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Research Article

Active and Passive Realization of Fractance Device of Order 1/2

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Active and passive realization of Fractance device of order 1/2 is presented. The crucial point in the realization of fractance device is finding the rational approximation of its impedance function. In this paper, rational approximation is obtained by using continued fraction expansion. The rational approximation thus obtained is synthesized as a ladder network. The results obtained have shown considerable improvement over the previous techniques.

1. INTRODUCTION

A system which is defined by fractional order differential equations is called as Fractional order System [1]. The significant advantage of Fractional order systems compared to integer order systems is that they are characterized by memory. Fractional order systems are characterized by infinite memory, whereas they are finite for integer order systems. Fractance Device, semi-infinite lossy Transmission Line, diffusion of heat into the semi-infinite solid, $PI^{\lambda}D^{\mu}$ controllers, and so forth are some of the examples of fractional order systems [1, 2]. For a semi infinite lossy transmission line current is related to applied voltage as, $I(s) = \sqrt{s}V(s)$. In the case of diffusion of heat into the semi-infinite solid, the temperature at the boundary of the surface is related to half integral of heat. However, in this paper, focus is made only on fractance device and its realization.

Fractance device is an electrical element which exhibits fractional order impedance properties. The Impedance of the fractance device is defined as

$$Z(j\omega) = (j\omega)^{\alpha}, \tag{1}$$

where ω is the angular frequency and α takes the values as -1, 0, 1 for capacitance, resistance, and the inductance, respectively [1]. Fractance device finds applications in Robotics, Hard Disk drives, signal processing circuits, fractional order control, and so forth [2–7]. The following are some of the important points about Fractance device.

(i) The phase angle is constant with frequency but depends only on the value of fractional order, α. Hence this device is also called as *constant phase angle device* or simply *fractor* [3].
(ii) Moderate characteristics between inductor, resistor, and capacitor can be obtained using fractance device.
(iii) By making use of an operational amplifier, a fractional order differentiation and integration can be accomplished easily [2].

Fractance device can be realized as either tree, chain, or a net grid type networks. Different recursive structure realizations were presented in [3, 4]. But, the disadvantage is hardware complexity [3, 4]. The second way of realizing fractance device is synthesizing network from the rational approximation describing its fractional order behavior. So, the key point in the realization of fractance device is finding the rational approximation of the fractional order operator. A rational approximation transfer function is characterized only by poles, whereas irrationality of fractional transfer functions gives a cut on the complex s-plane. Due to the irrationality, the fractional linear oscillations have a finite number of zeros [8]. The continued fractions approximation ignores this feature. So, in this paper, rational approximation for $\sqrt{s}$ is obtained using continued fraction expansion. Fractance device of order 1/2 is defined by the following volt-ampere characteristic:

$$Z(s) \approx \sqrt{\frac{R}{sC}} \approx \frac{k_0}{\sqrt{s}}, \tag{2}$$

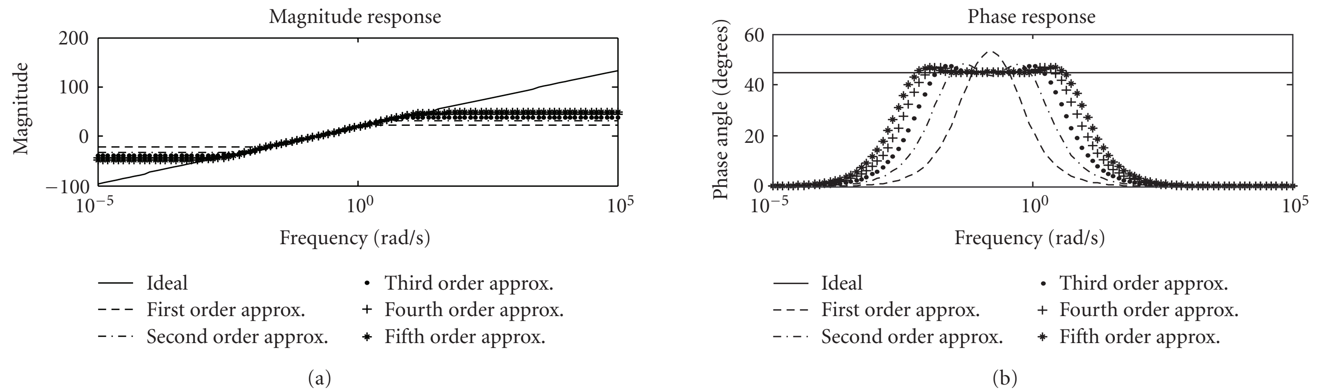

Figure 1: Comparison of magnitude and phase responses of rational approximation functions with ideal $\sqrt{s}$.

where $k_0 = \sqrt{R/C}$. By making use of well known Regular Newton Process, Carlson and Halijak [6] have obtained rational approximation of $1/\sqrt{s}$ as

$$H(s) = \frac{s^4 + 36s^3 + 126s^2 + 84s + 9}{9s^4 + 84s^3 + 126s^2 + 36s + 1}. \tag{3}$$

In [7], by approximating an irrational function with rational one, and fitting the original function in a set of logarithmically spaced points, Mastuda has obtained rational approximation of $1/\sqrt{s}$ as,

$$H(s) = \frac{0\cdot08549s^4 + 4\cdot877s^3 + 20\cdot84s^2 + 12\cdot995s + 1}{s^4 + 13s^3 + 20\cdot84s^2 + 4\cdot876s + 0\cdot08551}. \tag{4}$$

Oustaloup [2] has approximated the fractional differentiator operator s^α by a rational function and derived the following approximations:

$$\frac{1}{\sqrt{s}} = \frac{s^5 + 74\cdot97s^4 + 768\cdot5s^3 + 1218s^2 + 298\cdot5s + 10}{10s^{5+}298\cdot5s^4 + 1218s^3 + 768\cdot5s^2 + 74\cdot97s + 1},$$
$$\sqrt{s} = \frac{10s^5 + 298\cdot5s^4 + 1218s^3 + 768\cdot5s^2 + 74\cdot97s + 1}{s^5 + 74\cdot97s^4 + 768\cdot5s^3 + 1218s^2 + 298\cdot5s + 10}. \tag{5}$$

In this paper, a rational approximation for $\sqrt{s}$ is obtained using continued fraction expansion. The rational approximation thus obtained is synthesized. In Section 2, realization of fractance device is presented. Numerical Simulations are presented in Section 3. Finally, Conclusions were drawn in Section 4.

2. REALIZATION

We have the continued fraction expansion for $(1+x)^\alpha$ as [9]

$$(1+x)^\alpha = \frac{1}{1-}\,\frac{\alpha x}{1+}\,\frac{(1+\alpha)x}{2+}\,\frac{(1-\alpha)x}{3+}\,\frac{(2+\alpha)x}{2+}\,\frac{(2-\alpha)x}{5+------}. \tag{6}$$

The above continued fraction expansion converges in the finite complex s-plane, along the negative real axis from $x = -\infty$ to $x = -1$. Substituting $x = s - 1$ and taking number of terms of (6), the calculated rational approximations for $\sqrt{s}$ are presented in Table 1. In order to get the rational approximation of $1/\sqrt{s}$, the expressions has to be simply reversed.

Table 1: Rational approximations for $\sqrt{s}$.

S. NO	No. of terms	Rational approximation
1	2	$\frac{3s+1}{s+3}$
2	4	$\frac{5s^2+10s+1}{s^2+10s+5}$
3	6	$\frac{7s^3+35s^2+21s+1}{s^3+21s^2+35s+7}$
4	8	$\frac{9s^4+84s^3+126s^2+36s+1}{s^4+36s^3+126s^2+84s+9}$
5	10	$\frac{11s^5+165s^4+462s^3+330s^2+55s+1}{s^5+55s^4+330s^3+462s^2+165s+11}$

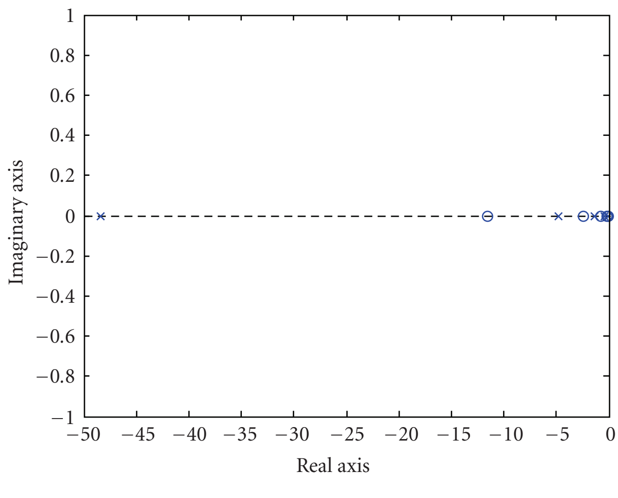

Figure 2: Pole-zero plot of $\sqrt{s}$.

Figures 1(a) and 1(b) compare the magnitude and phase responses of the rational approximations with the ideal one. It is observed from Figures 1 and 2 that fifth-order rational

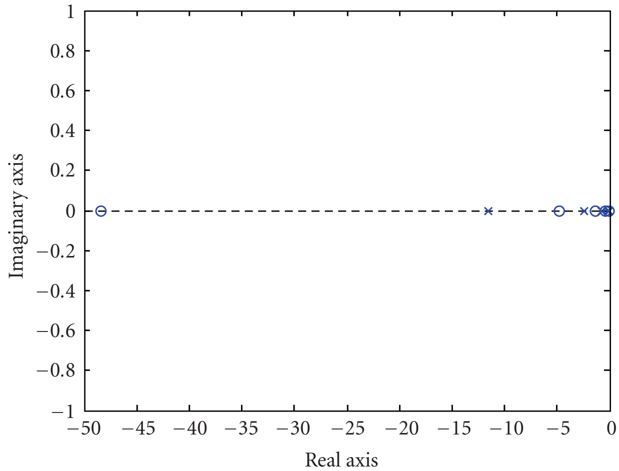

FIGURE 3: Pole-zero plot of $1/\sqrt{s}$.

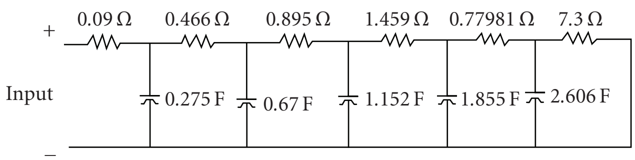

FIGURE 4: Passive realization of the fractance device.

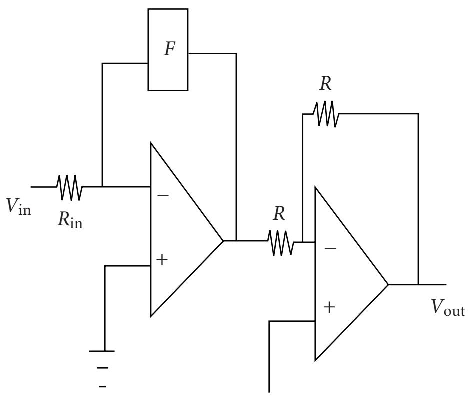

FIGURE 5: Active realization of the fractance device.

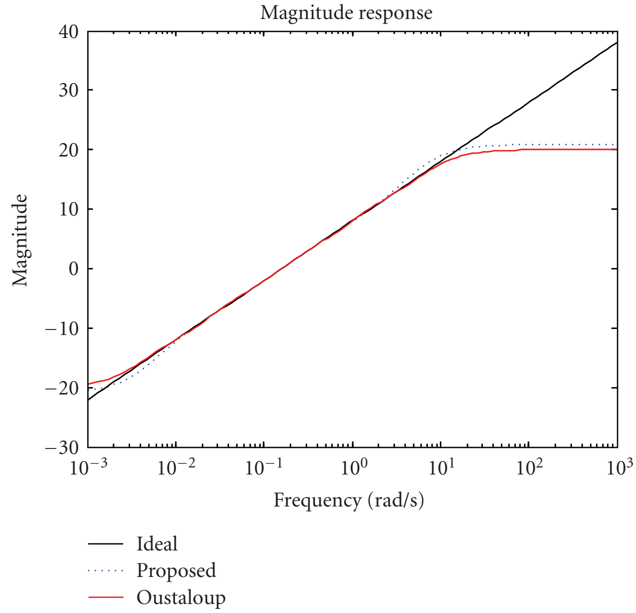

FIGURE 6: Magnitude response of $\sqrt{s}$.

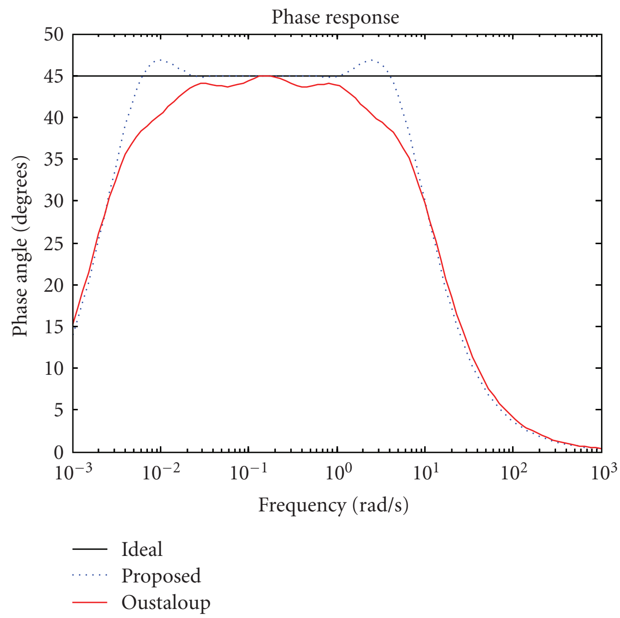

FIGURE 7: Phase response of $\sqrt{s}$.

approximation is best fit to ideal response up to certain range of frequencies. So,

$$\sqrt{s} = \frac{11s^5 + 165s^4 + 462s^3 + 330s^2 + 55s + 1}{s^5 + 55s^4 + 330s^3 + 462s^2 + 165s + 11},$$
$$\frac{1}{\sqrt{s}} = \frac{s^5 + 55s^4 + 330s^3 + 462s^2 + 165s + 11}{11s^5 + 165s^4 + 462s^3 + 330s^2 + 55s + 1}. \tag{7}$$

In order to check for the stability of the obtained rational approximations, pole-zero plot is drawn.

From Figures 2 and 3, it is evident that pole and zeros interlace on negative real axis making the system as stable one. So, the obtained rational approximation can be synthesized using RC or RL elements [10]. The realizaed active and passive networks are shown in Figures 5 and 4, respectively.

3. RESULTS

The following plots from Figures 6–11 compare the magnitude and phase responses for $s^{1/2}$ and $s^{-1/2}$ obtained using Oustaloup method and the proposed method.

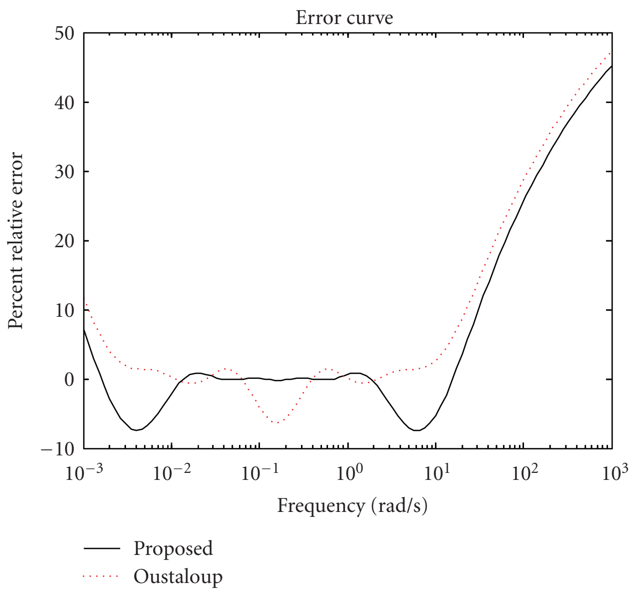

FIGURE 8: Error plot of $\sqrt{s}$.

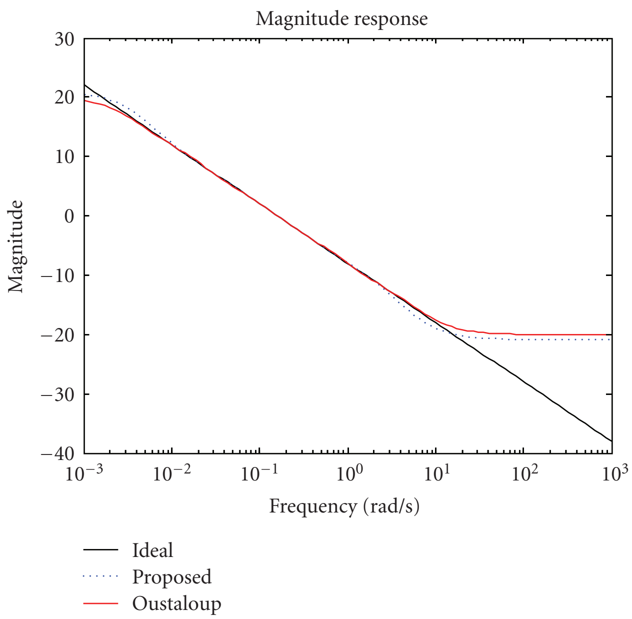

FIGURE 9: Magnitude response of $1/\sqrt{s}$.

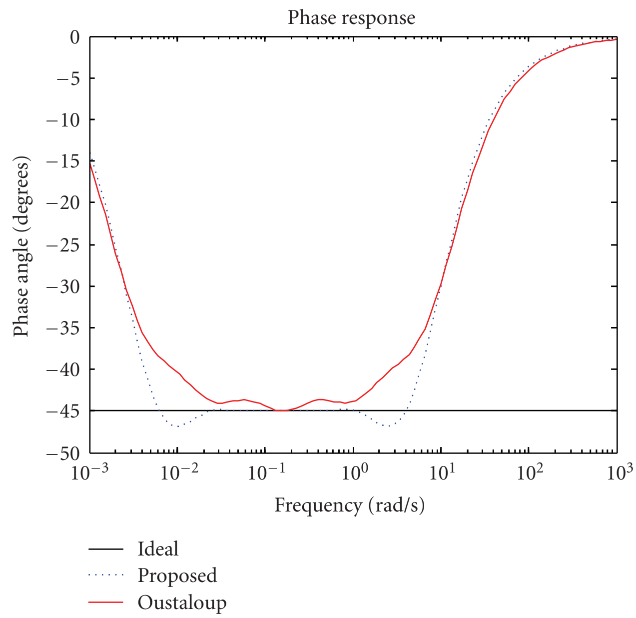

FIGURE 10: Phase response of $1/\sqrt{s}$.

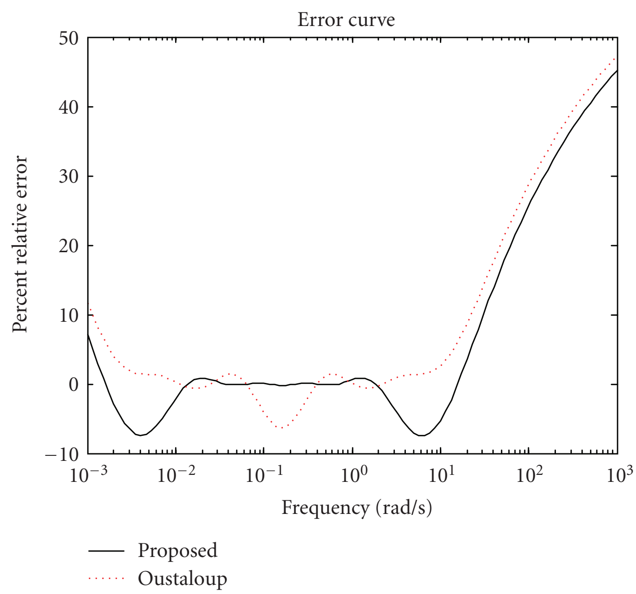

FIGURE 11: Error plot of $1/\sqrt{s}$.

4. CONCLUSIONS

Realization of fractance device of order 1/2 using continued fraction expansion is presented. From the results, it can be observed that the magnitude and phase responses have shown considerable improvement than compared to Oustaloup method. The percent relative error is almost zero for larger range of frequencies using proposed method. So, the proposed method can be used effectively for the realization like fractance device, fractional order differentiators, fractional order integrators, and so forth.

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