Consistency of isotropic modified Maxwell theory: Microcausality and unitarity

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Abstract

The Lorentz-violating isotropic modified Maxwell theory minimally coupled to standard Dirac theory is characterized by a single real dimensionless parameter which is taken to vanish for the case of the standard (Lorentz-invariant) theory. A finite domain of positive and negative values of this Lorentz-violating parameter is determined, in which microcausality and unitarity hold. The main focus of this article is on isotropic modified Maxwell theory, but similar results for an anisotropic nonbirefringent case are presented in the appendix.

Key words: Lorentz violation, quantum electrodynamics, microcausality, unitarity

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1. Introduction

There are two Lorentz-violating extensions of the standard theory of photons [1, 2, 3], which are both gauge invariant and power-counting renormalizable [4, 5]. The standard Lorentz-invariant Maxwell theory has a quadratic field strength term (F^2) in the Lagrange density and the first Lorentz-violating extension adds a CPT-odd Chern–Simons-type term ($m_{\mathrm{CS}}\,\widehat{k}\,A\,F$, with a fixed normalized "four-vector" $\widehat{k}^{\mu}$ and mass scale m_{CS}). The second Lorentz-violating extension adds another F^2 term, which has different contractions than those of the standard Maxwell term.

The consistency of the CPT-violating Maxwell–Chern–Simons (MCS) theory [6] has been studied in Ref. [7] and the result is that certain choices of the parameters (specifically, time-

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like $\widehat{k}^{\mu}$) lead to violation of microcausality and/or unitarity. The concern now is the consistency of the CPT-invariant modified Maxwell theory, in particular, the theory restricted to the isotropic sector.

The isotropic modified Maxwell theory is described by a single real dimensionless parameter $\widetilde{\kappa}_{\mathrm{tr}}$. The standard Lorentz-invariant Maxwell theory has $\widetilde{\kappa}_{\mathrm{tr}} = 0$. Positive values of $\widetilde{\kappa}_{\mathrm{tr}}$ have been derived from an underlying small-scale structure of spacetime in the long-wavelength limit of the photons [8, 9], so that isotropic modified Maxwell theory with small enough positive $\widetilde{\kappa}_{\mathrm{tr}}$ can be expected to be consistent. But the consistency of isotropic modified Maxwell theory for *negative* values of $\widetilde{\kappa}_{\mathrm{tr}}$ is an entirely open question. Furthermore, there are only partial results for $\widetilde{\kappa}_{\mathrm{tr}} \geq 0$ in the literature [10, 11], which makes it worthwhile to give a more or less comprehensive analysis of the isotropic case.

The outline of this article is as follows. A brief discussion of isotropic modified Maxwell theory is given in Sec. 2. The pure-photon theory is then extended by the introduction of a minimal coupling of this photon to a charged Dirac particle. In short, the theory considered is a particular modification of standard quantum electrodynamics, with a modified kinetic term of the photon in the action. The corresponding gauge-field propagator in a general axial gauge is then presented in Sec. 3. Microcausality (i.e., commutation of electric and magnetic field operators with certain spacelike separations) is established in Sec. 4, together with the global causality of the theory (e.g., absence of closed timelike loops). The reflection positivity of the Euclidean gauge-field propagator is demonstrated in Sec. 5 and the unitarity of the interacting theory is checked by the direct evaluation of the optical theorem for two processes. Concluding remarks are presented in Sec. 6. The results for an anisotropic nonbirefringent case are given in App. A.

2. Isotropic modified Maxwell theory

2.1. Action and nonbirefringent Ansatz

In this article, we consider modified Maxwell theory [4, 5, 12] which has an action given by

$$S_{\mathrm{modMax}} = \int_{\mathbb{R}^4} \mathrm{d}^4x\, \mathcal{L}_{\mathrm{modMax}}(x)\,, \tag{2.1a}$$

$$\mathcal{L}_{\mathrm{modMax}}(x) = -\frac{1}{4}\,\eta^{\mu\rho}\,\eta^{\nu\sigma}\,F_{\mu\nu}(x)F_{\rho\sigma}(x) - \frac{1}{4}\,\kappa^{\mu\nu\varrho\sigma}\,F_{\mu\nu}(x)F_{\varrho\sigma}(x)\,, \tag{2.1b}$$

where $F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$ is the field strength tensor of the $U(1)$ gauge field $A_\mu(x)$. The photons propagate over a flat Minkowski spacetime with global Cartesian coordinates $(x^\mu) = (x^0, \boldsymbol{x}) = (c\,t, x^1, x^2, x^3)$ and metric $g_{\mu\nu}(x) = \eta_{\mu\nu} \equiv \mathrm{diag}\,(1,\,-1,\,-1,\,-1)$. The fixed spacetime-independent background field $\kappa^{\mu\nu\varrho\sigma}$ in the second term of (2.1b) manifestly breaks Lorentz invariance.

If $\kappa^{\mu\nu\varrho\sigma}$ is taken to have a vanishing double trace, $\kappa^{\mu\nu}{}_{\mu\nu} = 0$, and to obey the same symmetries as the Riemann curvature tensor, the number of independent Lorentz-violating parameters is 19. Birefringence is controlled by 10 of these 19 parameters. We restrict our considerations to the nonbirefringent sector with 9 parameters, which is parameterized by the following *Ansatz* [13]:

$$\kappa^{\mu\nu\varrho\sigma} = \frac{1}{2}\left(\eta^{\mu\varrho}\,\widetilde{\kappa}^{\nu\sigma} - \eta^{\mu\sigma}\,\widetilde{\kappa}^{\nu\varrho} - \eta^{\nu\varrho}\,\widetilde{\kappa}^{\mu\sigma} + \eta^{\nu\sigma}\,\widetilde{\kappa}^{\mu\varrho}\right). \tag{2.2}$$

The constant 4×4 matrix $\widetilde{\kappa}^{\mu\nu}$ is symmetric and traceless. Here and in the following, natural units are used with $\hbar = c = 1$, where c corresponds to the maximal attainable velocity of the standard Dirac particles (see Sec. 2.3).

2.2. Restriction to the isotropic case

Next, restrict the nonbirefringent modified Maxwell theory to the isotropic sector which is characterized by a purely timelike four-vector ξ^μ in a preferred reference frame and a single real dimensionless parameter $\widetilde{\kappa}_{\mathrm{tr}}$:

$$\widetilde{\kappa}^{\mu\nu} = 2\,\widetilde{\kappa}_{\mathrm{tr}}\left(\xi^\mu\xi^\nu - \frac{1}{4}\,\xi^\lambda\xi_\lambda\,\eta^{\mu\nu}\right), \tag{2.3a}$$

$$(\xi^\mu) = (1,\,0,\,0,\,0)\,, \tag{2.3b}$$

$$(\widetilde{\kappa}^{\mu\nu}) = \frac{3}{2}\,\widetilde{\kappa}_{\mathrm{tr}}\,\mathrm{diag}\left(1,\,\frac{1}{3},\,\frac{1}{3},\,\frac{1}{3}\right). \tag{2.3c}$$

From (2.1)–(2.3), the Lagrange density becomes in terms of the standard electric field $E^i \equiv F^{i0}$ and magnetic field $B^i \equiv (1/2)\,\epsilon_{ijk}\,F^{jk}$:

$$\mathcal{L}^{\mathrm{isotropic}}_{\mathrm{modMax}}\big[c,\widetilde{\kappa}_{\mathrm{tr}}\big](x) = \frac{1}{2}\Big((1+\widetilde{\kappa}_{\mathrm{tr}})\,|\mathbf{E}(x)|^2 - (1-\widetilde{\kappa}_{\mathrm{tr}})\,|\mathbf{B}(x)|^2\Big)\,, \tag{2.4}$$

where the dependence on the fundamental constants c and $\widetilde{\kappa}_{\mathrm{tr}}$ has been made explicit on the left-hand side, which will be useful for the discussion of unitarity later.

The field equation of modified Maxwell theory,

$$M^{\mu\nu} A_\nu = 0\,, \quad M^{\mu\nu} \equiv k^\lambda k_\lambda\, \eta^{\mu\nu} - k^\mu k^\nu - 2\,\kappa^{\mu\rho\sigma\nu}\, k_\rho k_\sigma\,, \tag{2.5}$$

then give the following dispersion relation for the isotropic case:

$$\omega(k) = \mathcal{B}\, k\,, \quad \mathcal{B} \equiv \sqrt{\frac{1-\widetilde{\kappa}_{\rm tr}}{1+\widetilde{\kappa}_{\rm tr}}}\,, \tag{2.6}$$

in terms of the norm of the momentum three-vector $\mathbf{k}$, defined by $k \equiv |\mathbf{k}| \equiv (k_1)^2 + (k_2)^2 + (k_3)^2$. The additional constant $\mathcal{A} \equiv \mathcal{B}^{-1}$ has been used in Ref. [14], but, in the present article, we prefer to employ only the constant $\mathcal{B}$.

The dispersion relation (2.6) yields the following phase velocity of electromagnetic waves:

$$v_{\rm ph} \equiv \frac{\omega(k)}{k} = \mathcal{B}\,. \tag{2.7}$$

This phase velocity equals the group velocity,

$$v_{\rm gr} \equiv \left|\frac{\partial\omega(k)}{\partial\mathbf{k}}\right| = \mathcal{B}\,, \tag{2.8}$$

which implies that the shape of a wave package does not change with time. From the modified dispersion law (2.6), it is clear that the vacuum behaves like an effective medium with a refraction index

$$n \equiv \frac{k}{\omega(k)} = \mathcal{B}^{-1}\,, \tag{2.9}$$

which is frequency independent because $\mathcal{B}^{-1}$ is a constant. Hence, the vacuum of this particular Lorentz-violating photon theory does not show dispersion.

Unless stated otherwise, we henceforth restrict $\widetilde{\kappa}_{\rm tr}$ to the following half-open interval:

$$\widetilde{\kappa}_{\rm tr} \in I\,, \quad I \equiv (-1, 1]\,, \tag{2.10}$$

since for $\widetilde{\kappa}_{\rm tr} \notin I$ the dispersion relation (2.6) is a complex number, which renders the undamped propagation of electromagnetic waves impossible. The front velocity, which corresponds to the velocity of the high-frequency forerunners of electromagnetic waves [15], is given by

$$v_{\rm fr} \equiv \lim_{k\mapsto\infty} v_{\rm ph} = \mathcal{B}\,, \tag{2.11}$$

and is seen to be equal to both the phase and group velocity. For $\widetilde{\kappa}_{\rm tr} < 0$, the front velocity of light exceeds the maximum attainable velocity of the standard matter particles, $v_{\rm fr} > c \equiv 1$. This alerts us to the issue of causality, which will be discussed in Sec. 4.

2.3. Coupling to matter: Modified QED

For the coupling of photons to matter, we take the minimal coupling to standard (Lorentz-invariant) spin-$\frac{1}{2}$ Dirac particles with electric charge e and mass M. That is, the theory considered is a particular deformation of quantum electrodynamics (QED) [1, 2, 3] given by the following action:

$$S^{\text{isotropic}}_{\text{modQED}}\big[c,\widetilde{\kappa}_{\text{tr}},e,M\big] = S^{\text{isotropic}}_{\text{modMax}}\big[c,\widetilde{\kappa}_{\text{tr}}\big] + S_{\text{Dirac}}\big[c,e,M\big]\,, \tag{2.12}$$

with the modified-Maxwell term (2.1)–(2.4) for the gauge field $A_\mu(x)$ and the standard Dirac term for the spinor field $\psi(x)$,

$$S_{\text{Dirac}}\big[c,e,M\big] = \int_{\mathbb{R}^4} \mathrm{d}^4x\; \overline{\psi}(x)\Big(\gamma^\mu\big(\mathrm{i}\,\partial_\mu - eA_\mu(x)\big) - M\Big)\psi(x)\,, \tag{2.13}$$

with standard Dirac matrices γ^μ corresponding to the Minkowski metric $\eta^{\mu\nu}$. As mentioned before, the fundamental constant c may be operationally defined as the maximum attainable velocity of the Dirac particle. For further discussion on Lorentz violation and the role of different particle species, see, e.g., Refs. [16, 17, 18] and references therein.

3. Polarization sum and propagator

The polarization sum can be computed by solving the field equation (2.5) while respecting the normalization condition

$$\langle \mathbf{k},\lambda|\;:P^0:\;|\mathbf{k},\lambda\rangle = \langle \mathbf{k},\lambda|\int \mathrm{d}^3x\;:T^{00}:\;\;|\mathbf{k},\lambda\rangle \equiv \omega(\mathbf{k})\,, \tag{3.1}$$

where, as usual, the pair of colons stands for the normal ordering of operators and $|\mathbf{k},\lambda\rangle$ denotes a photon state with momentum three-vector $\mathbf{k}$ and polarization label λ. The T^{00} component of the energy-momentum tensor can be cast in the following form [5]:

$$\begin{aligned} T^{00} &= \frac{1}{2}\Big(|\mathbf{E}|^2 + |\mathbf{B}|^2\Big) - \kappa^{0j0k}E^jE^k + \frac{1}{4}\,\kappa^{jklm}\varepsilon^{jkp}\varepsilon^{lmq}\,B^pB^q \\ &= \frac{1}{2}\Big((1+\widetilde{\kappa}_{\text{tr}})\,|\mathbf{E}|^2 + (1-\widetilde{\kappa}_{\text{tr}})\,|\mathbf{B}|^2\Big)\,, \end{aligned} \tag{3.2}$$

with the three-dimensional totally antisymmetric Levi-Civita symbol ε^{ijk} and the electric and magnetic field components E^i and B^j. The final expression in (3.2) makes clear that, for $|\widetilde{\kappa}_{\text{tr}}| > 1$, the theory suffers from unavoidable instabilities if the coupling to matter (2.12) is taken into account (see, e.g., Ref. [19] for a general discussion of the energy-positivity condition).

Returning to the parameter domain (2.10), the solution of the field equation and the resulting energy-momentum tensor component (3.2) give the following expression for the polarization sum:

$$\begin{aligned}\Pi^{\mu\nu} &\equiv \sum_{\lambda=1,2} \overline{\big(\varepsilon^{(\lambda)}\big)}^{\mu} \big(\varepsilon^{(\lambda)}\big)^{\nu} \\ &= \frac{1}{1+\widetilde{\kappa}_{\mathrm{tr}}} \left(-\eta^{\mu\nu} - \frac{1}{|\mathbf{k}|^2}\, k^\mu k^\nu + \frac{\mathcal{B}}{|\mathbf{k}|} \big(k^\mu \xi^\nu + \xi^\mu k^\nu \big) + \frac{2\,\widetilde{\kappa}_{\mathrm{tr}}}{1+\widetilde{\kappa}_{\mathrm{tr}}}\, \xi^\mu \xi^\nu \right), \end{aligned} \tag{3.3}$$

where the sum runs over the two physical polarizations $\lambda \in \{1, 2\}$ with the polarization vectors $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$ being orthogonal to the momentum three-vector $\mathbf{k}$. Expression (3.3) for $\widetilde{\kappa}_{\mathrm{tr}} = 0$ reproduces the standard result [3].

For the study of causality and unitarity in the corresponding quantum theory we need the propagator $G_{\mu\nu}$, which is the inverse of the Green's function $(G^{-1})_{\mu\nu}$ in momentum space. This can be computed by solving the matrix equation $(G^{-1})^{\mu\nu} G_{\nu\lambda} = \mathrm{i}\,\delta^{\mu}{}_{\lambda}$. In the following, we choose the general axial gauge

$$n_\mu\, A^\mu(x) = 0\,, \tag{3.4}$$

with an arbitrary constant vector n_μ. The corresponding gauge-fixing (gf) condition can be added to the Lagrange density $\mathcal{L}_{\mathrm{modMax}}$ from (2.1) by use of the Lagrange multiplier ς:

$$\mathcal{L}_{\mathrm{gf}} = -\frac{1}{2\,\varsigma} \big(n_\mu\, A^\mu(x) \big)^2 . \tag{3.5}$$

By inverting the Green's function we obtain the gauge-field propagator in axial gauge:

$$\begin{aligned} G_{\nu\lambda}\Big|^{\mathrm{axial}} = -\mathrm{i} \Big\{ & a\,\eta_{\nu\lambda} + b\,k_\nu k_\lambda + c\,(k_\nu n_\lambda + n_\nu k_\lambda) \\ & + d\,\xi_\nu \xi_\lambda + e\,\big(k_\nu \xi_\lambda + \xi_\nu k_\lambda\big) + f\,n^\mu n^\lambda + g\,(n_\nu \xi_\lambda + \xi_\nu n_\lambda) \Big\}\, K\,, \end{aligned} \tag{3.6}$$

with the following expressions for the scalar propagator and coefficient functions:

$$K = \frac{1}{(1-\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2)\,k^2 + 2\,\widetilde{\kappa}_{\mathrm{tr}} (k\cdot\xi)^2}\,, \tag{3.7}$$

$$a = 1\,, \tag{3.8a}$$

$$b = \frac{1}{1+\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2}\,\frac{1}{(k\cdot n)^2}\Big\{\big(1+\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2\big)\,n^2 - 2\,\widetilde{\kappa}_{\mathrm{tr}}(\xi\cdot n)^2 + \varsigma\big(1+\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2\big)\Big[\big(1-\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2\big)\,k^2 + 2\,\widetilde{\kappa}_{\mathrm{tr}}(k\cdot\xi)^2\Big]\Big\}\,, \tag{3.8b}$$

$$c = -\frac{1}{k\cdot n}\,, \tag{3.8c}$$

$$d = -\frac{2\,\widetilde{\kappa}_{\mathrm{tr}}}{1+\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2}\,, \tag{3.8d}$$

$$e = \frac{2\,\widetilde{\kappa}_{\mathrm{tr}}}{1+\widetilde{\kappa}_{\mathrm{tr}}\,\xi^2}\,\frac{\xi\cdot n}{k\cdot n}\,, \tag{3.8e}$$

$$f = 0\,,\quad g = 0\,. \tag{3.8f}$$

4. Microcausality

4.1. Commutators of gauge potentials and physical fields

The notion of microcausality can be condensed to the statement that the commutator of two field operators $\Phi(x')$ and $\Phi(x'')$ must vanish for spacelike distances, specifically, $[\Phi(x'),\Phi(x'')] = 0$ for $(x'-x'')^2 < 0$. This assures that information can only propagate along or inside null-cones. Translation invariance of the modified Maxwell theory implies the following structure of the gauge-field commutator:

$$K_{\mu\nu}(x',x'') \equiv [A_\mu(x'),A_\nu(x'')] = [A_\mu(x'-x''),A_\nu(0)] = [A_\mu(x),A_\nu(0)]\,, \tag{4.1}$$

for $x_\mu \equiv x'_\mu - x''_\mu$. The corresponding result in momentum space must be of the form

$$K_{\mu\nu}(k) = \Xi_{\mu\nu}(k^0,\mathbf{k})\,\Big(\mathrm{i}D(k)\Big)\,, \tag{4.2}$$

where $\Xi_{\mu\nu}(k)$ respects the tensor structure of the commutator and $D(k)$ is a scalar commutator function.

The commutator $K_{\mu\nu}(k)$ can be computed either directly by Fourier decomposition of the gauge potential in positive and negative frequency parts or by extraction from the Feynman propagator. Both methods yield the same result:

$$\Xi_{\mu\nu} = (1+\widetilde{\kappa}_{\mathrm{tr}})\,\Pi_{\mu\nu}\,, \tag{4.3a}$$

$$D(k)^{-1} = (1+\widetilde{\kappa}_{\mathrm{tr}})\,k_0^2 - (1-\widetilde{\kappa}_{\mathrm{tr}})\,|\mathbf{k}|^2\,, \tag{4.3b}$$

where $\Pi_{\mu\nu}$ is the polarization sum (3.3). In fact, (3.3) gives $\Xi_{00} = \Xi_{0m} = \Xi_{m0} = 0$ for $m \in \{1, 2, 3\}$, so that only some of the purely spatial components of $K_{\mu\nu}$ may be nonzero.

By using (4.3) we can compute the commutators of the electric and magnetic fields in momentum space, which can then be transformed to configuration space. The results are given by

$$[E_i(x), E_j(0)] = (\partial_0^2\, \delta_{ij} - \mathcal{B}^2\, \partial_i \partial_j) \left(\mathrm{i}D(x)\right) , \tag{4.4a}$$

$$[E_i(x), B_j(0)] = \varepsilon_{ijk}\, \partial_0 \partial_k \left(\mathrm{i}D(x)\right) , \tag{4.4b}$$

$$[B_i(x), B_j(0)] = (\nabla^2\, \delta_{ij} - \partial_i \partial_j) \left(\mathrm{i}D(x)\right) . \tag{4.4c}$$

The scalar commutator function in configuration space can be written as follows

$$D(x) = \oint\limits_C \frac{\mathrm{d}k_0}{2\pi} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \, \frac{1}{(1 + \widetilde{\kappa}_{\mathrm{tr}})\, k_0^2 - (1 - \widetilde{\kappa}_{\mathrm{tr}})\, |\mathbf{k}|^2} \, \exp\left(\mathrm{i}k_0 x_0 + \mathrm{i}\, \mathbf{k} \cdot \mathbf{x}\right) , \tag{4.5}$$

where the poles are circled in the counterclockwise direction along a contour C. The evaluation of the four-dimensional integral (4.5) leads to the following expression:

$$D(x) = -\frac{1}{2\pi\sqrt{1 - \widetilde{\kappa}_{\mathrm{tr}}^2}} \, \mathrm{sgn}(\widetilde{x}_0)\, \delta\left((\widetilde{x}_0)^2 - |\mathbf{x}|^2\right) , \quad \widetilde{x}_0 \equiv \mathcal{B}\, x_0 \, , \tag{4.6}$$

with the sign function

$$\mathrm{sgn}(x) = \begin{cases} \;\;1 & \text{for} \quad x > 0 \\ \;\;0 & \text{for} \quad x = 0 \\ -1 & \text{for} \quad x < 0 \end{cases} . \tag{4.7}$$

The overall minus sign in (4.6) has its origin in the definition of the commutator function (4.5). In this definition, the first term in the exponential enters with a plus sign. This convention has been chosen to conform with Ref. [7] and is different from the one used in, for example, App. A1 of Ref. [2]. The commutators of the physical electric and magnetic fields in (4.4) are, of course, independent of this convention. For $\widetilde{\kappa}_{\mathrm{tr}} \mapsto 0$, these commutators are equal to the results of standard QED, first obtained by Jordan and Pauli [20, 1].

According to (4.6), the commutators (4.4) vanish if the distance in Minkowski spacetime corresponds to the modified null-cone,

$$(\widetilde{x}_0)^2 - |\mathbf{x}|^2 = 0\,. \tag{4.8}$$

Two observers are causally connected and can communicate by light signals if and only if their distance in Minkowski spacetime is given by (4.8). The same result holds in standard QED with the standard null-cone, $x_0^2 - |\mathbf{x}|^2 = 0$. Modified Maxwell theory only has the position of the null-cone shifted for a nonzero value of the Lorentz-violating parameter $\widetilde{\kappa}_{\mathrm{tr}}$.

Different from the commutators of Maxwell-Chern-Simons (MCS) theory [7], the commutators (4.4)–(4.6) vanish everywhere except on the null-cone, since the pure-photon sector of modified Maxwell theory is scale-invariant. MCS theory, on the other hand, is characterized by a mass scale, called m_{CS} in Sec. 1, which leads to nonvanishing commutators both on and inside the null-cone.

Returning to the isotropic modified Maxwell theory, consider now the interactions of photons and charged matter particles as given by the modified QED action (2.12) and take, for simplicity, a vanishing mass for the Dirac particle, $M = 0$. Then, the photon has a null-cone (4.8) and the Dirac particle a different one given by $x_0^2 - |\mathbf{x}|^2 = 0$. Intuitively, there are no causality problems to be expected from having these two different null-cones. There may, of course, be nonstandard interaction processes, for example, vacuum Cherenkov radiation for $\widetilde{\kappa}_{\mathrm{tr}} > 0$ and photon decay for $\widetilde{\kappa}_{\mathrm{tr}} < 0$ (see Ref. [14] for detailed calculations).

4.2. Wick rotation

For the analytic properties of the gauge-field propagator (3.6), the behavior of the propagator pole structure under Wick rotation is an important issue. The scalar part (3.7) of the propagator shows that by performing a Wick rotation the poles of the full propagator do not lie within the integration contour and that the Wick-rotated axes do not cross any poles.

But these properties hold only for $\widetilde{\kappa}_{\mathrm{tr}} \in (-1, 1]$. A Wick rotation $k_4 = -\mathrm{i}k_0$ from Minkowski spacetime to Euclidian space, for example, maps poles with positive real part in Minkowski spacetime to poles with negative imaginary part in Euclidian space or poles with positive imaginary part to poles with negative real part. Hence, an analytic continuation of the propagator from Minkowski spacetime to Euclidian space, or *vice versa*, is possible with the Wick rotation. The gauge-field propagator (3.6) is thus well-behaved for the above mentioned parameter domain. For $\widetilde{\kappa}_{\text{tr}} \notin (-1, 1]$, however, the k_0 poles in Minkowski spacetime lie on the imaginary axis, which implies that the corresponding energy becomes imaginary for this parameter domain.

4.3. Effective metric

Following up on earlier work about the coupling of Lorentz-violating theories to gravity [21], it has been shown in Refs. [22, 23] that the action of isotropic modified Maxwell theory from (2.1) can be cast in the following form:

$$S_{\text{modMax}}^{\text{isotropic}} = -\,(1-\widetilde{\kappa}_{\text{tr}}) \int\limits_{\mathbb{R}_4} \mathrm{d}^4x \; \frac{1}{4}\; \widetilde{\eta}^{\mu\varrho}\, \widetilde{\eta}^{\nu\sigma}\; F_{\mu\nu}F_{\varrho\sigma}\,, \tag{4.9}$$

with an effective metric

$$\widetilde{\eta}^{\mu\nu} = \eta^{\mu\nu} + \frac{2\,\widetilde{\kappa}_{\text{tr}}}{1-\widetilde{\kappa}_{\text{tr}}}\; \xi^{\mu}\xi^{\nu}\,, \tag{4.10}$$

for ξ^{μ} from (2.3b). The existence of such an effective metric has interesting implications.

First, recall that a spacetime $\mathcal{M}$ is said to be "stably causal" if and only if there exists a Lorentzian metric $g_{\mu\nu}(x)$ and a scalar function $\theta(x)$, defined everywhere on $\mathcal{M}$, so that $\nabla_{\mu}\theta \neq 0$ and $(\nabla_{\mu}\theta)(\nabla_{\nu}\theta)\, g^{\mu\nu} > 0$. If a spacetime is stably causal, it does not contain closed timelike or lightlike curves (cf. Sec. 6.4 of Ref. [24]).

For the isotropic case of modified Maxwell theory defined over Minkowski spacetime with standard global coordinates as given below (2.1), we can simply choose the globally defined scalar function $\theta(x)$ to be given by the time coordinate t. Then, $(\nabla_{\mu}t) = (1, 0, 0, 0) \neq 0$ and the effective metric (4.10) gives:

$$\widetilde{\eta}^{\mu\nu}\, \nabla_{\mu}t\; \nabla_{\nu}t = \frac{1+\widetilde{\kappa}_{\text{tr}}}{1-\widetilde{\kappa}_{\text{tr}}}\,, \tag{4.11}$$

which is positive for parameter $\widetilde{\kappa}_{\text{tr}} \in (-1, 1]$, where the value $\widetilde{\kappa}_{\text{tr}} = 1$ arises as the limit from below. As a result, there are no closed timelike or lightlike curves of the effective metric, along which the modified photons could propagate. This reflects the global causality of the theory considered, in particular, for the $\widetilde{\kappa}_{\text{tr}} < 0$ case mentioned in the last paragraph of Sec. 2.2. See, e.g., Refs. [25, 26] for further discussion on Lorentz violation and causality.

5. Unitarity

5.1. Reflection positivity: Simple test

Reflection positivity [27, 28] is an important property of the theory. It assures the existence of an analytic continuation of the Euclidian propagators to Minkowski propagators, such that the theory in Minkowski spacetime has a positive semi-definite Hermitian Hamiltonian H and, therefore, a unitary time evolution operator $\exp(-\mathrm{i}Ht)$.

Following the previous analysis of MCS theory [7], we restrict the general discussion of reflection positivity to the special case of reflection positivity of a Euclidian two-point function. Concretely, reflection positivity of the Euclidian two-point function corresponds to the following inequality:

$$\langle 0|\,\Theta\big(\phi(x_4,\mathbf{x})\big)\,\phi(x_4,\mathbf{x})\,|0\rangle \geq 0\,, \tag{5.1}$$

for a complex scalar field $\phi(x_4,\mathbf{x})$ in four-dimensional Euclidian space and the reflection operation Θ: $\phi(x_4,\mathbf{x}) \mapsto \phi^\dagger(-x_4,\mathbf{x})$.

With the Fourier decomposition of the scalar field operator, we can derive reflection positivity for the scalar Euclidian propagator $S_E(k_4,\mathbf{k})$:

$$S_E(x_4) \equiv \int_{\mathbb{R}^3} \mathrm{d}^3k \int\limits_{-\infty}^{+\infty} \mathrm{d}k_4\, \exp(-\mathrm{i}k_4x_4)\, S_E(k_4,\mathbf{k}) = \int \mathrm{d}^3k\, S_E(x_4,\mathbf{k}) \geq 0\,. \tag{5.2}$$

We can also derive the strong condition

$$S_E(x_4,\mathbf{k}) \geq 0\,, \tag{5.3}$$

but, for the present discussion, we focus on the weak condition (5.2).

5.2. Reflection positivity and unitarity

If the gauge-field propagator is coupled to physical sources, i.e., a conserved current $j^\mu(k)$, then it follows from current conservation (at the classical level) or the Ward identities (at the quantum level) that all terms of the propagator which contain a propagator four-momentum k_μ vanish by contraction with $j^\mu(k)$. Hence, what remains from the gauge-field propagator (3.6) after projecting on the physical subspace is the first term involving the metric tensor and the fourth term proportional to a bilinear combination of the fundamental "four-vector"

ξ^μ. Only these two terms describe the physical degrees of freedom. The pole structure of the propagator with respect to its momentum is of crucial importance for unitarity. The relevant pole structure is in the scalar part (3.7) of the propagator. [The term proportional to $\xi\,\xi$ has an additional pole at $\widetilde{\kappa}_{\mathrm{tr}} = -1$, which plays no role for our analysis and which we have excluded anyway by condition (2.10).] Hence, it is sufficient to restrict the unitarity analysis to the scalar part K of the propagator, given by (3.7).

Since we have shown in Sec. 4.2 that Wick rotation is possible, the scalar propagator part (3.7) is Wick-rotated to Euclidian space. The resulting Euclidian expression will be denoted by S_E. Recall, that a Wick rotation induces

$$x_4 = -\mathrm{i}x^0\,, \quad k_4 = -\mathrm{i}k^0\,. \tag{5.4}$$

With our conventions, $S_E(k_4,\mathbf{k})$ is then given by the negative of the Wick-rotated scalar propagator function:

$$S_E(k_4,\mathbf{k}) = S_E(k_4,|\mathbf{k}|) = \frac{1}{(1+\widetilde{\kappa}_{\mathrm{tr}})\,(k_4^2+|\mathbf{k}|^2) - 2\,\widetilde{\kappa}_{\mathrm{tr}}\,k_4^2} = \frac{1}{(1-\widetilde{\kappa}_{\mathrm{tr}})\,k_4^2 + (1+\widetilde{\kappa}_{\mathrm{tr}})\,|\mathbf{k}|^2}\,. \tag{5.5}$$

In order to show reflection positivity for the scalar part of the Euclidian propagator, the expression

$$S_E(x_4,|\mathbf{k}|) = \int_{-\infty}^{+\infty} \mathrm{d}k_4\, \exp(-\mathrm{i}k_4\,x_4)\, S_E(k_4,|\mathbf{k}|)\,, \tag{5.6}$$

needs to be examined. Performing the integrals yields the following results:

$$S_E(x_4,|\mathbf{k}|) = \frac{\pi}{\sqrt{1-\widetilde{\kappa}_{\mathrm{tr}}^2}}\,\frac{1}{|\mathbf{k}|}\, \exp\Big(-|x_4|\,\mathcal{B}^{-1}\,|\mathbf{k}|\Big)\,, \tag{5.7a}$$

$$S_E(x_4) = \frac{4\pi^2}{\sqrt{1-\widetilde{\kappa}_{\mathrm{tr}}^2}}\,\frac{\mathcal{B}^2}{x_4^2}\,. \tag{5.7b}$$

Both of these expressions for $S_E(x_4,|\mathbf{k}|)$ and $S_E(x_4)$ are manifestly larger than zero for $\widetilde{\kappa}_{\mathrm{tr}} \in (-1,1]$, where the value $\widetilde{\kappa}_{\mathrm{tr}} = 1$ arises as the limit from below. Hence, reflection positivity (5.2)–(5.3) is guaranteed for this parameter domain. In turn, this implies unitarity of the pure-photon sector, provided $\widetilde{\kappa}_{\mathrm{tr}} \in (-1,1]$.

For $\widetilde{\kappa}_{\mathrm{tr}} \notin [-1,1]$, the corresponding results are

$$S_E(x_4, |\mathbf{k}|) = -\frac{\pi}{\sqrt{\widetilde{\kappa}_{\mathrm{tr}}^2 - 1}}\,\frac{1}{|\mathbf{k}|}\,\sin\big(|x_4|\,\mathcal{C}\,|\mathbf{k}|\big)\,, \tag{5.8a}$$

$$S_E(x_4) = \frac{4\pi^3}{\sqrt{\widetilde{\kappa}_{\mathrm{tr}}^2 - 1}}\,\delta'\big(|x_4|\,\mathcal{C}\big)\,, \tag{5.8b}$$

with

$$\mathcal{C} \equiv \sqrt{\frac{\widetilde{\kappa}_{\mathrm{tr}} + 1}{\widetilde{\kappa}_{\mathrm{tr}} - 1}}\,. \tag{5.8c}$$

Both of these last expressions for $S_E(x_4, |\mathbf{k}|)$ and $S_E(x_4)$ are not manifestly positive because of the presence of the sine function and the delta function derivative.[1] As a result, unitarity is violated for this parameter choice, which is also obvious from the fact that the corresponding dispersion relation is imaginary.

More generally, it is clear that the pure-photon isotropic modified Maxwell theory (2.4) is unitary by the following simple argument (prefigured in the discussion of Sec. 4.3). As the electric field involves one derivative with respect to the spacetime coordinate $x^0 \equiv c\,t$, the Lagrange density (2.4) can be made to be proportional to the standard form (having $\widetilde{\kappa}_{\mathrm{tr}} = 0$) by the introduction of a rescaled velocity $c' \equiv c\,\mathcal{B}$, with constant $\mathcal{B} \equiv \sqrt{(1-\widetilde{\kappa}_{\mathrm{tr}})/(1+\widetilde{\kappa}_{\mathrm{tr}})}$ as defined in (2.6). Moreover, as the action always appears divided by the Planck constant $\hbar$, it is even possible to remove the remaining overall factor by the introduction of a rescaled constant $\hbar' \equiv \hbar/(1-\widetilde{\kappa}_{\mathrm{tr}})$.[2] Now, standard Maxwell–Jordan–Pauli photons (even with phase velocity c' and rescaled Planck constant $\hbar'$) have been proven to be unitary [3, 27, 28]. So, the outstanding issue is whether or not the unitary of the pure-photon isotropic modified Maxwell theory is affected by the standard minimal coupling (2.13) to matter (recall that this minimal coupling is governed by the gauge principle).

In that respect, it is highly relevant that two physical decay processes have already been calculated in Ref. [14]. The exact tree-level results for the corresponding decay rates were found to be well-behaved for parameter values in the domain (2.10). Clearly, this agrees with the conjecture that isotropic modified QED (2.12) is unitary for the proper values of the parameter $\widetilde{\kappa}_{\mathrm{tr}}$. Further evidence will be given in the next subsection.

---

[1]Equation (5.8b) is to be understood as acting on a test function. The sign of the resulting expression depends on the test function and need not be positive.

[2]Similar redefinitions bring the commutators (4.4) back to the standard Jordan–Pauli form [20].

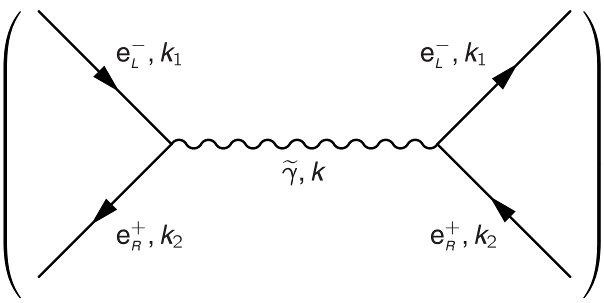
5.3. Optical theorem

In order to explicitly check the above statement about unitarity of isotropic modified QED (2.12) for the parameter domain $\widetilde{\kappa}_{\text{tr}} \in (-1, 1]$, we will consider two physical processes. The idea, now, is that the total cross section or decay width of a physical process is related to the imaginary part of the respective forward scattering amplitude via the optical theorem [29, 30]. The optical theorem follows directly from the unitarity of the S–matrix. Hence, if unitarity does not hold, this can be expected to show up as a violation of the optical theorem.

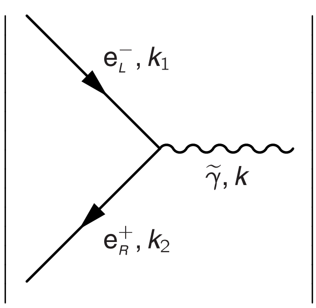
5.3.1. First process

Consider a process involving definite polarization states of the charged particles: pair creation of a left-handed electron and a right-handed positron, where the chirality conventions of Ref. [30] will be used. The optical theorem will be verified by comparing the imaginary part of the forward scattering amplitude to the total cross section for the production of a modified photon ($\widetilde{\gamma}$) from a left-handed electron (e_L^-) and a right-handed positron (e_R^+):

$2\,\mathrm{Im}$



$\stackrel{?}{=} \int \mathrm{d}\Pi_1$



^{2} , (5.9)

where $\mathrm{d}\Pi_1$ denotes the one-particle phase-space element of the modified photon $\widetilde{\gamma}$ in the final state.

Let us take massless fermions, so that the helicity of a particle is a physically well-defined property, that is, independent of the reference frame. (Recall that, for the case of massive fermions, chirality is not equal to helicity. In that case, left- and right-handed particles carry both parallel and antiparallel spins with respect to the momenta of the particles.) The assumption of massless particles leads to a conserved axial vector current: $\partial_\mu j_5^\mu(x) = 0$ with $j_5^\mu(x) = \overline{\psi}(x)\gamma^\mu\gamma_5\psi(x)$. As a result, we have $k_\mu j_5^\mu(k) = 0$ for a photon with momentum k_μ coupling to the current $j_5^\mu(k)$. Incidentally, we can neglect the anomalous nonconservation of the axial vector current [29, 30], because the possible additional terms in our calculation would be of higher order in the gauge coupling constant e.

The forward scattering amplitude $\mathcal{M}_1 \equiv \mathcal{M}(\mathrm{e}_L^- \mathrm{e}_R^+ \to \mathrm{e}_L^- \mathrm{e}_R^+)$ is then given by

$$\mathcal{M}_1 = e^2\, \overline{u}(k_1)\gamma^\nu \frac{\mathbb{1}-\gamma_5}{2} v(k_2)\overline{v}(k_2)\gamma^\mu \frac{\mathbb{1}-\gamma_5}{2} u(k_1) \\ \times \frac{1}{K^{-1}+\mathrm{i}\epsilon}\big(\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\,\xi_\mu \xi_\nu + e\,(k_\mu \xi_\nu + \xi_\mu k_\nu)\big)\,, \tag{5.10}$$

for the photon propagator of the isotropic modified Maxwell theory, that is, K, b, c, d, and e taking values from (3.7) and (3.8).

By introducing a four-dimensional integration over the momentum k^μ of the virtual photon we obtain:

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\, \delta^{(4)}(k_1+k_2-k)\, \mathcal{M}_1 = \\ = \int\limits_{-\infty}^{+\infty} \frac{\mathrm{d}k^0}{2\pi} \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta^{(4)}(k_1+k_2-k)\, e^2\, \overline{u}(k_1)\gamma^\nu \frac{\mathbb{1}-\gamma_5}{2} v(k_2)\overline{v}(k_2)\gamma^\mu \frac{\mathbb{1}-\gamma_5}{2} u(k_1) \\ \times \frac{1}{N}\, \frac{\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\,\xi_\mu \xi_\nu + e\,(k_\mu \xi_\nu + \xi_\mu k_\nu)}{(k^0 - \mathcal{B}|\mathbf{k}| + \mathrm{i}\epsilon)(k^0 + \mathcal{B}|\mathbf{k}| - \mathrm{i}\epsilon)}\,, \tag{5.11}$$

with

$$\frac{1}{N} \equiv \frac{1}{1+\widetilde{\kappa}_{\mathrm{tr}}}\,, \tag{5.12}$$

and $\mathcal{B}$ from (2.6).

For the imaginary part of the amplitude, only the propagator poles contribute, since Feynman's iϵ prescription only becomes important at the poles. These poles are given by $k^0 = +\mathcal{B}|\mathbf{k}| - \mathrm{i}\epsilon$ and $k^0 = -\mathcal{B}|\mathbf{k}| + \mathrm{i}\epsilon$, with a positive infinitesimal ϵ. The following holds for the propagator pole with a positive real part:

$$\frac{1}{k^0 - \mathcal{B}|\mathbf{k}| + \mathrm{i}\epsilon} = \mathcal{P}\frac{1}{k^0 - \mathcal{B}|\mathbf{k}|} - \mathrm{i}\pi\, \delta(k^0 - \mathcal{B}|\mathbf{k}|) = \mathcal{P}\frac{1}{k^0 - \omega} - \mathrm{i}\pi\, \delta(k^0 - \omega)\,, \tag{5.13}$$

where $\mathcal{P}$ denotes the principal value. The first term on the far right-hand-side of (5.13) is real, whereas the second one is imaginary and puts the virtual photon on-shell. For this reason, only the positive frequency pole can be physically relevant and, in order to obtain the imaginary part, all k^0 have to be replaced by the dispersion relation (2.6).

With the further notation $\widehat{\mathcal{M}}_1 \equiv \mathcal{M}(\mathrm{e}_L^- \mathrm{e}_R^+ \to \widetilde{\gamma})$, we finally get:

$$2\,\mathrm{Im}(\mathcal{M}_1) = -\int\limits_{-\infty}^{+\infty} \mathrm{d}k^0\, \delta(k^0 - \omega) \int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \delta^{(4)}(k_1+k_2-k)$$

$$
\begin{aligned}
&\times\, e^2\, \overline{u}(k_1)\gamma^\nu \frac{\mathbb{1}-\gamma_5}{2} v(k_2)\overline{v}(k_2)\gamma^\mu \frac{\mathbb{1}-\gamma_5}{2} u(k_1) \\
&\times\, \frac{1}{N}\, \frac{\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\,\xi_\mu\xi_\nu + e\,(k_\mu \xi_\nu + \xi_\mu k_\nu)}{k^0+\omega} \\
= -\int & \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\omega}\, \delta^{(4)}(k_1+k_2-k)\, e^2\, \overline{u}(k_1)\gamma^\nu \frac{\mathbb{1}-\gamma_5}{2} v(k_2)\overline{v}(k_2)\gamma^\mu \frac{\mathbb{1}-\gamma_5}{2} u(k_1) \\
&\times\, \frac{1}{N}\, \Big(\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\,\xi_\mu\xi_\nu + e\,(k_\mu \xi_\nu + \xi_\mu k_\nu)\Big) \\
= \int & \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\omega}\, \delta^{(4)}(k_1+k_2-k)\, (\widehat{\mathcal{M}}_1^\dagger)^\nu (\widehat{\mathcal{M}}_1)^\mu\, \frac{1}{N}\, \Big(-\eta_{\mu\nu} - d\,\xi_\mu\xi_\nu\Big) \\
= \int & \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\omega}\, \delta^{(4)}(k_1+k_2-k)\, (\widehat{\mathcal{M}}_1^\dagger)^\nu (\widehat{\mathcal{M}}_1)^\mu \left(\sum_{\lambda=1,2} \overline{(\varepsilon^{(\lambda)})}_\nu (\varepsilon^{(\lambda)})_\mu\right) \\
= \int & \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\omega}\, \delta^{(4)}(k_1+k_2-k)\, \sum_{\lambda=1,2} |\widehat{\mathcal{M}}_1|^2\,,
\end{aligned}
\tag{5.14}
$$

with the definition $\widehat{\mathcal{M}}_1(k) \equiv \varepsilon_\mu(k)\, \widehat{\mathcal{M}}_1^\mu(k)$ on the last line. In the third step, the Ward-identity has been used, so that all terms vanish for which the momentum k_μ is contracted with $(\widehat{\mathcal{M}}_1)^\mu$ or its Hermitian conjugate. Recall that the Ward-identity [3, 29, 30] reads

$$
k_\mu\, \mathcal{M}^\mu = 0\,,
\tag{5.15}
$$

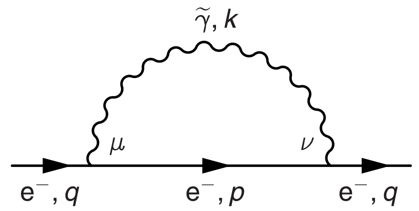
for a general matrix element $\mathcal{M}^\mu(k)$ to which an external photon [with polarization vector $\varepsilon_\mu(k)$ and momentum k_μ] couples. What remains in the fourth step of (5.14) is the polarization sum (3.3), since N corresponds to the normalization factor $1/(1+\widetilde{\kappa}_{\mathrm{tr}})$ and $-d$ to $2\widetilde{\kappa}_{\mathrm{tr}}/(1+\widetilde{\kappa}_{\mathrm{tr}})$ according to (3.8d).

The conclusion is that the imaginary part of the forward scattering amplitude of the process $\mathrm{e}^-_L \mathrm{e}^+_R \to \mathrm{e}^-_L \mathrm{e}^+_R$ is related to the total cross section for the annihilation process $\mathrm{e}^-_L \mathrm{e}^+_R \to \widetilde{\gamma}$. This results verifies the validity of the optical theorem, at least, for the process considered.

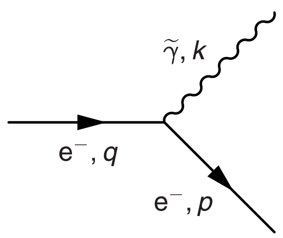
5.3.2. Second process

Next, consider the modified self-energy correction to the propagator of an electron. This time, we will not take a definite polarization state. The one-loop correction is then related to the total decay width for electron vacuum Cherenkov radiation [14], provided the optical theorem holds true. Specifically, we have to verify the relation

$2\,\mathrm{Im}\Bigg($



$\Bigg) \stackrel{?}{=} \displaystyle\int \mathrm{d}\Pi_2 \;\Bigg|$



$\Bigg|^2 \,, \qquad (5.16)$

where $\mathrm{d}\Pi_2$ is the two-particle phase-space element of the electron e^- and the modified photon $\widetilde{\gamma}$ in the final state.

In the following, we use the notations $\mathcal{M}_2 \equiv \mathcal{M}(\mathrm{e}^- \to \mathrm{e}^-)$ and $\widehat{\mathcal{M}_2} \equiv \mathcal{M}(\mathrm{e}^- \to \mathrm{e}^-\widetilde{\gamma})$. Again introducing an additional momentum integration, the amplitude (averaged over the spin of the incoming electron) is given by

$$\begin{aligned} \mathcal{M}_2 = \frac{\mathrm{i}}{2}\sum_{s_1}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\,\delta^{(4)}(p+k-q)\,e^2\,\overline{u}(q)\gamma^\nu\frac{\not{p}+m}{p^2-m^2+\mathrm{i}\epsilon}\gamma^\mu u(q) \\ \times\frac{1}{K^{-1}+\mathrm{i}\epsilon}\Big(\eta_{\mu\nu}+b\,k_\mu k_\nu+c\,(k_\mu n_\nu+n_\mu k_\nu)+d\,\xi_\mu\xi_\nu+e\,(k_\mu\xi_\nu+\xi_\mu k_\nu)\Big)\,. \end{aligned} \tag{5.17}$$

The above integral is power-counting divergent and must be replaced by, for example, the dimensionally-regulated version [3, 29, 30]. As our analysis only relies on complex function theory and Dirac algebra, we can simply keep this dimensional regularization implicit.

For the imaginary part of (5.17), the position of the propagator poles is, once more, of crucial importance. Equation (5.13) holds for the positive pole of the photon propagator. The denominator of the electron propagator can be written as

$$\frac{1}{p^2-m^2+\mathrm{i}\epsilon}=\frac{1}{\big(p^0-\sqrt{|\mathbf{p}|^2+m^2}+\mathrm{i}\epsilon\big)\big(p^0+\sqrt{|\mathbf{p}|^2+m^2}-\mathrm{i}\epsilon\big)}\,. \tag{5.18}$$

Furthermore, we obtain for the positive pole $p^0=\sqrt{|\mathbf{p}|^2+m^2}\equiv E$:

$$\frac{1}{p^0-E+\mathrm{i}\epsilon}=\mathcal{P}\frac{1}{p^0-E}-\mathrm{i}\pi\,\delta(p^0-E)\,. \tag{5.19}$$

With p^0 replaced by E and k^0 by ω as defined by (2.6), we then get the following result:

$$2\,\mathrm{Im}(\mathcal{M}_2)=-\frac{1}{4}\int_{-\infty}^{+\infty}\mathrm{d}k^0\,\delta(k^0-\omega)\int_{-\infty}^{+\infty}\mathrm{d}p^0\,\delta(p^0-E)$$

$$
\begin{aligned}
&\times \int \frac{\mathrm{d}^3k}{(2\pi)^3} \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, \delta^{(4)}(p+k-q) \sum_{s_1} e^2\, \overline{u}(q)\gamma^\nu \frac{\not{p}+m}{p^0+E}\gamma^\mu u(q) \\
&\times \frac{1}{N}\, \frac{\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\, \xi_\mu\xi_\nu + e\,(k_\mu\xi_\nu + \xi_\mu k_\nu)}{k^0+\omega} \\
= &-\frac{1}{4} \int \frac{\mathrm{d}^3k}{(2\pi)^3\, 2\omega} \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E}\, \delta^{(4)}(p+k-q) \sum_{s_1} e^2\, \overline{u}(q)\gamma^\nu(\not{p}+m)\gamma^\mu u(q) \\
&\times \frac{1}{N}\, \Big(\eta_{\mu\nu} + b\, k_\mu k_\nu + c\,(k_\mu n_\nu + n_\mu k_\nu) + d\, \xi_\mu\xi_\nu + e\,(k_\mu\xi_\nu + \xi_\mu k_\nu)\Big) \\
= &\frac{1}{4} \int \frac{\mathrm{d}^3k}{(2\pi)^3\, 2\omega} \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E}\, \delta^{(4)}(p+k-q) \\
&\qquad\qquad \times e^2\, \mathrm{Tr}\Big[(\not{q}+m)\gamma^\nu(\not{p}+m)\gamma^\mu\Big] \frac{1}{N}\, \Big(-\eta_{\mu\nu} - d\, \xi_\mu\xi_\nu\Big) \\
= &\frac{1}{4} \int \frac{\mathrm{d}^3k}{(2\pi)^3\, 2\omega} \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E}\, \delta^{(4)}(p+k-q) \\
&\qquad\qquad \times \left(\sum_{s_1,s_2} (\widehat{\mathcal{M}}_2^\dagger)^\nu (\widehat{\mathcal{M}}_2)^\mu\right) \left(\sum_{\lambda=1,2} \overline{(\varepsilon^{(\lambda)})}_\nu (\varepsilon^{(\lambda)})_\mu\right) \\
= &\int \frac{\mathrm{d}^3k}{(2\pi)^3\, 2\omega} \int \frac{\mathrm{d}^3p}{(2\pi)^3\, 2E}\, \delta^{(4)}(p+k-q)\, \frac{1}{4} \sum_{s_1,s_2} \sum_{\lambda=1,2} |\widehat{\mathcal{M}}_2|^2\,,
\end{aligned}
\tag{5.20}
$$

which verifies the optical theorem also for this process.

To summarize, only the position of the propagator poles and the existence of the Ward-identity are of importance for the validity of the optical theorem (see also the clear discussion of standard QED unitarity in Chap. 9 of Ref. [3]). The form of the matrix element itself (whether it is, for example, polarized or unpolarized) plays no role. Since both reflection positivity and the optical theorem have been verified in this section, we conclude that, most likely, unitarity of the modified QED theory (2.12) holds for $\widetilde{\kappa}_{\mathrm{tr}} \in (-1, 1]$. The only *caveat* we have is the assumed applicability of regularized Feynman–Dyson perturbation theory. But perturbation theory appears hold for modified QED [31], just as it holds for the standard Lorentz-invariant theory[3, 29, 30].

6. Discussion and outlook

In this article, the microcausality and unitarity of the isotropic modified Maxwell theory (2.4) have been established for numerical values of the "deformation parameter" $\widetilde{\kappa}_{\mathrm{tr}}$ lying in the domain (2.10).[3] In addition, strong evidence has been presented that these properties of the pure photon sector carry over to the modified QED theory (2.12) of photons minimally coupled to standard Lorentz-invariant Dirac particles. These results rely on Feynman–Dyson perturbation theory.

The next question is precisely which numerical value of the $\widetilde{\kappa}_{\rm tr}$ domain holds experimentally, where $\widetilde{\kappa}_{\rm tr} = 0$ corresponds to exact Lorentz invariance. Moreover, having a nonzero $\widetilde{\kappa}_{\rm tr}$ singles out a preferred frame of reference, in which the *Ansatz* "four-vector" ξ^μ from (2.3b) is purely timelike. We have no idea what the proper reference frame would be. Here, the reference frame is simply taken to correspond to the sun-centered celestial equatorial frame (SCCEF). Another possible choice would be the frame in which the cosmic microwave background is isotropic. The strategy is, first, to establish whether or not $\widetilde{\kappa}_{\rm tr}$ differs from zero and, then, to determine the relevant reference frame if the parameter is indeed nonzero.

Direct laboratory bounds on $|\widetilde{\kappa}_{\rm tr}|$ in the SCCEF range from the 10^{-2} level of the first experiment [32] to the 10^{-7} and 10^{-8} levels of the two most recent experiments [33, 34]. Indirect laboratory bounds are much stronger, ranging from the 10^{-11} level [35] to the 5×10^{-15} level [36]. Still better indirect earth-based bounds follow from the observation of ultra-high-energy-cosmic-ray (UHECR) primaries and TeV gamma-rays at the top of the Earth's atmosphere: $-0.9 \times 10^{-15} < \widetilde{\kappa}_{\rm tr} < 0.6 \times 10^{-19}$ at the two–σ level [14]. Future results on UHECRs and TeV gamma-rays may even improve this last two-sided bound by a factor 10^2 [37].

The tight experimental bounds on $\widetilde{\kappa}_{\rm tr}$ can perhaps be understood as implying the extreme smoothness of space, if $\widetilde{\kappa}_{\rm tr}$ arises as the excluded-spacetime-volume fraction of "defects" randomly embedded in flat Minkowski spacetime [9]. Specifically, calculations in simple models give a positive value for $\widetilde{\kappa}_{\rm tr}$ proportional to $(b/l)^4$, where b corresponds to the typical size of the defect (this size being obtained from measurements in the ambient flat spacetime) and l to the typical minimal length between the individual defects (again, from measurements in the ambient flat spacetime). Remark that, *a priori*, the excluded-spacetime-volume fraction $(b/l)^4$ can be of order unity, implying the same order of magnitude for the deformation parameter $\widetilde{\kappa}_{\rm tr}$ [9].[4]

[3] A similar domain has been established for a particular parity-even anisotropic case of nonbirefringent modified Maxwell theory in App. A.

[4] An alternative calculation of $\widetilde{\kappa}_{\rm tr}$ relies on anomalous effects and finds a positive value proportional to

This brings us, finally, to the structure of spacetime (and possibly the cosmological constant problem [38, 39]). In that respect, it is of direct relevance that the modified QED theory (2.12) can also be coupled to external gravitational fields. But, remarkably, modified QED cannot be coupled to dynamical gravitational fields [21, 22]. The main hurdle appears to be that the energy-momentum tensor $T_{\mu\nu}$ of isotropic modified QED has an antisymmetric part; see Eq. (2.11b) of Ref. [22] with ξ^μ from (2.3b) in this paper. The conclusion may be that either standard gravity rules out the particular theory (2.12) with explicit Lorentz violation or that the theory of gravity itself needs to be modified fundamentally.

A. Parity-even anisotropic case

A.1. Definition and dispersion relation

One particular anisotropic case of nonbirefringent modified Maxwell theory is characterized by a purely spacelike four-vector ξ^μ and a single Lorentz-violating parameter $\widetilde{\kappa}_{33}$:

$$\widetilde{\kappa}^{\mu\nu} = \frac{4}{3}\,\widetilde{\kappa}_{33}\left(\xi^\mu\xi^\nu - \frac{1}{4}\,\xi^\lambda\xi_\lambda\,\eta^{\mu\nu}\right), \tag{A.1a}$$

$$(\xi^\mu) = (0,\,0,\,0,\,1)\,, \tag{A.1b}$$

$$(\widetilde{\kappa}^{\mu\nu}) = \widetilde{\kappa}_{33}\ \mathrm{diag}\left(\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1\right). \tag{A.1c}$$

By choosing the momentum four-vector as

$$(k^\mu) = (\omega(\mathbf{k}),\,k_\perp,\,0,\,k_\parallel)\,,\quad k_\parallel = \mathbf{k}\cdot\boldsymbol{\xi}\,,\quad k_\perp = |\mathbf{k} - k_\parallel\,\boldsymbol{\xi}|\,,\quad \boldsymbol{\xi} = (0,\,0,\,1)\,, \tag{A.2}$$

we obtain the following dispersion relation from the field equation (2.5):

$$\omega(\mathbf{k}) = \sqrt{k_\perp^2 + \mathcal{D}^2\,k_\parallel^2}\,,\quad \mathcal{D} \equiv \sqrt{\frac{1-2\widetilde{\kappa}_{33}/3}{1+2\widetilde{\kappa}_{33}/3}}\,. \tag{A.3}$$

The case considered can be expressed in terms of the standard-model-extension (SME) parameters [12] with the help of the "translation dictionary" from [40]:

$$\widetilde{\kappa}_{\mathrm{tr}} = \frac{2}{9}\,\widetilde{\kappa}_{33}\,,\quad (\widetilde{\kappa}_{\mathrm{e}-})^{(11)} = \frac{4}{9}\,\widetilde{\kappa}_{33}\,,\quad (\widetilde{\kappa}_{\mathrm{e}-})^{(22)} = \frac{4}{9}\,\widetilde{\kappa}_{33}\,. \tag{A.4}$$

Hence, the anisotropic case considered in this appendix is a mixture of the three parity-even parameters $(\widetilde{\kappa}_{\mathrm{e}-})^{(11)}$, $(\widetilde{\kappa}_{\mathrm{e}-})^{(22)}$, and $\widetilde{\kappa}_{\mathrm{tr}}$.

---

the product of the fine-structure constant α and the affected-spacetime-volume fraction from "punctures" ($b = 0$) embedded in flat Minkowski spacetime [8]. Since, *a priori*, the affected–spacetime-volume fraction can be of order unity, the deformation parameter $\widetilde{\kappa}_{\mathrm{tr}}$ can then be of order $\alpha \sim 10^{-2}$.

A.2. Microcausality

The commutators of electric and magnetic fields can be computed just as for the isotropic case in Sec. 4.1. The results involve a particular tensor structure and a scalar commutator function (here, distinguished by a bar)

$$\overline{D}(x) = \oint\limits_C \frac{\mathrm{d}k_0}{2\pi} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{1}{(1+2\widetilde{\kappa}_{33}/3)\,(k_0^2-k_\perp^2)-(1-2\widetilde{\kappa}_{33}/3)\,k_\parallel^2} \exp\left(\mathrm{i}\,k_0\,x_0 + \mathrm{i}\,\mathbf{k}\cdot\mathbf{x}\right). \tag{A.5}$$

For the issue of microcausality, the properties of $\overline{D}(x)$ are important and the computation of the four-dimensional integral in (A.5) yields:

$$\overline{D}(x) = -\frac{1}{2\pi\sqrt{1-4\widetilde{\kappa}_{33}^2/9}}\,\mathrm{sgn}(x_0)\,\delta\left(x_0^2-x_\perp^2-x_\parallel^2\,/\,\mathcal{D}^2\right). \tag{A.6}$$

Hence, analogously to the isotropic case in Sec. 4.1, the commutator function vanishes everywhere except on the modified null-cone,

$$x_0^2-x_\perp^2-x_\parallel^2\,/\,\mathcal{D}^2 = 0\,. \tag{A.7}$$

As a result, microcausality is a property also for this particular anisotropic case of nonbirefringent modified Maxwell theory, provided

$$\frac{2}{3}\,\widetilde{\kappa}_{33} \in I\,, \quad I \equiv (-1,1]\,, \tag{A.8}$$

which matches the domain of the isotropic parameter (2.10). In fact, the formal structure of these two cases is similar — recall the definitions of the matrices $\widetilde{\kappa}^{\mu\nu}$ in Eqs. (2.3c) and (A.1c). Note also that these two cases of nonbirefringent modified Maxwell theory, for $\widetilde{\kappa}_{\mathrm{tr}} > 0$ and $\widetilde{\kappa}_{33} > 0$, can be induced from a single Lorentz-violating term in the fermionic action [41].

A.3. Reflection positivity and unitarity

The simple test of reflection positivity from Sec. 5.1 works just as for the isotropic case, since the scalar part of the Euclidian propagator (here, distinguished by a bar) is

$$\overline{S}_E(k_4,\mathbf{k}) = \frac{1}{(1+2\widetilde{\kappa}_{33}/3)\,(k_4^2+|\mathbf{k}|^2)-(4\widetilde{\kappa}_{33}/3)k_3^2} = \frac{1}{1+2\widetilde{\kappa}_{33}/3}\,\frac{1}{k_4^2+k_\perp^2+\mathcal{D}^2\,k_\parallel^2}\,. \tag{A.9}$$

Let us turn immediately to the strong reflection-positivity condition (5.3). The calculation of the corresponding integral gives:

$$\overline{S}_E(x_4,\mathbf{k}) = \int\limits_{-\infty}^{+\infty} \mathrm{d}k_4\,\exp(\mathrm{i}k_4x_4)\,\overline{S}_E(k_4,\mathbf{k}) = \frac{1}{1+2\widetilde{\kappa}_{33}/3}\int\limits_{-\infty}^{+\infty}\mathrm{d}k_4\,\frac{\exp(\mathrm{i}k_4x_4)}{k_4^2+\omega^2} =$$

$$= \frac{2}{1+2\widetilde{\kappa}_{33}/3}\int\limits_{0}^{\infty}\mathrm{d}k_4\,\frac{\cos(k_4x_4)}{k_4^2+\omega^2} = \frac{1}{1+2\widetilde{\kappa}_{33}/3}\,\frac{\pi}{\omega}\,\exp(-|x_4|\omega)\,, \tag{A.10}$$

with ω given by (A.3). Result (A.10), for $\widetilde{\kappa}_{33} > -3/2$, proves strong reflection positivity and unitarity can be expected to hold.

A.4. Discussion

As shown in this appendix, the pure-photon sector of the parity-even anisotropic non-birefringent modified Maxwell theory characterized by parameters (A.1) has microcausality and unitarity for the $\widetilde{\kappa}_{33}$ parameter domain (A.8). The same can be expected to hold for the modified QED theory (2.12) of photons minimally coupled to standard Lorentz-invariant Dirac particles. The question, now, is which numerical value of $\widetilde{\kappa}_{33}$ holds experimentally, where $\widetilde{\kappa}_{33} = 0$ corresponds to having exact Lorentz invariance.

The isolated Lorentz-violating parameters $(\widetilde{\kappa}_{\mathrm{e}-})^{(11)}$ and $(\widetilde{\kappa}_{\mathrm{e}-})^{(22)}$ are tightly bounded at the 10^{-17} level by direct laboratory experiments [34, 42, 43]. This implies that the experimental limit on the $\widetilde{\kappa}_{33}$ parameter of the particular case considered in this appendix is controlled by the less tight limits on $\widetilde{\kappa}_{\mathrm{tr}}$, which have already been discussed in the second paragraph of Sec. 6.

References

[1] W. Heitler, *The Quantum Theory of Radiation*, 3rd ed., Oxford University Press, London, 1954.

[2] J.M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, 2nd ed., Springer, New York, USA, 1976.

[3] M.J.G. Veltman, *Diagrammatica: The path to Feynman rules*, Cambridge University Press, Cambridge, England, 1994.

[4] S. Chadha and H.B. Nielsen, "Lorentz invariance as a low-energy phenomenon," Nucl. Phys. B 217 (1983) 125.

[5] D. Colladay and V.A. Kostelecký, "Lorentz-violating extension of the standard model," Phys. Rev. D 58 (1998) 116002, arXiv:hep-ph/9809521.

[6] S.M. Carroll, G.B. Field, and R. Jackiw, "Limits on a Lorentz- and parity-violating modification of electrodynamics," Phys. Rev. D 41 (1990) 1231.

[7] C. Adam and F.R. Klinkhamer, "Causality and CPT violation from an Abelian Chern–Simons-like term," Nucl. Phys. B 607 (2001) 247, arXiv:hep-ph/0101087.

[8] F.R. Klinkhamer and C. Rupp, "Spacetime foam, CPT anomaly, and photon propagation," Phys. Rev. D 70 (2004) 045020, arXiv:hep-th/0312032.

[9] S. Bernadotte and F.R. Klinkhamer, "Bounds on length scales of classical spacetime foam models," Phys. Rev. D 75 (2007) 024028, arXiv:hep-ph/0610216.

[10] R. Casana, M.M. Ferreira, A.R. Gomes, and P.R.D. Pinheiro, "Gauge propagator and physical consistency of the CPT-even part of the standard model extension," Phys. Rev. D 80 (2009) 125040, arXiv:0909.0544.

[11] R. Casana, M.M. Ferreira, A.R. Gomes, and F.E.P. Santos, "Gauge propagator of the nonbirefringent CPT-even sector of the standard model extension," arXiv:1010.2776.

[12] V.A. Kostelecký and M. Mewes, "Signals for Lorentz violation in electrodynamics," Phys. Rev. D 66 (2002) 056005, arXiv:hep-ph/0205211.

[13] Q.G. Bailey and V.A. Kostelecký, "Lorentz-violating electrostatics and magnetostatics," Phys. Rev. D 70 (2004) 076006, arXiv:hep-ph/0407252.

[14] F.R. Klinkhamer and M. Schreck, "New two-sided bound on the isotropic Lorentz-violating parameter of modified Maxwell theory," Phys. Rev. D 78 (2008) 085026, arXiv:0809.3217.

[15] L. Brillouin, *Wave Propagation and Group Velocity*, Academic, New York, USA, 1960.

[16] S.R. Coleman and S.L. Glashow, "High-energy tests of Lorentz invariance," Phys. Rev. D **59** (1999) 116008, arXiv:hep-ph/9812418.

[17] L. Freidel, J. Kowalski-Glikman, and L. Smolin, "2+1 gravity and doubly special relativity," Phys. Rev. D 69 (2004) 044001, arXiv:hep-th/0307085.

[18] C. Lämmerzahl and F.W. Hehl, "Riemannian light cone from vanishing birefringence in premetric vacuum electrodynamics," Phys. Rev. D 70 (2004) 105022, arXiv:gr-qc/0409072.

[19] V.A. Kostelecký and R. Lehnert, "Stability, causality, and Lorentz and CPT violation," Phys. Rev. D 63 (2001) 065008, arXiv:hep-th/0012060.

[20] P. Jordan and W. Pauli, "Zur Quantenelektrodynamik ladungsfreier Felder," Z. Phys. 47 (1928) 151.

[21] V.A. Kostelecký, "Gravity, Lorentz violation, and the standard model," Phys. Rev. D 69 (2004) 105009, arXiv:hep-th/0312310.

[22] G. Betschart, E. Kant, and F.R. Klinkhamer, "Lorentz violation and black-hole thermodynamics," Nucl. Phys. B 815 (2009) 198, arXiv:0811.0943.

[23] E. Kant, F.R. Klinkhamer, and M. Schreck, "Lorentz violation and black-hole thermodynamics: Compton scattering process," Phys. Lett. B 682 (2009) 316, arXiv:0909.0160.

[24] S.W. Hawking and G.F.R. Ellis, *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge, 1973.

[25] S. Liberati, S. Sonegom, and M. Visser, "Faster-than-c signals, special relativity, and causality," Annals

Phys. 298 (2002) 167, arXiv:gr-qc/0107091.

[26] S. Dubovsky, A. Nicolis, E. Trincherini, and G. Villadoro, "Microcausality in curved space-time," Phys. Rev. D 77 (2008) 084016, arXiv:0709.1483.

[27] K. Osterwalder and R. Schrader, "Axioms for Euclidean Green's functions. 1," Commun. Math. Phys. 31 (1973) 83; K. Osterwalder and R. Schrader, "Axioms for Euclidean Green's functions. 2," Commun. Math. Phys. 42 (1975) 281.

[28] I. Montvay and G. Münster, *Quantum Fields on a Lattice*, Cambridge University Press, Cambridge, England, 1994.

[29] C. Itzykson and J.B. Zuber, *Quantum Field Theory*, McGraw-Hill, New York, USA, 1980.

[30] M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory*, Addison–Wesley, Reading, USA, 1995.

[31] V.A. Kostelecký, C.D. Lane, and A.G.M. Pickering, "One-loop renormalization of Lorentz-violating electrodynamics," Phys. Rev. D 65 (2002) 056006, arXiv:hep-th/0111123.

[32] H.E. Ives and G.R. Stilwell, "An experimental study of the rate of a moving atomic clock," J. Opt. Soc. Am. 28 (1938) 215.

[33] S. Reinhardt *et al.*, "Test of relativistic time dilation with fast optical atomic clocks at different velocities," Nature Phys. 3 (2007) 861.

[34] M.A. Hohensee, P.L. Stanwix, M.E. Tobar, S.R. Parker, D.F. Phillips, and R.L. Walsworth, "Improved constraints on isotropic shift and anisotropies of the speed of light using rotating cryogenic sapphire oscillators," Phys. Rev. D 82 (2010) 076001, arXiv:1006.1376.

[35] M. A. Hohensee, R. Lehnert, D.F. Phillips, and R.L. Walsworth, "Limits on isotropic Lorentz violation in QED from collider physics," Phys. Rev. D 80 (2009) 036010, arXiv:0809.3442.

[36] B. Altschul, "Bounding isotropic Lorentz violation using synchrotron losses at LEP," Phys. Rev. D 80 (2009) 091901, arXiv:0905.4346.

[37] F.R. Klinkhamer, "Potential sensitivities to Lorentz violation from nonbirefringent modified Maxwell theory of Auger, HESS, and CTA," Phys. Rev. D 82 (2010) 105024, arXiv:1008.1967.

[38] S. Weinberg, "The cosmological constant problem," Rev. Mod. Phys. 61 (1989) 1.

[39] F.R. Klinkhamer and G.E. Volovik, "Towards a solution of the cosmological constant problem," JETP Lett. 91 (2010) 259, arXiv:0907.4887.

[40] F.R. Klinkhamer and M. Risse, "Addendum: Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified Maxwell theory," Phys. Rev. D 77 (2008) 117901, arXiv:0806.4351.

[41] M. Gomes, J.R. Nascimento, A.Y. Petrov, and A.J. da Silva, "Aether-like Lorentz-breaking actions," Phys. Rev. D 81 (2010) 045018, arXiv:0911.3548.

[42] C. Eisele, A.Y. Nevsky, and S. Schiller, "Laboratory test of the isotropy of light propagation at the 10^{-17} level," Phys. Rev. Lett. 103 (2009) 090401.

[43] S. Herrmann, A. Senger, K. Mohle, M. Nagel, E. Kovalchuk, and A. Peters, "Rotating optical cavity experiment testing Lorentz invariance at the 10^{-17} level," Phys. Rev. D 80 (2009) 105011, arXiv:1002.1284.