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A BRIEF DISCUSSION ON ENERGY CONSIDERATIONS FOR GaAs PARABOLIC QUANTUM DOTS

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The hamiltonian operator corresponding to a GaAs parabolic quantum dot is formulated by analyzing carefully the terms of this operator. In this formulation, certain plasma aspects are discussed with respect to the hamiltonian. Our study is referred to the absence of electric and magnetic fields.

Keywords: Hamiltonian operator; GaAs parabolic quantum dot; Plasma

1. INTRODUCTION

Quasi-zero-dimensional electron systems (quantum dots) present a great interest in the context of semiconducting nanostructures since these systems constitute a crucial element to conceive new electronic devices as well as to give rise to a considerable impact on existing devices [1–3]. But in spite of a large amount of experimental data collected to date, the basic physical mechanisms underlying this subject are not well understood so that important research efforts in theoretical work are needed. Moreover, new physical phenomena are expected to be explored referring to a number of possibilities in the technology field. In the following, we will present a brief discussion tending to clarify the basic aspects related to energy involved in a GaAs parabolic quantum dot in the absence of electric and magnetic fields.

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2. THEORY

Consider a quantum dot of electrons in the conduction band of n-type GaAs in the absence of electric and magnetic fields. Since this dot can be conceived as an artificial atom, we assume spherical symmetry for the dot so that the behaviour of the electrons within the dot is governed by the non-relativistic time-independent Schrödinger equation where the hamiltonian operator $\hat{H}$ reads:

$$\hat{H} \equiv \frac{1}{2m_n^*}\sum_{\alpha=1}^{N}\hat{p}_{r_\alpha}^2 + \frac{1}{2}m_n^*\omega_0^2\sum_{\alpha=1}^{N}\hat{r}_\alpha^2 + \frac{e^2}{4\pi\varepsilon}\sum_{\alpha<\beta}^{N}|\vec{r}_\alpha - \vec{r}_\beta|^{-1} \qquad (1)$$

where N is the number of electrons inside the dot, m_n^* stands for electron effective mass, ω_0 is the angular frequency of oscillation of the dot, $\hat{p}_{r_\alpha}$ and $\hat{r}_\alpha$ stand for radial momentum and position operators respectively, e denotes electron charge, and ε is permittivity (in our case, GaAs permittivity $\varepsilon \approx 12\,\varepsilon_0$, ε_0 being the permittivity of vaccum).

Putting $\hat{H}$ in a more explicit form, formula (1) becomes:

$$\hat{H} \equiv -\frac{\hbar^2}{2m_n^*}\sum_{\alpha=1}^{N}\left(\frac{d^2}{dr_\alpha^2} + \frac{2}{r_\alpha}\frac{d}{dr_\alpha}\right) + \frac{1}{2}m_n^*\omega_0^2\sum_{\alpha=1}^{N}r_\alpha^2 + \frac{e^2}{4\pi\varepsilon}\sum_{\alpha<\beta}^{N}|\vec{r}_\alpha - \vec{r}_\beta|^{-1} \qquad (2)$$

The first term of the right-hand side of expression (2) is the kinetic term. The second term represents the potential energy associated with the electrons in the dot as harmonic oscillators; this energy corresponds to the confining parabolic potential in the dot. Finally, the third term represents the Coulomb potential energy of interaction. With respect to the second term, notice the relevance of ω_0 which is given by the following semiclassical relationship $E_0 = (1/2)m_n^*\omega_0^2 r_0^2$ so that $\omega_0 = r_0^{-1}(2E_0/m_n^*)^{1/2}$ where r_0 is the radius of the dot and E_0 is the ground-state energy of an electron in the dot namely $E_0 = (3/2)\hbar\omega_0$; then, from the above formulae one gets $E_0 = 9\hbar^2/(2m_n^* r_0^2)$, $\omega_0 = 3\hbar/(m_n^* r_0^2)$. At this point, for a typical dot with $r_0 = 50$ nm and $N = 200$, and by using $\omega_0 = 3\hbar/(m_n^* r_0^2)$, $m_n^* \approx 0.067 \times 9.11 \times 10^{-31}$Kg, one obtains $\omega_0 \approx 2.07 \times 10^{12}$ rad/s, $n = N/((4/3)\pi r_0^3) \approx 3.82 \times 10^{17}\text{cm}^{-3}$ by assuming uniform carrier distribution. On the other hand, note that the third term of the right-hand side of formula (2) corresponds to electrostatic repulsion among electrons; electrostatic attraction between electrons and holes has not been included since we consider electrons in the conduction band. In addition, notice that the plasma frequency $\nu_0 = (\omega_0/2\pi) \approx 300\,\mathrm{GHz}$ corresponds, of course, to the ground state of the system (electrons in the valence band); in order to determine the oscillation frequency of the electron plasma at the bottom of the conduction band, we have $\omega_c = (E_c - E_0)/\hbar$ where E_c denotes energy corresponding to this band bottom. Finally, we want to remark that the electron density value $n \approx 3.82 \times 10^{17}\,\mathrm{cm}^{-3}$ corresponds to heavily doped n-type GaAs so that the sample considered above is degenerate.

3. CONCLUDING REMARKS

We have developed a qualitative discussion to clarify some aspects of problems that require considerable research efforts to elucidate them. The physics and technology of quantum dots constitute a very exciting field that may be considered as an incipient area with a number of aspects which are not well understood despite the great number of experimental data collected to date. Some work has been developed last decade in order to solve several open questions (see, for example, refs. [4–6]) but the number of remaining problems is considerably large. In particular, with respect to the above exposition, an interesting problem can be enunciated: the thermodynamic limit with respect to massive dots of an amount of electrons appreciably superior to $N = 200$.

References

[1] Smith III, T. P. (1990). *Surf. Sci.*, **229**, 239.
[2] Merkt, U. (1990). *Advances in Solid State Physics*, **30**, 77.
[3] Grado-Caffaro, M. A. and Grado-Caffaro, M., *Optik* (to be published).
[4] Bryant, G. W. (1988). *Surf. Sci.*, **196**, 596.
[5] Kash, K. (1990). *J. Luminescence*, **46**, 69.
[6] Grado-Caffaro, M. A. and Grado-Caffaro, M. (unpublished results, 1999).