Neutrino oscillations trigger a minimal length

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In this paper, we investigate the effect of the presence of a quantum gravity induced minimal length in neutrino oscillation probabilities. Neutrinos can propagate freely over large distances without interacting with matter. Therefore, minimal length effects could pile up beyond detectable thresholds. After determining a modified survival probability, we find that the deviations from the standard oscillations are too small to be detected for typical physical parameters and data from the MINOS experiment. On the other hand, due to an enhancement in the oscillation length, modified oscillations are observable over longer propagations. In particular for a reduced fundamental scale, one could detect significant deviations already for solar system distances.

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Quantum gravity has been drawing the attention of the scientific community for thirty years, but it still escapes to be fully understood and explained. If on the theoretical side there is some progress due to the presence of at least two viable candidate theories, string theory and loop quantum gravity, what seems even more puzzling is the absence of any quantum gravity data up to now [1]. The difficulty in getting experimental evidences is connected to the extreme tininess of the length scale $L_P \approx 1.6 \times 10^{-33}$ cm at which quantum gravity signals are supposed to appear. In the absence of extradimensions, the energy required to probe a scale like L_P would be at least 16 orders of magnitude higher than that reachable by the LHC, the world's largest and highest-energy particle accelerator ever built, currently operating at CERN Labs in Geneva [2]. Against this background we propose an alternative way to overcome difficulties in getting quantum gravity evidence. We start by considering an effective theory for quantum gravity. In Physics an effective theory is an approximate theory that includes a reduced number of degrees of freedom to describe physical phenomena occurring at a chosen length scale, while ignoring substructures and degrees of freedom at shorter distances. The requirement for an effective theory is that it must faithfully approximate the full theory at least in the chosen regime. A key point about quantum gravity is a long-held belief about the emergence of a minimal length. Indeed L_P is not only the scale at which quantum gravity effects should show up, but also the smallest scale for the resolution of position measurements. This idea is dated back to early times of quantum gravity [3] and lies at the basis of all current formulations [4]. As a result, instead of embarking in the interesting but difficult problem of studying testable effects by means of a full formulation of quantum gravity, we will focus on the specific feature of the emergence of a minimal length. To this purpose there exist many effective approaches to implement a minimal length and give rise to a model of quantum spacetime (see for instance generalized uncertainty principle (GUP) approaches [5] and noncommutative geometry (NCG) approaches [6]). In recent years an original formulation based on coordinate coherent state NCG has been successfully employed to improve classical curvature singularities in black hole spacetimes [7] and provide a reliable description of the fractal structure of the universe at the Planck scale [8]. The key feature of this approach is the possibility of implementing the nonlocal character of NCG, by an effective deformation of conventional field equations. The starting point is the ultimate fate of the classical point-like object in NCG. In a series of papers [9] it has been shown that the noncommutative smearing effect can be represented by the action of a nonlocal operator $e^{\ell^2 \Delta_x}$ which spreads the point-like Dirac delta into a Gaussian distribution, i.e.

$$e^{\ell^2 \Delta_x} \delta(\vec{x}) = \rho_\theta(\vec{x}) = \frac{1}{(4\pi\ell^2)^{d/2}} e^{-\vec{x}^2/4\ell}, \qquad (1)$$

where Δ_x is the Laplacian operator on d dimensional Euclidean manifold and ℓ is the NCG induced minimal length. In addition, it has been shown that primary corrections to any field equation in the presence of a noncommutative background can be obtained by replacing the conventional point like source term (matter sector) with a Gaussian distribution, while keeping differential operators (geometry sector) formally unchanged. Roughly speaking one can average noncommuative fluctuations of the manifold and work with the resulting mean values. This affects the integration measure in the Euclidean momentum space representation

$$1 = \int d^n p \; e^{-\ell^2 \vec{p}^2/2} \; |p\rangle \langle p| \qquad (2)$$

which becomes squeezed in the UV region. We notice that higher momenta are more strongly suppressed compared to conventional GUP deformations. In addition, we overcome conventional difficulties emerging from expansions of product of functions in the Moyal $\star$-product approach to NCG: in doing this, at any truncation at a desired order one actually destroys the nonlocal character of the theory. Against this background our deformation in (2) contains an infinite number of terms and it is intrinsically nonlocal. We see that the exponential damping term cannot be expanded for small ℓ. This is the key point at the basis of the UV completeness of this approach as recently underlined in [10]. In the above formulation the minimal length is not set *a priori*. In a four dimensional universe one expects ℓ to be of the order of the Planck length $\ell \sim L_P$. In the case of a higher dimensional spacetime, one can exploit limits on the deviation of Newton's law to get bounds on the fundamental energy scale $E_f \equiv \ell^{-1} \gtrsim 1$ TeV, i.e. $\ell \lesssim 10^{-17}$ cm [11].

As a second step, we propose to study neutrino oscillations as accurate phenomenon to test quantum gravity effects hereby described. Highly energetic particles probe the structure of spacetime to microscopic scales and should be able to feel effects induced by a minimal length. For this reason neutrino propagation has already been the subject of investigations for possible tests of modified dispersion relations [12] and modified de Broglie relations [13]. In this paper we propose to use them for testing NCG as a reliable effective formulation of quantum gravity. Even if the expected deviations are very small, we have two supporting reasons for employing neutrino oscillations as a viable test:

1. Since neutrinos can propagate freely over astronomic distances without interacting with matter, they provide a natural probe for the specific effect coming from the minimal length.

2. The propagation over long distances will pile up the minimal length effect beyond detectable thresholds.

This explains the current interest in quantum gravity effects within the IceCube collaboration [14].

The most common interpretation of neutrino oscillations is that neutrinos do not propagate in a flavor eigenstate but in a mass eigenstate. Neutrinos are not massless, as assumed in the Standard Model, but have a nonvanishing mass. The basis change from the flavour eigenbasis to the mass eigenbasis is parameterized by a unitary 3×3 -matrix U (the Pontecorvo-Maki-Nakagawa-Sakata matrix) and is characterized by three mixing angles $\theta_{12},\ \theta_{13},\ \theta_{23}$ and a CP-violating phase δ_{CP}:

$$|\nu_k\rangle = \sum_{\alpha=1}^{3} U_{k\alpha}\, |\nu_\alpha\rangle\,,$$

where greek indices stand for flavour eigenstates while roman indices stand for mass eigenstates. Since the free Hamilton operator is diagonal in the mass eigenbasis we have

$$|\nu_k(t)\rangle = \exp(-iE_k t)\, |\nu_k\rangle\,. \tag{3}$$

The oscillation propability for a flavour change from flavour α to flavour β is therefore given by

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |\langle \nu_\beta | \nu_\alpha(t)\rangle|^2 \\ &= \sum_{k,j=1}^{3} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} e^{-i(E_k - E_j)t}. \end{aligned}$$

Here, we can approximate

$$E_k - E_j = \sqrt{p^2 + m_k^2} - \sqrt{p^2 + m_j^2} \approx \frac{\Delta m^2_{kj}}{2E}, \tag{4}$$

as well as $t \approx L$. The quantities appearing in the exponential, $L_o = \frac{2E}{\Delta m^2}$, L and $\phi = 2\pi \frac{L}{L_o}$, are called oscillation length, propagation length and oscillation phase respectively. For large values of the phase, the oscillation becomes more and more rapid and the observable effect is washed out. As a result clear signals arise for small oscillation phase only. However, for oscillations to occur, the oscillation phase should not be too small, either. This shows that the oscillation length should not be too big with respect to the propagation length. From data available in [15], the oscillation length $\nu_\mu \leftrightarrow \nu_e$ is too large for these oscillations to be detected in earthbound experiments. In general, if for any propagation length L

$$\frac{L}{E} < 1000 \frac{\text{km}}{\text{GeV}}, \tag{5}$$

the oscillation $\nu_\mu \leftrightarrow \nu_e$ gives oscillation probabilities which may be neglected compared to experimental uncertainties [16]. If this condition is met, the oscillation reduces to a two-flavour problem. In this case the basis change from flavour eigenbasis to mass eigenbasis can be characterized by a single angle

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{6}$$

The transition probability then simplifies to

$$P(\nu_\alpha \to \nu_\beta) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \tag{7}$$

From that we find the survival probability $P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$.

We are now ready to implement ℓ, the minimal length, by following the coherent state approach. As shown in [17], we can incorporate deformations of momentum space integration measure by means of a modified form for the plane wave in (8). As a result we have

$$|\nu_k(t)\rangle = \exp(-\ell^2 E_k^2/2) \exp(-iE_k t)\, |\nu_k\rangle\,. \tag{8}$$

The interpretation of the above expression is the following. For small energies $E_k \ll E_f = \ell^{-1}$ the damping term $\exp(-\ell^2 E_k^2/2) \approx 1$ and therefore the neutrino propagation is unaffected. However for $E_k \sim E_f = \ell^{-1}$ the background spacetime where the propagation occurs switches from a smooth differential manifold to a "rough" quantum manifold. The minimal length is the response of the manifold when it is probed by high energy neutrinos. As a consequence neutrino propagation is strongly suppressed.

It follows that, for the survival probability in the two-flavour case we now find

$$P_\ell(\nu_\alpha \to \nu_\alpha) = \\ 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4E}\exp(-\ell^2 E^2)L\right). \tag{9}$$

As a third step, we test the result (9), by using data from the MINOS neutrino experiment. Since MINOS measures neutrinos in an energy range up to ~ 10 GeV and has a propagation length of 735 km, condition (5) is fulfilled and we can work with the two-flavour model. For the preliminary data published this year (see Fig.1),

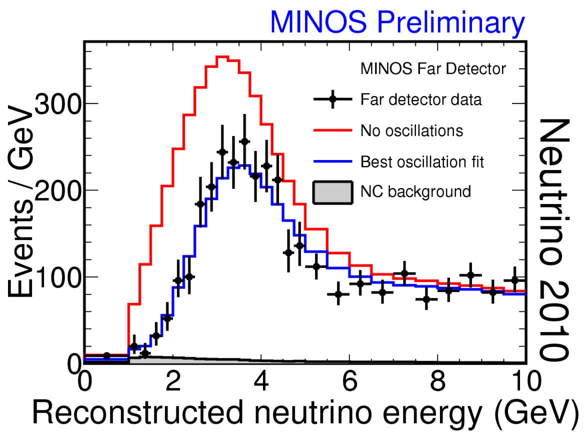

FIG. 1: Measured energy distribution at the far detector of the MINOS experiment, taken from [18].

the best fit values for the oscillation parameters are given by $\Delta m^2 = 2.35^{+0.11}_{-0.08} \cdot 10^{-3}\text{eV}^2$ and $\sin^2(2\theta) > 0.91$.

In Fig. 2 we see the corrections due to (9). We find that deviations from expected neutrino fluxes are small for the realistic case of $E_f = 1$ TeV. In Fig. 2, we drew additional curves corresponding to cases of relevant deviations from the classical case. However, these cases occur for inconsistent values of the minimal length ℓ. A χ^2 test confirms that the earthbound neutrino oscillation experiments cannot provide indications about quantum gravity minimal lengths: the weak result only implies $E_f > 1$ GeV.

At this point, we look for experimental conditions which could lead to the detection of some effect in the near future. To this purpose we test the parameter space for baseline and energy range of neutrino experiments by considering a significant departure from classical oscillation probabilities, i.e.

$$\Delta P \equiv |P(\alpha \to \beta) - P_\ell(\alpha \to \beta)| > 0.1. \tag{10}$$

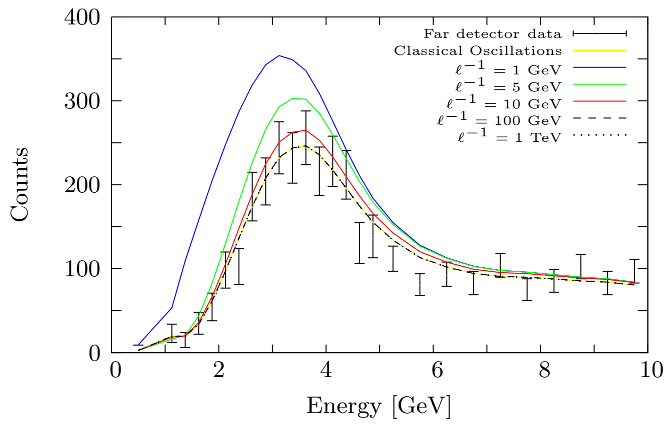

FIG. 2: Expected neutrino flux for the noncommutative model with different fundamental scales, the classical curve and the curve for $\ell^{-1} = 1$ TeV are coincident.

Parameter	Value
θ_{12}	0.5839
θ_{13}	0.1001
θ_{23}	0.7854
Δm^2_{12}	$7.65 \cdot 10^{-5}\text{eV}^2$
Δm^2_{13}	$2.4 \cdot 10^{-3}\text{eV}^2$
Δm_{23}	$2.4 \cdot 10^{-3}\text{eV}^2$
δ_{CP}	0

TABLE I: Oscillation parameters used in three-flavour analysis of parameter space.

Fig. (3) shows regions in which the above condition in the L-E-plane for a $\nu_\mu \to \nu_\mu$ transition in the three-flavour model is fulfilled, with oscillation parameters as in table I. Measuring neutrino oscillations in these parameter regions should lead to an observable increase or decrease of detected ν_μ neutrinos in comparison to the standard oscillations. As we are only looking at the difference in oscillation probabilities, these results are only weakly dependent on the values of the mixing angles as further calculations confirm. We see that already at distances within the solar system a significant effect could be measurable. For instance, in the case $E_f = 1$ TeV detectable effects would show up for a source on the Moon emitting neutrinos with an energy of 400 GeV to 650 GeV (see Fig. 3).

Concerning the oscillation phase, in the minimal length scenario we have

$$\phi = 2\pi\frac{L}{L_{o\ell}} = \frac{4\pi E}{\Delta m^2}\exp(-\ell^2 E^2) \tag{11}$$

which means that coherence is maintained for much higher energies than in the standard oscillation frame-

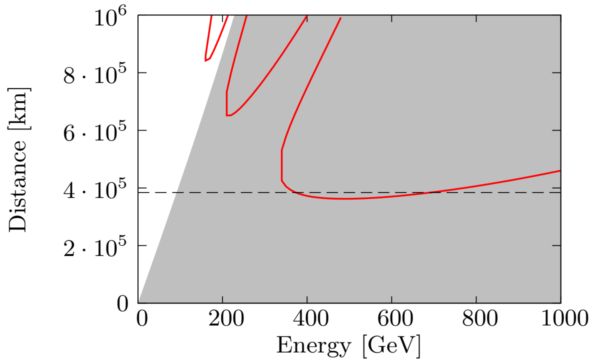

FIG. 3: Parameter space for three-flavour oscillations. The grey area shows the parameter space in which oscillations are coherent and therefore observable. The bounded areas show regions in which $\Delta P > 0.1$. The specific example with $L = 3.84 \cdot 10^5$ km is indicated. A hypothetical experiment could measure a significant difference in the oscillations in an energy band between 400 GeV and 650 GeV.

work. If we require the oscillation phase to be ~ 1, we obtain the grey region in Fig. 3. This confirms that for Earth-Moon distances oscillations are observable.

These results have been obtained by implementing an effective quantum gravity minimal length by means of a specific approach to NCG. However our results leave no room for surprises, since the exponential damping is the strongest correction one can have for neutrino oscillation. Further terms due to additional quantum gravity effects might occur, but they would be sub-leading, i.e. unable to significantly modify our conclusions.

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