

Variational Approach to Yang–Mills Theory with non-Gaussian Wave Functionals

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Abstract. A general method for treating non-Gaussian wave functionals in quantum field theory is presented and applied to the Hamiltonian approach to Yang–Mills theory in Coulomb gauge in order to include a three-gluon kernel in the exponential of the vacuum wave functional. The three-gluon vertex is calculated using the propagators found in the variational approach with a Gaussian trial wave functional as input.

Keywords: Coulomb gauge, variational techniques

PACS: 11.10.Ef, 12.38.Aw, 12.38.Lg

INTRODUCTION

Over the last few years, there have been substantial efforts devoted to a variational solution of the Yang–Mills Schrödinger equation in Coulomb gauge [1, 2, 3]. In this approach, using Gaussian type wave functionals, minimization of the energy density results in the so-called gap equation for the inverse equal-time gluon propagator. This equation has been solved analytically in the ultraviolet [2] and in the infrared [4], and numerically in the full momentum regime [2, 3]. One finds an inverse gluon propagator which in the UV behaves like the photon energy but diverges in the IR, signalling confinement. The obtained propagator also compares favourably with the available lattice data. There are, however, deviations in the mid-momentum regime (and minor ones in the UV) which can be attributed to the missing gluon loop, which escapes the Gaussian wave functionals. These deviations are presumably irrelevant for the confinement properties, which are dominated by the ghost loop (which is fully included under the Gaussian ansatz), but are believed to be important for a correct description of spontaneous breaking of chiral symmetry [5].

In this talk, we present a generalization of the variational approach to the Hamiltonian formulation of Yang–Mills theory [2] to non-Gaussian wave functionals. The expectation value of the Hamilton operator can be expressed in terms of the variational kernels occurring in the ansatz through Dyson–Schwinger equations (DSEs). The three-gluon vertex and the effects of the gluon loop on the gluon propagator are investigated.

NON-GAUSSIAN WAVE FUNCTIONALS

In the Hamiltonian approach to Yang–Mills theory in Coulomb gauge, the vacuum expectation value (VEV)

of an operator depending on the transverse gauge field is given by

$$\langle K[A] \rangle = \int \mathcal{D}A J_A |\psi[A]|^2 K[A], \quad (1)$$

where $J_A = \text{Det}(G_A)$ is the Faddeev–Popov determinant, $G_A = (-\partial\hat{D})^{-1}$ is the inverse Faddeev–Popov operator, and $\psi[A]$ is the vacuum wave functional. In Eq. (1), the functional integration runs over transverse field configurations $\partial_i A_i^a = 0$ and is restricted to the first Gribov region. Writing the vacuum functional as

$$|\psi[A]|^2 =: \exp\{-S[A]\}, \quad (2)$$

one can derive DSEs from the functional identity

$$0 = \int \mathcal{D}A \frac{\delta}{\delta A} \{J_A e^{-S[A]} K[A]\}. \quad (3)$$

In this talk, we consider a functional of the form

$$S[A] = \omega A^2 + \frac{1}{3!} \gamma_3 A^3, \quad (4)$$

where ω and γ_3 are variational functions. A vacuum functional containing also a quartic term is discussed in Ref. [6]. With the explicit form Eq. (4) for the vacuum functional, the DSEs are the usual DSEs of Landau gauge Yang–Mills theory, however, in $D = 3$ dimensions and with the bare vertices of the usual Yang–Mills action replaced by the variational kernels. For the gluon $\langle AA \rangle =: 1/(2\Omega)$ and ghost propagator $\langle G_A \rangle$ these equations are shown in Fig. 1. It should be stressed that these Hamiltonian DSEs are not equations of motion in the usual sense, but rather relations between the Green functions and the so far undetermined variational kernels.

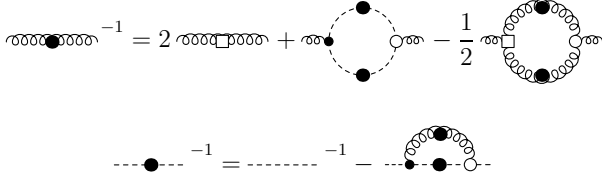


FIGURE 1. Dyson–Schwinger equation for the gluon (top) and ghost propagator (bottom). Here and in the following, small filled dots represent propagators, small empty dots vertex functions, and empty boxes the variational kernels.

VARIATIONAL APPROACH

The Yang–Mills Hamilton operator in Coulomb gauge reads [7]

$$H_{\text{YM}} = \int \left[-\frac{1}{2} J_A^{-1} \frac{\delta}{\delta A} J_A \frac{\delta}{\delta A} + \frac{1}{2} B^2 \right] - \frac{g^2}{2} \int J_A^{-1} \left(\hat{A} \frac{\delta}{\delta A} \right) J_A F_A \left(\hat{A} \frac{\delta}{\delta A} \right), \quad (5)$$

where B is the non-abelian magnetic field, \hat{A} is the gauge field in the adjoint representation of the colour group, and $F_A = G_A(-\partial^2)G_A$ is the Coulomb interaction kernel. The vacuum energy is evaluated as VEV of the Hamilton operator Eq. (5) with the vacuum state defined by Eqs. (2) and (4). By using the DSEs stemming from the identity Eq. (3), the energy density can be written as a functional of the variational kernels,

$$\langle H_{\text{YM}} \rangle = E[\omega, \gamma_3]. \quad (6)$$

By using a skeleton expansion, the vacuum energy can be written at the desired order of loops. Confining ourselves to two loops, the variation of the vacuum energy Eq. (6) with respect to the kernel γ_3 fixes the latter to

$$\gamma_{ijk}^{abc}(\vec{p}, \vec{q}, \vec{k}) = 2ig f^{abc} \times \frac{\delta_{ij}(p-q)_k + \delta_{jk}(q-k)_i + \delta_{ki}(k-p)_j}{\Omega(\vec{p}) + \Omega(\vec{q}) + \Omega(\vec{k})}. \quad (7)$$

Equation (7) is reminiscent of the lowest-order perturbative result [8], with the perturbative gluon energy $|\vec{p}|$ replaced by the non-perturbative one $\Omega(\vec{p})$.

Combining the gluon DSE with the variational equation for the two-gluon kernel ω , one arrives at the gap equation for the gluon propagator

$$\Omega(\vec{p})^2 = \vec{p}^2 + \chi(\vec{p})^2 + I_C(\vec{p}) - I_G(\vec{p}). \quad (8)$$

The explicit expressions for the loop terms can be found in Ref. [6]. With a Gaussian wave functional, only the ghost loop $\chi(\vec{p})$ and the contribution $I_C(\vec{p})$ of the

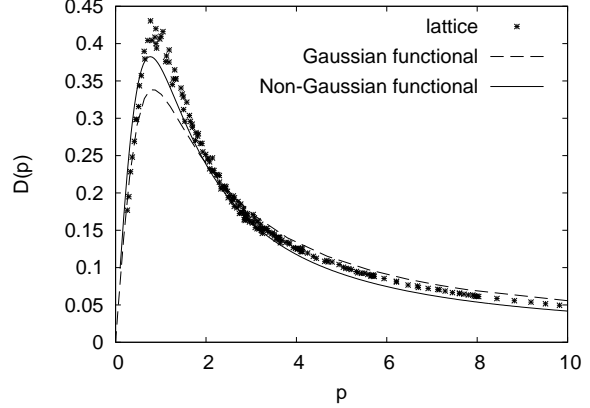


FIGURE 2. Gluon propagator obtained with a Gaussian (dashed line) and a non-Gaussian functional (straight line), compared to the lattice data from Ref. [9].

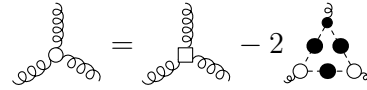


FIGURE 3. Truncated DSE for the three-gluon vertex, under the assumption of ghost dominance.

Coulomb kernel appear in the gap equation. The presence of the gluon loop $I_G(\vec{p})$ in the gap equation (8) modifies the UV behaviour and allows us to extract, from the non-renormalization of the ghost-gluon vertex, the correct first coefficient of the β function. The solution of the full set of coupled integral equations for the ghost and gluon propagators and for the Coulomb form factor (see Ref. [6]) is in progress. Here, in order to estimate the size of the gluon-loop contribution to the gluon propagator, we use the gluon and ghost propagators obtained with a Gaussian wave functional [3] to calculate the gluon loop. The result is shown in Fig. 2, together with lattice data from Ref. [9]. The agreement between the continuum and the lattice results is improved in the mid-momentum regime by the inclusion of the gluon loop, i.e. the three-gluon vertex, as observed also in Landau gauge [10]. The mismatch in the UV is a consequence of the approximations involved, and should disappear when the full system of coupled equations is solved.

THREE-GLUON VERTEX

The truncated DSE for the three-gluon vertex Γ_3 under the assumption of ghost dominance is represented diagrammatically in Fig. 3. Possible tensor decompositions of the three-gluon vertex are given in Ref. [11]. Here, for sake of illustration, we confine ourselves to the form factor corresponding to the tensor structure of the bare

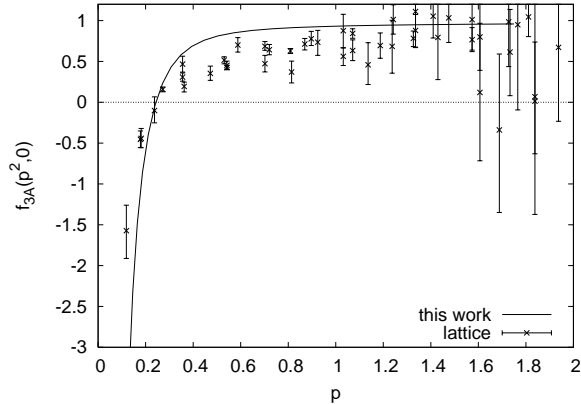


FIGURE 4. Form factor f_{3A} of the three-gluon vertex for orthogonal momenta and comparison to lattice data for the $d = 3$ Landau-gauge vertex [12]. The momentum scale is arbitrary and has been adjusted to make the sign change occur at the same point. The lattice data are shown by courtesy of A. Maas.

three-gluon vertex

$$f_{3A} := \frac{\Gamma_3 \cdot \Gamma_3^{(0)}}{\Gamma_3^{(0)} \cdot \Gamma_3^{(0)}}, \quad (9)$$

where $\Gamma_3^{(0)}$ is the perturbative vertex, given by Eq. (7) with $\Omega(\vec{p})$ replaced by $|\vec{p}|$. Furthermore, we consider a particular kinematic configuration, where two momenta have the same magnitude

$$\vec{p}_1^2 = \vec{p}_2^2 = p^2, \quad \vec{p}_1 \cdot \vec{p}_2 = cp^2. \quad (10)$$

To evaluate the form factor $f_{3A}(p^2, c)$, we use the ghost and gluon propagators obtained with a Gaussian wave functional [3] as input. The IR analysis of the equation for $f_{3A}(p^2, c)$ [Eq. (9)] performed in Ref. [4] shows that this form factor should behave as a power law in the IR, with an exponent three times the one of the ghost dressing function; this is confirmed by our numerical solution [6]. The result for the scalar form factor f_{3A} for orthogonal momenta, $f(p^2, 0)$, is shown in Fig. 4, together with lattice results for $d = 3$ Landau gauge Yang–Mills theory. Our result and the lattice data compare favourably in the low-momentum regime. In particular, in both studies, the sign change of the form factor occurs roughly at the same momentum where the gluon propagator has its maximum. (The scale in Fig. 4 is arbitrary.)

CONCLUSIONS

We have presented a method to treat non-Gaussian wave functionals in the Hamiltonian formulation of quantum field theory. By means of Dyson–Schwinger techniques,

the expectation value of the Hamiltonian is expressed in terms of kernels occurring in the exponent of the vacuum wave functional. These kernels are then determined by minimizing the vacuum energy density. We have estimated the three-gluon vertex by using the propagators found with a Gaussian wave functional as input. The result compares fairly well to the available lattice data obtained in $d = 3$ Landau gauge. The gap equation for the gluon propagator contains the gluon loop, which was missed in previous variational approaches with a Gaussian wave functional. The gluon loop gives a substantial contribution in the mid-momentum regime while leaving the IR sector unchanged, and it also provides the correct asymptotic UV behaviour of the gluon propagator in accord with perturbation theory [8]. The presently developed approach allows a systematic treatment of correlators in the Hamiltonian formulation of a field theory, and opens up a wide range of applications. In particular, it allows us to extend the variational approach from pure Yang–Mills theory to full QCD.

ACKNOWLEDGMENTS

The authors are grateful to J. M. Pawłowski and P. Watson for useful discussions, and to A. Maas for providing the lattice data from Ref. [12]. They also thank the organizers for the interesting conference, and acknowledge financial support by the DFG under contracts No. Re856/6-2,3 and by the Cusanuswerk.

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