Gauge invariant approach to low-spin anomalous conformal currents and shadow fields

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Conformal low-spin anomalous currents and shadow fields in flat space-time of dimension greater than or equal to four are studied. Gauge invariant formulation for such currents and shadow fields is developed. Gauge symmetries are realized by involving the Stueckelberg fields. Gauge invariant differential constraints for anomalous currents and shadow fields and realization of global conformal symmetries are obtained. Gauge invariant two-point vertices for anomalous shadow fields are also obtained. In Stueckelberg gauge frame, these gauge invariant vertices become the standard two-point vertices of CFT. Light-cone gauge two-point vertices of the anomalous shadow fields are derived. AdS/CFT correspondence for anomalous currents and shadow fields and the respective normalizable and non-normalizable solutions of massive low-spin AdS fields is studied. The bulk fields are considered in modified de Donder gauge that leads to decoupled equations of motion. We demonstrate that leftover on-shell gauge symmetries of bulk massive fields correspond to gauge symmetries of boundary anomalous currents and shadow fields, while the modified (Lorentz) de Donder gauge conditions for bulk massive fields correspond to differential constraints for boundary anomalous currents and shadow fields.

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I. INTRODUCTION

In space-time of dimension $d \geq 4$, fields of CFT can be separated into two groups: conformal currents and shadow fields. Field having Lorentz algebra spin s and conformal dimension $\Delta = s + d - 2$, is referred to as conformal current with canonical dimension, while field having Lorentz algebra spin s and conformal dimension $\Delta > s + d - 2$ is referred to as anomalous conformal current. Accordingly, field having Lorentz algebra spin s and conformal dimension $\Delta = 2 - s$, is referred to as shadow field with canonical dimension[1], while field having Lorentz algebra spin s and conformal dimension $\Delta < 2 - s$ is referred to as anomalous shadow field.

In Refs.[8, 9], we developed the gauge invariant (Stueckelberg) approach to conformal currents and shadow fields having canonical conformal dimensions. The purpose of this paper is to develop gauge invariant approach to anomalous conformal currents and shadow fields. The examples of spin-1 and spin-2 conformal fields demonstrate all characteristic features of our approach. In this paper, because these examples are important in their own right, we discuss spin-1 and spin-2 anomalous conformal currents and shadow fields. Arbitrary spin anomalous conformal currents and shadow fields will be considered in forthcoming publication. Our approach can be summarized as follows.

i) Starting with field content of the standard formulation of anomalous conformal currents (and anomalous shadow fields), we introduce auxiliary fields, i.e., we extend space of fields entering the standard CFT. We note that these auxiliary fields are similar to the ones used in gauge invariant formulation of massive fields which involves Stueckelberg fields.

ii) On the extended space of currents (and shadow fields), we introduce differential constraints, gauge transformations, and conformal algebra transformations. These differential constraints are invariant under the gauge transformations and the conformal algebra transformations.

iii) The gauge symmetries and the differential constraints make it possible to match our approach and the standard one, i.e., by appropriate gauge fixing of the Stueckelberg fields and by solving differential constraints we obtain standard formulation of anomalous conformal currents and shadow fields.

We apply our approach to the study of AdS/CFT correspondence between massive AdS fields and corresponding boundary anomalous conformal currents and shadow fields. We demonstrate that normalizable modes of massive AdS fields are related to anomalous conformal currents, while non-normalizable modes of massive AdS fields are related to anomalous shadow fields. In the earlier literature, the correspondence between non-normalizable bulk modes and shadow fields was studied in Ref.[10] (for spin-1 fields) and in Ref.[11] (for spin-2 fields). To our knowledge, AdS/CFT correspondence between normalizable massive modes and anomalous conformal currents has not been considered in the earlier literature. As compared to the studies in Refs.[10, 11], our approach involves large amount of gauge symmetries. Therefore results of these references are obtained from ones in this paper by using some particular gauge condition, which we refer to as Stueckeleberg gauge fixing. Perhaps, one of the

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[1] It is the shadow fields having canonical dimension that are used to discuss conformal invariant equations of motion and Lagrangian formulations (see e.g. Refs.[1–6]. In earlier literature, discussion of shadow field dualities may be found in Ref.[7].

main advantage of our approach is that this approach gives easy access to the study of AdS/CFT correspondence in light-cone gauge frame. This is very important for future application of our approach to studying string/gauge theory dualities because one expects that string theory in AdS/Ramond-Ramond background can be quantized only in light-cone gauge.

Our approach to the study of AdS/CFT correspondence can be summarized as follows.

i) We use CFT adapted gauge invariant approach to AdS field dynamics developed in Ref.[12]. For spin-1 and spin-2 massive AdS fields we use the respective modified Lorentz gauge and modified de Donder gauges. Remarkable property of these gauges is that they lead to the simple *decoupled* bulk equations of motion which can be solved in terms of Bessel function and this simplifies considerably study of AdS/CFT correspondence. Also using these gauges, we demonstrate that the two-point gauge invariant vertex of the anomalous shadow field does indeed emerge from massive AdS field action when it is evaluated on solution of the Dirichlet problem. AdS field action evaluated on solution of the Dirichlet problem will be referred to as effective action in this paper.

ii) The number of boundary gauge fields involved in our gauge invariant approach to anomalous conformal current (or anomalous shadow field) coincides with the number of bulk massive gauge AdS fields involved in gauge invariant Stueckelberg approach to massive field. Note however that, instead of standard gauge invariant approach to massive field, we use CFT adapted formulation of massive spin AdS field theory developed in Ref.[12][2].

iii) Our modified Lorentz gauge (for spin-1 massive AdS field) and modified de Donder gauge (for spin-2 massive AdS field) turn out to be related to the differential constraints we obtained in the framework of gauge invariant approach to anomalous conformal currents and shadow fields.

iv) *Leftover on-shell* gauge symmetries of massive bulk AdS fields are related to the gauge symmetries of boundary anomalous conformal currents (or anomalous shadow fields).

The rest of the paper is organized as follows.

In Sec. II, we summarize the notation used in this paper.

In Secs. III and IV, we start with the respective examples of spin-1 anomalous conformal current and shadow field. We illustrate our gauge invariant approach to describing anomalous conformal current and shadow field. For spin-1 anomalous shadow field we obtain the gauge invariant two point vertex. Also we discuss how our gauge invariant approach is related to the standard approach to CFT. Using our gauge invariant approach we obtain light-cone gauge description of spin-1 anomalous conformal current and shadow field.

Secs. V and VI are devoted to spin-2 anomalous conformal current and spin-2 anomalous shadow field respectively. In these Sections we generalize results of Secs. III and IV to the case of spin-2 anomalous conformal current and shadow field.

In Sec. VII, we discuss two-point current-shadow field interaction vertex.

Sec. IX is devoted to the study of AdS/CFT correspondence for massive spin-1 bulk AdS field and boundary spin-1 anomalous conformal current and shadow field while in Sec.X we extend results of Sec.IX to the case of spin-2 fields.

We collect various technical details in two appendices. In Appendices A and B we present details of the derivation of CFT adapted gauge invariant Lagrangian for the respective spin-1 and spin-2 massive AdS fields.

II. PRELIMINARIES

A. Notation

Our conventions are as follows. x^a denotes coordinates in d-dimensional flat space-time, while ∂_a denotes derivatives with respect to x^a, $\partial_a \equiv \partial/\partial x^a$. Vector indices of the Lorentz algebra $so(d-1,1)$ take the values $a, b, c, e = 0, 1, \ldots, d-1$. We use mostly positive flat metric tensor η^{ab}. To simplify our expressions we drop η_{ab} in scalar products, i.e., we use $X^a Y^a \equiv \eta_{ab} X^a Y^b$. Throughout this paper we use operators constructed out of the derivatives and coordinates,

$$\Box = \partial^a \partial^a \,, \qquad x\partial \equiv x^a \partial^a \,, \qquad x^2 = x^a x^a \,. \tag{2.1}$$

Sometimes we use light-cone frame. In the light-cone frame, space-time coordinates are decomposed as $x^a = x^+, x^-, x^i$, where light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$ and x^+ is taken to be a light-cone time. $so(d-2)$ algebra vector indices take values $i, j = 1, \ldots, d-2$. We adopt the conventions:

$$\partial^i = \partial_i \equiv \partial/\partial x^i \,, \qquad \partial^\pm = \partial_\mp \equiv \partial/\partial x^\mp \,. \tag{2.2}$$

B. Global conformal symmetries

In d-dimensional flat space-time, the conformal algebra $so(d,2)$ consists of translation generators P^a, dilatation generator D, conformal boost generators K^a, and generators of

[2] We note also that the number of gauge transformation parameters involved in our gauge invariant approach to anomalous current (or anomalous shadow field) coincides with the number of gauge transformation parameters of bulk massive gauge AdS field involved in the standard gauge invariant approach to massive field.

the $so(d-1,1)$ Lorentz algebra J^{ab}. We assume the following normalization for commutators of the conformal algebra:

$$[D,P^a]=-P^a\,,\qquad [P^a,J^{bc}]=\eta^{ab}P^c-\eta^{ac}P^b, \quad (2.3)$$

$$[D,K^a]=K^a\,,\qquad [K^a,J^{bc}]=\eta^{ab}K^c-\eta^{ac}K^b, \quad (2.4)$$

$$[P^a,K^b]=\eta^{ab}D-J^{ab}\,, \quad (2.5)$$

$$[J^{ab},J^{ce}]=\eta^{bc}J^{ae}+3\text{ terms}\,. \quad (2.6)$$

Let ϕ denotes conformal current (or shadow field) in flat space-time of dimension $d\geq 4$. Under conformal algebra transformations the ϕ transforms as

$$\delta_{\hat{G}}\phi=\hat{G}\phi\,, \quad (2.7)$$

where realization of the conformal algebra generators $\hat{G}$ in terms of differential operators takes the form

$$P^a=\partial^a\,, \quad (2.8)$$

$$J^{ab}=x^a\partial^b-x^b\partial^a+M^{ab}\,, \quad (2.9)$$

$$D=x\partial+\Delta\,, \quad (2.10)$$

$$K^a=K^a_{\Delta,M}+R^a\,, \quad (2.11)$$

and we use the notation

$$K^a_{\Delta,M}\equiv-\frac{1}{2}x^2\partial^a+x^aD+M^{ab}x^b\,. \quad (2.12)$$

In (2.9)-(2.11), Δ is operator of conformal dimension, M^{ab} is spin operator of the Lorentz algebra. Action of M^{ab} on fields of the Lorentz algebra is well known and for rank-2 tensor, vector, and scalar fields considered in this paper is given by

$$M^{ab}\phi^{ce}=\eta^{ae}\phi^{cb}+\eta^{ac}\phi^{be}-(a\leftrightarrow b)\,, \quad (2.13)$$

$$M^{ab}\phi^{c}=\eta^{ac}\phi^{b}-(a\leftrightarrow b)\,, \quad (2.14)$$

$$M^{ab}\phi=0\,. \quad (2.15)$$

These relations implies the following transformations of the fields,

$$K^a_{\Delta,M}\phi^{bc}=K^a_{\Delta}\phi^{bc}+M^{abf}\phi^{fc}+M^{acf}\phi^{bf}\,, \quad (2.16)$$

$$K^a_{\Delta,M}\phi^{b}=K^a_{\Delta}\phi^{b}+M^{abf}\phi^{f}\,, \quad (2.17)$$

$$K^a_{\Delta,M}\phi=K^a_{\Delta}\phi\,, \quad (2.18)$$

where we use the notation

$$K^a_{\Delta}\equiv-\frac{1}{2}x^2\partial^a+x^a(x\partial+\Delta)\,, \quad (2.19)$$

$$M^{abc}\equiv\eta^{ab}x^c-\eta^{ac}x^b\,. \quad (2.20)$$

In (2.11), R^a is operator depending, in general, on derivatives with respect to space-time coordinates[3] and not depending on space-time coordinates x^a. In standard formulation of conformal currents and shadow fields, the operator R^a is equal to zero, while in gauge invariant approach that we develop in this paper, the operator R^a is non-trivial. This implies that, in the framework of gauge invariant approach, complete description of the conformal currents and shadow fields requires, among other things, finding the operator R^a.

[3] For conformal currents and shadow fields studied in this paper, the operator R^a does not depend on derivatives. Dependence on derivatives of R^a appears e.g., in ordinary-derivative approach to conformal fields [13].

III. SPIN-1 ANOMALOUS CONFORMAL CURRENT

In this section we develop gauge invariant approach to spin-1 conformal current. Besides gauge invariant formulation we discuss two gauge conditions which can be used for studying the anomalous conformal currents - Stueckelberg gauge and light-cone gauge. We would like to discuss these gauges because of the following reasons.

i) It turns out that the Stueckelberg gauge reduces our approach to the standard formulation of CFT. Therefore the use of the Stueckelberg gauge allows us to demonstrate how the standard approach to anomalous conformal currents is obtained from our gauge invariant approach.

ii) Motivation for considering the light-cone gauge frame cames from conjectured duality of the SYM theory and the theory of the superstring in AdS background [14]. By analogy with flat space, we expect that a quantization of the Green-Schwarz AdS superstring [15] will be straightforward only in the light-cone gauge [16, 17]. Therefore it seems that from the stringy perspective of AdS/CFT correspondence, light-cone approach to CFT is the fruitful direction to go.

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-1 anomalous conformal current in flat space of dimension $d\geq 4$ we use one vector field $\phi^a_{\rm cur,0}$ and two scalar fields $\phi_{\rm cur,1}$, $\phi_{\rm cur,-1}$:

$$\phi^a_{\rm cur,0}\,,\qquad \phi_{\rm cur,-1}\,,\qquad \phi_{\rm cur,1}\,. \quad (3.1)$$

The fields $\phi^a_{\rm cur,0}$ and $\phi_{\rm cur,\pm1}$ transform in the respective vector and scalar irreps of the Lorentz algebra $so(d-1,1)$. We note that fields (3.1) have the conformal dimensions

$$\Delta_{\phi^a_{\rm cur,0}}=\frac{d}{2}+\kappa\,,\qquad \Delta_{\phi_{\rm cur,\pm1}}=\frac{d}{2}+\kappa\pm1\,, \quad (3.2)$$

where parameter κ is restricted to be positive $\kappa>0$. In the framework of AdS/CFT correspondence, κ is related to the mass parameter m of spin-1 AdS massive field as

$$\kappa\equiv\sqrt{m^2+\frac{(d-2)^2}{4}}\,. \quad (3.3)$$

We now introduce the following differential constraint:

$$\partial^a \phi^a_{\mathrm{cur},0} + r^{00}_z \Box \phi_{\mathrm{cur},-1} + r^{00}_\zeta \phi_{\mathrm{cur},1} = 0\,, \quad (3.4)$$

$$r^{00}_z \equiv \Big(\frac{2\kappa + d - 2}{4\kappa}\Big)^{1/2}, \quad (3.5)$$

$$r^{00}_\zeta \equiv \Big(\frac{2\kappa - d + 2}{4\kappa}\Big)^{1/2}. \quad (3.6)$$

One can make sure that this constraint is invariant under gauge transformations

$$\delta\phi^a_{\mathrm{cur},0} = \partial^a \xi_{\mathrm{cur},0} \quad (3.7)$$

$$\delta\phi_{\mathrm{cur},-1} = -r^{00}_z \xi_{\mathrm{cur},0}\,, \quad (3.8)$$

$$\delta\phi_{\mathrm{cur},1} = -r^{00}_\zeta \Box \xi_{\mathrm{cur},0}\,, \quad (3.9)$$

where $\xi_{\mathrm{cur},0}$ is a gauge transformation parameter.

To complete our gauge invariant formulation we provide realization of the operator R^a on space of gauge fields (3.1),

$$R^a \phi^b_{\mathrm{cur},0} = -2\kappa r^{00}_z \eta^{ab} \phi_{\mathrm{cur},-1}\,, \quad (3.10)$$

$$R^a \phi_{\mathrm{cur},-1} = 0\,, \quad (3.11)$$

$$R^a \phi_{\mathrm{cur},1} = -2\kappa r^{00}_\zeta \phi^a_{\mathrm{cur},0}\,. \quad (3.12)$$

Using (3.10)-(3.12), we make sure that constraint (3.4) is invariant under transformations of the conformal algebra (2.7).

B. Stueckelberg gauge frame

We now discuss spin-1 anomalous conformal current in the Stueckelberg gauge frame. From (3.8), we see that the scalar field $\phi_{\mathrm{cur},-1}$ transforms as Stueckelberg field, i.e., this field can be gauged away via Stueckelberg gauge fixing,

$$\phi_{\mathrm{cur},-1} = 0. \quad (3.13)$$

Using this gauge in constraint (3.4), we see that the remaining scalar field $\phi_{\mathrm{cur},1}$ can be expressed in terms of the vector field $\phi^a_{\mathrm{cur},0}$,

$$\phi_{\mathrm{cur},1} = -\frac{1}{r^{00}_\zeta} \partial^a \phi^a_{\mathrm{cur},0}\,, \quad (3.14)$$

i.e., making use of the gauge symmetry and differential constraint (3.4) we reduce field content of our approach (3.1) to the one in the standard approach. In other words, the gauge symmetry and differential constraint make it possible to match our approach and the standard one, i.e., by gauge fixing of the Stueckelberg field (3.13) and by solving differential constraint (3.4) we obtain standard formulation of spin-1 anomalous conformal current[4].

C. Light-cone gauge frame

For spin-1 anomalous conformal current, the light-cone gauge frame is achieved through the use of differential constraint (3.4) and light-cone gauge condition. Using the gauge symmetry of the spin-1 anomalous conformal current (3.7), we impose light-cone gauge on the field $\phi^a_{\mathrm{cur},0}$,

$$\phi^+_{\mathrm{cur},0} = 0\,. \quad (3.15)$$

Using this gauge in differential constraint (3.4), we find

$$\phi^-_{\mathrm{cur},0} = -\frac{\partial^j}{\partial^+}\phi^j_{\mathrm{cur},0} - \frac{r^{00}_z}{\partial^+}\Box\phi_{\mathrm{cur},-1} - \frac{r^{00}_\zeta}{\partial^+}\phi_{\mathrm{cur},1}\,. \quad (3.16)$$

We see that we are left with vector field $\phi^i_{\mathrm{cur},0}$ and two scalar fields $\phi_{\mathrm{cur},\pm1}$. These fields constitute the field content of the light-cone gauge frame.

IV. SPIN-1 ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-1 anomalous shadow field in space of dimension $d \geq 4$ we use one vector field $\phi^a_{\mathrm{sh},0}$ and two scalar fields $\phi_{\mathrm{sh},-1}$, $\phi_{\mathrm{sh},1}$:

$$\phi^a_{\mathrm{sh},0}\,, \qquad \phi_{\mathrm{sh},-1}\,, \qquad \phi_{\mathrm{sh},1}\,. \quad (4.1)$$

The fields ϕ^a_{sh} and $\phi_{\mathrm{sh},\pm1}$ transform in the respective vector and scalar representations of the Lorentz algebra $so(d-1,1)$. We note that these fields have the conformal dimensions

$$\Delta_{\phi^a_{\mathrm{sh},0}} = \frac{d}{2} - \kappa\,, \qquad \Delta_{\phi_{\mathrm{sh},\pm1}} = \frac{d}{2} - \kappa \pm 1\,, \quad (4.2)$$

where $\kappa > 0$. In the framework of AdS/CFT correspondence, the κ is related to the mass parameter m of spin-1 AdS massive field by relation (3.3).

We now introduce the following differential constraint:

$$\partial^a \phi^a_{\mathrm{sh},0} + r^{00}_\zeta \Box \phi_{\mathrm{sh},-1} + r^{00}_z \phi_{\mathrm{sh},1} = 0\,, \quad (4.3)$$

[4] As in standard approach to CFT, our currents can be considered either as fundamental field degrees of freedom or as composite operators. At the group theoretical level, we study in this paper, this distinction is immaterial. Discussion of interesting methods for building conformal currents as composite operators may be found in Refs.[18, 19].

where r_z^{00}, r_ζ^{00} are given in (3.5),(3.6). We make sure that constraint (4.3) is invariant under gauge transformations

$$\delta\phi^a_{\mathrm{sh},0} = \partial^a \xi_{\mathrm{sh},0} \tag{4.4}$$

$$\delta\phi_{\mathrm{sh},-1} = -r_\zeta^{00}\xi_{\mathrm{sh},0}\,. \tag{4.5}$$

$$\delta\phi_{\mathrm{sh},1} = -r_z^{00}\Box\xi_{\mathrm{sh},0}\,, \tag{4.6}$$

where $\xi_{\mathrm{sh},0}$ is a gauge transformation parameter.

To complete our gauge invariant formulation of spin-1 anomalous shadow field we provide realization of the operator R^a on space of gauge fields (4.1),

$$R^a\phi^b_{\mathrm{sh},0} = 2\kappa r_\zeta^{00}\eta^{ab}\phi_{\mathrm{sh},-1}\,, \tag{4.7}$$

$$R^a\phi_{\mathrm{sh},-1} = 0\,, \tag{4.8}$$

$$R^a\phi_{\mathrm{sh},1} = 2\kappa r_z^{00}\phi^a_{\mathrm{sh},0}\,. \tag{4.9}$$

We proceed with the discussion of the two-point vertex for the anomalous spin-1 shadow field. The gauge invariant two-point vertex we find takes the form

$$\Gamma = \int d^dx_1 d^dx_2 \Gamma_{12}\,, \tag{4.10}$$

$$\Gamma_{12} = \frac{\phi^a_{\mathrm{sh},0}(x_1)\phi^a_{\mathrm{sh},0}(x_2)}{2|x_{12}|^{2\kappa+d}} + \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\mathrm{sh},\lambda}(x_1)\phi_{\mathrm{sh},\lambda}(x_2)\,, \tag{4.11}$$

$$\omega_1 = \frac{1}{2\kappa(2\kappa+d-2)}\,, \tag{4.12}$$

$$\omega_{-1} = 2(\kappa+1)(2\kappa+d)\,, \tag{4.13}$$

$$|x_{12}|^2 \equiv x^a_{12}x^a_{12}\,, \qquad x^a_{12} = x^a_1 - x^a_2\,. \tag{4.14}$$

One can check that this vertex is invariant under the gauge transformations of the spin-1 anomalous shadow field given in (4.4)-(4.6). Also, we check that the vertex is invariant under the conformal algebra transformations.

The kernel of the vertex Γ is related to a two-point correlation function of the spin-1 anomalous conformal current. In our approach, the spin-1 anomalous conformal current is described by gauge fields given in (3.1). Therefore, in order to discuss the correlation function of the anomalous conformal current in a proper way, we should impose a gauge condition on the gauge fields in (3.1).[5] We have considered anomalous conformal current in Stueckelberg gauge frame and light-cone gauge frame. This is to say that correlation function of anomalous conformal current in Stueckelberg gauge frame and light-cone gauge frame can be obtained from two-point vertex Γ taken in the respective Stueckelberg gauge frame and light-cone gauge frame. To this end we now discuss the anomalous shadow field in Stueckelberg gauge and light-cone gauge frames.

B. Stueckelberg gauge frame

For spin-1 anomalous shadow field, the Stueckelberg gauge frame is achieved through the use of differential constraint (4.3) and the Stueckelberg gauge condition.

From (4.5), we see that the scalar field $\phi_{\mathrm{sh},-1}$ transforms as Stueckelberg field, i.e., this field can be gauged away via Stueckelberg gauge fixing,

$$\phi_{\mathrm{sh},-1} = 0\,. \tag{4.15}$$

Using this gauge in (4.3) we see that the remaining scalar field $\phi_{\mathrm{sh},1}$ can be expressed in terms of the vector field $\phi^a_{\mathrm{sh},0}$,

$$\phi_{\mathrm{sh},1} = -\frac{1}{r_z^{00}}\partial^a\phi^a_{\mathrm{sh},0}\,. \tag{4.16}$$

Thus, we see that the use of the gauge symmetry and differential constraint (4.3) makes it possible to match field content of our approach and the one in the standard approach to spin-1 anomalous conformal current.

We proceed with the discussion of Stueckelberg gauge fixed two-point vertex of the spin-1 anomalous shadow field, i.e. we relate our vertex (4.10) with the one in the standard approach to CFT. To this end we note that vertex of the standard approach to CFT is obtained from our gauge invariant vertex (4.10) by using Stueckelberg gauge condition (4.15) and solution to differential constraint (4.16) in (4.10). Doing so and ignoring the total derivative, we find that two-point density Γ_{12} (4.11) takes the following form:

$$\Gamma_{12}^{\mathrm{Stuck.g.fram}} = k_1\Gamma_{12}^{\mathrm{stand}}\,, \tag{4.17}$$

$$\Gamma_{12}^{\mathrm{stand}} = \frac{\phi^a_{\mathrm{sh}}(x_1)O^{ab}_{12}\phi^b_{\mathrm{sh}}(x_2)}{|x_{12}|^{2\kappa+d}}\,, \tag{4.18}$$

$$O^{ab}_{12} \equiv \eta^{ab} - \frac{2x^a_{12}x^b_{12}}{|x_{12}|^2}\,, \tag{4.19}$$

$$k_1 \equiv \frac{2\kappa+d}{2(2\kappa+d-2)}\,, \tag{4.20}$$

where $\Gamma_{12}^{\mathrm{stand}}$ (4.18) stands for the two-point vertex of the spin-1 anomalous shadow field in the standard approach to

[5] We note that, in the gauge invariant approach, correlation functions of the conformal current can be studied without gauge fixing. To do that one needs to construct gauge invariant field strengths for the gauge potentials $\phi^a_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},\pm1}$. Study of field strengths for the conformal currents is beyond the scope of this paper. Recent interesting discussion of method for building field strengths may be found in Refs.[20, 21].

CFT. From (4.17), we see that our gauge invariant vertex taken to be in the Stueckelberg gauge frame coincides, up to normalization factor k_1, with the two-point vertex in the standard approach to CFT.

As we have demonstrated in Sec.III B, in the Stueckelberg gauge frame, we are left with vector field $\phi^a_{\rm cur,0}$. Two-point correlation function of this vector field is given by the kernel of vertex $\Gamma^{\rm stand}$ (4.18),

$$\langle \phi^a_{\rm cur,0}(x_1), \phi^b_{\rm cur,0}(x_2)\rangle = \frac{O^{ab}_{12}}{|x_{12}|^{2\kappa+d}}\,. \tag{4.21}$$

C. Light-cone gauge frame

For spin-1 anomalous shadow field, the light-cone gauge frame is achieved through the use of the light-cone gauge and differential constraint (4.3). Taking into account the gauge transformation of the field $\phi^a_{\rm sh,0}$ (4.4), we impose the light-cone gauge,

$$\phi^+_{\rm sh,0} = 0\,. \tag{4.22}$$

Using this gauge in differential constraint (4.3) we obtain solution for $\phi^-_{\rm sh}$,

$$\phi^-_{\rm sh,0} = -\frac{\partial^j}{\partial^+}\phi^j_{\rm sh,0} - \frac{r^{00}_z}{\partial^+}\phi_{\rm sh,1} - \frac{r^{00}_\zeta}{\partial^+}\Box\phi_{\rm sh,-1}\,. \tag{4.23}$$

We see that we are left with vector field $\phi^i_{\rm sh,0}$ and the scalar fields $\phi_{\rm sh,\pm1}$. These fields constitute the field content of the light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, the scalar fields $\phi_{\rm sh,\pm1}$ become independent field D.o.F in the light-cone gauge frame.

Using (4.22) in (4.11) leads to light-cone gauge fixed vertex

$$\Gamma^{\rm (l.c.)}_{12} = \frac{\phi^i_{\rm sh,0}(x_1)\phi^i_{\rm sh,0}(x_2)}{2|x_{12}|^{2\kappa+d}} + \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\rm sh,\lambda}(x_1)\phi_{\rm sh,\lambda}(x_2)\,, \tag{4.24}$$

where ω_λ are given in (4.12),(4.13). As in the case of gauge invariant vertex (4.11), light-cone vertex (4.24) is diagonal with respect to the fields $\phi^i_{\rm sh,0}$ and $\phi_{\rm sh,\pm1}$. Note however that, in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the fields which are not subject to any constraints.

The kernel of the light-cone vertex gives two-point correlation function of spin-1 anomalous conformal current taken to be in the light-cone gauge. Defining two-point correlation functions of the fields $\phi^i_{\rm cur,0}$, $\phi_{\rm cur,\pm1}$ in a usual way,

$$\langle \phi^i_{\rm cur,0}(x_1), \phi^j_{\rm cur,0}(x_2)\rangle = \frac{\delta^2\Gamma^{\rm (l.c.)}}{\delta\phi^i_{\rm sh,0}(x_1)\delta\phi^j_{\rm sh,0}(x_2)}\,, \tag{4.25}$$

$$\langle \phi_{\rm cur,\lambda}(x_1), \phi_{\rm cur,\lambda}(x_2)\rangle = \frac{\delta^2\Gamma^{\rm (l.c.)}}{\delta\phi_{\rm sh,-\lambda}(x_1)\delta\phi_{\rm sh,-\lambda}(x_2)}\,, \tag{4.26}$$

$\lambda=\pm1$, and using (4.24), we obtain the two-point light-cone gauge correlation functions of the spin-1 anomalous conformal current,

$$\langle \phi^i_{\rm cur,0}(x_1), \phi^j_{\rm cur,0}(x_2)\rangle = \frac{\delta^{ij}}{|x_{12}|^{2\kappa+d}}\,, \tag{4.27}$$

$$\langle \phi_{\rm cur,\lambda}(x_1), \phi_{\rm cur,\lambda}(x_2)\rangle = \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}}\,, \tag{4.28}$$

$\lambda=\pm1$, where ω_λ are given in (4.12),(4.13).

V. SPIN-2 ANOMALOUS CONFORMAL CURRENT

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-2 anomalous conformal current in flat space of dimension $d\geq 4$ we use one rank-2 tensor field, two vector fields, and three scalar fields,

$$\begin{gathered}\phi^{ab}_{\rm cur}\,,\\ \phi^a_{\rm cur,-1}\,,\qquad \phi^a_{\rm cur,1}\,,\\ \phi_{\rm cur,-2}\,,\qquad \phi_{\rm cur,0}\,,\qquad \phi_{\rm cur,2}\,.\end{gathered} \tag{5.1}$$

The fields $\phi^{ab}_{\rm cur}$, $\phi^a_{\rm cur,\pm1}$ and $\phi_{\rm cur,0}$, $\phi_{\rm cur,\pm2}$ transform in the respective rank-2 tensor, vector and scalar representations of the Lorentz algebra $so(d-1,1)$. Note that the tensor field $\phi^{ab}_{\rm cur,0}$ is not traceless. We note that fields (5.1) have the conformal dimensions

$$\begin{aligned}\Delta_{\phi^{ab}_{\rm cur,0}} &= \frac{d}{2}+\kappa\,,\\ \Delta_{\phi^a_{\rm cur,\lambda}} &= \frac{d}{2}+\kappa+\lambda\,,\qquad \lambda=\pm1\,,\\ \Delta_{\phi_{\rm cur,\lambda}} &= \frac{d}{2}+\kappa+\lambda\,,\qquad \lambda=0,\pm2\,,\end{aligned} \tag{5.2}$$

where the parameter κ is considered to be positive $\kappa>0$. In the framework of AdS/CFT correspondence κ is related to the mass parameter m of spin-2 AdS massive field as[6]

$$\kappa = \sqrt{m^2+\frac{d^2}{4}}\,. \tag{5.3}$$

We now introduce the following differential constraints:

$$\partial^b\phi^{ab}_{\rm cur,0} - \frac{1}{2}\partial^a\phi^{bb}_{\rm cur,0} + r^{00}_z\Box\phi^a_{\rm cur,-1} + r^{00}_\zeta\phi^a_{\rm cur,1} = 0\,, \tag{5.4}$$

[6] Parameter κ for spin-2 field (5.3) should not be confused with the one for spin-1 field (3.3).

$$\partial^a \phi^a_{\mathrm{cur},-1} + \frac{1}{2} r^{00}_z \phi^{aa}_{\mathrm{cur},0} + \sqrt{2} r^{01}_z \Box \phi_{\mathrm{cur},-2} + r^{01}_\zeta \phi_{\mathrm{cur},0} = 0\,, \tag{5.5}$$

$$\partial^a \phi^a_{\mathrm{cur},1} + \frac{1}{2} r^{00}_\zeta \Box \phi^{aa}_{\mathrm{cur},0} + r^{10}_z \Box \phi_{\mathrm{cur},0} + \sqrt{2} r^{10}_\zeta \phi_{\mathrm{cur},2} = 0\,, \tag{5.6}$$

$$r^{00}_z \equiv \Big(\frac{2\kappa + d}{4\kappa}\Big)^{1/2}, \tag{5.7}$$

$$r^{10}_z \equiv \Big(\frac{(2\kappa + d)(\kappa - 1)d}{4\kappa(\kappa + 1)(d - 2)}\Big)^{1/2}, \tag{5.8}$$

$$r^{01}_z \equiv \Big(\frac{2\kappa + d - 2}{4(\kappa - 1)}\Big)^{1/2}, \tag{5.9}$$

$$r^{00}_\zeta \equiv \Big(\frac{2\kappa - d}{4\kappa}\Big)^{1/2}, \tag{5.10}$$

$$r^{10}_\zeta \equiv \Big(\frac{2\kappa - d + 2}{4(\kappa + 1)}\Big)^{1/2}, \tag{5.11}$$

$$r^{01}_\zeta \equiv \Big(\frac{(2\kappa - d)(\kappa + 1)d}{4\kappa(\kappa - 1)(d - 2)}\Big)^{1/2}. \tag{5.12}$$

One can make sure that these differential constraints are invariant under the gauge transformations

$$\delta \phi^{ab}_{\mathrm{cur},0} = \partial^a \xi^b_{\mathrm{cur},0} + \partial^b \xi^a_{\mathrm{cur},0} + \frac{2 r^{00}_z}{d - 2} \eta^{ab} \Box \xi_{\mathrm{cur},-1} + \frac{2 r^{00}_\zeta}{d - 2} \eta^{ab} \xi_{\mathrm{cur},1}\,, \tag{5.13}$$

$$\delta \phi^a_{\mathrm{cur},-1} = \partial^a \xi_{\mathrm{cur},-1} - r^{00}_z \xi^a_{\mathrm{cur},0}\,, \tag{5.14}$$

$$\delta \phi^a_{\mathrm{cur},1} = \partial^a \xi_{\mathrm{cur},1} - r^{00}_\zeta \Box \xi^a_{\mathrm{cur},0}\,, \tag{5.15}$$

$$\delta \phi_{\mathrm{cur},-2} = -\sqrt{2} r^{01}_z \xi_{\mathrm{cur},-1}\,, \tag{5.16}$$

$$\delta \phi_{\mathrm{cur},0} = -r^{01}_\zeta \Box \xi_{\mathrm{cur},-1} - r^{10}_z \xi_{\mathrm{cur},1}\,, \tag{5.17}$$

$$\delta \phi_{\mathrm{cur},2} = -\sqrt{2} r^{10}_\zeta \Box \xi_{\mathrm{cur},1}\,, \tag{5.18}$$

where $\xi^a_{\mathrm{cur},0}$, $\xi_{\mathrm{cur},\pm 1}$ are gauge transformation parameters.

To complete our gauge invariant formulation we find realization of the operator R^a on space of gauge fields (5.1),

$$R^a \phi^{bc}_{\mathrm{cur},0} = -2\kappa r^{00}_z (\eta^{ab} \phi^c_{\mathrm{cur},-1} + \eta^{ac} \phi^b_{\mathrm{cur},-1}) + \frac{4(\kappa - 1) r^{00}_z}{d - 2} \eta^{bc} \phi^a_{\mathrm{cur},-1}\,, \tag{5.19}$$

$$R^a \phi^b_{\mathrm{cur},-1} = -2\sqrt{2}(\kappa - 1) r^{01}_z \eta^{ab} \phi_{\mathrm{cur},-2}\,, \tag{5.20}$$

$$R^a \phi^b_{\mathrm{cur},1} = -r^{00}_\zeta (2\kappa \phi^{ab}_{\mathrm{cur},0} + \eta^{ab} \phi^{cc}_{\mathrm{cur},0}) - 2(\kappa + 1) r^{10}_z \eta^{ab} \phi_{\mathrm{cur},0}\,, \tag{5.21}$$

$$R^a \phi_{\mathrm{cur},-2} = 0\,, \tag{5.22}$$

$$R^a \phi_{\mathrm{cur},0} = -2(\kappa - 1) r^{01}_\zeta \phi^a_{\mathrm{cur},-1}\,, \tag{5.23}$$

$$R^a \phi_{\mathrm{cur},2} = -2\sqrt{2}(\kappa + 1) r^{10}_\zeta \phi^a_{\mathrm{cur},1}\,. \tag{5.24}$$

Using (5.19)-(5.24), we check that constraints (5.4)-(5.6) are invariant under conformal algebra transformations (2.7).

B. Stueckelberg gauge frame

For spin-2 anomalous conformal current, the Stueckelberg gauge frame is achieved through the use of differential constraints (5.4)-(5.6) and Stueckelberg gauge condition. From (5.14),(5.16),(5.17), we see that the vector field $\phi^a_{\mathrm{cur},-1}$ and the scalar fields $\phi_{\mathrm{cur},-2}$, $\phi_{\mathrm{cur},0}$ transform as Stueckelberg fields, i.e., these fields can be gauged away via Stueckelberg gauge fixing,

$$\phi^a_{\mathrm{cur},-1} = 0\,, \qquad \phi_{\mathrm{cur},-2} = 0\,, \qquad \phi_{\mathrm{cur},0} = 0\,. \tag{5.25}$$

Using gauge conditions (5.25) in constraint (5.5) we find that the field $\phi^{ab}_{\mathrm{cur},0}$ becomes traceless, while using gauge conditions (5.25) in constraints (5.4),(5.6) we find that the remaining vector field $\phi^a_{\mathrm{cur},1}$ and the scalar field $\phi_{\mathrm{cur},2}$ can be expressed in terms of the rank-2 tensor field $\phi^{ab}_{\mathrm{cur},0}$,

$$\phi^{aa}_{\mathrm{cur},0} = 0\,, \tag{5.26}$$

$$\phi^a_{\mathrm{cur},1} = -\frac{1}{r^{00}_\zeta} \partial^b \phi^{ab}_{\mathrm{cur},0}\,, \tag{5.27}$$

$$\phi_{\mathrm{cur},2} = \frac{1}{\sqrt{2} r^{00}_\zeta r^{10}_\zeta} \partial^a \partial^b \phi^{ab}_{\mathrm{cur},0}\,. \tag{5.28}$$

Relations (5.25)-(5.28) provide complete description of the Stueckelberg gauge frame for the spin-2 anomalous conformal current. We note that the traceless rank-2 tensor $\phi^{ab}_{\mathrm{cur},0}$ can be identified with the one in the standard approach to CFT.

Thus, we see that the Stueckelberg gauge symmetries and the differential constraints make it possible to match our approach and the standard one, i.e., by gauge fixing of the Stueckelberg fields (5.25) and by solving differential constraints (5.4)-(5.6) we obtain the standard formulation of spin-1 anomalous conformal current.

C. Light-cone gauge frame

For spin-2 anomalous conformal current, the light-cone gauge frame is achieved through the use differential constraints (5.4)-(5.6) and light-cone gauge condition.

Using the gauge transformations of the fields $\phi^{ab}_{\mathrm{cur},0}$, $\phi^a_{\mathrm{cur},\pm 1}$ (5.13)-(5.15) we impose the light-cone gauge,

$$\phi^{+a}_{\mathrm{cur},0} = 0\,, \qquad \phi^+_{\mathrm{cur},\lambda} = 0\,, \qquad \lambda = \pm 1\,. \tag{5.29}$$

Plugging this gauge in differential constraints (5.4)-(5.6), we find

$$\phi^{ii}_{\rm cur,0}=0\,, \tag{5.30}$$

$$\phi^{-i}_{\rm cur,0}=-\frac{\partial^j}{\partial^+}\phi^{ij}_{\rm cur,0}-\frac{r^{00}_z}{\partial^+}\Box\phi^i_{\rm cur,-1}-\frac{r^{00}_\zeta}{\partial^+}\phi^i_{\rm cur,1}\,, \tag{5.31}$$

$$\begin{aligned}\phi^{--}_{\rm cur,0}=&\frac{\partial^i\partial^j}{\partial^+\partial^+}\phi^{ij}_{\rm cur,0}+\frac{2r^{00}_z\partial^i}{\partial^+\partial^+}\Box\phi^i_{\rm cur,-1}+\frac{2r^{00}_\zeta\partial^i}{\partial^+\partial^+}\phi^i_{\rm cur,1}\\&+\frac{\sqrt{2}r^{00}_zr^{10}_z}{\partial^+\partial^+}\Box^2\phi_{\rm cur,-2}+\frac{\sqrt{2}r^{00}_\zeta r^{01}_\zeta}{\partial^+\partial^+}\phi_{\rm cur,2}\\&+\frac{r^{00}_zr^{01}_\zeta+r^{00}_\zeta r^{10}_z}{\partial^+\partial^+}\Box\phi_{\rm cur,0}\,,\end{aligned} \tag{5.32}$$

$$\phi^{-}_{\rm cur,-1}=-\frac{\partial^j}{\partial^+}\phi^j_{\rm cur,-1}-\frac{\sqrt{2}r^{01}_z}{\partial^+}\Box\phi_{\rm cur,-2}-\frac{r^{01}_\zeta}{\partial^+}\phi_{\rm cur,0}\,, \tag{5.33}$$

$$\phi^{-}_{\rm cur,1}=-\frac{\partial^j}{\partial^+}\phi^j_{\rm cur,1}-\frac{r^{10}_z}{\partial^+}\Box\phi_{\rm cur,0}-\frac{\sqrt{2}r^{10}_\zeta}{\partial^+}\phi_{\rm cur,2}\,. \tag{5.34}$$

We see that we are left with $so(d-2)$ algebra traceless rank-2 tensor field, two vector fields, and three scalar fields,

$$\begin{gathered}\phi^{ij}_{\rm cur,0}\,,\\ \phi^i_{\rm cur,-1}\,,\qquad \phi^i_{\rm cur,1}\,,\\ \phi_{\rm cur,-2}\,,\qquad \phi_{\rm cur,0}\,,\qquad \phi_{\rm cur,2}\,.\end{gathered} \tag{5.35}$$

Fields in (5.35) constitute the field content of the light-cone gauge frame.

VI. SPIN-2 ANOMALOUS SHADOW FIELD

A. Gauge invariant formulation

To discuss gauge invariant formulation of spin-2 anomalous shadow field in flat space of dimension $d\geq 4$ we use one rank-2 tensor field, two vector fields, and three scalars fields,

$$\begin{gathered}\phi^{ab}_{\rm sh,0}\,,\\ \phi^a_{\rm sh,-1}\,,\qquad \phi^a_{\rm sh,1}\,,\\ \phi_{\rm sh,-2}\,,\qquad \phi_{\rm sh,0}\,,\qquad \phi_{\rm sh,2}\,.\end{gathered} \tag{6.1}$$

The fields $\phi^{ab}_{\rm sh,0}$, $\phi^a_{\rm sh,\pm1}$ and $\phi_{\rm sh,0}$, $\phi_{\rm sh,\pm2}$ transform in the respective rank-2 tensor, vector and scalar representations of the Lorentz algebra $so(d-1,1)$. Note that the field $\phi^{ab}_{\rm sh,0}$ is not traceless. Conformal dimensions of the fields are given by

$$\Delta_{\phi^{ab}_{\rm sh,0}}=\frac{d}{2}-\kappa\,,$$

$$\Delta_{\phi^a_{\rm sh,\lambda}}=\frac{d}{2}-\kappa+\lambda\,,\qquad \lambda=\pm1\,, \tag{6.2}$$

$$\Delta_{\phi_{\rm sh,\lambda}}=\frac{d}{2}-\kappa+\lambda\,,\qquad \lambda=0,\pm2\,, \tag{6.3}$$

where $\kappa>0$. In the framework of AdS/CFT correspondence, the κ is related to the mass parameter m of spin-2 AdS massive field by relation (5.3).

We now introduce the following differential constraints:

$$\partial^b\phi^{ab}_{\rm sh,0}-\frac{1}{2}\partial^a\phi^{bb}_{\rm sh,0}+r^{00}_\zeta\Box\phi^a_{\rm sh,-1}+r^{00}_z\phi^a_{\rm sh,1}=0\,, \tag{6.4}$$

$$\partial^a\phi^a_{\rm sh,-1}+\frac{1}{2}r^{00}_\zeta\phi^{aa}_{\rm sh,0}+\sqrt{2}r^{10}_\zeta\Box\phi_{\rm sh,-2}+r^{10}_z\phi_{\rm sh,0}=0, \tag{6.5}$$

$$\partial^a\phi^a_{\rm sh,1}+\frac{1}{2}r^{00}_z\Box\phi^{aa}_{\rm sh,0}+r^{01}_\zeta\Box\phi_{\rm sh,0}+\sqrt{2}r^{01}_z\phi_{\rm sh,2}=0\,, \tag{6.6}$$

where the parameters r^{mn}_ζ and r^{mn}_z are given in (5.7)-(5.12). One can make sure that these constraints are invariant under gauge transformations

$$\begin{aligned}\delta\phi^{ab}_{\rm sh,0}=&\partial^a\xi^b_{\rm sh,0}+\partial^b\xi^a_{\rm sh,0}\\&+\frac{2r^{00}_z}{d-2}\eta^{ab}\xi_{\rm sh,1}+\frac{2r^{00}_\zeta}{d-2}\eta^{ab}\Box\xi_{\rm sh,-1}\,,\end{aligned} \tag{6.7}$$

$$\delta\phi^a_{\rm sh,-1}=\partial^a\xi_{\rm sh,-1}-r^{00}_\zeta\xi^a_{\rm sh,0}\,, \tag{6.8}$$

$$\delta\phi^a_{\rm sh,1}=\partial^a\xi_{\rm sh,1}-r^{00}_z\Box\xi^a_{\rm sh,0}\,, \tag{6.9}$$

$$\delta\phi_{\rm sh,-2}=-\sqrt{2}r^{10}_\zeta\xi_{\rm sh,-1}\,, \tag{6.10}$$

$$\delta\phi_{\rm sh,0}=-r^{01}_\zeta\xi_{\rm sh,1}-r^{10}_z\Box\xi_{\rm sh,-1}\,, \tag{6.11}$$

$$\delta\phi_{\rm sh,2}=-\sqrt{2}r^{01}_z\Box\xi_{\rm sh,1}\,, \tag{6.12}$$

where $\xi^a_{\rm sh,0}$, $\xi_{\rm sh,\pm1}$ are gauge transformation parameters.

We then find that a realization of the operator R^a on fields (6.1) takes the following form:

$$\begin{aligned}R^a\phi^{bc}_{\rm sh,0}=&2\kappa r^{00}_\zeta(\eta^{ab}\phi^c_{\rm sh,-1}+\eta^{ac}\phi^b_{\rm sh,-1})\\&-\frac{4(\kappa+1)r^{00}_\zeta}{d-2}\eta^{bc}\phi^a_{\rm sh,-1}\,,\end{aligned} \tag{6.13}$$

$$R^a\phi^b_{\rm sh,-1}=2\sqrt{2}(\kappa+1)r^{10}_\zeta\eta^{ab}\phi_{\rm sh,-2}\,, \tag{6.14}$$

$$\begin{aligned}R^a\phi^b_{\rm sh,1}=&r^{00}_z(2\kappa\phi^{ab}_{\rm sh,0}-\eta^{ab}\phi^{cc}_{\rm sh,0})\\&+2(\kappa-1)r^{01}_\zeta\eta^{ab}\phi_{\rm sh,0}\,,\end{aligned} \tag{6.15}$$

$$R^a\phi_{\rm sh,-2}=0\,, \tag{6.16}$$

$$R^a\phi_{\rm sh,0}=2(\kappa+1)r^{10}_z\phi^a_{\rm sh,-1}\,, \tag{6.17}$$

$$R^a\phi_{\rm sh,2}=2\sqrt{2}(\kappa-1)r^{01}_z\phi^a_{\rm sh,1}\,. \tag{6.18}$$

Using (6.13)-(6.18), we check that constraints (6.4)-(6.6) are invariant under transformations of the conformal algebra.

We proceed with the discussion of two-point vertex for spin-2 anomalous shadow field. The gauge invariant two-point vertex we find takes the form given (4.10), where the two-point density Γ_{12} is given by

$$\begin{aligned}\Gamma_{12} &= \frac{1}{4|x_{12}|^{2\kappa+d}}\Big(\phi^{ab}_{\rm sh,0}(x_1)\phi^{ab}_{\rm sh,0}(x_2) \\ &\qquad -\frac{1}{2}\phi^{aa}_{\rm sh,0}(x_1)\phi^{bb}_{\rm sh,0}(x_2)\Big) \\ &+ \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi^{a}_{\rm sh,\lambda}(x_1)\phi^{a}_{\rm sh,\lambda}(x_2) \\ &+ \sum_{\lambda=0,\pm2}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\rm sh,\lambda}(x_1)\phi_{\rm sh,\lambda}(x_2)\,, \qquad (6.19)\end{aligned}$$

$$\omega_1 = \frac{1}{2\kappa(2\kappa+d-2)}\,, \qquad (6.20)$$

$$\omega_{-1} = 2(\kappa+1)(2\kappa+d)\,, \qquad (6.21)$$

$$\omega_2 = \frac{1}{4\kappa(\kappa-1)(2\kappa+d-2)(2\kappa+d-4)}\,, \qquad (6.22)$$

$$\omega_{-2} = 4(\kappa+1)(\kappa+2)(2\kappa+d)(2\kappa+d+2)\,. \qquad (6.23)$$

We check that this vertex is invariant under gauge transformations of the spin-2 anomalous shadow field (6.7)-(6.12). Also, we check that the vertex is invariant under conformal algebra transformations. Remarkable feature of the vertex is its diagonal form with respect to the gauge fields entering field content (6.1).

B. Stueckelberg gauge frame

For spin-2 anomalous shadow field, the Stueckelberg gauge frame is achieved though the use of differential constraints (6.4)-(6.6) and Stueckelberg gauge condition.

From gauge transformation given in (6.8),(6.10),(6.11), we see that the vector field $\phi^a_{\rm sh,-1}$ and the scalar fields $\phi_{\rm sh,-2}$, $\phi_{\rm sh,0}$ transform as Stueckelberg fields, i.e., these fields can be gauged away via Stueckelberg gauge fixing,

$$\phi^a_{\rm sh,-1} = 0\,, \qquad \phi_{\rm sh,-2} = 0\,, \qquad \phi_{\rm sh,0} = 0\,. \qquad (6.24)$$

Using gauge conditions (6.24) in constraint (6.5) we find that the field $\phi^{ab}_{\rm sh,0}$ becomes traceless, while using gauge conditions (6.24) in constraints (6.4),(6.6) we find that the remaining vector field $\phi^a_{\rm sh,1}$ and the scalar field $\phi_{\rm sh,2}$ can be expressed in terms of the rank-2 tensor field $\phi^{ab}_{\rm sh,0}$,

$$\phi^{aa}_{\rm sh,0} = 0, \qquad (6.25)$$

$$\phi^a_{\rm sh,1} = -\frac{1}{r^{00}_z}\partial^b\phi^{ab}_{\rm sh,0}\,, \qquad (6.26)$$

$$\phi_{\rm sh,2} = \frac{1}{\sqrt{2}r^{00}_z r^{01}_z}\partial^a\partial^b\phi^{ab}_{\rm sh,0}\,. \qquad (6.27)$$

Relations (6.24)-(6.27) provide the complete description of the Stueckelberg gauge frame for the spin-2 anomalous shadow field.

Plugging (6.24)-(6.27) in (6.19) and ignoring the total derivative, we find that our two-point density Γ_{12} (6.19) takes the following form:

$$\Gamma^{\rm Stuck.g.fram}_{12} = k_2\Gamma^{\rm stand}_{12}\,, \qquad (6.28)$$

$$\Gamma^{\rm stand}_{12} = \phi^{a_1a_2}_{\rm sh,0}(x_1)\frac{O^{a_1b_1}_{12}O^{a_2b_2}_{12}}{|x_{12}|^{2\kappa+d}}\phi^{b_1b_2}_{\rm sh,0}(x_2)\,, \qquad (6.29)$$

$$k_2 \equiv \frac{2\kappa+d+2}{4(2\kappa+d-2)}\,, \qquad (6.30)$$

where O^{ab}_{12} is defined in (4.19), while $\Gamma^{\rm stand}_{12}$ (6.29) stands for the two-point vertex of the spin-2 anomalous shadow field in the standard approach to CFT. From (6.28), we see that our gauge invariant vertex taken to be in the Stueckelberg gauge frame coincides, up to normalization factor k_2, with the two-point vertex in the standard approach to CFT.

The kernel of vertex $\Gamma^{\rm stand}$ (6.29) defines two-point correlation function of the spin-2 conformal conformal current taken to be in the Stueckelberg gauge frame.

C. Light-cone gauge frame

For spin-2 anomalous shadow field, the light-cone gauge frame is achieved through the use of differential constraints (6.4)-(6.6) and light-cone gauge. Taking into account the gauge transformations of the fields $\phi^{ab}_{\rm sh,0}$, $\phi^a_{\rm sh,\pm1}$ given in (6.7)-(6.9), we impose the light-cone gauge condition,

$$\phi^{+a}_{\rm sh,0} = 0\,, \qquad \phi^{+}_{\rm sh,\lambda} = 0\,, \qquad \lambda = \pm1\,. \qquad (6.31)$$

Plugging this gauge condition in constraints (6.4)-(6.6), we find

$$\phi^{ii}_{\rm sh,0} = 0\,, \qquad (6.32)$$

$$\phi^{-i}_{\rm sh,0} = -\frac{\partial^j}{\partial^+}\phi^{ij}_{\rm sh,0} - \frac{r^{00}_\zeta}{\partial^+}\Box\phi^i_{\rm sh,-1} - \frac{r^{00}_z}{\partial^+}\phi^i_{\rm sh,1}\,, \qquad (6.33)$$

$$\begin{aligned}\phi^{--}_{\rm sh,0} &= \frac{\partial^i\partial^j}{\partial^+\partial^+}\phi^{ij}_{\rm sh,0} + \frac{2r^{00}_\zeta\partial^i}{\partial^+\partial^+}\Box\phi^i_{\rm sh,-1} + \frac{2r^{00}_z\partial^i}{\partial^+\partial^+}\phi^i_{\rm sh,1} \\ &+ \frac{\sqrt{2}r^{00}_\zeta r^{10}_\zeta}{\partial^+\partial^+}\Box^2\phi_{\rm sh,-2} + \frac{\sqrt{2}r^{00}_z r^{01}_z}{\partial^+\partial^+}\phi_{\rm sh,2} \\ &+ \frac{r^{00}_z r^{01}_\zeta + r^{00}_\zeta r^{10}_z}{\partial^+\partial^+}\Box\phi_{\rm sh,0}\,, \qquad (6.34)\end{aligned}$$

$$\phi^{-}_{\rm sh,-1} = -\frac{\partial^j}{\partial^+}\phi^{j}_{\rm sh,-1} - \frac{\sqrt{2}r^{10}_\zeta}{\partial^+}\Box\phi_{\rm sh,-2} - \frac{r^{10}_z}{\partial^+}\phi_{\rm sh,0}\,, \qquad (6.35)$$

$$\phi^{-}_{\rm sh,1} = -\frac{\partial^j}{\partial^+}\phi^j_{\rm sh,1} - \frac{r^{01}_\zeta}{\partial^+}\Box\phi_{\rm sh,0} - \frac{\sqrt{2}r^{01}_z}{\partial^+}\phi_{\rm sh,2}\,. \qquad (6.36)$$

We see that we are left with the $so(d-2)$ algebra traceless rank-2 tensor field, two vector fields and three scalar fields,

$$\begin{gathered}\phi^{ij}_{\rm sh,0}\,,\\ \phi^i_{\rm sh,-1}\,,\qquad \phi^i_{\rm sh,1}\,,\\ \phi_{\rm sh,-2}\,,\qquad \phi_{\rm sh,0}\,,\qquad \phi_{\rm sh,2}\,.\end{gathered} \qquad (6.37)$$

Fields in (6.37) constitute a field content of the spin-2 anomalous shadow field in light-cone gauge frame. Note that, in contrast to the Stueckelberg gauge frame, the vector fields and the scalar fields become independent field D.o.F in the light-cone gauge frame.

Using (6.31),(6.32) in (6.19) leads to light-cone gauge fixed vertex

$$\begin{aligned}\Gamma^{\rm l.c.}_{12} &= \frac{1}{4|x_{12}|^{2\kappa+d}}\phi^{ij}_{\rm sh,0}(x_1)\phi^{ij}_{\rm sh,0}(x_2)\\ &+ \sum_{\lambda=\pm1}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi^i_{\rm sh,\lambda}(x_1)\phi^i_{\rm sh,\lambda}(x_2)\\ &+ \sum_{\lambda=0,\pm2}\frac{\omega_\lambda}{2|x_{12}|^{2\kappa+d-2\lambda}}\phi_{\rm sh,\lambda}(x_1)\phi_{\rm sh,\lambda}(x_2)\,,\end{aligned} \qquad (6.38)$$

where ω_λ are defined in (6.20)-(6.23). We see that, as in the case of gauge invariant vertex (6.19), light-cone vertex (6.38) is diagonal with respect to the fields entering the field content of light-cone gauge frame (6.37). Note however that, in contrast to the gauge invariant vertex, the light-cone vertex is constructed out of the fields (6.37) which are not subject to any differential constraints.

The kernel of light-cone vertex (6.38) gives two-point correlation function of the spin-2 anomalous conformal current taken to be in the light-cone gauge. Defining two-point correlation functions of the anomalous conformal currents (5.35) as the second functional derivative of Γ with respect to the anomalous shadow field (6.37)

$$\langle\phi^{ij}_{\rm cur,0}(x_1),\phi^{kl}_{\rm cur,0}j(x_2)\rangle \equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi^{ij}_{\rm sh,0}(x_1)\delta\phi^{kl}_{\rm sh,0}(x_2)}\,, \qquad (6.39)$$

$$\langle\phi^{i}_{\rm cur,\lambda}(x_1),\phi^{j}_{\rm cur,\lambda}(x_2)\rangle \equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi^{i}_{\rm sh,-\lambda}(x_1)\delta\phi^{j}_{\rm sh,-\lambda}(x_2)}\,, \qquad (6.40)$$

$$\langle\phi_{\rm cur,\lambda}(x_1),\phi_{\rm cur,\lambda}(x_2)\rangle \equiv \frac{\delta^2\Gamma^{(\rm l.c.)}}{\delta\phi_{\rm sh,-\lambda}(x_1)\delta\phi_{\rm sh,-\lambda}(x_2)}\,, \qquad (6.41)$$

we obtain

$$\langle\phi^{ij}_{\rm cur,0}(x_1),\phi^{kl}_{\rm cur,0}(x_2)\rangle = \frac{1}{|x_{12}|^{2\kappa+d}}\Pi^{ij;kl}\,, \qquad (6.42)$$

$$\langle\phi^{i}_{\rm cur,\lambda}(x_1),\phi^{j}_{\rm cur,\lambda}(x_2)\rangle = \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}}\delta^{ij}\,, \qquad (6.43)$$

$$\langle\phi_{\rm cur,\lambda}(x_1),\phi_{\rm cur,\lambda}(x_2)\rangle = \frac{\omega_{-\lambda}}{|x_{12}|^{2\kappa+d+2\lambda}}\,, \qquad (6.44)$$

where ω_λ are defined in (6.20)-(6.23) and we use the notation

$$\Pi^{ij;kl} = \frac{1}{2}\Big(\delta^{ik}\delta^{jl} + \delta^{il}\delta^{jk} - \frac{2}{d-2}\delta^{ij}\delta^{kl}\Big)\,. \qquad (6.45)$$

VII. TWO POINT CURRENT-SHADOW FIELD INTERACTION VERTEX

We now discuss two-point current-shadow field interaction vertex. In the gauge invariant approach, interaction vertex is determined by requiring the vertex to be invariant under both gauge transformations of currents and shadow fields. Also, the interaction vertex should be invariant under conformal algebra transformations.

Spin-1. We begin with spin-1 fields. Let us consider the following vertex:

$$\mathcal{L} = \phi^a_{\rm cur,0}\phi^a_{\rm sh,0} + \phi_{\rm cur,-1}\phi_{\rm sh,1} + \phi_{\rm cur,1}\phi_{\rm sh,-1}\,. \qquad (7.1)$$

Denoting the left hand side of (4.3) by $C_{\rm sh}$ we find that under gauge transformations of the current (3.7)-(3.9) the variation of vertex (7.1) takes the form (up to total derivative)

$$\delta_{\xi_{\rm cur,0}}\mathcal{L} = -\xi_{\rm cur,0}C_{\rm sh}\,. \qquad (7.2)$$

From this expression, we see that the vertex $\mathcal{L}$ is invariant under gauge transformations of the current provided the shadow field satisfies differential constraint (4.3). Denoting the left hand side of (3.4) by $C_{\rm cur}$ we find that under gauge transformations of the shadow field (4.4)-(4.6) the variation of vertex (7.1) takes the form (up to total derivative)

$$\delta_{\xi_{sh}}\mathcal{L} = -\xi_{\rm sh,0}C_{\rm cur}\,, \qquad (7.3)$$

i.e., the vertex $\mathcal{L}$ is invariant under gauge transformations of the shadow field provided the current satisfies differential constraint (3.4).

Making use of the realization of the conformal algebra symmetries obtained in the Sections III,IV we check that vertex $\mathcal{L}$ (7.1) is invariant under the conformal algebra transformations.

Spin-2. We proceed with spin-2 fields. One can make sure that the following vertex

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\phi^{ab}_{\rm cur,0}\phi^{ab}_{\rm sh,0} - \frac{1}{4}\phi^{aa}_{\rm cur,0}\phi^{bb}_{\rm sh,0}\\ &+ \sum_{\lambda=\pm1}\phi^a_{\rm cur,\lambda}\phi^a_{\rm sh,-\lambda} + \sum_{\lambda=0,\pm2}\phi_{\rm cur,\lambda}\phi_{\rm sh,-\lambda}\end{aligned} \qquad (7.4)$$

is invariant under gauge transformations of the spin-2 shadow field (6.7)-(6.12) provided the spin-2 current satisfies differential constraints (5.4)-(5.6). Vertex (7.4) is also invariant under

gauge transformations of the spin-2 current (5.13)-(5.18) provided the spin-2 shadow field satisfies differential constraints (6.4)-(6.6). Using the representation for generators of the conformal algebra obtained in the Sections V,VI we check that vertex $\mathcal{L}$ (7.4) is invariant under the conformal algebra transformations.

VIII. ADS/CFT CORRESPONDENCE. PRELIMINARIES

We now apply our results to the study of AdS/CFT correspondence for free massive AdS fields and boundary anomalous conformal currents and shadow fields. To this end we use the gauge invariant CFT adapted description of AdS massive fields and modified Lorentz and de Donder gauges found in Ref.[12]. *It is the use of our fields and the modified Lorentz and de Donder gauges that leads to decoupled form of gauge fixed equations of motion and surprisingly simple Lagrangian*[7]. Owing these properties of our fields and the modified (Lorentz) de Donder gauge, we simplify significantly the computation of the effective action[8]. The modified (Lorentz) de Donder gauge is invariant under the on-shell leftover gauge symmetries of bulk AdS fields. Note that, in our approach, we have gauge symmetries not only at AdS side but also at the boundary CFT. Thus, in the framework of our approach, study of AdS/CFT correspondence implies matching of:

i) Lorentz (de Donder) gauge condition for bulk massive and differential constraints for boundary anomalous conformal currents and shadow fields;

ii) leftover on-shell gauge symmetries for bulk massive fields and gauge symmetries of boundary anomalous conformal currents and shadow fields;

iii) global symmetries of bulk massive fields and global symmetries of boundary anomalous conformal currents and shadow fields;

iv) effective action evaluated on solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and boundary two-point gauge invariant vertex for anomalous shadow field.

Global symmetries in CFT adapted approach. Relativistic symmetries of the AdS_{d+1} field dynamics are described by the $so(d,2)$ algebra. In d-dimensional space, global symmetries of anomalous conformal currents and shadow fields are also described by the $so(d,2)$ algebra. To discuss global symmetries of anomalous conformal currents and shadow fields we have used conformal basis of the $so(d,2)$ algebra (see (2.3)-(2.6)). Therefore for application to the study of AdS/CFT correspondence, it is convenient to realize the relativistic bulk $so(d,2)$ algebra symmetries by using basis of the conformal algebra. Most convenient way to achieve conformal basis realization of bulk $so(d,2)$ symmetries is to use Poincaré parametrization of AdS space[9],

$$ds^2 = \frac{1}{z^2}(dx^a dx^a + dz\, dz)\,. \tag{8.1}$$

In this parametrization, the $so(d,2)$ algebra transformations of the massive arbitrary spin AdS field ϕ take the form $\delta_{\hat{G}}\phi = \hat{G}\phi$, where realization of the $so(d,2)$ algebra generators $\hat{G}$ in terms of differential operators is given by

$$P^a = \partial^a\,, \tag{8.2}$$

$$J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}\,, \tag{8.3}$$

$$D = x\partial + \Delta\,, \qquad \Delta = z\partial_z + \frac{d-1}{2}\,, \tag{8.4}$$

$$K^a = K^a_{\Delta,M} + R^a\,, \tag{8.5}$$

$$K^a_{\Delta,M} = -\frac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b\,, \tag{8.6}$$

$$R^a = R^a_{(0)} + R^a_{(1)}\,, \tag{8.7}$$

$$R^a_{(1)} = -\frac{1}{2}z^2\partial^a\,. \tag{8.8}$$

Operator $R^a_{(0)}$ (8.7) does not depend on boundary coordinates x^a, boundary derivatives ∂^a, and derivative with respect to radial coordinate, ∂_z. Operator $R^a_{(0)}$ acting on spin D.o.F. depends only on radial coordinate z. Thus, we see all that is required to complete description of AdS field dynamics global symmetries is to find realization of the operator $R^a_{(0)}$ on space of gauge AdS fields.

AdS/CFT correspondence for spin-0 anomalous current and normalizable modes of scalar massive AdS field[10]. Because use of modified Lorentz (de Donder) gauge makes study of AdS/CFT correspondence for spin-1 (spin-2) field similar to the one for scalar field we begin with brief review of the AdS/CFT correspondence for the scalar field.

Action and Lagrangian for the massive scalar field in AdS_{d+1} background take the form[11]

$$S = \int d^d x dz\, \mathcal{L}\,, \tag{8.9}$$

[7] Our massive gauge fields are obtained from gauge fields used in the standard gauge invariant approach to massive fields by the invertible transformation. Details of the transformation may be found in Appendices A,B. Discussion of interesting methods for solving AdS field equations of motion without gauge fixing may be found in Refs.[22, 23].

[8] We remind that the bulk action evaluated on solution of the Dirichlet problem is referred to as effective action in this paper.

[9] In our approach only $so(d-1,1)$ symmetries are realized manifestly. The $so(d,2)$ symmetries could be realized manifestly by using ambient space approach (see e.g. [24–26])

[10] Also see Refs.[27].

[11] From now on we use, unless otherwise specified, the Euclidian signature.

$$\mathcal{L} = \tfrac{1}{2}\sqrt{|g|}\left(g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\Phi^2\right) . \qquad (8.10)$$

In terms of the canonical normalized field ϕ defined by relation $\Phi = z^{\frac{d-1}{2}}\phi$ the Lagrangian takes the form (up to total derivative)

$$\mathcal{L} = \frac{1}{2}|d\phi|^2 + \frac{1}{2}|\mathcal{T}_{\nu-\frac{1}{2}}\phi|^2 , \qquad (8.11)$$

$$\mathcal{T}_\nu \equiv \partial_z + \frac{\nu}{z} , \qquad (8.12)$$

$$\nu = \sqrt{m^2 + \frac{d^2}{4}} . \qquad (8.13)$$

Equations of motion obtained from Lagrangian (8.11) take the form

$$\Box_\nu\phi = 0 , \qquad (8.14)$$

$$\Box_\nu \equiv \Box + \partial_z^2 - \frac{1}{z^2}(\nu^2 - \frac{1}{4}) . \qquad (8.15)$$

Normalizable solution of equation (8.14) is given by

$$\phi(x,z) = U_\nu^{\rm sc}\phi_{\rm cur}(x) , \qquad (8.16)$$

$$U_\nu^{\rm sc} \equiv h_\nu\sqrt{zq}J_\nu(zq)q^{-(\nu+\frac{1}{2})} , \qquad (8.17)$$

$$h_\nu \equiv 2^\nu\Gamma(\nu+1) , \qquad q^2 \equiv \Box , \qquad (8.18)$$

where J_ν stands for the Bessel function. The asymptotic behavior of solution (8.16) is given by

$$\phi(x,z) \stackrel{z\to 0}{\longrightarrow} z^{\nu+\frac{1}{2}}\phi_{\rm cur}(x) , \qquad (8.19)$$

i.e., we see that spin-0 current $\phi_{\rm cur}$ is indeed boundary value of the normalizable solution.

In the case under consideration, we have no gauge symmetries and gauge conditions. Therefore all that is required to complete AdS/CFT is to match bulk global symmetries of AdS field $\phi(x,z)$ and boundary global symmetries of the current $\phi_{\rm cur}(x)$. Global symmetries on AdS side and CFT side are described in (8.2)-(8.8) and (2.8)-(2.11) respectively. We see that the Poincaré symmetries match automatically. Using the notation $D_{_{AdS}}$ and $D_{_{CFT}}$ to indicate the respective realizations of D-symmetry on the bulk fields (8.4) and the conformal currents (2.10) we obtain the relation

$$D_{_{AdS}}\phi(x,z) = U_\nu D_{_{CFT}}\phi_{\rm cur}(x) , \qquad (8.20)$$

where the expressions for $D_{_{CFT}}$ corresponding to $\phi_{\rm cur}$ can be obtained from (2.10) by using $\Delta = \frac{d}{2} + \nu$ with ν given in (8.13). Thus, D-symmetries of $\phi(x,z)$ and $\phi_{\rm cur}(x)$ also match. To match the K^a-symmetries in (2.11) and (8.5) we note that the respective operators $R^a_{(0)}$ and R^a act trivially, $R^a_{(0)}\phi(x,z) = 0$, $R^a\phi_{\rm cur}(x) = 0$ and make sure that K^a-symmetries also match.

AdS/CFT correspondence for spin-0 shadow field and non-normalizable modes of scalar massive AdS field. Following the procedure in Ref.[28], we note that non-normalizable solution of equations (8.14) with the Dirichlet problem corresponding to boundary shadow scalar field $\phi_{\rm sh}(x)$ takes the form

$$\phi(x,z) \;=\; \sigma\int d^dy\, G_\nu(x-y,z)\phi_{\rm sh}(y) , \qquad (8.21)$$

$$G_\nu(x,z) = \frac{c_\nu z^{\nu+\frac{1}{2}}}{(z^2+|x|^2)^{\nu+\frac{d}{2}}} , \qquad (8.22)$$

$$c_\nu \equiv \frac{\Gamma(\nu+\frac{d}{2})}{\pi^{d/2}\Gamma(\nu)} . \qquad (8.23)$$

To be flexible, we use normalization factor σ in (8.21). For the case of scalar field, commonly used normalization in (8.21) is achieved by setting $\sigma = 1$. Asymptotic behaviors of Green function (8.22) and solution (8.21) are well known,

$$G_\nu(x,z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+\frac{1}{2}}\delta^d(x) , \qquad (8.24)$$

$$\phi(x,z) \stackrel{z\to 0}{\longrightarrow} z^{-\nu+\frac{1}{2}}\sigma\phi_{\rm sh}(x) . \qquad (8.25)$$

From (8.25), we see that our solution has indeed asymptotic behavior corresponding to the shadow scalar field.

Using equations of motion (8.14) in bulk action (8.9) with Lagrangian (8.11) we obtain the effective action given by[12]

$$-S_{\rm eff} \;=\; \int d^dx\,\mathcal{L}_{\rm eff}\Big|_{z\to 0} , \qquad (8.26)$$

$$\mathcal{L}_{\rm eff} \;=\; \frac{1}{2}\phi\mathcal{T}_{\nu-\frac{1}{2}}\phi . \qquad (8.27)$$

Plugging solution of the Dirichlet problem (8.21) into (8.26), (8.27), we obtain the effective action

$$-S_{\rm eff} = \nu c_\nu\sigma^2\int d^dx_1 d^dx_2\frac{\phi_{\rm sh}(x_1)\phi_{\rm sh}(x_2)}{|x_{12}|^{2\nu+d}} . \qquad (8.28)$$

Using the commonly used value of σ, $\sigma = 1$, in (8.28), we obtain the properly normalized effective action found in Refs.[29, 30]. Interesting novelty of our computation of $S_{\rm eff}$ is that we use Fourier transform of the Green function. Details of our computation may be found in Appendix C in Ref.[9].

IX. ADS/CFT CORRESPONDENCE FOR SPIN-1 FIELDS

We now discuss AdS/CFT correspondence for bulk massive spin-1 AdS field and boundary spin-1 anomalous conformal current and shadow field. To this end we are going to

[12] Following commonly used setup, we consider solution of the Dirichlet problem which tends to zero as $z \to \infty$. Therefore, in (8.26), we ignore contribution to $S_{\rm eff}$ when $z = \infty$.

use CFT adapted gauge invariant Lagrangian and the modified Lorentz gauge condition [12][13]. Because our approach is closely related with gauge invariant approach to massive field we start with brief review of the latter approach.

Gauge invariant approach to spin-1 massive field in AdS_{d+1} space. In gauge invariant approach, spin-1 massive field is described by fields

$$\Phi^A\,, \qquad \Phi\,, \tag{9.1}$$

which transform in the respective vector and scalar representations of $so(d,1)$ algebra. In Lorentzian signature, Lagrangian given by

$$e^{-1}\mathcal{L} \;=\; -\frac{1}{4}F^{AB}F^{AB} - \frac{1}{2}F^{A}F^{A}\,, \tag{9.2}$$

$$F^{AB} \equiv \mathcal{D}^A\Phi^B - \mathcal{D}^B\Phi^A\,, \tag{9.3}$$

$$F^{A} \equiv \mathcal{D}^A\Phi + m\Phi^A\,, \tag{9.4}$$

is invariant under gauge transformations

$$\delta\Phi^A = \mathcal{D}^A\Xi\,, \qquad \delta\Phi = -m\Xi\,. \tag{9.5}$$

Details of our notation may be found in Appendix A. Lagrangian (9.2) can be cast into the form which is more convenient for our purposes,

$$e^{-1}\mathcal{L} = \frac{1}{2}\Phi^A(\mathcal{D}^2 - m^2 + d)\Phi^A + \frac{1}{2}\Phi(\mathcal{D}^2 - m^2)\Phi + \frac{1}{2}C_{\rm st}^2\,, \tag{9.6}$$

$$C_{\rm st} \equiv \mathcal{D}^C\Phi^C + m\Phi\,. \tag{9.7}$$

A. CFT adapted gauge invariant approach to massive spin-1 field in AdS_{d+1}

In our approach, the massive spin-1 AdS field is described by fields

$$\phi^a\,, \qquad \phi_{-1}\,, \qquad \phi_1 \tag{9.8}$$

which are the respective vector and scalar fields of the $so(d)$ algebra. Fields in (9.8) are related by invertible transformation with fields in (9.1) (see Appendix A). CFT adapted gauge invariant action and Lagrangian for field (9.8) take the form,

$$S \;=\; \int d^dxdz\,\mathcal{L}\,, \tag{9.9}$$

$$\mathcal{L} \;=\; \frac{1}{2}|d\phi^a|^2 + \frac{1}{2}|\mathcal{T}_{\kappa-\frac{1}{2}}\phi^a|^2 + \frac{1}{2}\sum_{\lambda=\pm1}\Big(|d\phi_\lambda|^2 + |\mathcal{T}_{\kappa-\frac{1}{2}+\lambda}\phi_\lambda|^2\Big) - \frac{1}{2}C^2\,, \tag{9.10}$$

$$C \equiv \partial^a\phi^a + r_\zeta^{00}\mathcal{T}_{\kappa+\frac{1}{2}}\phi_1 + r_z^{00}\mathcal{T}_{-\kappa+\frac{1}{2}}\phi_{-1}\,, \tag{9.11}$$

where $\mathcal{T}_\nu$ is given in (8.12), while κ and r_z^{00}, r_ζ^{00} are defined in (3.3) and (3.5),(3.6) respectively. Lagrangian (9.10) is invariant under gauge transformations

$$\delta\phi^a = \partial^a\xi\,, \tag{9.12}$$

$$\delta\phi_{-1} = r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}}\xi\,, \tag{9.13}$$

$$\delta\phi_1 = r_\zeta^{00}\mathcal{T}_{-\kappa-\frac{1}{2}}\xi\,, \tag{9.14}$$

where ξ is a gauge transformation parameter. Details of the derivation of Lagrangian (9.10) from the one in (9.6) may be found in Appendix A.

Gauge invariant equations of motion obtained from Lagrangian (9.10) take the form

$$\Box_\kappa\phi^a - \partial^aC = 0\,, \tag{9.15}$$

$$\Box_{\kappa-1}\phi_{-1} - r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}}C = 0\,, \tag{9.16}$$

$$\Box_{\kappa+1}\phi_1 - r_\zeta^{00}\mathcal{T}_{-\kappa-\frac{1}{2}}C = 0\,, \tag{9.17}$$

where the operator $\Box_\nu$ is given in (8.15).

Global symmetries in CFT adapted approach. General form of realization of global symmetries for arbitrary spin AdS field was given in (8.2)-(8.5). All that is required to complete description of the global symmetries is to find realization of the operator $R^a_{(0)}$ on space of gauge fields. For the case of massive spin-1 field, realization of the operator $R^a_{(0)}$ on space of gauge fields (9.8) is given by

$$R^a_{(0)}\phi^b = z\eta^{ab}r_\zeta^{00}\phi_1 + z\eta^{ab}r_z^{00}\phi_{-1}\,, \tag{9.18}$$

$$R^a_{(0)}\phi_{-1} = -zr_z^{00}\phi^a\,, \tag{9.19}$$

$$R^a_{(0)}\phi_1 = -zr_\zeta^{00}\phi^a\,. \tag{9.20}$$

Modified Lorentz gauge. Modified Lorentz gauge is defined to be

$$C = 0\,, \qquad \text{modified Lorentz gauge}\,, \tag{9.21}$$

where C is given in (9.11). Using this gauge condition in equations of motion (9.15)-(9.17) gives simple gauge fixed equations of motion,

$$\Box_\kappa\phi^a = 0\,, \tag{9.22}$$

$$\Box_{\kappa+\lambda}\phi_\lambda = 0\,, \qquad \lambda = \pm1\,. \tag{9.23}$$

[13] For spin-1 massless field modified Lorentz gauge was found in Ref.[31], while for massless arbitrary spin field the modified de Donder gauge was discovered in Ref.[32].

Thus, we see that the gauge fixed equations of motion are decoupled.

We note that modified Lorentz gauge and gauge-fixed equations have leftover on-shell gauge symmetry. Namely, modified Lorentz gauge (9.21) and gauge-fixed equations (9.22), (9.23) are invariant under gauge transformations given in (9.12)-(9.14) provided the gauge transformation parameter satisfies the equation

$$\Box_\kappa \xi = 0\,. \tag{9.24}$$

B. AdS/CFT correspondence for anomalous current and normalizable modes of massive AdS field

We now ready to discuss AdS/CFT correspondence for spin-1 massive AdS field and spin-1 anomalous conformal current. We begin with analysis of normalizable solution of equations (9.22),(9.23). The normalizable solution of equations (9.22),(9.23) takes the form

$$\phi^a(x,z) = U_\kappa \phi^a_{\mathrm{cur},0}(x)\,, \tag{9.25}$$

$$\phi_{-1}(x,z) = -U_{\kappa-1}\phi_{\mathrm{cur},-1}(x)\,, \tag{9.26}$$

$$\phi_1(x,z) = U_{\kappa+1}\phi_{\mathrm{cur},1}(x)\,, \tag{9.27}$$

$$U_\nu \equiv h_\kappa \sqrt{zq} J_\nu(zq) q^{-(\nu+\frac{1}{2})}\,, \tag{9.28}$$

$$h_\kappa \equiv 2^\kappa \Gamma(\kappa+1)\,, \qquad q^2 \equiv \Box\,. \tag{9.29}$$

The asymptotic behavior of solution (9.25)-(9.27) is given by

$$\phi^a(x,z) \overset{z\to 0}{\longrightarrow} z^{\kappa+\frac{1}{2}}\phi^a_{\mathrm{cur},0}(x)\,, \tag{9.30}$$

$$\phi_{-1}(x,z) \overset{z\to 0}{\longrightarrow} -2\kappa z^{\kappa-\frac{1}{2}}\phi_{\mathrm{cur},-1}(x)\,, \tag{9.31}$$

$$\phi_1(x,z) \overset{z\to 0}{\longrightarrow} \frac{z^{\kappa+\frac{3}{2}}}{2(\kappa+1)}\phi_{\mathrm{cur},1}(x)\,. \tag{9.32}$$

From (9.30)-(9.32), we see that $\phi^a_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},\pm 1}$ are indeed boundary values of the normalizable solution.

In the r.h.s. (9.25)-(9.27) we use the notation $\phi^a_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},\pm 1}$ since we are going to demonstrate that these boundary values are indeed the anomalous conformal currents entering our gauge invariant formulation in the Sec.III. Namely one can prove the following statements:

i) For normalizable solution (9.25)-(9.27) modified Lorentz gauge condition (9.21) leads to differential constraint (3.4) of the spin-1 anomalous conformal current.

ii) *Leftover on-shell* gauge transformations (9.12)-(9.14) of normalizable solution (9.25)-(9.27) lead to gauge transformations (3.7)-(3.9) of the spin-1 anomalous conformal current.

iii) Global $so(d,2)$ symmetries of the normalizable massive spin-1 modes in AdS_{d+1} become global $so(d,2)$ conformal symmetries of the anomalous conformal spin-1 current.

These statements can easily be proved by using the following relations for the operator U_ν:

$$\mathcal{T}_{\nu-\frac{1}{2}} U_\nu = U_{\nu-1}\,, \tag{9.33}$$

$$\mathcal{T}_{-\nu-\frac{1}{2}} U_\nu = -U_{\nu+1}\Box\,, \tag{9.34}$$

$$\mathcal{T}_{-\nu+\frac{1}{2}}(zU_\nu) = -zU_{\nu+1}\Box + 2U_\nu\,, \tag{9.35}$$

which, in turn, can be obtained by using the following well-known identities for the Bessel function:

$$\mathcal{T}_\nu J_\nu = J_{\nu-1}\,, \qquad \mathcal{T}_{-\nu} J_\nu = -J_{\nu+1}\,. \tag{9.36}$$

Matching of bulk modified Lorentz gauge and boundary constraint. As an illustration we demonstrate how constraint for the anomalous conformal current (3.4) can be obtained from modified Lorentz gauge condition (9.21). To this end, adapting relations (9.33) and (9.34) for the respective $\nu = \kappa+1$ and $\nu = \kappa-1$ we obtain the relations

$$\mathcal{T}_{\kappa+\frac{1}{2}} U_{\kappa+1} = U_\kappa\,, \qquad \mathcal{T}_{-\kappa+\frac{1}{2}} U_{\kappa-1} = -U_\kappa\Box\,. \tag{9.37}$$

Plugging solutions ϕ^a, $\phi_{\pm 1}$ (9.25)-(9.27) in C (9.11) and using (9.37) we obtain the relation

$$C = U_\kappa C_{\mathrm{cur}}\,, \tag{9.38}$$

where C_{cur} stands for left hand side of (3.4). From (9.38) we see that our modified Lorentz gauge condition $C=0$ (9.21) leads indeed to differential constraint for the anomalous conformal current (3.4).

Matching of bulk and boundary gauge symmetries. As second illustration, we demonstrate how gauge transformations of the anomalous conformal current (3.7)-(3.9) can be obtained from leftover on-shell gauge transformations of massive AdS field (9.12)-(9.14)[14]. To this end we note that the respective normalizable solution of equation for gauge transformation parameter (9.24) takes the form

$$\xi(x,z) = U_\kappa \xi_{\mathrm{cur},0}(x)\,. \tag{9.39}$$

Plugging (9.25) and (9.39) in (9.12) we see that (9.12) leads indeed to (3.7). To match boundary gauge transformations (3.8) and bulk gauge transformation (9.13) we plug solution for ξ (9.39) in bulk gauge transformation (9.13) and adapt relation (9.33) for $\nu=\kappa$ to obtain

$$\delta\phi_{-1}(x,z) \;=\; r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}} U_\kappa \xi_{\mathrm{cur},0}(x)$$

[14] Transformations given in (9.12)-(9.14) are off-shell gauge transformations. Leftover on-shell gauge transformation are obtained from (9.12)-(9.14) by using gauge transformation parameter which satisfies equation (9.24).

$$= U_{\kappa-1} r_z^{00} \xi_{\mathrm{cur},0}(x) \tag{9.40}$$

on the one hand. On the other hand relation (9.26) implies

$$\delta\phi_{-1}(x,z) = -U_{\kappa-1}\delta\phi_{\mathrm{cur},-1}(x)\,. \tag{9.41}$$

Comparing (9.40) and (9.41) we see that the boundary gauge transformations (3.8) and bulk gauge transformation (9.13) match. In the same way, using (9.34), one can make sure that the remaining boundary gauge transformation (3.9) and bulk gauge transformation (9.14) also match.

Matching of bulk and boundary global symmetries. We proceed to comparison of bulk and boundary global symmetries. To this end we note that representation for generators given in (8.2)-(8.5) is valid for gauge invariant theory of AdS field. This to say that our modified Lorentz gauge respects the Poicaré and dilatation symmetries, but break K^a-symmetries. In other words, expressions for generators P^a, J^{ab} and D given in (8.2)-(8.4) are still valid for the gauge-fixed AdS fields, while expression for the generator K^a (8.5) should be modified to restore conformal boost symmetries for the gauge-fixed AdS fields. Therefore let us first to demonstrate matching of the Poincaré and dilatation symmetries. What is required is to demonstrate matching of the $so(d,2)$ algebra generators for bulk AdS fields given in (8.2)-(8.4) and ones for boundary anomalous conformal current given in (2.8)-(2.10). As for generators of the Poincaré algebra, P^a, J^{ab}, they already coincide on both sides (see formulas (2.8), (2.9) and the respective formulas (8.2),(8.3)). Next, consider the dilatation generator D. Here we need explicit form of solution to bulk theory equations of motion given in (9.25)-(9.27). Using the notation $D_{_{AdS}}$ and $D_{_{CFT}}$ to indicate the respective realizations of the dilatation generator D on bulk fields (8.4) and bulk currents (2.10) we obtain the relations

$$D_{_{AdS}}\phi^{ab}(x,z) = U_\kappa D_{_{CFT}}\phi^{ab}_{\mathrm{cur},0}(x)\,, \tag{9.42}$$

$$D_{_{AdS}}\phi_{-1}(x,z) = -U_{\kappa-1} D_{_{CFT}}\phi_{\mathrm{cur},-1}(x)\,, \tag{9.43}$$

$$D_{_{AdS}}\phi_{1}(x,z) = U_{\kappa+1} D_{_{CFT}}\phi_{\mathrm{cur},1}(x)\,, \tag{9.44}$$

where the expressions for $D_{_{CFT}}$ corresponding to $\phi^{ab}_{\mathrm{cur},0}$, $\phi_{\mathrm{cur},-1}$, $\phi_{\mathrm{cur},1}$ can be obtained from (2.10) and the respective conformal dimensions given in (3.2). Thus, the generators $D_{_{AdS}}$ and $D_{_{CFT}}$ also match.

We now turn to matching of the conformal boost K^a-symmetries. Matching of the K^a-symmetries requires analysis of some subtleties of our gauge fixing for AdS field. We now discuss these subtleties.

As we have already said our modified Lorentz gauge breaks the K^a-symmetries. To demonstrate this we note that K^a transformations of gauge fields are given by

$$K^a\phi^b = K^a_\Delta\phi^b + M^{abe}\phi^e + z\eta^{ab} r^{00}_\zeta \phi_1 + z\eta^{ab} r^{00}_z \phi_{-1} - \frac{1}{2} z^2 \partial^a \phi^b\,, \tag{9.45}$$

$$K^a\phi_1 = K^a_\Delta\phi_1 - z r^{00}_\zeta \phi^a - \frac{1}{2} z^2 \partial^a \phi_1\,, \tag{9.46}$$

$$K^a\phi_{-1} = K^a_\Delta\phi_{-1} - z r^{00}_z \phi^a - \frac{1}{2} z^2 \partial^a \phi_{-1}\,, \tag{9.47}$$

where K^a_Δ and M^{abc} are defined in (2.19),(2.20), while Δ is given in (8.4). Using these transformation rules we find that C (9.11) transforms as

$$K^a C = K^a_{\Delta+1} C - \frac{1}{2} z^2 \partial^a C - 2\phi^a\,, \tag{9.48}$$

i.e., we see that the modified Lorentz gauge condition $C=0$ is not invariant w.r.t. K^a transformations,

$$K^a C\,\big|_{C=0} = -2\phi^a\,. \tag{9.49}$$

This implies that generator K^a given in (8.5) should be modified to restore the conformal boost symmetries (K^a symmetries) of the gauge-fixed AdS field theory. In order to restore these broken K^a symmetries we should, following standard procedure, add compensating gauge transformations to maintain the K^a symmetries. Thus, in order to find improved K^a transformations of the gauge-fixed AdS fields (9.8) we start with the generic global K^a transformations (9.45)-(9.47) supplemented by the appropriate compensating gauge transformation

$$K^a_{\mathrm{impr}}\phi^b = K^a\phi^b + \partial^b \xi^{K^a}\,, \tag{9.50}$$

$$K^a_{\mathrm{impr}}\phi_{-1} = K^a\phi_{-1} + r^{00}_z \mathcal{T}_{\kappa-\frac{1}{2}}\xi^{K^a}\,, \tag{9.51}$$

$$K^a_{\mathrm{impr}}\phi_{1} = K^a\phi_{1} + r^{00}_\zeta \mathcal{T}_{-\kappa-\frac{1}{2}}\xi^{K^a}\,, \tag{9.52}$$

where ξ^{K^a} stands for parameter of compensating gauge transformation. Computing K^a_{impr} transformation of C

$$K^a_{\mathrm{impr}} C = K^a_{\Delta+1} C - \frac{1}{2} z^2 \partial^a C - 2\phi^a + \Box_\kappa \xi^{K^a}\,, \tag{9.53}$$

and requiring the K^a_{impr}-transformation to maintain the gauge condition $C=0$,

$$K^a_{\mathrm{impr}} C\,\big|_{C=0} = 0\,, \tag{9.54}$$

we get equation for ξ^{K^a}

$$\Box_\kappa \xi^{K^a} - 2\phi^a = 0\,. \tag{9.55}$$

Thus, we obtain the non-homogeneous second-order differential equation for the compensating gauge transformation parameter ξ^{K^a}. Plugging normalizable solution solution (9.25) in (9.55) we obtain the equation

$$\Box_\kappa \xi^{K^a}(x,z) = 2U_\kappa \phi^a_{\mathrm{cur},0}(x)\,. \tag{9.56}$$

Solution to this equations is found to be

$$\xi^{K^a}(x,z) = zU_{\kappa+1}\phi^a_{\text{cur},0}(x)\,. \tag{9.57}$$

Plugging this solution in (9.50)-(9.52), we obtain the K^a_{impr}-transformations of gauge fixed fields. We then make sure that these K^a_{impr}-transformations lead to the conformal boost transformations for the anomalous conformal current given in (2.7),(2.11) with operator R^a defined in (3.10)-(3.12).

C. AdS/CFT correspondence for anomalous shadow field and non-normalizable mode of massive AdS field.

We proceed to discussion of AdS/CFT correspondence for bulk massive spin-1 AdS field and boundary spin-1 anomalous shadow field.

Matching of effective action and boundary two-point vertex. In order to find bulk effective action S_{eff} we should, following standard strategy, solve bulk equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into bulk action. Using gauge invariant equations of motion (9.15)-(9.17) in bulk action (9.9), we obtain the following effective action:

$$S_{\text{eff}} = -\int d^dx\,\mathcal{L}_{\text{eff}}\Big|_{z\to 0}\,, \tag{9.58}$$

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\phi^a\mathcal{T}_{\kappa-\frac{1}{2}}\phi^a + \frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda\mathcal{T}_{\kappa-\frac{1}{2}+\lambda}\phi_\lambda - \frac{1}{2}(r^{00}_z\phi_{-1} + r^{00}_\zeta\phi_1)C\,. \tag{9.59}$$

As we have already seen, use of modified Lorentz gauge considerably simplify equations of motion. Now, using modified Lorentz gauge (9.21) in (9.59), we obtain

$$\mathcal{L}_{\text{eff}}\Big|_{C=0} = \frac{1}{2}\phi^a\mathcal{T}_{\kappa-\frac{1}{2}}\phi^a + \frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda\mathcal{T}_{\kappa-\frac{1}{2}+\lambda}\phi_\lambda\,, \tag{9.60}$$

i.e. we see that $\mathcal{L}_{\text{eff}}$ is also simplified. In order to find S_{eff} we should solve gauge fixed equations of motion (9.22),(9.23) with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into (9.60). We now discuss solution of equations of motion (9.22),(9.23).

Because gauge fixed equations of motion (9.22),(9.23) are similar to the ones for scalar AdS field (8.14) we can simply apply result in Sec. VIII. This is to say that solution of equations (9.22),(9.23) with the Dirichlet problem corresponding to the spin-1 anomalous shadow field takes the form

$$\phi^a(x,z) = \sigma_{1,0}\int d^dy\,G_\kappa(x-y,z)\phi^a_{\text{sh},0}(y)\,, \tag{9.61}$$

$$\phi_{-1}(x,z) = \sigma_{0,-1}\int d^dy\,G_{\kappa-1}(x-y,z)\phi_{\text{sh},1}(y)\,, \tag{9.62}$$

$$\phi_1(x,z) = \sigma_{0,1}\int d^dy\,G_{\kappa+1}(x-y,z)\phi_{\text{sh},-1}(y)\,, \tag{9.63}$$

$$\sigma_{1,0} = 1\,, \tag{9.64}$$

$$\sigma_{0,-1} = -\frac{1}{2(\kappa-1)}\,, \tag{9.65}$$

$$\sigma_{0,1} = 2\kappa\,, \tag{9.66}$$

where the Green function is given in (8.22).

Using asymptotic behavior of the Green function G_ν given in (8.24), we find the asymptotic behavior of our solution

$$\phi^a(x,z) \overset{z\to0}{\longrightarrow} z^{-\kappa+\frac{1}{2}}\phi^a_{\text{sh},0}(x)\,, \tag{9.67}$$

$$\phi_{-1}(x,z) \overset{z\to0}{\longrightarrow} -\frac{z^{-\kappa+\frac{3}{2}}}{2(\kappa-1)}\phi_{\text{sh},1}(x)\,, \tag{9.68}$$

$$\phi_1(x,z) \overset{z\to0}{\longrightarrow} 2\kappa z^{-\kappa-\frac{1}{2}}\phi_{\text{sh},-1}(x)\,. \tag{9.69}$$

From these expressions, we see that our solution has indeed asymptotic behavior corresponding to the spin-1 anomalous shadow field. Note that because the solution has non-integrable asymptotic behavior (9.67)-(9.69), such solution is referred to as the non-normalizable solution in the literature.

We now explain the choice of the normalization factors $\sigma_{1,0}$, $\sigma_{0,\pm1}$ in (9.64)-(9.66). The choice of $\sigma_{1,0}$ is a matter of convention. Following commonly used convention, we set this normalization factor to be equal to 1. The remaining normalization factors $\sigma_{0,\pm1}$ are then determined uniquely by requiring that the modified Lorentz gauge condition for the massive spin-1 AdS field (9.21) be amount to the differential constraint for the spin-1 anomalous shadow field (4.3). With the choice made in (9.64)-(9.66) we find the relations

$$\partial^a\phi^a = \int d^dy\,G_\kappa(x-y,z)\partial^a\phi^a_{\text{sh},0}(y)\,, \tag{9.70}$$

$$\mathcal{T}_{-\kappa+\frac{1}{2}}\phi_{-1} = \int d^dy\,G_\kappa(x-y,z)\phi_{\text{sh},1}(y), \tag{9.71}$$

$$\mathcal{T}_{\kappa+\frac{1}{2}}\phi_1 = \int d^dy\,G_\kappa(x-y,z)\Box\phi_{\text{sh},-1}(y)\,. \tag{9.72}$$

From these relations and (9.11), we see that our choice of $\sigma_{1,0}$, $\sigma_{1,\pm1}$ (9.64)-(9.66), allows us to match modified Lorentz gauge for the spin-1 AdS field (9.21) and differential constraint for the spin-1 anomalous shadow field given in (4.3). We note the helpful relations for the Green function which we use for the derivation of relations (9.71),(9.72),

$$\mathcal{T}_{-\kappa+\frac{1}{2}}G_{\kappa-1} = -2(\kappa-1)G_\kappa,\,, \tag{9.73}$$

$$\mathcal{T}_{\kappa+\frac{1}{2}}G_{\kappa+1} = \frac{1}{2\kappa}\Box G_\kappa\,, \tag{9.74}$$

where $G_\nu \equiv G_\nu(x-y,z)$.

All that remains to obtain $S_{\rm eff}$ is to plug solution of the Dirichlet problem for AdS fields, (9.61)-(9.63) into (9.58), (9.60). Using general formula given in (8.28), we obtain

$$-S_{\rm eff}=2\kappa c_\kappa\Gamma\,, \tag{9.75}$$

where κ and c_κ are defined in (3.3),(8.23) respectively and Γ is gauge invariant two-point vertex of the spin-1 anomalous shadow field given in (4.10),(4.11).

Thus we see that *imposing the modified Lorentz gauge on the massive spin-1 AdS field and computing the bulk action on the solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field we obtain the gauge invariant two-point vertex of the spin-1 anomalous shadow field*.

Because in the literature $S_{\rm eff}$ is expressed in terms of two-point vertex taken in the Stueckelberg gauge frame, $\Gamma^{\rm stand}$ (4.18), we use (4.17) and represent our result (9.75) as

$$-S_{\rm eff}=\frac{\kappa(2\kappa+d)}{2\kappa+d-2}c_\kappa\Gamma^{\rm stand}\,. \tag{9.76}$$

This relation was obtained in Ref.[10]. Note that we have obtained more general relation given in (9.75), while relation (9.76) is obtained from (9.75) by using the Stueckelberg gauge frame. We note that for the systematical study of AdS/CFT correspondence it is important to know the normalization factor in front of $\Gamma^{\rm stand}$ (9.76). Our normalization factor coincides with the one found in Ref.[10][15].

Matching of bulk and boundary gauge symmetries. Modified Lorentz gauge (9.21) and gauge-fixed equations (9.22),(9.23) are invariant under gauge transformations given in (9.12)-(9.14) provided the gauge transformation parameter satisfies the equation (9.24). Non-normalizable solution to this equation is given by

$$\xi(x,z)=\int d^dy\,G_\kappa(x-y,z)\xi_{\rm sh}(y)\,. \tag{9.77}$$

We now note that, on the one hand, plugging (9.77) in (9.12)-(9.14) and using relations (9.73),(9.74) we represent on-shell gauge transformations of $\phi^a(x,z)$, $\phi_{-1}(x,z)$ and $\phi_1(x,z)$ as

$$\delta\phi^a=\int d^dy\,G_\kappa(x-y,z)\partial^a\xi_{\rm sh}(y)\,, \tag{9.78}$$

$$\delta\phi_{-1}=\frac{r^{00}_z}{2(\kappa-1)}\int d^dy\,G_{\kappa-1}(x-y,z)\Box\xi_{\rm sh}(y)\,, \tag{9.79}$$

$$\delta\phi_1=-2\kappa r^{00}_\zeta\int d^dy\,G_{\kappa+1}(x-y,z)\xi_{\rm sh}(y)\,. \tag{9.80}$$

On the other hand, relations (9.61)-(9.63) imply

$$\delta\phi^a(x,z)=\sigma_{1,0}\int d^dy\,G_\kappa(x-y,z)\delta\phi^a_{{\rm sh},0}(y)\,, \tag{9.81}$$

$$\delta\phi_{-1}(x,z)=\sigma_{1,-1}\int d^dy\,G_{\kappa-1}(x-y,z)\delta\phi_{{\rm sh},1}(y)\,, \tag{9.82}$$

$$\delta\phi_1(x,z)=\sigma_{1,1}\int d^dy\,G_{\kappa+1}(x-y,z)\delta\phi_{{\rm sh},-1}(y)\,. \tag{9.83}$$

Comparing (9.78)-(9.80) with (9.81)-(9.83) we see that the on-shell leftover gauge symmetries of solution of the Dirichlet problem for AdS spin-1 massive field amount to the gauge symmetries of the spin-1 anomalous shadow field.

Following procedure in Sec.IX B one can make sure that global bulk and boundary symmetries match. It is this *matching of the bulk on-shell leftover gauge symmetries of the solution to Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action coincides with the gauge invariant two-point vertex for the boundary anomalous shadow field (see (9.75))*.

X. ADS/CFT CORRESPONDENCE FOR SPIN-2 FIELDS.

Before discussing AdS/CFT correspondence for massive spin-2 AdS field and spin-2 anomalous conformal current and shadow field we present our CFT adapted gauge invariant approach to spin-2 massive AdS field. Because our approach is closely related with gauge invariant approach to massive field we start with brief review of the latter approach.

Gauge invariant approach to spin-2 massive field in AdS_{d+1} space. In gauge invariant approach, spin-2 massive field is described by gauge fields

$$\Phi^{AB}\,,\qquad \Phi^A\,,\qquad \Phi\,, \tag{10.1}$$

which transform in the respective rank-2 tensor, vector and scalar representations of $so(d,1)$ algebra. In Lorentzian signature, Lagrangian found in Ref.[33] takes the form[16]

$$\begin{aligned}\frac{1}{e}\mathcal{L}\ =\ &\frac{1}{4}\Phi^{AB}E_{_{EH}}\Phi^{AB}+\frac{1}{2}\Phi^AE_{_{Max}}\Phi^A+\frac{1}{2}\Phi\mathcal{D}^2\Phi\\ &+\ m\Phi^A(\mathcal{D}^B\Phi^{BA}-\mathcal{D}^A\Phi^{BB})+f\Phi\mathcal{D}^A\Phi^A\\ &-\ \frac{m^2-2}{4}\Phi^{AB}\Phi^{AB}+\frac{m^2+d-2}{4}\Phi^{AA}\Phi^{BB}\\ &+\ \frac{fm}{2}\Phi^{AA}\Phi-\frac{d}{2}\Phi^A\Phi^A+\frac{(d+1)m^2}{2(d-1)}\Phi^2\,,\end{aligned} \tag{10.2}$$

[15] Computation of $S_{\rm eff}$ for spin-1 massless field may be found in Ref.[30] and, in the framework of our approach, in Ref.[9].

[16] Recent interesting discussion of massive AdS fields may be found in [34].

$$f \equiv \left(\frac{2d}{d-1}m^2 + 2d\right)^{1/2}, \qquad (10.3)$$

where the respective second-derivative Einstein-Hilbert and Maxwell operators $E_{_{EH}}$, $E_{_{Max}}$ are given by

$$E_{_{EH}}\Phi^{AB} = \mathcal{D}^2\Phi^{AB} - \mathcal{D}^A\mathcal{D}^C\Phi^{CB} - \mathcal{D}^B\mathcal{D}^C\Phi^{CA} + \mathcal{D}^A\mathcal{D}^B\Phi^{CC} + \eta^{AB}(\mathcal{D}^C\mathcal{D}^E\Phi^{CE} - \mathcal{D}^2\Phi^{CC}), \quad (10.4)$$

$$E_{_{Max}}\Phi^A = \mathcal{D}^2\Phi^A - \mathcal{D}^A\mathcal{D}^B\Phi^B. \qquad (10.5)$$

Lagrangian (10.2) is invariant under gauge transformations

$$\delta\Phi^{AB} = \mathcal{D}^A\Xi^B + \mathcal{D}^B\Xi^A + \frac{2m}{d-1}\eta^{AB}\Xi, \qquad (10.6)$$

$$\delta\Phi^A = \mathcal{D}^A\Xi - m\Xi^A, \qquad (10.7)$$

$$\delta\Phi = -f\Xi, \qquad (10.8)$$

where Ξ^A, Ξ are gauge transformation parameters. In Ref.[12], we found new representation for Lagrangian (10.2),

$$\begin{aligned}\frac{1}{e}\mathcal{L} &= \frac{1}{4}\Phi^{AB}(\mathcal{D}^2 - m^2 + 2)\Phi^{AB} \\ &- \frac{1}{8}\Phi^{AA}(\mathcal{D}^2 - m^2 - 2d + 4)\Phi^{BB} \\ &+ \frac{1}{2}\Phi^A(\mathcal{D}^2 - m^2 - d)\Phi^A + \frac{1}{2}\Phi(\mathcal{D}^2 - m^2 - 2d)\Phi \\ &+ \frac{1}{2}C^A_{\rm st}C^A_{\rm st} + \frac{1}{2}C^2_{\rm st}, \end{aligned} \qquad (10.9)$$

$$C^A_{\rm st} = \mathcal{D}^B\Phi^{BA} - \frac{1}{2}\mathcal{D}^A\Phi^{BB} + m\Phi^A, \qquad (10.10)$$

$$C_{\rm st} = \mathcal{D}^A\Phi^A + \frac{m}{2}\Phi^{AA} + f\Phi. \qquad (10.11)$$

From (10.9), we see that it is the use of quantities $C^A_{\rm st}$ and $C_{\rm st}$ that simplifies the structure of the gauge invariant Lagrangian. We note also that the relations $C^A_{\rm st} = 0$, $C_{\rm st} = 0$ define standard de Donder gauge condition for massive spin-2 field[17].

Interrelation of gauge invariant Lagrangian and Pauli-Fierz Lagrangian. As is well know spin-2 field can be described by the Pauli-Fierz Lagrangian given by

$$\begin{aligned}\frac{1}{e}\mathcal{L}_{_{PF}} &= \frac{1}{4}\Phi^{AB}_{_{PF}}(E_{_{EH}}\Phi_{_{PF}})^{AB} - \frac{m^2-2}{4}\Phi^{AB}_{_{PF}}\Phi^{AB}_{_{PF}} \\ &+ \frac{m^2+d-2}{4}\Phi^{AA}_{_{PF}}\Phi^{BB}_{_{PF}}, \end{aligned} \qquad (10.12)$$

where $\Phi^{AB}_{_{PF}}$ is rank-2 tensor field of $so(d,1)$ algebra. Pauli-Fierz Lagrangian can be obtained from gauge invariant Lagrangian (10.2) in obvious way. Namely, gauge transformations (10.7),(10.8) allow us to gauge away the fields Φ^A and Φ. Doing so and identifying rank-2 tensor field in (10.2) with $\Phi^{AB}_{_{PF}}$ we get the Pauli-Fierz Lagrangian from gauge invariant Lagrangian (10.2),

$$\mathcal{L}_{_{PF}} = \mathcal{L}|_{\Phi^{AB}\equiv\Phi^{AB}_{_{PF}},\,\Phi^A=0,\,\Phi=0}. \qquad (10.13)$$

For the case of flat space, it is well known that the gauge invariant Lagrangian can be obtained from Pauli-Fierz Lagrangian. It turns out that this interrelation is still to be valid in AdS space too. Namely, inserting the following representation of Pauli-Fierz field in terms of the gauge fields (10.1)

$$\begin{aligned}\Phi^{AB}_{_{PF}} &= \Phi^{AB} + \frac{1}{m}(\mathcal{D}^A\Phi^B + \mathcal{D}^B\Phi^A) \\ &+ \frac{2}{mf}\mathcal{D}^A\mathcal{D}^B\Phi + \frac{2m}{(d-1)f}\eta^{AB}\Phi, \end{aligned} \qquad (10.14)$$

into Pauli-Fierz Lagrangian (10.9) we obtain gauge invariant Lagrangian (10.2)[18].

A. CFT adapted gauge invariant approach to spin-2 massive field in AdS_{d+1}

We now discuss our CFT adapted approach to massive spin-2 AdS field. For details of the derivation of the CFT adapted gauge invariant Lagrangian see Appendix B.

In our approach, the massive spin-2 field is described by gauge fields

$$\begin{array}{ccc} & \phi^{ab}, & \\ \phi^a_{-1}, & & \phi^a_1, \\ \phi_{-2}, & \phi_0, & \phi_2. \end{array} \qquad (10.15)$$

The fields ϕ^{ab}, $\phi^a_{\pm1}$ and ϕ_0, $\phi_{\pm2}$ are the respective rank-2 tensor, vector and scalar fields of the $so(d)$ algebra. The CFT adapted gauge invariant Lagrangian for these fields takes the form [12]

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}|d\phi^{ab}|^2 - \frac{1}{8}|d\phi^{aa}|^2 + \frac{1}{4}|\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{ab}|^2 - \frac{1}{8}|\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{aa}|^2 \\ &+ \frac{1}{2}\sum_{\lambda=\pm1}\left(|d\phi^a_\lambda|^2 + |\mathcal{T}_{\kappa-\frac{1}{2}+\lambda}\phi^a_\lambda|^2\right)\end{aligned}$$

[17] Recent discussion of the *standard* de Donder-Feynman gauge for massless fields may be found in Refs.[35–37]. To our knowledge explicit form of $C^A_{\rm st}$, $C_{\rm st}$ (10.10),(10.11) has not been discussed in the earlier literature.

[18] To our knowledge formula (10.14) is new and has not been discussed in the earlier literature. For $4d$ flat space formula (10.14) was given in Ref.[38], while for flat space with $d > 4$ was given in Ref.[8].

$$+\ \frac{1}{2}\sum_{\lambda=0,\pm 2}\Big(|d\phi_\lambda|^2+|\mathcal{T}_{\kappa-\frac{1}{2}+\lambda}\phi_\lambda|^2\Big)$$
$$-\ \frac{1}{2}C^aC^a-\frac{1}{2}C_1C_1-\frac{1}{2}C_{-1}C_{-1}\,, \qquad (10.16)$$

where we use the notation

$$C^a \equiv \partial^b\phi^{ab}-\frac{1}{2}\partial^a\phi^{bb} + r_z^{00}\mathcal{T}_{-\kappa+\frac{1}{2}}\phi^a_{-1}+r_\zeta^{00}\mathcal{T}_{\kappa+\frac{1}{2}}\phi^a_1\,, \qquad (10.17)$$

$$C_1 \equiv \partial^a\phi^a_1-\frac{1}{2}r_\zeta^{00}\mathcal{T}_{-\kappa-\frac{1}{2}}\phi^{aa} + r_z^{10}\mathcal{T}_{-\kappa-\frac{1}{2}}\phi_0+\sqrt{2}r_\zeta^{10}\mathcal{T}_{\kappa+\frac{3}{2}}\phi_2\,, \qquad (10.18)$$

$$C_{-1} \equiv \partial^a\phi^a_{-1}-\frac{1}{2}r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{aa} + \sqrt{2}r_z^{01}\mathcal{T}_{-\kappa+\frac{3}{2}}\phi_{-2}+r_\zeta^{01}\mathcal{T}_{\kappa-\frac{1}{2}}\phi_0\,, \qquad (10.19)$$

and $\mathcal{T}_\nu$ is given in (8.12), while κ and r_z^{mn}, r_ζ^{mn} are defined in (5.3) and (5.7)-(5.12) respectively. Lagrangian (10.16) is invariant under the gauge transformations

$$\delta\phi^{ab}=\partial^a\xi^b+\partial^b\xi^a + \frac{2r_\zeta^{00}}{d-2}\eta^{ab}\mathcal{T}_{\kappa+\frac{1}{2}}\xi_1+\frac{2r_z^{00}}{d-2}\eta^{ab}\mathcal{T}_{-\kappa+\frac{1}{2}}\xi_{-1}\,, \qquad (10.20)$$

$$\delta\phi^a_{-1}=\partial^a\xi_{-1}+r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}}\xi^a\,, \qquad (10.21)$$

$$\delta\phi^a_1=\partial^a\xi_1+r_\zeta^{00}\mathcal{T}_{-\kappa-\frac{1}{2}}\xi^a\,, \qquad (10.22)$$

$$\delta\phi_{-2}=\sqrt{2}r_z^{01}\mathcal{T}_{\kappa-\frac{3}{2}}\xi_{-1}\,, \qquad (10.23)$$

$$\delta\phi_0=r_z^{10}\mathcal{T}_{\kappa+\frac{1}{2}}\xi_1+r_\zeta^{01}\mathcal{T}_{-\kappa+\frac{1}{2}}\xi_{-1}\,, \qquad (10.24)$$

$$\delta\phi_2=\sqrt{2}r_\zeta^{10}\mathcal{T}_{-\kappa-\frac{3}{2}}\xi_1\,, \qquad (10.25)$$

where ξ^a, $\xi_{\pm1}$ are gauge transformation parameters.

Gauge invariant equations of motion obtained from Lagrangian (10.16) take the form

$$\Box_\kappa\phi^{ab}-\partial^aC^b-\partial^bC^a -\frac{2r_z^{00}\eta^{ab}}{d-2}\mathcal{T}_{-\kappa+\frac{1}{2}}C_{-1}-\frac{2r_\zeta^{00}\eta^{ab}}{d-2}\mathcal{T}_{\kappa+\frac{1}{2}}C_1=0\,, \qquad (10.26)$$

$$\Box_{\kappa-1}\phi^a_{-1}-\partial^aC_{-1}-r_z^{00}\mathcal{T}_{\kappa-\frac{1}{2}}C^a=0\,, \qquad (10.27)$$

$$\Box_{\kappa+1}\phi^a_1-\partial^aC_1-r_\zeta^{00}\mathcal{T}_{-\kappa-\frac{1}{2}}C^a=0\,, \qquad (10.28)$$

$$\Box_{\kappa-2}\phi_{-2}-\sqrt{2}r_z^{01}\mathcal{T}_{\kappa-\frac{3}{2}}C_{-1}=0\,, \qquad (10.29)$$

$$\Box_\kappa\phi_0-r_\zeta^{01}\mathcal{T}_{-\kappa+\frac{1}{2}}C_{-1}-r_z^{10}\mathcal{T}_{\kappa+\frac{1}{2}}C_1=0\,, \qquad (10.30)$$

$$\Box_{\kappa+2}\phi_2-\sqrt{2}r_\zeta^{10}\mathcal{T}_{-\kappa-\frac{3}{2}}C_1=0\,, \qquad (10.31)$$

where $\Box_\nu$ is defined in (8.15). We see that the gauge invariant equations of motion are coupled.

Global symmetries. We now discuss realization of global symmetries on space of gauge fields (10.15). The realization of the of global symmetries in terms of differential operators is already given in (8.2)-(8.8). All that remains to complete description of global symmetries is to find realization of the operator $R^a_{(0)}$ on space of gauge fields (10.15). Action of the operator $R^a_{(0)}$ on space of gauge fields (10.15) is found to be,

$$R^a_{(0)}\phi^{bc}=zr_\zeta^{00}(\eta^{ab}\phi^c_1+\eta^{ac}\phi^b_1-\frac{2\eta^{bc}}{d-2}\phi^a_1) + zr_z^{00}(\eta^{ab}\phi^c_{-1}+\eta^{ac}\phi^b_{-1}-\frac{2\eta^{bc}}{d-2}\phi^a_{-1}), \qquad (10.32)$$

$$R^a_{(0)}\phi^b_1=-zr_\zeta^{00}\phi^{ab}+z\eta^{ab}(\sqrt{2}r_\zeta^{10}\phi_2+r_z^{10}\phi_0), \qquad (10.33)$$

$$R^a_{(0)}\phi^b_{-1}=-zr_z^{00}\phi^{ab}+z\eta^{ab}(\sqrt{2}r_z^{01}\phi_{-2}+r_\zeta^{01}\phi_0), \qquad (10.34)$$

$$R^a_{(0)}\phi_2=-z\sqrt{2}r_\zeta^{10}\phi^a_1\,, \qquad (10.35)$$

$$R^a_{(0)}\phi_0=-zr_z^{10}\phi^a_1-zr_\zeta^{01}\phi^a_{-1}\,, \qquad (10.36)$$

$$R^a_{(0)}\phi_{-2}=-z\sqrt{2}r_z^{01}\phi^a_{-1}\,. \qquad (10.37)$$

Modified de Donder gauge. Modified de Donder gauge is defined to be

$$C^a=0,\quad C_{-1}=0,\quad C_1=0,\quad \text{modified de Donder gauge.} \qquad (10.38)$$

Using this gauge in equations of motion (10.26)-(10.31) gives the surprisingly simple gauge fixed equations of motion,

$$\Box_\kappa\phi^{ab}=0\,, \qquad (10.39)$$

$$\Box_{\kappa+\lambda}\phi^a_\lambda=0\,,\qquad \lambda=\pm1\,, \qquad (10.40)$$

$$\Box_{\kappa+\lambda}\phi_\lambda=0\,,\qquad \lambda=0,\pm2\,. \qquad (10.41)$$

We see that the gauge fixed equations are decoupled.

Modified de Donder gauge and gauge-fixed equations have leftover on-shell gauge symmetry. Namely, modified de Donder gauge (10.38) and gauge-fixed equations (10.39)-(10.41) are invariant under gauge transformations given in (10.20)-(10.25) provided the gauge transformation parameters satisfy the equations

$$\Box_\kappa\xi^a=0\,,\qquad \Box_{\kappa+\lambda}\xi_\lambda=0\,,\qquad \lambda=\pm1\,. \qquad (10.42)$$

B. AdS/CFT correspondence for anomalous current and normalizable modes of massive AdS field

We now ready to discuss AdS/CFT correspondence for bulk massive spin-2 AdS field and boundary spin-2 anoma-

lous conformal current[19]. To this end we use, as before, our CFT adapted approach to AdS field dynamics and modified de Donder gauge.

First of all we note that the normalizable solution of equations of motion (10.39)-(10.41) is given by

$$\phi^{ab}(x,z) = U_\kappa \phi^{ab}_{\rm cur,0}(x)\,, \tag{10.43}$$

$$\phi^{a}_{-1}(x,z) = -U_{\kappa-1} \phi^{a}_{\rm cur,-1}(x)\,, \tag{10.44}$$

$$\phi^{a}_{1}(x,z) = U_{\kappa+1} \phi^{a}_{\rm cur,1}(x)\,, \tag{10.45}$$

$$\phi_{-2}(x,z) = U_{\kappa-2} \phi_{\rm cur,-2}(x)\,, \tag{10.46}$$

$$\phi_{0}(x,z) = -U_{\kappa} \phi_{\rm cur,0}(x)\,, \tag{10.47}$$

$$\phi_{2}(x,z) = U_{\kappa+2} \phi_{\rm cur,2}(x)\,, \tag{10.48}$$

where U_ν is defined in (9.28). From these relations, we find the asymptotic behavior of the normalizable solution

$$\phi^{ab}(x,z) \stackrel{z\to 0}{\longrightarrow} z^{\kappa+\frac{1}{2}} \phi^{ab}_{\rm cur,0}(x)\,, \tag{10.49}$$

$$\phi^{a}_{-1}(x,z) \stackrel{z\to 0}{\longrightarrow} -2\kappa z^{\kappa-\frac{1}{2}} \phi^{a}_{\rm cur,-1}(x)\,, \tag{10.50}$$

$$\phi^{a}_{1}(x,z) = \stackrel{z\to 0}{\longrightarrow} \frac{z^{\kappa+\frac{3}{2}}}{2(\kappa+1)} \phi^{a}_{\rm cur,1}(x)\,, \tag{10.51}$$

$$\phi_{-2}(x,z) = \stackrel{z\to 0}{\longrightarrow} 4\kappa(\kappa+1) z^{\kappa-\frac{3}{2}} \phi_{\rm cur,-2}(x)\,, \tag{10.52}$$

$$\phi_{0}(x,z) = \stackrel{z\to 0}{\longrightarrow} -z^{\kappa+\frac{1}{2}} \phi_{\rm cur,0}(x)\,, \tag{10.53}$$

$$\phi_{2}(x,z) = \stackrel{z\to 0}{\longrightarrow} \frac{z^{\kappa+\frac{5}{2}}}{4\kappa(\kappa-1)} \phi_{\rm cur,2}(x)\,. \tag{10.54}$$

From (10.49)-(10.54), we see that the fields $\phi^{ab}_{\rm cur,0}$, $\phi^{a}_{\rm cur,\pm1}$, $\phi_{\rm cur,0}$, $\phi_{\rm cur,\pm2}$ are indeed boundary values of the normalizable solution.

In the r.h.s. (10.43)-(10.48), we use the notation $\phi^{ab}_{\rm cur,0}$, $\phi^{a}_{\rm cur,\pm1}$, $\phi_{\rm cur,0}$, $\phi_{\rm cur,\pm2}$ because these boundary values turn out to be the gauge fields entering our gauge invariant formulation of spin-2 anomalous conformal currents in the Sec.V A. Namely, one can prove the following statements:

i) *Leftover on-shell* gauge transformations (10.20)-(10.25) of normalizable solution (10.43)-(10.48) lead to gauge transformations (5.13)-(5.18) of the anomalous conformal current.

ii) For normalizable solution (10.43)-(10.48), modified de Donder gauge condition (10.38) leads to differential constraints (5.4)-(5.6) of the anomalous conformal current.

iii) Global $so(d,2)$ bulk symmetries of the normalizable massive spin-2 modes in AdS_{d+1} become global $so(d,2)$ boundary conformal symmetries of the spin-2 anomalous conformal current.

These statements can be proved following procedure we demonstrated for the case of spin-1 fields in Sec.IX B. Therefore to avoid repetitions we briefly discuss some necessary details.

Matching of bulk and boundary gauge symmetries. To much gauge symmetries we analyze leftover on-shell gauge symmetries which are described by solutions of equations given in (10.42). Normalizable solution to these equations takes the form,

$$\xi^{a}(x,z) = U_\kappa \xi^{a}_{\rm cur,0}(x)\,, \tag{10.55}$$

$$\xi_{-1}(x,z) = -U_{\kappa-1} \xi_{\rm cur,-1}(x)\,, \tag{10.56}$$

$$\xi_{1}(x,z) = U_{\kappa+1} \xi_{\rm cur,1}(x)\,. \tag{10.57}$$

Plugging these solutions and solution for equations of motion for AdS fields (10.43)-(10.48) into bulk gauge transformations (10.20)-(10.25) one can make sure that these leftover on-shell bulk gauge transformations amount to boundary gauge transformations of the spin-2 anomalous conformal current given in (5.13)-(5.18).

Matching of bulk de Donder gauge and boundary differential constraints. All that is required is to plug solution for equations of motion of AdS fields (10.43)-(10.48) into the modified de Donder gauge and use relations for the operator U_ν given in (9.33),(9.34). Doing so, we make sure that modified de Donder gauge (10.38) amounts to differential constrains (5.4)-(5.6).

Matching of bilk and boundary global symmetries. Matching of bulk and boundary Poincaé symmetries is obvious. Using conformal dimensions for spin-2 anomalous current given in (5.2), solution for bulk fields in (10.43)-(10.48), and bulk dilatation operator (8.4) we make sure that dilatation bulk and boundary symmetries also match. What is no-trivial is to match conformal boost symmetries (K^a-symmetries). The reason for this is that the modified de Donder gauge breaks bulk K^a-symmetries. In order to restore these broken K^a-symmetries we should, following standard procedure, add compensating gauge transformations to maintain the K^a-symmetries,

$$K^a_{\rm impr} = K^a + \delta_{\xi^{K^a}}\,. \tag{10.58}$$

The compensating gauge transformation parameters can as usually be found by requiring improved transformation (10.58) to maintain the modified de Donder gauge (10.38),

$$K^a_{\rm impr} C^b = 0, \quad K^a_{\rm impr} C_{-1} = 0, \quad K^a_{\rm impr} C_1 = 0\,. \tag{10.59}$$

One can make sure that equations (10.59) amount to the following equations for the compensating gauge transformation

[19] To our knowledge AdS/CFT correspondence for bulk massive spin-2 AdS field and boundary spin-2 anomalous conformal current has not studied in the literature.

parameters,

$$\Box_\kappa \xi^{bK^a} = 2\phi^{ab} - \eta^{ab}\phi^{cc}\,, \tag{10.60}$$

$$\Box_{\kappa-1} \xi^{K^a}_{-1} = 2\phi^a_{-1}\,, \tag{10.61}$$

$$\Box_{\kappa+1} \xi^{K^a}_{1} = 2\phi^a_{1}\,. \tag{10.62}$$

Using (10.43)-(10.45) we find solution for the compensating gauge transformation parameters,

$$\xi^{bK^a}(x,z) = zU_{\kappa+1}(\phi^{ab}_{\mathrm{cur},0}(x) - \frac{1}{2}\eta^{ab}\phi^{cc}_{\mathrm{cur},0})\,, \tag{10.63}$$

$$\xi^{K^a}_{-1}(x,z) = -zU_\kappa \phi^a_{\mathrm{cur},-1}(x)\,, \tag{10.64}$$

$$\xi^a_1(x,z) = zU_{\kappa+2}\phi^a_{\mathrm{cur},1}(x)\,. \tag{10.65}$$

where operator U_ν is given in (9.28). Using these compensating gauge transformation parameters in improved bulk K_{impr}-symmetries (10.58) we make sure that these K^a_{impr}-symmetries amount to K^a-symmetries of spin-2 anomalous conformal current given in (2.11) and (5.19)-(5.24).

C. AdS/CFT correspondence for anomalous shadow field and non-normalizable mode of massive AdS field.

We proceed to discussion of AdS/CFT correspondence for bulk massive spin-2 AdS field and boundary spin-2 anomalous shadow field.

Matching of effective action and boundary two-point vertex. In order to find S_{eff} we should solve equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into action. Using equations of motion (10.26)-(10.31) in bulk action (9.9) with Lagrangian (10.16), we obtain boundary effective action (9.58) with $\mathcal{L}_{\mathrm{eff}}$ given by

$$\begin{aligned}
\mathcal{L}_{\mathrm{eff}} \;=\; & \frac{1}{4}\phi^{ab}\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{ab} - \frac{1}{8}\phi^{aa}\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{bb} \\
& + \frac{1}{2}\sum_{\lambda=\pm1}\phi^a_\lambda \mathcal{T}_{\kappa+\lambda-\frac{1}{2}}\phi^a_\lambda \\
& + \frac{1}{2}\sum_{\lambda=0,\pm2}\phi_\lambda \mathcal{T}_{\kappa+\lambda-\frac{1}{2}}\phi_\lambda\,, \\
& - \frac{1}{2}(r^{00}_z\phi^a_{-1} + r^{00}_\zeta \phi^a_1)C^a \\
& + (\frac{r^{00}_z}{4}\phi^{aa} - \frac{r^{01}_z}{\sqrt{2}}\phi_{-2} - \frac{r^{01}_\zeta}{2}\phi_0)C_{-1} \\
& + (\frac{r^{00}_\zeta}{4}\phi^{aa} - \frac{r^{10}_z}{2}\phi_0 - \frac{r^{10}_\zeta}{\sqrt{2}}\phi_2)C_1\,.
\end{aligned} \tag{10.66}$$

As we have demonstrated use of the modified de Donder gauge considerable simplifies the equations of motion. Now using modified de Donder gauge (10.38) in (10.66), we obtain

$$\begin{aligned}
\mathcal{L}_{\mathrm{eff}}\Big|_{\substack{C^a=0\\ C_{\pm1}=0}} \;=\; & \frac{1}{4}\phi^{ab}\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{ab} - \frac{1}{8}\phi^{aa}\mathcal{T}_{\kappa-\frac{1}{2}}\phi^{bb} \\
& + \frac{1}{2}\sum_{\lambda=\pm1}\phi^a_\lambda \mathcal{T}_{\kappa+\lambda-\frac{1}{2}}\phi^a_\lambda \\
& + \frac{1}{2}\sum_{\lambda=0,\pm2}\phi_\lambda \mathcal{T}_{\kappa+\lambda-\frac{1}{2}}\phi_\lambda\,,
\end{aligned} \tag{10.67}$$

i.e. we see that $\mathcal{L}_{\mathrm{eff}}$ is also considerably simplified. In order to find S_{eff} we should solve equations of motion (10.39)-(10.41) with the Dirichlet problem corresponding to the boundary anomalous shadow field and plug the solution into $\mathcal{L}_{\mathrm{eff}}$. To this end we discuss solution of equations of motion (10.39)-(10.41).

As before our equations of motion take decoupled form and similar to the equations of motion for the massive scalar AdS field. Therefore we can apply the procedure described in Sec. VIII. Doing so, we obtain solution of equation (10.39)-(10.41) with the Dirichlet problem corresponding to the spin-2 anomalous shadow field,

$$\phi^{ab}(x,z) = \sigma_{2,0}\int d^dy\, G_\kappa(x-y,z)\phi^{ab}_{\mathrm{sh},0}(y)\,, \tag{10.68}$$

$$\begin{aligned}
\phi^a_\lambda(x,z) = \sigma_{1,\lambda}\int d^dy\, G_{\kappa+\lambda}(x-y,z)\phi^a_{\mathrm{sh},-\lambda}(y)\,, \\
\lambda = \pm1\,,
\end{aligned} \tag{10.69}$$

$$\begin{aligned}
\phi_\lambda(x,z) = \sigma_{0,\lambda}\int d^dy\, G_{\kappa+\lambda}(x-y,z)\phi_{\mathrm{sh},-\lambda}(y)\,, \\
\lambda = 0,\pm2\,,
\end{aligned} \tag{10.70}$$

$$\sigma_{2,0} = 1\,, \tag{10.71}$$

$$\sigma_{1,-1} = -\frac{1}{2(\kappa-1)}\,, \tag{10.72}$$

$$\sigma_{1,1} = 2\kappa\,, \tag{10.73}$$

$$\sigma_{0,-2} = \frac{1}{4(\kappa-1)(\kappa-2)}\,, \tag{10.74}$$

$$\sigma_{0,0} = -1\,, \tag{10.75}$$

$$\sigma_{0,2} = 4\kappa(\kappa+1)\,, \tag{10.76}$$

where the Green function G_ν is given in (8.22), while κ is defined in (5.3). Choice of normalization factor $\sigma_{2,0}$ (10.71) is a matter of convention. The remaining normalization factors given in (10.72)-(10.76) are uniquely determined by requiring that modified de Donder gauge (10.38) be amount to the differential constraints for the spin-2 anomalous shadow field.

Using asymptotic behavior of the Green function given in (8.24), we find the asymptotic behavior of our solution

$$\phi^{ab}(x,z) \;\stackrel{z\to0}{\longrightarrow}\; z^{-\kappa+\frac{1}{2}}\phi^{ab}_{\mathrm{sh},0}(x)\,, \tag{10.77}$$

$$\phi^a_{-1}(x,z) \stackrel{z\to 0}{\longrightarrow} -\frac{z^{-\kappa+\frac{3}{2}}}{2(\kappa-1)}\phi^a_{\rm sh,1}(x)\,, \qquad (10.78)$$

$$\phi^a_{1}(x,z) \stackrel{z\to 0}{\longrightarrow} 2\kappa z^{-\kappa-\frac{1}{2}}\phi^a_{\rm sh,-1}(x)\,, \qquad (10.79)$$

$$\phi_{-2}(x,z) \stackrel{z\to 0}{\longrightarrow} \frac{z^{-\kappa+\frac{5}{2}}}{4(\kappa-1)(\kappa-2)}\phi_{\rm sh,2}(x)\,, \qquad (10.80)$$

$$\phi_{0}(x,z) \stackrel{z\to 0}{\longrightarrow} -z^{-\kappa+\frac{1}{2}}\phi_{\rm sh,0}(x)\,, \qquad (10.81)$$

$$\phi_{2}(x,z) \stackrel{z\to 0}{\longrightarrow} 4\kappa(\kappa+1)z^{-\kappa-\frac{3}{2}}\phi_{\rm sh,-2}(x)\,. \qquad (10.82)$$

From these expressions, we see that our solution (10.68)-(10.70) has indeed asymptotic behavior corresponding to the spin-2 anomalous shadow field.

Finally, to obtain the effective action we plug solution of the Dirichlet problem for AdS fields, (10.68)-(10.70) into (9.58), (10.67). Using general formula given in (8.28), we obtain

$$-S_{\rm eff} = 2\kappa c_\kappa \Gamma\,, \qquad (10.83)$$

where κ and c_κ are defined in (5.3) and (8.23) respectively and Γ is gauge invariant two-point vertex of the spin-2 anomalous shadow field given in (4.10),(6.19).

Thus, *using modified de Donder gauge for massive spin-2 AdS field and computing the bulk action on solution of equations of motion with the Dirichlet problem corresponding to the boundary anomalous shadow field we obtain the gauge invariant two-point vertex of spin-2 anomalous shadow field*.

Because in the literature $S_{\rm eff}$ is expressed in terms of two-point vertex taken in the Stueckelberg gauge frame, $\Gamma^{\rm stand}_{12}$ (6.29), we use (6.28) to represent our result (10.83) as

$$-S_{\rm eff} = \frac{\kappa(2\kappa+d+2)}{2(2\kappa+d-2)}c_\kappa\Gamma^{\rm stand}\,. \qquad (10.84)$$

This relation was obtained in Ref.[11]. Note that we have obtained more general relation given in (10.83), while relation (10.84) is obtained from (10.83) by using the Stueckelberg gauge frame. The fact that $S_{\rm eff}$ is related to $\Gamma^{\rm stand}$ is expected because of the conformal symmetry. What is important for the systematical study of AdS/CFT correspondence is the computation of the normalization factor in front of $\Gamma^{\rm stand}$ (10.84). We note that our normalization factor in (10.84) coincides with the one found in Ref.[11][20].

Matching of bulk and boundary gauge symmetries. Modified de Donder gauge (10.38) and gauge-fixed equations (10.39)-(10.41) are invariant under gauge transformations given in (10.20)-(10.25) provided the gauge transformation parameters satisfy the equations (10.42). Non-normalizable solution to equations (10.42) is given by

$$\xi^a(x,z) = \int d^dy\, G_\kappa(x-y,z)\xi_{\rm sh,0}(y)\,, \qquad (10.85)$$

$$\xi_\lambda(x,z) = \sigma_{1,\lambda}\int d^dy\, G_{\kappa+\lambda}(x-y,z)\xi_{\rm sh,-\lambda}(y)\,, \qquad (10.86)$$

$\lambda = \pm 1$, where $\sigma_{1,\pm 1}$ are given in (10.72),(10.73). Using solution in (10.85),(10.86) and following procedure described for spin-1 field in Sec.IX C one can prove that the on-shell leftover gauge symmetries of solution of the Dirichlet problem for AdS spin-2 massive field amount to the gauge symmetries of the spin-2 anomalous shadow field.

Following procedure in Sec.IX B one can make sure that global bulk and boundary symmetries match. It is this *matching of the bulk on-shell leftover gauge symmetries of the solution to Dirichlet problem and bulk global symmetries and the respective boundary gauge symmetries of the anomalous shadow field and boundary global symmetries that explains why the effective action coincides with the gauge invariant two-point vertex for the boundary anomalous shadow field (see (10.83))*.

Comparing our results for spin-1 and spin-2 fields given in (9.75) and (10.83) respectively we see that our approach gives uniform description of the interrelation between the effective action of massive fields and two-point gauge invariant vertex of shadow field. Note however that value of κ for spin-1 field (3.3) should not be confused with the one for spin-2 field (5.3). For the case of arbitrary spin-s field, the κ was found in Refs.[12, 42],

$$\kappa = \sqrt{m^2 + \big(s+\frac{d-4}{2}\big)^2}\,. \qquad (10.87)$$

All that is required to generalize the relation (10.83) to the case of arbitrary spin-s fields is to plug κ (10.87) in (10.83). Detailed study of arbitrary spin fields will be given in forthcoming publication.

XI. CONCLUSIONS

In this paper, we extend the gauge invariant Stueckelberg approach to CFT initiated in Refs.[8, 9] to the study of anomalous conformal currents and shadow fields. In the framework of AdS/CFT correspondence the anomalous conformal currents and shadow fields are related with massive fields of AdS string. It is well known that all Lorentz covariant approaches to string field theory involve large amount of Stueckelberg fields and the corresponding gauge symmetries (see e.g. [43]). Because our approach to anomalous conformal currents and shadow fields also involves Stueckelberg

[20] Computation of $S_{\rm eff}$ for spin-2 massless field may be found in Refs.[39–41]. In the framework of our approach, $S_{\rm eff}$ was studied in Ref.[12].

fields we believe that our approach will be helpful to understand string/gauge theory duality better. Note also that we obtain gauge invariant vertex for anomalous shadow fields which provides quick and easy access to light-cone gauge vertex. In the framework of AdS/CFT correspondence this vertex is related to AdS field action evaluated on solution of the Dirichlet problem. Because on expects that quantization of AdS superstring is straightforward only in light-cone gauge we believe that our light-cone gauge vertex will also be helpful in various studies of AdS/CFT duality. The results obtained should have a number of the following interesting applications and generalizations.

(i) In this paper we considered the gauge invariant approach for spin-1 and spin-2 anomalous conformal currents and shadow fields. It would be interesting to generalize our approach to the case of arbitrary spin anomalous conformal currents and shadow fields.

(ii) In this paper we studied the two-point gauge invariant vertex of anomalous shadow fields. Generalization of our approach to the case of 3-point and 4-point gauge invariant vertices will give us the possibility to the study of various applications of our approach along the lines of Refs.[44–46]

(iii) Because our modified de Donder gauge leads to considerably simplified analysis of AdS field dynamics we believe that this gauge might also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed e.g. in Refs.[47, 48].

iv) BRST approach is one of powerful approaches to analysis of various aspects of relativistic dynamics (see e.g. Refs.[49]-[54]. We think that extension of this approach to the case anomalous conformal currents and shadow field should be relatively straightforward.

v) In the last years, there were interesting developments in studying the mixed symmetry fields [55]-[59]. It would be interesting to apply methods developed in these references to studying anomalous conformal currents and shadow field. There are other various interesting approaches in the literature which could be used to discuss gauge invariant formulation of anomalous conformal currents and shadow fields. This is to say that various recently developed interesting formulations in terms of unconstrained fields in flat space may be found in Refs.[60]-[62] .

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Appendix A: Derivation of CFT adapted Lagrangian for massive spin-1 field in AdS_{d+1}

In this Appendix, we explain some details of the derivation of the CFT adapted gauge invariant Lagrangian for massive spin-1 fields given in (9.10). Presentation in this Appendix is given by using Lorentzian signature. Euclidean signature Lagrangian in Sec.IX A, is obtained from the Lorentzian signature Lagrangian by simple substitution $\mathcal{L} \to -\mathcal{L}$.

Spin-1 massive field. We use field Φ^A carrying flat Lorentz algebra $so(d,1)$ vector indices $A, B = 0, 1, \ldots, d-1, d$. The field Φ^A is related with field carrying the base manifold indices Φ^μ, $\mu = 0, 1, \ldots, d$, in a standard way $\Phi^A = e^A_\mu \Phi^\mu$, where e^A_μ is vielbein of AdS_{d+1} space. For the Poincaré parametrization of AdS_{d+1} space (8.1), vielbein $e^A = e^A_\mu dx^\mu$ and Lorentz connection, $de^A + \omega^{AB} \wedge e^B = 0$, are given by

$$e^A_\mu = \frac{1}{z}\delta^A_\mu \,, \qquad \omega^{AB}_\mu = \frac{1}{z}(\delta^A_z \delta^B_\mu - \delta^B_z \delta^A_\mu) \,, \tag{A1}$$

where δ^A_μ is Kronecker delta symbol. We use a covariant derivative with the flat indices $\mathcal{D}^A$,

$$\mathcal{D}_A \equiv e^\mu_A \mathcal{D}_\mu \,, \qquad \mathcal{D}^A = \eta^{AB}\mathcal{D}_B \,, \tag{A2}$$

where e^μ_A is inverse of AdS vielbein, $e^A_\mu e^\mu_B = \delta^A_B$ and η^{AB} is flat metric tensor. With choice made in (A1), the covariant derivative takes the form

$$\mathcal{D}^A \Phi^B = \hat{\partial}^A \Phi^B + \delta^B_z \Phi^A - \eta^{AB}\Phi^z \,, \quad \hat{\partial}^A \equiv z\partial^A \,, \tag{A3}$$

where we adopt the following conventions for the derivatives and coordinates: $\partial^A = \eta^{AB}\partial_B$, $\partial_A = \partial/\partial x^A$, $x^A \equiv \delta^A_\mu x^\mu$, $x^A = x^a, x^d, x^d \equiv z$.

In arbitrary parametrization of AdS, Lagrangian of the massive spin-1 field is given in (9.6). We now use the Poincaré parametrization of AdS and introduce the following quantity:

$$\mathbf{C} \equiv \mathcal{D}^C \Phi^C + m\Phi + 2\Phi^z \,. \tag{A4}$$

We note that it is the relation $\mathbf{C} = 0$ that defines the modified Lorentz gauge. Using the relations (up to total derivative)

$$\begin{aligned} e\Phi^A \mathcal{D}^2 \Phi^A &= e\Big(\Phi^A(\Box_{0\,AdS} - 1)\Phi^A \\ &+ 4\Phi^z \mathbf{C} + (d-7)\Phi^z\Phi^z - 4m\Phi\Phi^z\Big) \,, \end{aligned} \tag{A5}$$

$$e\Phi\mathcal{D}^2\Phi = e\Phi\Box_{0\,AdS}\Phi \tag{A6}$$

$$C^2_{\rm st} = \mathbf{C}^2 - 4\Phi^z\mathbf{C} + 4\Phi^z\Phi^z \,, \tag{A7}$$

$$\Box_{0\,AdS} \equiv z^2(\Box + \partial_z^2) + (1-d)z\partial_z \,, \tag{A8}$$

we represent Lagrangian (9.6) and $\mathbf{C}$ (A4) as

$$e^{-1}\mathcal{L} = \frac{1}{2}\Phi^A(\Box_{0\,AdS} - m^2 + d - 1)\Phi^A$$

$$+\ \frac{1}{2}\Phi(\Box_{0\,AdS}-m^2)\Phi$$

$$+\ \frac{d-3}{2}\Phi^z\Phi^z-2m\Phi\Phi^z+\frac{1}{2}\mathbf{C}^2\,, \qquad \text{(A9)}$$

$$\mathbf{C}\ =\ \hat{\partial}^A\Phi^A+(2-d)\Phi^z+m\Phi\,. \qquad \text{(A10)}$$

Using canonically normalized fields $\widetilde{\Phi}^A$, $\widetilde{\Phi}$ and C defined by

$$\Phi^A=z^{\frac{d-1}{2}}\widetilde{\Phi}^A\,, \qquad \Phi=z^{\frac{d-1}{2}}\widetilde{\Phi}\,, \qquad \mathbf{C}=z^{\frac{d+1}{2}}C\,, \qquad \text{(A11)}$$

we obtain

$$\mathcal{L}\ =\ \frac{1}{2}\widetilde{\Phi}^A\Big(\Box+\partial_z^2-\frac{1}{z^2}(m^2+\frac{d^2-1}{4}+1-d)\Big)\widetilde{\Phi}^A$$

$$+\ \frac{1}{2}\widetilde{\Phi}\Big(\Box+\partial_z^2-\frac{1}{z^2}(m^2+\frac{d^2-1}{4})\Big)\widetilde{\Phi}$$

$$+\ \frac{d-3}{2z^2}\widetilde{\Phi}^z\widetilde{\Phi}^z-\frac{2m}{z^2}\widetilde{\Phi}^z\widetilde{\Phi}+\frac{1}{2}C^2\,, \qquad \text{(A12)}$$

$$C\ =\ \partial^A\widetilde{\Phi}^A+\frac{3-d}{2z}\widetilde{\Phi}^z+\frac{m}{z}\widetilde{\Phi}\,. \qquad \text{(A13)}$$

In terms of $so(d-1,1)$ tensorial components of the field Φ^A given by Φ^a, Φ^z, Lagrangian (A12) and C (A13) take the form

$$\mathcal{L}\ =\ \mathcal{L}_1+\mathcal{L}_0+\frac{1}{2}C^2\,, \qquad \text{(A14)}$$

$$\mathcal{L}_1=\frac{1}{2}\widetilde{\Phi}^a\hat{K}_0\widetilde{\Phi}^a \qquad \text{(A15)}$$

$$\mathcal{L}_0=\frac{1}{2}\widetilde{\Phi}^z\hat{K}_{3-d}\widetilde{\Phi}^z+\frac{1}{2}\widetilde{\Phi}\hat{K}_{d-1}\widetilde{\Phi}-\frac{2m}{z^2}\widetilde{\Phi}^z\widetilde{\Phi}\,, \qquad \text{(A16)}$$

$$C=\partial^a\widetilde{\Phi}^a+\mathcal{T}_{\frac{3-d}{2}}\widetilde{\Phi}^z+\frac{m}{z}\widetilde{\Phi}\,, \qquad \text{(A17)}$$

$$\hat{K}_\omega=\Box+\partial_z^2-\frac{1}{z^2}(\kappa^2-\frac{1}{4}+\omega)\,, \qquad \text{(A18)}$$

where κ and $\mathcal{T}_\nu$ are defined in (3.3) and (8.12) respectively. In terms of fields (9.8) defined by

$$\widetilde{\Phi}^a=\phi^a\,, \qquad \text{(A19)}$$

$$\widetilde{\Phi}^z=r_z^{00}\phi_{-1}+r_\zeta^{00}\phi_1\,, \qquad \text{(A20)}$$

$$\widetilde{\Phi}=-r_\zeta^{00}\phi_{-1}+r_z^{00}\phi_1\,, \qquad \text{(A21)}$$

where r_z^{00}, r_ζ^{00} are defined in (3.5),(3.6), we cast $\mathcal{L}_1$, $\mathcal{L}_0$ (A15),(A16) into the form

$$\mathcal{L}_1=\frac{1}{2}\phi^a\Box_\kappa\phi^a\,, \qquad \text{(A22)}$$

$$\mathcal{L}_0=\frac{1}{2}\sum_{\lambda=\pm1}\phi_\lambda\Box_{\kappa+\lambda}\phi_\lambda\,, \qquad \text{(A23)}$$

while C (A17) takes desired form given in (9.11). Noticing the relation

$$\mathcal{T}^\dagger_{\nu-\frac{1}{2}}\mathcal{T}_{\nu-\frac{1}{2}}=-\partial_z^2+\frac{1}{z^2}(\nu^2-\frac{1}{4})\,, \qquad \text{(A24)}$$

and taking into account expressions for $\mathcal{L}_1$ (A22), $\mathcal{L}_0$ (A23), we see that Lagrangian (A14) takes the form of the CFT adapted gauge invariant Lagrangian (9.10).

Lagrangian (9.6) is invariant under gauge transformations (9.5). Making the rescaling $\Xi=z^{(d-3)/2}\xi$, we check that these gauge transformations lead to the ones given in (9.12)-(9.14).

Appendix B: Derivation of CFT adapted Lagrangian for massive spin-2 field in AdS_{d+1}

We now present some details of the derivation of the CFT adapted gauge invariant Lagrangian and the respective gauge transformations of massive spin-2 fields given in (10.16) and (10.20)-(10.25).

In arbitrary parametrization of AdS, Lagrangian for the massive spin-2 field is given in (10.9). We now use the Poincaré parametrization of AdS and introduce the following quantities

$$\mathbf{C}^A\equiv C^A_{\rm st}+2\Phi^{zA}-\delta^A_z\Phi^{BB}\,, \qquad \text{(B1)}$$

$$\mathbf{C}\equiv C_{\rm st}+2\Phi^z\,. \qquad \text{(B2)}$$

We note that it is the relations $\mathbf{C}^A=0$, $\mathbf{C}=0$ that define the modified de Donder gauge. Using the relations (up to total derivative)

$$\frac{1}{4}e\Phi^{AB}\mathcal{D}^2\Phi^{AB}=e\Big(\frac{1}{4}\Phi^{AB}(\Box_{0\,AdS}-2)\Phi^{AB}$$

$$+\frac{d-5}{2}\Phi^{zA}\Phi^{zA}+2\Phi^{zz}\Phi^{AA}-\frac{d}{4}\Phi^{AA}\Phi^{BB}$$

$$+2\Phi^{zA}\mathbf{C}^A-\Phi^{AA}\mathbf{C}^z-2m\Phi^{zA}\Phi^A+m\Phi^{AA}\Phi^z\Big), \qquad \text{(B3)}$$

$$\frac{1}{2}C^A_{\rm st}C^A_{\rm st}=\frac{1}{2}\mathbf{C}^A\mathbf{C}^A-2\Phi^{zA}\mathbf{C}^A+\Phi^{AA}\mathbf{C}^z$$

$$+2\Phi^{zA}\Phi^{zA}-2\Phi^{zz}\Phi^{AA}+\frac{1}{2}\Phi^{AA}\Phi^{BB}\,, \qquad \text{(B4)}$$

$$e\Phi^A\mathcal{D}^2\Phi^A=e\Big(\Phi^A(\Box_{0\,AdS}-1)\Phi^A$$

$$+4\Phi^z\mathbf{C}+(d-7)\Phi^z\Phi^z-2m\Phi^{AA}\Phi^z-4f\Phi\Phi^z\Big), \qquad \text{(B5)}$$

$$C^2_{\rm st}=\mathbf{C}^2-4\Phi^z\mathbf{C}+4\Phi^z\Phi^z\,, \qquad \text{(B6)}$$

where $\Box_{0\,AdS}$ is given in (A8), we represent Lagrangian (10.9) and $\mathbf{C}^A$, $\mathbf{C}$ (B1),(B2) as

$$e^{-1}\mathcal{L}\ =\ \frac{1}{4}\Phi^{AB}(\Box_{0\,AdS}-m^2)\Phi^{AB}$$

$$-\ \frac{1}{8}\Phi^{AA}(\Box_{0\,AdS}-m^2)\Phi^{BB}$$

$$+\ \frac{d-1}{2}\Phi^{zA}\Phi^{zA}-2m\Phi^{zA}\Phi^{A}$$
$$+\ \frac{1}{2}\Phi^{A}(\Box_{_{0\,AdS}}-m^2-d-1)\Phi^{A}$$
$$+\ \frac{d-3}{2}\Phi^{z}\Phi^{z}-2f\Phi\Phi^{z}$$
$$+\ \frac{1}{2}\Phi(\Box_{_{0\,AdS}}-m^2-2d)\Phi$$
$$+\ \frac{1}{2}\mathbf{C}^{A}\mathbf{C}^{A}+\frac{1}{2}\mathbf{C}\mathbf{C}\,, \tag{B7}$$

$$\mathbf{C}^{A}\ =\ \hat{\partial}^{B}\Phi^{AB}-\frac{1}{2}\hat{\partial}^{A}\Phi^{BB}+(1-d)\Phi^{zA}+m\Phi^{A}\,, \tag{B8}$$

$$\mathbf{C}\ =\ \hat{\partial}^{A}\Phi^{A}+(2-d)\Phi^{z}+\frac{m}{2}\Phi^{AA}+f\Phi\,. \tag{B9}$$

Using canonically normalized fields and quantities $\tilde{\mathbf{C}}^{A}$, $\tilde{\mathbf{C}}$,

$$\Phi^{AB}=z^{\frac{d-1}{2}}\tilde{\Phi}^{AB}\,,\quad \Phi^{A}=z^{\frac{d-1}{2}}\tilde{\Phi}^{A}\,,\quad \Phi=z^{\frac{d-1}{2}}\tilde{\Phi}\,, \tag{B10}$$

$$\mathbf{C}^{A}=z^{\frac{d+1}{2}}\tilde{\mathbf{C}}^{A}\,,\qquad \mathbf{C}=z^{\frac{d+1}{2}}\tilde{\mathbf{C}}\,, \tag{B11}$$

we obtain

$$\mathcal{L}\ =\ \frac{1}{4}\tilde{\Phi}^{AB}\hat{K}_0\tilde{\Phi}^{AB}-\frac{1}{8}\tilde{\Phi}^{AA}\hat{K}_0\tilde{\Phi}^{BB}$$
$$+\ \frac{1}{2}\tilde{\Phi}^{A}\hat{K}_{d+1}\tilde{\Phi}^{A}+\frac{1}{2}\tilde{\Phi}\hat{K}_{2d}\tilde{\Phi}$$
$$+\ \frac{d-1}{2z^2}\tilde{\Phi}^{zA}\tilde{\Phi}^{zA}-\frac{2m}{z^2}\tilde{\Phi}^{zA}\tilde{\Phi}^{A}+\frac{d-3}{2z^2}\tilde{\Phi}^{z}\tilde{\Phi}^{z}$$
$$-\ \frac{2f}{z^2}\tilde{\Phi}^{z}\tilde{\Phi}+\frac{1}{2}\tilde{\mathbf{C}}^{A}\tilde{\mathbf{C}}^{A}+\frac{1}{2}\tilde{\mathbf{C}}\tilde{\mathbf{C}}\,, \tag{B12}$$

$$\tilde{\mathbf{C}}^{a}\ =\ \partial^{b}\tilde{\Phi}^{ab}-\frac{1}{2}\partial^{a}\tilde{\Phi}^{BB}+\mathcal{T}_{-\frac{d-1}{2}}\tilde{\Phi}^{za}+\frac{m}{z}\tilde{\Phi}^{a}\,, \tag{B13}$$

$$\tilde{\mathbf{C}}^{z}\ =\ \partial^{a}\tilde{\Phi}^{za}-\frac{1}{2}\mathcal{T}_{\frac{d-1}{2}}\tilde{\Phi}^{BB}+\mathcal{T}_{-\frac{d-1}{2z}}\tilde{\Phi}^{zz}+\frac{m}{z}\tilde{\Phi}^{z}\,, \tag{B14}$$

$$\tilde{\mathbf{C}}\ =\ \partial^{a}\tilde{\Phi}^{a}+\mathcal{T}_{-\frac{d-3}{2}}\tilde{\Phi}^{z}+\frac{m}{2z}\tilde{\Phi}^{AA}+\frac{f}{z}\tilde{\Phi}\,, \tag{B15}$$

where κ and $\hat{K}_{\omega}$ are defined in (5.3) and (A18) respectively. In terms of new fields defined by the relations

$$\phi^{ab}=\tilde{\Phi}^{ab}+\frac{1}{d-2}\eta^{ab}\tilde{\Phi}^{zz}\,, \tag{B16}$$

$$\phi^{za}=\tilde{\Phi}^{za}\,,\qquad \phi^{a}=\tilde{\Phi}^{a}\,, \tag{B17}$$

$$\phi^{zz}=\frac{u}{2}\,\tilde{\Phi}^{zz}\,,\qquad \phi^{z}=\tilde{\Phi}^{z}\,,\qquad \phi=\tilde{\Phi}\,, \tag{B18}$$

Lagrangian $\mathcal{L}$ (B12) and $\tilde{\mathbf{C}}^{A}$, $\tilde{\mathbf{C}}$ (B13)-(B15) take the form

$$\mathcal{L}=\mathcal{L}_2+\mathcal{L}_1+\mathcal{L}_0+\frac{1}{2}\tilde{\mathbf{C}}^{A}\tilde{\mathbf{C}}^{A}+\frac{1}{2}\tilde{\mathbf{C}}\tilde{\mathbf{C}}\,, \tag{B19}$$

$$\mathcal{L}_2\ =\ \frac{1}{4}\phi^{ab}\hat{K}_0\phi^{ab}-\frac{1}{8}\phi^{aa}\hat{K}_0\phi^{bb}\,, \tag{B20}$$

$$\mathcal{L}_1\ =\ \frac{1}{2}\phi^{za}\hat{K}_{1-d}\phi^{za}+\frac{1}{2}\phi^{a}\hat{K}_{1+d}\phi^{a}-\frac{2m}{z^2}\phi^{za}\phi^{a}\,, \tag{B21}$$

$$\mathcal{L}_0\ =\ \frac{1}{2}\phi^{zz}\hat{K}_{4-2d}\phi^{zz}+\frac{1}{2}\phi^{z}\hat{K}_{4}\phi^{z}+\frac{1}{2}\phi\hat{K}_{2d}\phi$$
$$-\ \frac{2g}{z^2}\phi^{zz}\phi^{z}-\frac{2f}{z^2}\phi^{z}\phi\,, \tag{B22}$$

$$\tilde{\mathbf{C}}^{a}\ =\ \partial^{b}\phi^{ab}-\frac{1}{2}\partial^{a}\phi^{bb}+\mathcal{T}_{\frac{1-d}{2}}\phi^{za}+\frac{m}{z}\phi^{a}\,, \tag{B23}$$

$$\tilde{\mathbf{C}}^{z}\ =\ \partial^{a}\phi^{za}-\frac{1}{2}\mathcal{T}_{\frac{d-1}{2}}\phi^{aa}+u\mathcal{T}_{\frac{3-d}{2}}\phi^{zz}+\frac{m}{z}\phi^{z}\,, \tag{B24}$$

$$\tilde{\mathbf{C}}=\partial^{a}\phi^{a}+\mathcal{T}_{\frac{3-d}{2}}\phi^{z}+\frac{m}{2z}\phi^{aa}-\frac{g}{(d-2)z}\phi^{zz}+\frac{f}{z}\phi\,, \tag{B25}$$

$$g\equiv m\Big(2\frac{d-2}{d-1}\Big)^{1/2}\,,\qquad u\equiv\Big(2\frac{d-1}{d-2}\Big)^{1/2}\,, \tag{B26}$$

where f is defined in (10.3). We proceed as follows.
i) First, we note that $\mathcal{L}_2$ (B20) can be represented as

$$\mathcal{L}_2=\frac{1}{4}\phi^{ab}\Box_{\kappa}\phi^{ab}-\frac{1}{8}\phi^{aa}\Box_{\kappa}\phi^{bb}\,, \tag{B27}$$

where κ and $\Box_{\kappa}$ are given in (5.3) and (8.15) respectively.
ii) Introducing vector fields $\phi^{a}_{\pm1}$ by the orthogonal transformation

$$\phi^{za}=r^{00}_{z}\phi^{a}_{-1}+r^{00}_{\zeta}\phi^{a}_{1}\,, \tag{B28}$$

$$\phi^{a}=-r^{00}_{\zeta}\phi^{a}_{-1}+r^{00}_{z}\phi^{a}_{1}\,, \tag{B29}$$

where r^{00}_{z}, r^{00}_{ζ} are given in (5.7),(5.10) we cast $\mathcal{L}_1$ (B21) into the form

$$\mathcal{L}_1=\frac{1}{2}\sum_{\lambda=\pm1}\phi^{a}_{\lambda}\Box_{\kappa+\lambda}\phi^{a}_{\lambda}\,. \tag{B30}$$

We note that inverse of the transformation (B28),(B29) is given by

$$\phi^{a}_{-1}=r^{00}_{z}\phi^{za}-r^{00}_{\zeta}\phi^{a}\,, \tag{B31}$$

$$\phi^{a}_{1}=r^{00}_{\zeta}\phi^{za}+r^{00}_{z}\phi^{a}\,. \tag{B32}$$

iii) Introducing scalar fields ϕ_0, $\phi_{\pm2}$ by the orthogonal transformation

$$\phi^{zz}=s_{11}\phi_{-2}+s_{12}\phi_{0}+s_{13}\phi_{2}\,, \tag{B33}$$

$$\phi^{z}=s_{21}\phi_{-2}+s_{22}\phi_{0}+s_{23}\phi_{2}\,, \tag{B34}$$

$$\phi=s_{31}\phi_{-2}+s_{32}\phi_{0}+s_{33}\phi_{2}\,, \tag{B35}$$

$$s_{11}=\Big(\frac{(2\kappa+d)(2\kappa+d-2)(d-2)}{16\kappa(\kappa-1)(d-1)}\Big)^{1/2}\,, \tag{B36}$$

$$s_{12}=\Big(\frac{(2\kappa+d)(2\kappa-d)d}{8(\kappa^2-1)(d-1)}\Big)^{1/2}, \tag{B37}$$

$$s_{13}=\Big(\frac{(2\kappa-d)(2\kappa-d+2)(d-2)}{16\kappa(\kappa+1)(d-1)}\Big)^{1/2}, \tag{B38}$$

$$s_{21}=-\Big(\frac{(2\kappa-d)(2\kappa+d-2)}{8\kappa(\kappa-1)}\Big)^{1/2}, \tag{B39}$$

$$s_{22}=\Big(\frac{d(d-2)}{4(\kappa^2-1)}\Big)^{1/2}, \tag{B40}$$

$$s_{23}=\Big(\frac{(2\kappa+d)(2\kappa-d+2)}{8\kappa(\kappa+1)}\Big)^{1/2}, \tag{B41}$$

$$s_{31}=\Big(\frac{(2\kappa-d)(2\kappa-d+2)d}{16\kappa(\kappa-1)(d-1)}\Big)^{1/2}, \tag{B42}$$

$$s_{32}=-\Big(\frac{(2\kappa+d-2)(2\kappa-d+2)(d-2)}{8(\kappa^2-1)(d-1)}\Big)^{1/2}, \tag{B43}$$

$$s_{33}=\Big(\frac{(2\kappa+d)(2\kappa+d-2)d}{16\kappa(\kappa+1)(d-1)}\Big)^{1/2}, \tag{B44}$$

we cast $\mathcal{L}_0$ (B22) into the form

$$\mathcal{L}_0=\frac{1}{2}\sum_{\lambda=-2,0,2}\phi_\lambda\Box_{\kappa+\lambda}\phi_\lambda\,. \tag{B45}$$

For the readers convenience, we note that inverse of the transformation (B33)-(B35) is given by

$$\phi_{-2}=s_{11}\phi^{zz}+s_{21}\phi^{z}+s_{31}\phi\,, \tag{B46}$$

$$\phi_{0}=s_{12}\phi^{zz}+s_{22}\phi^{z}+s_{32}\phi\,, \tag{B47}$$

$$\phi_{2}=s_{13}\phi^{zz}+s_{23}\phi^{z}+s_{33}\phi\,. \tag{B48}$$

iv) Representing $\tilde{\mathbf{C}}^a$, $\tilde{\mathbf{C}}^z$, $\tilde{\mathbf{C}}$ in terms of the vector fields $\phi^a_{\pm1}$ and the scalar fields ϕ_0, $\phi_{\pm2}$ and introducing C^a, $C_{\pm1}$ by relations

$$C^a=\tilde{\mathbf{C}}^a\,, \tag{B49}$$

$$C_1=r^{00}_\zeta\tilde{\mathbf{C}}^z+r^{00}_z\tilde{\mathbf{C}}\,, \tag{B50}$$

$$C_{-1}=r^{00}_z\tilde{\mathbf{C}}^z-r^{00}_\zeta\tilde{\mathbf{C}}\,, \tag{B51}$$

we find that these C^a, $C_{\pm1}$ take the form given in (10.17)-(10.19). We note the helpful relation

$$\tilde{\mathbf{C}}^A\tilde{\mathbf{C}}^A+\tilde{\mathbf{C}}\tilde{\mathbf{C}}=C^aC^a+C_{-1}C_{-1}+C_1C_1\,. \tag{B52}$$

v) Making use of relation (A24) and taking into account expressions for $\mathcal{L}_2$ (B27), $\mathcal{L}_1$ (B30), $\mathcal{L}_0$ (B45) and formula (B52), we see that Lagrangian (B19) takes the form of the CFT adapted gauge invariant Lagrangian (10.16).

We now present some details of the derivation of gauge transformation given in (10.20)-(10.25). Lagrangian (10.9) is invariant under gauge transformations given in (10.6)-(10.8). In terms of canonically normalized fields (B10) these gauge transformations take the form

$$\delta\tilde{\Phi}^{ab}=\partial^a\xi^b+\partial^b\xi^a-\frac{2}{z}\eta^{ab}\xi^z+\frac{2m\eta^{ab}}{(d-1)z}\xi\,, \tag{B53}$$

$$\delta\tilde{\Phi}^{za}=\partial^a\xi^z+\mathcal{T}_{\frac{d-1}{2}}\xi^a\,, \tag{B54}$$

$$\delta\tilde{\Phi}^{zz}=2\mathcal{T}_{\frac{d-3}{2}}\xi^z+\frac{2m}{(d-1)z}\xi\,, \tag{B55}$$

$$\delta\tilde{\Phi}^{a}=\partial^a\xi-\frac{m}{z}\xi^a\,, \tag{B56}$$

$$\delta\tilde{\Phi}^{z}=\mathcal{T}_{\frac{d-3}{2}}\xi-\frac{m}{z}\xi^z\,, \tag{B57}$$

$$\delta\tilde{\Phi}=-\frac{f}{z}\xi\,. \tag{B58}$$

In terms of fields defined in (B16)-(B18), gauge transformations (B53)-(B58) take the form

$$\delta\phi^{ab}=\partial^a\xi^b+\partial^b\xi^a+\frac{2\eta^{ab}}{d-2}\mathcal{T}_{-\frac{d-1}{2}}\xi^z+\frac{2m\eta^{ab}}{d-2}\xi, \tag{B59}$$

$$\delta\phi^{za}=\partial^a\xi^z+\mathcal{T}_{\frac{d-1}{2}}\xi^a\,, \tag{B60}$$

$$\delta\phi^{zz}=u\mathcal{T}_{\frac{d-3}{2}}\xi^z+\frac{mu}{(d-1)z}\xi\,, \tag{B61}$$

$$\delta\phi^{a}=\partial^a\xi-\frac{m}{z}\xi^a\,, \tag{B62}$$

$$\delta\phi^{z}=\mathcal{T}_{\frac{d-3}{2}}\xi-\frac{m}{z}\xi^z\,, \tag{B63}$$

$$\delta\phi=-\frac{f}{z}\xi\,. \tag{B64}$$

Introducing new gauge transformation parameters by the orthogonal transformation

$$\xi^z=r^{00}_z\xi_{-1}+r^{00}_\zeta\xi_1\,, \tag{B65}$$

$$\xi=-r^{00}_\zeta\xi_{-1}+r^{00}_z\xi_1\,, \tag{B66}$$

and using the vector fields $\phi^a_{\pm1}$ and the scalar fields ϕ_0, $\phi_{\pm2}$, we find that gauge transformations (B59)-(B64) take desired form given in (10.20)-(10.25).

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