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On global solar dynamo simulations

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Global dynamo simulations solving the equations of magnetohydrodynamics (MHD) have been a tool of astrophysicists who try to understand the magnetism of the Sun for several decades now. During recent years many fundamental issues in dynamo theory have been studied in detail by means of local numerical simulations that simplify the problem and allow the study of physical effects in isolation. Global simulations, however, continue to suffer from the age-old problem of too low spatial resolution, leading to much lower Reynolds numbers and scale separation than in the Sun. Reproducing the internal rotation of the Sun, which plays a crucual role in the dynamo process, has also turned out to be a very difficult problem. In the present paper the current status of global dynamo simulations of the Sun is reviewed. Emphasis is put on efforts to understand how the large-scale magnetic fields, i.e. whose length scale is greater than the scale of turbulence, are generated in the Sun. Some lessons from mean-field theory and local simulations are reviewed and their possible implications to the global models are discussed. Possible remedies to some of the current issues of the solar simulations are put forward.

1 Introduction

The large-scale magnetic field of the Sun varies quasiperiodically in time and space: the amount of sunspots varies with an average period of 11 years whereas the period of the magnetic field itself is 22 years. Sunspots appear on a latitude strip ± 40 degrees away from the equator, with spots appearing at high latitudes in the beginning of the cycle and progressively closer to the equator as the cycle advances. Explaining this activity has been one of the principal goals of solar physicists since the first detection of magnetic fields in the Sun by Hale (1908) and Hale et al. (1919).

Nowadays it is generally accepted that the magnetic field of the Sun is maintained by a dynamo residing within, or just below, the convection zone which occupies roughly the outer third of solar radius. The problem is that the flows within the solar convection zone are highly turbulent. Especially in the early days of dynamo theory, computational capabilities were very limited rendering direct solutions of the MHD equations impossible.

The first successful models of solar magnetism were based on a statistical description of turbulent eddies under the influence of rotation and their interaction with large-scale shear (Parker 1955). Work along similar lines evolved into a rigorous mathematical theory, now often referred to as turbulent mean-field dynamo theory, where the separation of small and large scales plays a crucial role (e.g. Moffatt 1978; Parker 1979; Krause & Rädler 1980; Rüdiger & Hollerbach 2004). In turbulent mean-field dynamo theory, large-scale magnetic fields are maintained by the combined action of helical turbulence (α-effect) and large-scale shear flow against turbulent diffusion. Mean-field models cabaple of reproducing the main features of solar observations have existed for decades (e.g. Parker 1955; Steenbeck & Krause 1969; Köhler 1973; see Ossendrijver 2003 for a recent review).

Mean-field models rely on parametrizations of turbulence, such as the α-effect and turbulent diffusion, that we refer to as turbulent transport coefficients. In the absence of observational data or methods to extract them from direct numerical simulations, the turbulent transport coefficients had to be computed from ill-defined approximations. Such procedure often involves a number of free parameters that can be tuned in the mean-field models, which is obviously not a satisfactory state of affairs. Only very recently has an efficient method for computing turbulent transport coefficients from simulations surfaced in the form of the so-called test-field method (Schrinner et al. 2005, 2007).

As the computing power increased, attempts to model solar magnetism by solving the equations of magnetohydrodynamics directly, started to surface (e.g. Gilman 1983; Glatzmaier 1985). More sophisticated simulations have continued to appear ever since (e.g. Brun et al. 2004; Browning et al. 2006; Ghizaru et al. 2010). However, none of the current models can reproduce the main features of solar magnetic activity (see, e.g. Miesch & Toomre 2009). Another aspect that the simulations still struggle with is the internal rotation of the Sun: most simulations produce angular velocity profiles that are dominated by the Taylor–Proudman balance and cylindrical isocontour whereas the

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in the Sun the contours are more conical (e.g. Thompson et al. 2003). Furthermore, the shear layers close to the top and at the base of the convection zone, both of which have been suggested as the locations of the solar dynamo (e.g. Parker 1993; Brandenburg 2005), cannot yet be reproduced numerically in a self-consistent manner.

The problems that direct simulations are facing today are most likely caused by the fact that the parameter regime that is accessible by simulations is still too far removed from solar conditions. Unfortunately, realistic values of the Rayleigh and Reynolds numbers are not likely to be reached any time soon which means that the models need to be improved in a more clever way if progress is to be made. This could include more sophisticated subgrid-scale models and boundary conditions, and numerical techniques to increase resolution in places where it is most needed. In this paper some of these issues are discussed and possible remedies are suggested.

The paper is organised as follows: Sect. 2 summarizes the main numerical issues encountered in global dynamo simulations. In Sects. 3 and 4 possible guidance from turbulent mean-field theory and local simulations are discussed, respectively. Sections 5 and 6 summarize the current state of global solar dynamo simulations and their possible caveats. Final thoughts are given in Sect. 7.

2 Numerical challenges

Here it is assumed that stellar interiors can be dealt within the scope of the MHD approximation and that the gas obeys the equation of state of ideal gas. These assumptions are quite likely violated in the very uppermost and lowermost parts of the solar convection zone where radiation becomes important, but here we assume their effects to be minor for large-scale dynamos. Even then a realistic model of the solar and stellar dynamos must overcome three major numerical challenges: (i) the small molecular diffusivities lead to immense Rayleigh and Reynolds numbers, (ii) the time scales of thermal relaxation are far removed from the turnover time of the turbulence, and (iii) the convection zones of stars are extremely stratified with more than 20 pressure scale heights. Some dimensionless parameters relevant for the Sun are listed in Table 1 (see also Ossendrijver et al. 2003; Brandenburg & Subramanian 2005).

The only way to address issue (i) and to reach realistic Rayleigh and Reynolds numbers is to radically increase the resolution of the simulations. Given that in the Sun the fluid Reynolds number is of the order of 10^{12}, $\mathrm{Re}^{3/4} \approx 10^9$ grid points per direction would be needed for all physically relevant scales to be resolved (e.g. Robinson & Chan 2001). The largest global simulations to date can afford of the order of 10^3 grid points per direction (Miesch et al. 2008). Even if the computing power continues to increase at the current rate, it will take decades before sufficient resolution can be reached. Furthermore, the thermal and magnetic Prandtl numbers (Pr and Pm) are much smaller than unity, leading to much larger lenght scales for the temperature and magnetic field than that of the velocity. For example, in the Sun the smallest scale of velocity is 10^7 times smaller than that of the temperature. This implies that numerical resolution of at least 10^7 grid points is needed to resolve both scales in the same model. Similar, although somewhat less extreme, contrast is encountered with the magnetic fields.

The second issue concerns the vastly varying time scales involved in the solar convection zone: the turnover time of convection cells on the surface of the Sun is of the order of minutes whereas the period of the magnetic cycle is 22 years. However, the most severe issue is due to the thermal relaxation (Kelvin–Helmholtz) time scale which is of the order of a 10^7 years for the Sun. This means that the energy flux flowing through the convection zone is small in comparison to the internal energy. Furthermore, this leads to a very small Mach number in the bulk of the convection zone. In such cases the time step in the simulations is determined by the large sound speed at the base of the convection zone and not by the dynamical velocity. This issue can be alleviated by the use of the anelastic approximation (e.g. Gough 1969; Brun et al. 2004) which, however, breaks down near the surface. Currently no global models are capable of dealing with both the small Mach number flows in the deep layers and transonic flows near the surface.

Issue (iii) arises due to the immense density stratification and leads to similar problems as in (i): a minimum number of grid points, of the order of five, is required to resolve a pressure scale height H_P. Close to the surface of the Sun $H_\mathrm{P} \approx 100$ km so we could get away with a grid resolution of 20 km. Given that the depth of the solar convection zone is $2 \cdot 10^5$ km, a minimum of 10^4 grid points is required to resolve this. Such resolution is not quite within the grasp of simulations as of yet. However, using a non-uniform grid in the radial direction (e.g. Chan & Sofia 1986; Robinson & Chan 2001) can alleviate this issue.

In summary, very few parameters can have their realistic values in global simulations (cf. Table 1). Possibly the only exception is the Coriolis, i.e. inverse Rossby, number, which spans from roughly 10 at the base of the convection zone to 10^{-3} near the surface. However, it is not possible to cover this range in a single model either. If the large-scale dynamo of the Sun is driven by a turbulent dynamo relying on helical turbulence arising from the interaction of rotation and stratified turbulence (see below), then it might not be a problem that we cannot reach realistic Rayleigh and Reynolds numbers. Currently the best hope is that as long as Ra, Re, and Rm are sufficiently high as to produce vigorous turbulence, and the rotational influence is correctly modelled, the main aspects of solar magnetism can be captured.

3 Guidance from mean-field theory

It is useful to make a small recourse into theory in order to have an idea when a large-scale dynamo can be expected to be excited. In mean-field dynamo theory the evolution of

Table 1 Summary of some dimensionless parameters in the Sun and in typical simulations. The last column denotes whether the simulations capture the solar regime (+) or not (–). Here g is the acceleration due to gravity, d is the typical scale of turbulence, δ is the superadiabaticity, ν is the viscosity, η is the magnetic diffusivity, H_P is the pressure scale height, u is a typical velocity, χ is the thermal diffusivity, whereas p and c_s are the pressure and sound speed, respectively, and Ω is the rotation rate.

Parameter	Sun	Simulations	Comparability
$\mathrm{Ra} = gd^4\delta/(\nu\chi H_\mathrm{P})$	10^{20}	10^7	–
$\mathrm{Re} = ud/\nu$	10^{12}	$< 10^4$	–
$\mathrm{Rm} = ud/\eta$	10^9	$< 10^4$	–
$\mathrm{Pr} = \nu/\chi$	10^{-7}	0.01	–
$\mathrm{Pm} = \nu/\eta$	$10^{-6}\ldots10^{-4}$	10^{-3}	–
$N_\mathrm{P} = \ln(p_\mathrm{base}/p_\mathrm{top})$	20	≈ 5	–
$\mathrm{Ma} = u/c_s$	$10^{-4}\ldots1$	$10^{-4}\ldots1$	–/+
$\delta = \nabla - \nabla_\mathrm{ad}$	$10^{-8}\ldots0.1$	$10^{-8}\ldots0.1$	–/+
$\mathrm{Ta} = 4\Omega^2d^4/\nu^2$	$10^{19}\ldots10^{27}$	10^8	–
$\mathrm{Co} = 2\Omega d/u$	$10^{-3}\ldots10$	$10^{-3}\ldots10$	+

the large-scale magnetic field is governed by the averaged induction equation

$$\frac{\partial\overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla}\times(\overline{\boldsymbol{U}}\times\overline{\boldsymbol{B}} + \overline{\boldsymbol{\mathcal{E}}} - \eta\mu_0\overline{\boldsymbol{J}}), \tag{1}$$

where the overbars denote a suitable average, $\boldsymbol{U}$, $\boldsymbol{B}$, and $\boldsymbol{J} = \mu_0^{-1}\boldsymbol{\nabla}\times\boldsymbol{B}$ are the velocity, magnetic field, and current density, respectively. Furthermore, η is the magnetic diffusivity and μ_0 is the vacuum permeability. The extra term in comparison to the standard induction equation is the electromotive force

$$\overline{\boldsymbol{\mathcal{E}}} = \overline{\boldsymbol{u}\times\boldsymbol{b}}, \tag{2}$$

where $\boldsymbol{u} = \boldsymbol{U} - \overline{\boldsymbol{U}}$ and $\boldsymbol{b} = \boldsymbol{B} - \overline{\boldsymbol{B}}$ are the fluctuations of velocity and magnetic field, respectively. Given that the large-scale field $\overline{\boldsymbol{B}}$ varies slowly in space and time, $\overline{\boldsymbol{\mathcal{E}}}$ can be written in terms of the large-scale quantities where turbulent transport coefficients describe the effects of turbulence on the large scales (e.g. Krause & Rädler 1980):

$$\overline{\mathcal{E}}_i = \alpha_{ij}\overline{B}_j + \eta_{ijk}\frac{\partial\overline{B}_j}{\partial x_k} + \cdots, \tag{3}$$

where α_{ij} and η_{ijk} are second and third rank tensors, respectively, and the dots indicate the possibility to take higher order derivatives into account.

In simple systems, such as homogeneous, isotropic turbulence, the first term on the rhs of Eq. (3) describes the α-effect whereas the second term is responsible for turbulent diffusion:

$$\overline{\boldsymbol{\mathcal{E}}} = \alpha\overline{\boldsymbol{B}} - \eta_\mathrm{t}\mu_0\overline{\boldsymbol{J}}, \tag{4}$$

where α and η_t are scalars (Steenbeck et al. 1966). In the high conductivity limit these scalars are given by

$$\alpha = -\tfrac{1}{3}\tau_\mathrm{c}\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}, \quad \eta_\mathrm{t} = \tfrac{1}{3}\tau_\mathrm{c}\overline{\boldsymbol{u}^2}, \tag{5}$$

where τ_c is the correlation time of turbulence and $\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}}$ is the kinetic helicity. In more realistic systems α is no longer directly proportional to kinetic helicity (e.g. Rädler 1980), although it is still an often used proxy.

In the absence of shear, the α-effect alone is able to overcome turbulent diffusion and excite a large-scale dynamo. This can be quantified by requiring that a dimensionless dynamo number

$$D_\alpha = \frac{\alpha d}{\eta_\mathrm{t}}, \tag{6}$$

where d is the spatial extent of the system (e.g. the radius of the Sun or the depth of the convection zone), exceeds a threshold value. At the same time, the magnetic Reynolds number has to exceed a critical value. However, for large-scale dynamos this is typically of the order of unity and thus not an issue for the Sun (e.g. Krause & Rädler 1980; Brandenburg 2009; Käpylä et al. 2010b). Another essential ingredient is the separation of scales: the turbulence must have a scale smaller by a factor of few than the system size for the dynamo to work. Numerical simulations in idealised setups (Brandenburg 2001) have shown such α^2-dynamos exist but it is not likely that this type of dynamo is the main contributor to solar magnetism. This is because most α^2-dynamos produce non-oscillatory solutions, although non-uniform α-profiles can excite oscillatory modes as well (e.g. Baryshnikova & Shukurov 1987; Rüdiger et al. 2003; Stefani & Gerbeth 2003).

When shear is present, not only is the dynamo easier to excite, but the solutions often exhibit oscillatory solutions or dynamo waves. The direction of propagation of such waves in $\alpha\Omega$-dynamos is determined by the sign of the product of radial shear and the α-effect (e.g. Yoshimura 1975). According to symmetry considerations the simplest form of the α-effect in a rotating stratified atmosphere of a star is given by (e.g. Krause & Rädler 1980):

$$\alpha_{ij} = \alpha_1\delta_{ij}\hat{\boldsymbol{G}}\cdot\hat{\boldsymbol{\Omega}} + \alpha_2(\hat{G}_i\hat{\Omega}_j + \hat{G}_j\hat{\Omega}_i), \tag{7}$$

where $\hat{\boldsymbol{G}}$ and $\hat{\boldsymbol{\Omega}}$ denote the unit vectors along the direction of inhomogeneity (e.g. turbulence intensity or density stratification due to gravity) and rotation, respectively. This suggests that α is positive (negative) in Northern (Southern) hemisphere in the Sun. The early dynamo models postulated (e.g. Parker 1955; Köhler 1973) a positive α in the Northern hemisphere and negative radial shear in the convection zone which produces an equatorward migrating dynamo wave.

Helioseismology, however, has revealed that the regions of negative radial shear in the solar convection zone are situated in the tachocline at high latitudes and in the surface shear layer in the outermost five per cent of solar radius. The realization that it is actually quite difficult to obtain equatorward migrating activity with $\alpha\Omega$-dynamos is sometimes referred to as the 'dynamo dilemma' (Parker 1987). However, the profile and magnitude of the α-effect and turbulent diffusivity in the solar convection zone are rather poorly known.

During recent years the importance of magnetic helicity conservation has been realized in the nonlinear saturation of large-scale dynamos (e.g. Brandenburg & Subramanian 2005 and references therein). More specifically, if magnetic field lines are confined within the object, magnetic helicity can change only due to microscopic magnetic diffusion leading to extremely long saturation time scales (see e.g. Brandenburg 2001). The Sun, however, is not a closed system and can shed the small-scale magnetic helicity e.g. by coronal mass ejections. Thus it is probably important to design simulation setups so that magnetic helicity can escape without hindering the growth of the large-scale magnetic fields.

4 Lessons from comparisons of theory and local simulations

Early local simulations of turbulent convection failed to generate appreciable large-scale magnetic fields (e.g. Nordlund et al. 1992; Brandenburg et al. 1996) although all the necessary ingredients (turbulence, rotation, and stratification) for an α^2-dynamo were present. Around the same time theoretical studies and supporting numerical simulations suggested that generating large-scale magnetic fields becomes all but impossible in the regime of large magnetic Reynolds number (Cattaneo & Vainshtein 1991; Vainshtein & Cattaneo 1992; Cattaneo & Hughes 1996). Furthermore, convection simulations yielded conflicting results for the α-effect, suggesting values close to theoretical expectations (Brandenburg et al. 1990; Ossendrijver et al. 2001,2002; Käpylä et al. 2006a) or close to zero (Cattaneo & Hughes 2006; Hughes & Cattaneo 2008), further adding to the confusion.

The early simulations all lacked an important ingredient, namely large-scale shear flow. Adding sufficiently strong shear indeed excites a large-scale dynamo (Käpylä et al. 2008; Hughes & Proctor 2009), similarly as in non-helically forced turbulence simulations (Yousef et al. 2008a,b; Brandenburg et al. 2008). However, the origin of the large-scale fields still remained controversial due to the widely differing estimates of α (see, e.g. Hughes & Proctor 2009). Here the test-field method (Schrinner et al. 2005, 2007) comes to the rescue: with it, all of the relevant turbulent transport coefficients, including the turbulent magnetic diffusivity, can be computed from the simulations. Performing such analysis to the simulations of Käpylä et al. (2008) it turns out that the large-scale dynamos in the presence of shear cannot be accounted for by the α-effect alone, but other turbulent mean-field effects, such as the $\overline{\boldsymbol{\Omega}} \times \overline{\boldsymbol{J}}$ (Rädler 1969,1980) and shear–current effects (Rogachevskii & Kleeorin 2003,2004), also contribute (Käpylä et al. 2009b).

Similarly, the test-field results indicate that increasing the rotation rate decreases turbulent diffusion and increases α, suggesting that large-scale α^2-dynamo action becomes possible at sufficiently rapid rotation (Käpylä et al. 2009b). This was indeed realized by direct simulations in the same parameter regime (Käpylä et al. 2009a; see also Jones & Roberts 2000; Rotvig & Jones 2002). On the other hand, the previously obtained small values α (e.g. Cattaneo & Hughes 2006; Hughes & Cattaneo 2008; Hughes & Proctor 2009) turn out to be artefacts of the so-called imposed field method which does not take the inhomogeneities of the large-scale field into consideration (Käpylä et al. 2010a, see also Hubbard et al. 2009).

Another aspect that has been studied mainly using local simulations is the nonlinear saturation of large-scale dynamos. In particular, it is of great interest to study what happens to the saturation level of the large-scale magnetic field when magnetic helicity fluxes are either allowed or suppressed. It turns out that open boundaries allow saturation on a dynamical timescale and large-scale field strengths around equivipartition with the turbulence (Käpylä et al. 2010b). When the flux is suppressed, the large-scale field strength decreases steeply as a function of the magnetic Reynolds number. In some cases this might explain why no large-scale dynamo is seen with periodic or perfectly conducting boundaries (Tobias et al. 2008) whereas the same system with magnetically open boundaries shows a strong large-scale field (Käpylä et al. 2008).

The main lesson from comparisons of theory and local simulations is that the predictions of mean-field theory need to be taken seriously: most of the results of direct simulations can be reproduced qualitatively and many also quantitatively by mean-field models using turbulent transport coefficients from corresponding test-field runs (e.g. Käpylä et al. 2009b; Gressel 2010). Furthermore, local simulations have shown that the resulting large-scale dynamo is sensitive to the magnetic boundary conditions which in many cases can be understood in terms of magnetic helicity conservation.

5 Global simulations

We consider here three classes of models that solve the equations of magnetohydrodynamics without the mean-field approximations: forced turbulence simulations where convection and large-scale flows are omitted, rapidly rotating convection simulations, and models that endeavour to reproduce the Sun the best possible way permitted by the available resources. We consider each of these cases separately.

5.1 Idealised forced turbulence simulations

It is often useful to study highly idealised systems where the turbulence is driven by a body force instead of stratified convection, and where different physics can be added or removed by hand. In such setups it is possible to test simple ideas and to see whether the predictions from mean-field theory can be realised, although such simulations may have a limited applicability to the real Sun. Another advantageous aspect of such models is that by virtue of their simplicity, they are much easier to control and analyze than simulations with convection.

One example of such idealised simulations, reproducing results of mean-field theory and supplying possible hints as to how the Sun is working was recently reported by Mitra et al. (2010). In this model, helically forced turbulence produces an α-profile that changes sign at the equator. Since large-scale flows are omitted, this system can only host an α^2-dynamo. Given that the forcing is sufficiently helical, a dynamo which produces large-scale magnetic fields, is excited. The remarkable aspect of this dynamo is that it produces equatorward migrating active regions in the absence of shear. Such configurations have previously been obtained only in mean-field models of α^2-dynamos (e.g. Baryshnikova & Shukurov 1987; Rädler & Bräuer 1987). Although this process is an unlikely main driver of equatorward migration in the Sun, it may still contribute to the observed activity.

5.2 Simulations of rapidly rotating stars

Photometric observations suggest that stars rotating even much faster than the Sun are likely to have a comparable absolute differential rotation $\Delta\Omega = \Omega_{\rm equator} - \Omega_{\rm pole}$ (e.g. Korpi & Tuominen 2003 and references therein). On the other hand, the magnitude of the α-effect is, to first order, proportional to the rotation rate. Furthermore, test-field simulations indicate that turbulent diffusion decreases as a function of rotation (Käpylä et al. 2009b). Combined, these very crude estimates suggest that the dynamo number, proportional to $\alpha\Delta\Omega$, in a rapidly rotating star greater than in the Sun. At face value this seems to indicate that exciting a turbulent large-scale dynamo in a simulation with faster than solar rotation should be easier than with solar values.

There are some indications that this conjecture has some validity, see e.g. the studies of Brown et al. (2007, 2010) and Käpylä et al. (2010c). These simulations exhibit a solar-like rotation profile with a fast equator and slow poles but also non-axissymmetric nests of convection near the equator (e.g. Busse 2002; Brown et al. 2008) which are not observed at least in the Sun. Meridional flows are concentrated in a number of small cells. The radial gradient of Ω near the equator is positive, whereas at higher latitudes differential rotation is much weaker. Negative (positive) kinetic helicity in the Northern (Southern) hemisphere suggests a positive (negative) α-effect (e.g. Käpylä et al. 2010c). According to mean-field theory a poleward propagating dynamo wave should appear in regions close to the equator, which is indeed realized in the simulations. However, the α-effect has not been measured directly in any of the studies reported so far. The large-scale magnetic fields tends to fill most of the convection zone suggesting that a distributed dynamo is at operation. If an overshoot layer is included, a non-oscillating field resides in the stable layer below the convection zone (Käpylä et al. 2010c). The large-scale magnetic field is also to a fairly high degree axissymmetric (e.g. Brown et al. 2010).

Although increasing the rotation rate above the solar value makes it easier to excite a dynamo in the simulations, other issues arise: observational studies suggest that the magnetic activity of rapidly rotating stars appears in the form of strong non-axissymmetric structures at very high latitudes (e.g. Berdyugina & Tuominen 1998), which possibly show similar magnetic cycles as the Sun (e.g. Jetsu et al. 1993) in which the high-latitude spots alternate in strength. Such configurations can be obtained from mean-field models (e.g. Elstner & Korhonen 2005) but not in direct simulations as of yet.

5.3 Solar simulations

The first successful convection-driven dynamo simulations producing large-scale magnetic fields were performed already by Gilman (1983) and Glatzmaier (1985). The rotation profile of these simulations was solar-like, i.e. equator rotating faster than the poles, and a positive radial gradient of Ω was found near the equator. Thus the activity belts migrated towards the poles in contradiction to the Sun. Although these studies demonstrated the possbility of large-scale dynamo action, many parameters were not exatcly solar-like. For example, Gilman (1983) used the Boussinesq approximation and a rotation rate that is likely greater than in the Sun.

Later studies have refined these models further, using the anelastic approximation in conjunction with thermal stratification computed from solar structure models, and accurate physical parameters such as the solar rotation rate and luminosity (e.g. Elliott et al. 1999; Brun & Toomre 2002; Brun et al. 2004; Miesch et al. 2000,2006,2008) but often omitting an overshoot layer and the shear layers near the surface and at the bottom of the solar convection zone.

Majority of the studies quoted above present hydrodynamical simulations which concentrate on the study of differential rotation. The problem there is that even for solar rotation rates the resulting rotation profile is dominated by the Taylor–Proudman balance leading to cylindrical isocontours of Ω (e.g. Brun & Toomre 2002). This is the so-called 'Taylor number puzzle' encountered earlier in mean-field models (e.g. Brandenburg et al. 1991). It turns out that reproducing the solar interior rotation self-consistently is very hard indeed and none of the current simulations are able to do this. However, the solutions are sensitive to boundary conditions: fixing the energy flux on the outer boundary appears to alleviate the Taylor–Proudman constraint (Elliott et al. 1999). Furthermore, imposing a temperature difference of around 10 K on the lower boundary leads to a thermal wind contribution that has a similar effect (Miesch et al. 2006). Such temperature gradients occur in the simulations naturally but they are apparently not strong enough. However, as the imposed latitudinal variation of temperature is transmitted to the convection zone by thermal diffusivity, the efficiency of the forcing diminishes as the resolution is increased and diffusivity lowered (e.g. Miesch et al. 2008). So a more robust method of sustaining a latitudinal temperature gradient is likely to be needed.

The dynamo action of such solar simulations was studied by Brun et al. (2004). However, in their simulations no appreciable large-scale magnetic fields were found: a strong magnetic field is obtained but the large-scale field is only of the order of a few per cent of the total. This suggests that a fluctuation dynamo is excited but the large-scale dynamo is subcritical or suppressed. These results are puzzling, given that the simulations claim to use real solar parameters. Taken at face value the results suggest that there is no turbulent mean-field dynamo within the solar convection zone. However, a number of ingredients are still missing, the most important of which concern the rotation profiles realised in the simulations.

More specifically, it is currently not possible to reproduce the surface shear layer or the tachocline at the base of the solar convection zone with direct numerical simulations. Especially the tachocline has been considered to host the solar dynamo. Introducing a tachocline by hand does indeed enable a large-scale dynamo (Browning et al. 2006), although the field is mostly confined in the overshoot layer and does not show reversals of polarity. In a more recent study, a similar model did show oscillatory behaviour (Ghizaru et al. 2010). It is not clear why the latter shows oscillations while the former does not, although it is possible that the earlier simulation was simply not ran long enough. However, even in the study of Ghizaru et al. (2010) the activity is at too high latitudes and the migration of the activity belts towards the equator is not very pronounced.

6 Possible missing ingredients

6.1 Insufficient resolution

The most obvious defect of all current simulations is the lack of numerical resolution. This is also the most difficult problem to solve because ultimately only bigger computers and codes that scale well in them give a proper solution. However, in the meantime the current setups need to be optimised to take full advantage of the resources available. This includes increasing the resolution near the surface where the pressure scale height is small by means of a non-uniform grid.

A more radical solution is to omit certain parts of the star in the models in order to increase the resolution in the remaining areas. This is motivated by the fact that the large-scale magnetic activity in the Sun is concentrated near the equator and that the sunspots appear more or less independent of longitude. These observations suggest that the essential ingredients of the solar dynamo could be captured by modelling only the relevant latitudes and a reduced longitudinal extent. Simulations in such 'wedge' geometry have been used in the past (e.g. Robinson & Chan 2001; DeRosa & Hurlburt 2003) and more recent convection simulations (Käpylä et al. 2010c) seem to compare well with results in full spheres (e.g. Brown et al. 2010).

6.2 Unresolved effects of turbulence

It is clear from Table 1 that it is not possible to resolve all the physically relavant turbulent scales in current simulations. It is also not obvious what effect these scales would have on the resolved scales. However, it is in principle possible to take these effects into account by applying suitable subgrid-scale models. Furthermore, the subgrid-scale models need to be validated by comparing their results with local numerical simulations (e.g. Snellman et al. 2009; Garaud et al. 2010).

For example, the Taylor–Proudman balance could be alleviated if the anisotropy of turbulent heat transport due to rotation is taken into account (e.g. Kitchatinov et al. 1994). This effect has been successfully used in hydrodynamical mean-field models (e.g. Durney & Roxburgh 1971; Brandenburg et al. 1992; Rüdiger et al. 2005; Küker & Rüdiger 2008) but not so far in three dimensional simulations. Similar modelling could be adopted for the Reynolds stress and electromotive force as well. In particular the non-diffusive part of the Reynolds stress, often referred to as the Λ-effect (e.g. Rüdiger 1989), is likely to be important in sustaining the surface shear layer. On the other hand, the lack of large-scale magnetic field in solar simulation without overshoot (Brun et al. 2004) could indicate that the relevant scale for the α-effect is not resolved which could be remedied by introducing it via a subgrid-scale model.

6.3 Surface shear layer

The idea that the solar dynamo resides close to the surface arises from sunspot observations which are consistent with the picture that the spots are initially formed at a depth of around $r = 0.95R_{\odot}$, which coincides with the lower part of the surface shear layer. The surface dynamo idea has recently been revived in the paper of Brandenburg (2005) who demonstrated that non-helical turbulence with radial and latitudinal shear leads to a large-scale dynamo and structures that resemble bipolar regions. In the Sun an α-effect is also likely to be present in these depths so conceivably an oscillatory dynamo could be obtained or the direction of the dynamo wave reversed near the surface (e.g. Käpylä et al. 2006b).

None of the current solar simulations, however, capture the surface shear layer even in the most stratified and highest resolution runs performed so far (Miesch et al. 2008). This can be due to still insufficient density stratification and scale sepration rendering the Λ-effect ineffective near the surface. Another possible reason is that shear layer is sensitive to the outer boundary condition of the simulations.

6.4 Tachocline

The studies of Browning et al. (2006) and Ghizaru et al. (2010) have highlighted the importance of the tachocline for the dynamo. The problem here is that it is not yet possible to form a tachocline self-consistently in the simulations so it has to be enforced by some method. This is because the diffusivities in the current simulations are still so large that the differential rotation from the convection zone diffuses into the stable layer. This could be countered by lowering the diffusion coefficients below the convection zone but this is likely to cause numerical issues. Another issue is the stability of the tachocline with respect to magnetic fields: while certain dynamo models assume that the field in the tachocline needs to be of the order of 10^5 Gauss (e.g. Dikpati & Charbonneau 1999), other studies indicate that the tachocline becomes unstable already for field strenghts that are two orders of magnitude lower (e.g. Arlt et al. 2005).

6.5 Meridional circulation

Another large-scale flow component in the Sun is the meridional flow. Observations indicate that the flow is poleward at the surface with a magnitude of around 10 m s^{-1}. Hydrodynamic mean-field models indicate that the flow consists of a single counter-clockwise cell (e.g. Rempel 2005). The return flow should reside near the base of the convection zone with a magnitude of 1 m s^{-1}. If the solar dynamo also resided in the deep layers of the convection zone, the deep return flow could in principle change the direction of the dynamo wave. This flow configuration would also be essential for the flux-transport dynamo model to work (e.g. Dikpati & Charbonneau 1999). However, current simulations do not show a clear single cell configuration but rather a large number of smaller cells (e.g. Brun et al. 2004). Furthermore, helioseismology is currently unable to provide observational confirmation of the structure of the meridional flow within the solar convection zone.

7 Conclusions

Current global simulations of solar magnetism struggle with several aspects of the observed activity: firstly, it is difficult to obtain a large-scale field at all without a tachocline (Brun et al. 2004). Secondly, when a tachocline is imposed, there is a large-scale field but it is non-oscillating (Browning et al. 2006) or the activity belts are at too high latitudes (Ghizaru et al. 2010). Most of the problems can be associated with insufficient numerical resolution due to which some of the physically relevant scales of turbulence are not resolved, and prevents the self-consistent generation of a tachocline of the surface shear layer.

Relatively little can be done to overcome the resolution issue, although simulations in the 'wedge' geometry (Mitra et al. 2009, 2010; Käpylä et al. 2010) promise to deliver some benefits. However, a perhaps more promising alternative is to introduce improved subgrid-scale models, capturing also non-diffusive effects of turbulence, into the simulations. However, this is also likely to be a long and rocky road because the subgrid-scale models should be validated as rigorously as possible before their use.

Furthermore, our current understanding of the existing simulations that are capable of large-scale dynamo action (e.g. Browning et al. 2006; Brown et al. 2010; Käpylä et al. 2010; Ghizaru et al. 2010) is still quite insufficient. For example, there are only enlightened guesses of the α-effect based on the sign of kinetic helicity, and even less is known of turbulent transport coefficients related to other dynamo mechanisms such as the $\overline{\boldsymbol{\Omega}} \times \overline{\boldsymbol{J}}$ and shear-current effects or turbulent diffusivity. Therefore the current simulations should be analyzed in greater detail, e.g. with the help of test-field methods and corresponding mean-field models, in order to find out which effects are responsible for the dynamo and where it is situated. Such analysis could also help to understand what is missing from the simulations in comparison to the Sun.

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