and the metric as $A = \frac{R^2}{2\pi\alpha'}\tilde{A}$ and $g_{\alpha\beta} = R^2\tilde{g}_{\alpha\beta}$. We also rescale the coordinates as

$$u = \frac{U}{R} , \quad t = \frac{X_0}{R} , \quad \vec{x} = \frac{\vec{X}}{R} , \quad \tau = \frac{X_4}{R} . \qquad (5)$$

Assuming that the gauge field is constant on the S^4, we obtain an effective 5-dimensional action,

$$\begin{aligned} S/c = & -\int_{M_4\times\mathbb{R}} dtd^3xdu\, u^{\frac{1}{4}}\sqrt{-\det(\tilde{g}_{\alpha\beta} + \tilde{F}_{\alpha\beta})} \\ &+ \alpha\int_{M_4\times\mathbb{R}} dtd^3xdu\, \epsilon^{\mu_1\mu_2\mu_3\mu_4\mu_5}\tilde{A}_{\mu_1}\tilde{F}_{\mu_2\mu_3}\tilde{F}_{\mu_4\mu_5} . \end{aligned} \qquad (6)$$

with the 5-dimensional metric,

$$\begin{aligned} ds^2 = u^{\frac{3}{2}}\left(-f(u)dt^2 + d\vec{x}^2 + d\tau^2\right) + \frac{1}{u^{\frac{3}{2}}f(u)}du^2, \\ f(u) = 1 - \frac{u_T^3}{u^3}, \quad u_T = \left(\frac{4\pi}{3}\right)^2 R^2T^2 . \end{aligned} \qquad (7)$$

The Chern-Simons coupling α is fixed to be

$$\alpha = \frac{3}{4}, \qquad (8)$$

and the factor c is

$$c = \frac{8\pi^2}{3}T_{D8}N_f g_s^{-1}R^9. \qquad (9)$$

Note that, modulo the overall factor c, the action (6) depends only on u_T.

If the kinetic term for the gauge field were of the Maxwell form $\tilde{F}^2$, the electric field strength could be made arbitrarily high by raising the baryon density, and any non-zero value of the Chern-Simons coupling would induce an instability of the type discovered in [1]. With the DBI action, there is an upper bound for the field strength, and it requires a more careful analysis to determine whether the instability takes place.

Let us consider a background configuration with non-zero $\tilde{A}_0 = \tilde{A}_0(u)$. The equation of motion gives,

$$\tilde{E}(u) = \frac{\tilde{\rho}}{\sqrt{\tilde{\rho}^2 + u^5}} , \qquad (10)$$

where $\tilde{E} = -\tilde{F}_{tu} = \partial_u\tilde{A}_0$. The integration constant $\tilde{\rho}$ will be identified as a rescaled value of the quark density

ρ ($= N_c$ times the baryon density). As advertised in the introduction, this solution with finite quark density is regular everywhere on the brane. Note that $\tilde{\rho} \to \infty$ gives $\tilde{E} \to 1$. We choose the gauge potential $\tilde{A}_0(u)$ so that it vanishes on the horizon. Since the chemical potential $\tilde{\mu}$ is given by the asymptotic value of $\tilde{A}_0$ at $u \to \infty$, we have

$$\tilde{\mu} = \tilde{A}_0(u = \infty) = \int_{u_T}^{\infty} du \frac{\tilde{\rho}}{\sqrt{\tilde{\rho}^2 + u^5}} \,. \tag{11}$$

Let us perturb it as $\tilde{F} \to \tilde{F} + \delta\tilde{F}$. To find an onset of the instability, we look for a static normalizable solution in the linearized equation for $\delta\tilde{F}$,

$$\partial_u \left[\frac{u^{\frac{5}{2}} f(u)}{\sqrt{1 - \tilde{E}(u)^2}} \delta\tilde{F}_{ui} \right] - u^{-\frac{1}{2}} \sqrt{1 - \tilde{E}(u)^2} \partial_j \delta\tilde{F}_{ij} - 2\alpha \epsilon_{ijk} \tilde{E}(u) \delta\tilde{F}_{jk} = 0 \tag{12}$$

By applying $\epsilon_{ijk}\partial_j$ to the k-th component of the above equation and using the expression (10) for $E(u)$, we find

$$\partial_u \left[f(u) \sqrt{\tilde{\rho}^2 + u^5} \partial_u \delta\tilde{F}_i \right] + \frac{u^2}{\sqrt{\tilde{\rho}^2 + u^5}} \partial_j \partial_j \delta\tilde{F}_i - 4\alpha \frac{\tilde{\rho}}{\sqrt{\tilde{\rho}^2 + u^5}} \epsilon_{ijk} \partial_j \delta\tilde{F}_k = 0 \,, \tag{13}$$

where $\delta\tilde{F}_i = \frac{1}{2}\epsilon_{ijk}\delta\tilde{F}_{jk}$. In the Fourier mode $\delta F_i = v_i e^{-ik_j x^j} \phi(u)$ with the polarization v_i being an eigenvector of $\epsilon_{ijk}k_j$ with an eigenvalue $ik = i|\vec{k}|$ (we can also consider $-ik$ for an eigenvalue with the same result), $\phi(u)$ obeys a second order ordinary differential equation,

$$\left[-\frac{d}{du} f(u) \sqrt{\tilde{\rho}^2 + u^5} \frac{d}{du} + \frac{-4\alpha\tilde{\rho}k + u^2 k^2}{\sqrt{\tilde{\rho}^2 + u^5}} \right] \phi(u) = 0. \tag{14}$$

At the horizon $u = u_T$, we use the in-going boundary condition for static waves.

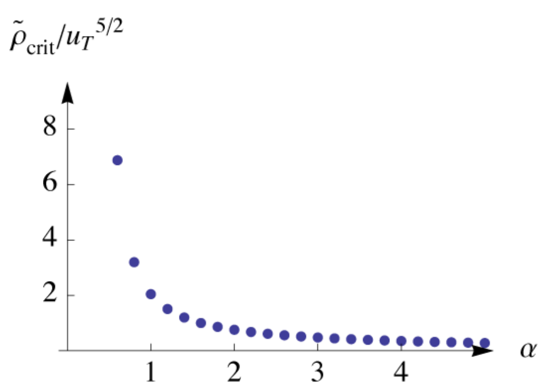

FIG. 1. The critical quark density $\tilde{\rho}$ as a function of the Chern-Simons coupling α.

We solved the linearized equation (14) numerically for general values of the Chern-Simons coupling α. For each value of the Chern-Simons coupling $\alpha > 1/4$, we found a critical value of $\tilde{\rho}$ above which the instability takes place. Figure 1 depicts the critical quark density $\tilde{\rho}_{\rm crit}$ as a function of α. We note that $\tilde{\rho}_{\rm crit}$ diverges as $\alpha \to 1/4$.

We can also show analytically that $\alpha = 1/4$ is the limiting value of the Chern-Simons coupling. Let us rescale variables as

$$\bar{u} = \tilde{\rho}^{-\frac{2}{5}} u, \quad \bar{k} = \tilde{\rho}^{-\frac{1}{5}} k, \tag{15}$$

and take the limit $\tilde{\rho} \to \infty$ in the linearized equation (14). We find

$$\left[-\frac{d}{d\bar{u}} \sqrt{1 + \bar{u}^5} \frac{d}{d\bar{u}} + \frac{-4\alpha\bar{k} + \bar{u}^2\bar{k}^2}{\sqrt{1 + \bar{u}^5}} \right] \tilde{\phi}(\bar{u}) = 0. \tag{16}$$

We have verified that a solution to this equation approaches the solution to (14) in the sense of the $\mathcal{L}^2$ measure. From the numerical evaluation of (16), we find that the momentum $\bar{k}$ with non-trivial normalizable solutions tends to infinity as we take $\tilde{\rho} \to \infty$ and α approaches the limiting value. Anticipating this, we take $\bar{k} \to \infty$ in (16) while keeping $v = \sqrt{\bar{k}}\bar{u}$ and obtain,

$$\left(-\frac{d^2}{dv^2} - 4\alpha + v^2 \right) \tilde{\phi}(\bar{u}) = 0. \tag{17}$$

This can be solved by the harmonic oscillator ground state $\phi(v) = e^{-v^2/2}$ with $\alpha = 1/4$.

In the quark-gluon plasma phase, the Chern-Simons coupling on the worldvolume theory is $\alpha = 3/4$ and is above the limiting value of $1/4$. At this value of α, the critical quark density is numerically evaluated as

$$\tilde{\rho}_{\rm crit} = 3.714 \; u_T^{\frac{5}{2}}. \tag{18}$$

Let us express the critical quark density in the original set of variables. The quark density ρ is defined by a variation of the Lagrangian density with respect to $E = \partial_u A_0$. In the above, we rescale the action by the factor c given by (9) and the gauge field is rescaled as $A = \frac{R^2}{2\pi\alpha'}\tilde{A}$. We should also remember that we rescaled our spacetime coordinates by R. The physical quark density ρ is then related to $\tilde{\rho}$ discussed in the above as

$$\rho = c \left(\frac{R^2}{2\pi\alpha'} \right)^{-1} \frac{\tilde{\rho}}{R^3} = \frac{2}{3(2\pi)^5} \frac{N_f}{g_s} \frac{R^4}{l_s^7} \tilde{\rho}. \tag{19}$$

Substituting (18) into this, the critical quark density at $\alpha = 3/4$ is given as

$$\rho_{\rm crit} = c_0 N_f N_c (g_s N_c l_s)^2 T^5, \tag{20}$$

where $c_0 = 3.714(2/3)^6\pi^3 \approx 10$.

It is important to make sure that we can ignore backreaction of this quark density to the bulk geometry. One way to see this is to note that the critical baryon density is given by dividing the quark density $\rho_{\rm critial}$ by N_c. The result is proportional to $N_f(g_sN_c)^2T^5$, but it does not have any power of N_c. Since the baryons can be thought of as D4 branes wrapping S^4 [13, 14], their backreaction becomes significant only when their density scales as N_c or more. Thus, the backreaction is negligible provided $N_f \ll N_c$. Another way to see this is to evaluate the energy density due to the electric field using the action (6) and show that it is proportional to N_f/g_s times some power of g_sN_c. This is the same scaling behavior as the tension of the N_f 8-branes.

It is an interesting exercise to express the critical density in terms of QCD quantities. The string parameters g_s and l_s are related to the Yang-Mills coupling g_{YM} and the Kaluza-Klein scale M_{KK} for the compactification circle S^1_c as $g^2_{YM} = 4\pi^2 g_s l_s/L$ and $M_{KK} = 2\pi/L$, where L is the circumference of S^1_c [15]. The critical baryon density can then be written as,

$$\frac{\rho_{\rm crit}}{N_c} = \frac{c_0 N_f}{4\pi^2}\frac{\lambda^2}{M^2_{KK}}T^5, \tag{21}$$

where $\lambda = g^2_{YM}N_c$ is the 't Hooft coupling. The constants M_{KK} and λ can be determined by fitting, for example, with the pion decay constant and the mass of the ρ-meson, as $M_{KK} = 949$ MeV and $\lambda = g^2_{YM}N_c = 16.6$ [3]. The deconfinement temperature, where the thermal cycle S^1_T becomes contractible, is at $M_{KK}/2\pi = 151$ MeV. Interestingly, this turns out to be close the critical temperature expected for the quark-gluon plasma. If we substitute $T = 150$ MeV in (21), for example, the critical baryon density comes out as,

$$\frac{\rho_{\rm crit}}{N_c} \approx 0.8 N_f \ {\rm fm}^{-3}. \tag{22}$$

For $N_f = 2$, this is about 10 times the nucleon density in atomic nuclei.

END-POINT OF THE PHASE TRANSITION

Let us study the full non-linear equations to identify the end-point of the instability. Following [2], we make the following ansatz:

$$\begin{aligned} \tilde{A}_t &= a(u) \\ \tilde{A}_x + i\tilde{A}_y &= h(u)e^{-ikz} \ , \end{aligned} \tag{23}$$

with the other gauge field components vanishing. They obey

$$\begin{aligned} &\partial_u\left[\frac{u\sqrt{u^3+k^2h(u)^2}a'(u)}{\sqrt{1-a'(u)^2+f(u)h'(u)^2}}\right] + 4\alpha k h(u)h'(u) = 0 \\ &\partial_u\left[\frac{uf(u)\sqrt{u^3+k^2h(u)^2}h'(u)}{\sqrt{1-a'(u)^2+f(u)h'(u)^2}}\right] + 4\alpha k a'(u)h(u) \\ &\qquad - \frac{k^2uh(u)\sqrt{1-a'(u)^2+f(u)h'(u)^2}}{\sqrt{u^3+k^2h(u)^2}} = 0. \end{aligned} \tag{24}$$

We assume that the embedding coordinate τ is constant, and this assumption is consistent with the equations of motion. The first equation can be integrated easily, and gives us the quark density analogously to the homogeneous solution.

$$\frac{u\sqrt{u^3+k^2h(u)^2}a'(u)}{\sqrt{1-a'(u)^2+f(u)h'(u)^2}} + 2\alpha k h(u)^2 = \tilde{\rho}\ . \tag{25}$$

Using this expression, the second equation becomes

$$\begin{aligned} &K(u)\partial_u\left(K(u)f(u)h'(u)\right) - k^2u^2h(u) \\ &\qquad +4k\alpha h(u)(\tilde{\rho} - 2k\alpha h(u)^2) = 0\ , \end{aligned} \tag{26}$$

where

$$K(u) = \sqrt{\frac{\tilde{\rho}^2 + u^5 + kh(u)^2(ku^2 - 4\tilde{\rho}\alpha + 4k\alpha^2h(u)^2}{1+f(u)h'(u)^2}}\ . \tag{27}$$

The equation (26) can be solved numerically. Figure 2 shows the range of momenta where non-linear static solutions exist for a sample case of $\tilde{\rho} = 5u_T^{5/2}$. The vertical axis h_0 is an initial value of $h(u)$: $h_0 = h(u_T)$. Since we have a family of solutions parametrized by the momentum k, we look for the one which minimizes the free energy density $\mathcal{F}$, given by

$$\mathcal{F}(\rho) = \mu\rho + \int du\,\mathcal{L}_E, \tag{28}$$

where $\mathcal{L}_E$ is the DBI Lagrangian plus the Chern-Simons term. Note that the free energy $\mathcal{F}$ is a function of ρ, and not μ.

We have identified the momentum with the lowest value of the free energy, and the expectation value of the current operator $\langle\tilde{J}\rangle$ dual to $h(u)$ can be read off from the asymptotic behavior of the normalizable solutions. That is, the current corresponding to (23) has x and y components $\tilde{J}_x + i\tilde{J}_y = \tilde{J}e^{-ikz}$.

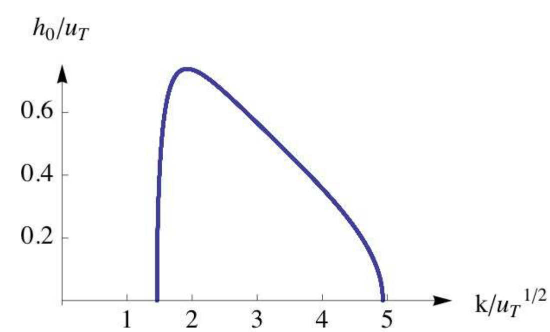

FIG. 2. Static normalizable solutions exist along this curve in the h_0-k plane, when $\alpha = \frac{3}{4}$ and $\tilde{\rho} = 5u_T^{5/2}$. The minimum free energy occurs at $k = 2.35u_T^{1/2}$, which is slightly larger than k for the maximum value of h_0.

At the critical density given by (20), the instability begins to occur at the momentum $k = 2.39u_T^{1/2}$, which in the original coordinates is given by $k/R \approx 10T$. If we set $T = 150$ MeV, the momentum is about 1.5 GeV, and the corresponding wave length is 0.8 fm.

Figure 3 shows the relation between $\tilde{\rho}$ and $\langle \tilde{J} \rangle$. Note that there is a critical value of $\tilde{\rho}$ below which there is no spatially modulated solution. In terms of the original coordinates in (1), the dual current $\langle J \rangle$ is given by

$$\frac{\langle J \rangle}{N_c} = \frac{\pi}{4}\left(\frac{2}{3}\right)^6 \frac{\lambda^2}{M_{KK}^2} N_f T^5 \cdot \frac{\langle \tilde{J} \rangle}{u_T^{5/2}} \,. \tag{29}$$

So far, we have focused on the dynamics on the D8 brane worldvolume. The analysis on the $\overline{\text{D8}}$ branes is identical except that the Chern-Simons coupling has the opposite sign due to the CPT invariance. There are separate gauge fields A_L and A_R on the D8 and $\overline{\text{D8}}$ branes, respectively, and they cause the instability above the critical charge density. The baryon vector current is dual to $(A_L + A_R)$ and the axial current is dual to $(A_L - A_R)$. The baryon charge density turns on the same amount of chemical potentials for both A_L and A_R. Above the critical baryon density, the instability will take place on both branes, each of which will settle in a configuration carrying a momentum of the size $|k|/R \approx 10T$. However, directions of the momenta on the D8 and $\overline{\text{D8}}$ branes can be different, and the currents J_L and J_R dual to A_L and A_R can carry momenta in different directions. Thus, in the spatially modulated phase, both vector and axial baryon currents are generated on the boundary.

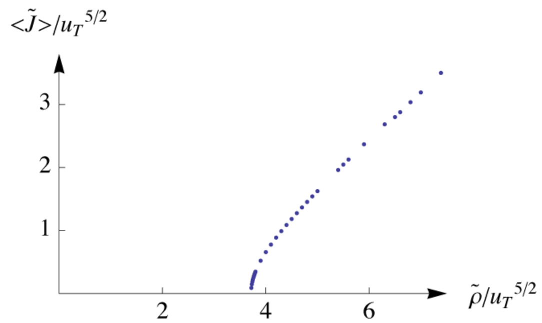

FIG. 3. The expectation value of the dual current operator $\langle \tilde{J} \rangle$ as a function of $\tilde{\rho}$ at $\alpha = \frac{3}{4}$.

ACKNOWLEDGMENTS

We would like to thank Makoto Natsuume, Dam Son, Shigeki Sugimoto, and Chen-Pin Yeh for stimulating discussions. H.O. thanks the Aspen Center for Physics and the Simons Center for Geometry and Physics, and C.S.P. thanks the IPMU for their hospitalities, where parts of this work were carried out.

This work is supported in part by DOE grant DE-FG03-92-ER40701 and by the World Premier International Research Center Initiative of MEXT. H.O. is also supported in part by a Grant-in-Aid for Scientific Research (C) 20540256 of JSPS. C.S.P. is also supported in part by DOE grant DE-FG02-04ER41286.

[1] S. Nakamura, H. Ooguri and C. S. Park, “Gravity Dual of Spatially Modulated Phase,” Phys. Rev. D **81**, 044018 (2010) [arXiv:0911.0679 [hep-th]].

[2] H. Ooguri and C. S. Park, “Holographic End-Point of Spatially Modulated Phase Transition,” arXiv:1007.3737 [hep-th].

[3] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. **113**, 843 (2005) [arXiv:hep-th/0412141].

[4] H. Hata, T. Sakai, S. Sugimoto and S. Yamato, “Baryons from instantons in holographic QCD,” Prog. Theor. Phys. **117**, 1157 (2007) [arXiv:hep-th/0701280].

[5] O. Bergman, G. Lifschytz and M. Lippert, “Holographic Nuclear Physics,” JHEP **0711**, 056 (2007) [arXiv:0708.0326 [hep-th]].

[6] O. Aharony, J. Sonnenschein and S. Yankielowicz, “A holographic model of deconfinement and chiral symmetry restoration,” Annals Phys. **322**, 1420 (2007) [arXiv:hep-th/0604161].

[7] S. Kobayashi, D. Mateos, S. Matsuura, R. C. Myers and R. M. Thomson, “Holographic phase transitions at finite baryon density,” JHEP **0702**, 016 (2007) [arXiv:hep-th/0611099].

[8] S. Nakamura, Y. Seo, S. J. Sin and K. P. Yogendran, “Baryon-charge Chemical Potential in AdS/CFT,” Prog. Theor. Phys. **120**, 51 (2008) [arXiv:0708.2818 [hep-th]].

[9] D. Mateos, S. Matsuura, R. C. Myers and R. M. Thomson, “Holographic phase transitions at finite chemical potential,” JHEP **0711**, 085 (2007) [arXiv:0709.1225 [hep-th]].

[10] E. Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories,” Adv. Theor. Math. Phys. **2**, 505 (1998) [arXiv:hep-th/9803131].

[11] S. K. Domokos and J. A. Harvey, “Baryon number-induced Chern-Simons couplings of vector and axial-vector mesons in holographic QCD,” Phys. Rev. Lett. **99**, 141602 (2007) [arXiv:0704.1604 [hep-ph]].

[12] W.-Y. Chuang, S.-H. Dai, S. Kawamoto, F.-L. Lin and C.-P. Yeh, “Dynamical Instability of Holographic QCD at Finite Density,” arXiv:1004.0162 [hep-th].

[13] E. Witten, “Baryons and branes in anti de Sitter space,” JHEP **9807**, 006 (1998) [arXiv:hep-th/9805112].

[14] D. J. Gross and H. Ooguri, “Aspects of large N gauge theory dynamics as seen by string theory,” Phys. Rev. D **58**, 106002 (1998) [arXiv:hep-th/9805129].

[15] M. Kruczenski, D. Mateos, R. C. Myers and D. J. Winters, “Towards a holographic dual of large-N(c) QCD,” JHEP **0405**, 041 (2004) [arXiv:hep-th/0311270].