DWT-Based Antenna Selection for Correlated MIMO Channels

Mohamed I. As-Saafani, Ehab F. Badran, Darwish A. Mohamed, and Shawki Shaaban

***Abstract*—**In this paper, we address the antenna selection problem in spatially correlated multiple-input multiple-output (MIMO) channels. To reduce the severe performance degradation of the traditional antenna selection scheme in correlated channels, we propose to embed discrete Wavelet transform (DWT) operations in the RF chains. The resulting system shows a significant improvement in for the capacity of spatial multiplexing schemes that reaching 5b/s/Hz, while requiring only a minor hardware overhead and low computational complexity.

***Keywords*—**Antenna selection, Discrete Wavelet Transform, MIMO systems, spatial fading.

I. INTRODUCTION

WIRELESS communications systems show large performance improvements when using MIMO systems [1], [2]. Industrially, there is a major drawback for those systems. This drawback is the requirement for multiple RF chains (one for each antenna element), which leads to high implementation costs. For this reason, researches [3] and [4], have suggested antenna selection algorithms that optimally choose a subset of the available transmit and/or receive antennas, and process the signals associated with those antennas. This allows us to combine a large number of low-cost antenna elements (e.g., patch or dipole antennas) with a small number of (high-cost) RF chains, allowing to maximally benefiting from the multiple antenna diversities within the RF cost constraint.

These antenna selection schemes work well for the uncorrelated MIMO channels (e.g., i.i.d. Rayleigh fading at each antenna element). However, most practically occurring cellular channels exhibit fading correlation due to a non-uniform power azimuth spectrum at the base station [5]—[7]. In [8], these antenna selection algorithms were observed in the correlated channels, the results showed a noticeable degradation in the performance. In [9] and [10], Molisch *et al* addressed such problem for spatially fading correlated channels.

Sometimes, there are non-available features of observed information signals at certain domain. Hence, the transformation of these signals from such domain to other domain may obtain more features. Molisch *et al* proposed embedded operations (DFT or FFT) in RF chains by a Butler matrix [9] and [10]. Our proposed technique employs DWT in which both space and time features are obtained. Our proposed scheme (DWT-based antenna selection) provides a significant improvement in the capacity of the spatial multiplexing, requires only a minor hardware overhead, and provides lower computational complexity than those proposed in [9] and [10].

The rest of the paper is organized as follows; section 2 describes the system model and the assumptions about the propagation channel. Next, we describe our new proposed technique for spatial multiplexing schemes. Simulation results and the performance evaluation of our proposed technique are shown in section 4. A summary and conclusions wrap up this paper.

II. CHANNEL MODEL

Fig. 1 shows the block diagram of the considered system. We denote M_T and M_R as the number of transmit and receive antenna elements respectively, and $\mathbf{H}$ is the $M_R \times M_T$ transfer matrix of the MIMO channel. We consider the case where the transmitter does not have channel state information (CSI), and the available transmitter power is equally distributed among the M_T transmit antennas. For equal-power transmission, the outage capacity of a MIMO channel is given by [1], [2]

$$C(\mathbf{H}) = \log_2\left[\det\left(\mathbf{I}_{M_R} + \tfrac{\rho}{M_T}\mathbf{H}\mathbf{H}^H\right)\right] \quad bps/Hz. \tag{1}$$

where ρ is the SNR per receive branch, $(\bullet)^H$ denotes the Hermitian transpose, $\mathbf{I}_{M_R}$ is the $M_R \times M_R$ identity matrix, det(•) stands for the matrix determinant. We assume uncorrelated zero-mean noise samples present at the receiver arms.

The channel fading is assumed to be block-fading, which remains fixed over a block of symbols and then changes to a new independent realization. Following the spatial channel model in [11], the $M_R \times M_T$ channel matrix is given by

$$\mathbf{H} = \mathbf{R}^{1/2}\,\mathbf{W}\,\mathbf{T}^{1/2} \tag{2}$$

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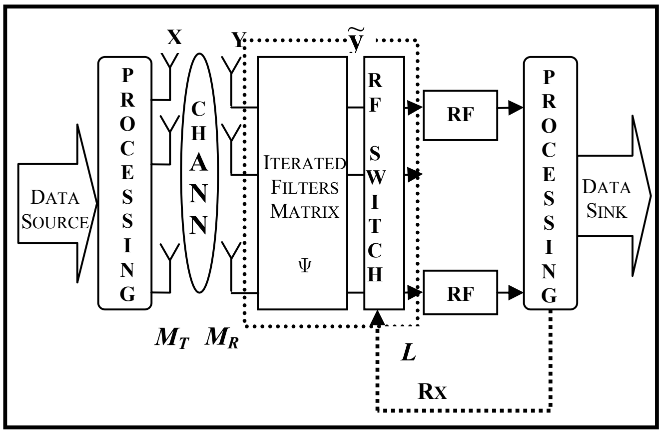


Fig. 1 Proposed DWT-based antenna selection scheme

where $\mathbf{W}$ is an $M_R \times M_T$ Rayleigh fading matrix with i.i.d. complex Gaussian entries $\sim N_c\,(0;1)$. The matrices $\mathbf{T}$ and $\mathbf{R}$ describe the $M_T \times M_T$ and $M_R \times M_R$ transmit and receive correlations, respectively. However, we consider the case of receive correlation only (BS side) and assume that $\mathbf{T} = \mathbf{I}_{M_T}$. Such an assumption is reasonable for the uplink of a cellular system since there are usually enough scatterers around the MS while at the BS the angular spread is small due to the high attitude. PAS (Power Azimuth Spectrum) of the AoA (Angle-of-Arrival) is assumed to be Gaussian distributed around the mean values: $\theta = \theta_o + \varepsilon;\ \varepsilon \sim N\,(0,\ \sigma_r^2)$. ULA is assumed at the receiver (BS) with the relative spacing (with respect to the carrier wavelength) is denoted as d_r. With small angle spread, the *(m,n)th* element of the receive correlation matrix is given by [12]

$$\mathbf{R}_{m,n} \approx e^{-j2\pi(n-m)d_r\cos\theta_r}\, e^{-\frac{1}{2}\left[2\pi(n-m)d_r\sigma_r\sin\theta_r\right]^2} \tag{3}$$

III. PROPOSED DWT-BASED ANTENNA SELECTION

The main objective is reducing the severe performance degradation of the traditional antenna selection scheme for spatially correlated MIMO channels. Our new proposed scheme, as shown in Fig. 1, is based on the wavelets decomposition operations [13] embedded in the RF chains by the iterated filters. Let us next give an intuitive argument for the use of the DWT. The output of the DWT can be regarded as "beams" oriented into different directions in space. Each beam implicitly has a beamforming gain proportional to the dimension of the DWT, which is M_R. In a strongly correlated channel, the scheme just picks the strongest beams, and is thus as good as MRC (maximal-ratio-combining). When the PAS is uniform, also DWT has an effect on the performance.

The new proposed scheme can be realized in a simple, low-cost way by placing a Wavelet matrix (iterated filters realize circular convolutions) between the antenna elements and the RF switch. For such, we send all received observation streams through a (spatial) wavelet transform before selection and down conversion. This can be implemented easily by means of a praymid algorithm of the iterated filters [13], which performs a DWT in the RF domain. The M_R point DWT matrix is illustrated by the $M_R \times M_R$ matrix $\mathbf{\Psi}$ as shown in Fig. 1, which is the form of Haar wavelets. $\mathbf{\Psi}$ is completely independent of the channel state. When $\mathbf{\Psi}$ is inserted before selection, the antenna selection is performed on the $M_R \times M_T$ virtual channel $\mathbf{\Psi H}$. Meanwhile, the noises $\mathbf{n}$ are also multiplied by $\mathbf{\Psi}$, which results in a (different) vector of i.i.d. Gaussian noise variables. The linear system model of the transmitted signal vector $\mathbf{x}$ and the wavelet transformed received signal vector $\widetilde{\mathbf{y}}$ can be modified as

$$\widetilde{\mathbf{y}} = \mathbf{\Psi H x} + \mathbf{\Psi n} \tag{4}$$

The capacity of a MIMO system using all antenna elements is given by (1). Our new scheme is now represented in the following mathematical description; the antenna selection is performed on the virtual correlated channel $\mathbf{\Psi H}$.

$$C_{DWT-Selection} = \max_{\widetilde{\mathrm{H}} \in S(\mathbf{\Psi H})} \left(\log_2 \left[\det \left(\mathbf{I}_{L_R} + \frac{\rho}{M_T} \widetilde{\mathbf{H}} \widetilde{\mathbf{H}}^H \right) \right] \right) \tag{5}$$

where $\widetilde{\mathbf{H}}$ is the $L_R \times M_T$ selected antenna matrix. If the singular value decomposition (SVD) is applied on the above equation, we can sense an increase in the singular values along with the rank of the channel matrix. These increments, directly, contribute to the effective degree of freedom (EDOF) of the channel and hence the performance.

On the other hand, the modified representation for the spatial structure of the virtual correlation channel is

$$\mathbf{\Psi H} = \mathbf{\Psi R^{1/2} W T^{1/2}} \tag{6}$$

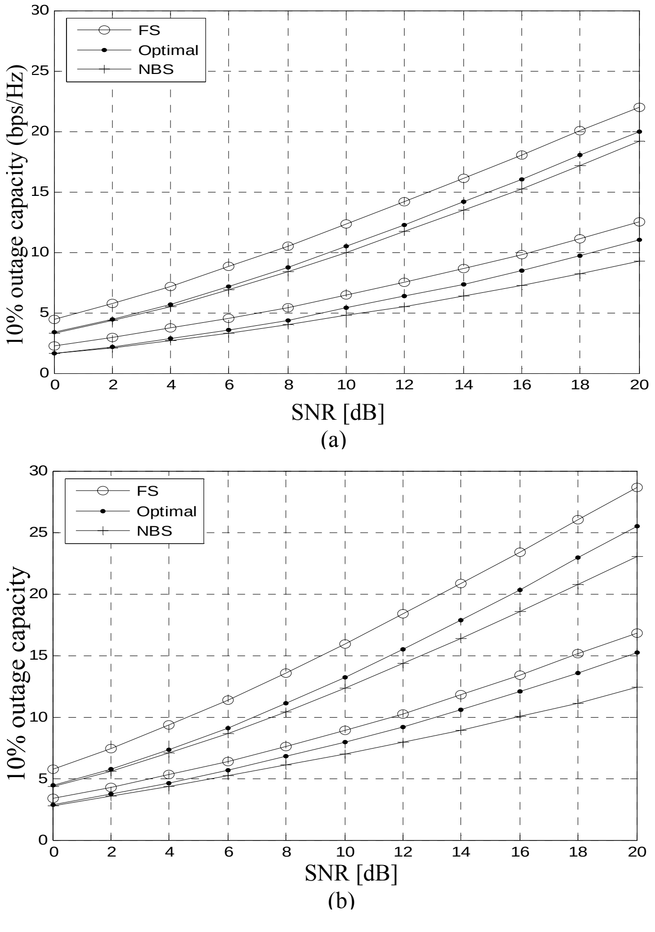
DWT matrix serves as asymptotically (as the number of antenna elements goes to infinity) optimal eigenmodes for the channel matrix, and thus the elements of the correlation matrix $\mathbf{R}$ constitute samples of the underlying spectral representation (due to the wavelets orthogonalization) and are hence uncorrelated. In addition, a significant drawback of the channel correlation matrix $\mathbf{H}$ is the size. DWT can be employed to reduce the computational complexity of the correlated fading matrix ($\mathbf{\Psi H}$).

IV. SIMULATION RESULTS

In this section, we present simulation results that validate our theoritical approach. For comparison, we show the performance of the DWT-based antenna selection in MIMO system with the performance of DFT/FFT-based antenna selection [9] and [10]. The spatial fading correlated channel and i.i.d channel are considered. The optimal and norm-based selection (NBS) algorithms [4] are considered in our simulations. Since receive antenna selection without any embedded operation, actually, incurs a loss in capacity, we also plot the capacity of the full-complexity system FS (no selection) for comparison purposes. Furthermore, FS and the pure optimal selection (without any embedded operation) will be used in our simulations for evaluation purposes.

For computer simulations (Monte Carlo simulations), we created 10,000 random realizations of mobile radio channels. For a given realization, each receive antenna branch has a range of SNR = [0 : 20] dB. In the following, MIMO system ($M_R \times M_T$) will be applied with $M_R = 8$ elements and $M_T = 3$ elements. ($M_R \times M_T$; L_R) is used to represent the MIMO systems with antenna selection and 10% outage capacity is considered. The first three levels of wavelets decomposition are considered and obtained in our simulations. We adopt the correlated channel model described in [11]. The relative antenna separation being 0.5. Varying the element spacing is not option, thus the angular spread is the only factor that influences the fading correlation and hence capacity. We also assume the "broadside" case as defined in [11].

Fig. 2 shows the performance of FS, optimal and NBS with our proposed DWT (Fig. 2(b)) compared without DWT (Fig. 2(a)). Two cases are cosidered: upper panel (i.i.d case) and lower panel (correlated case: $\sigma_r = 2^o$). Table I summurizes the performance evaluation of the proposed system.



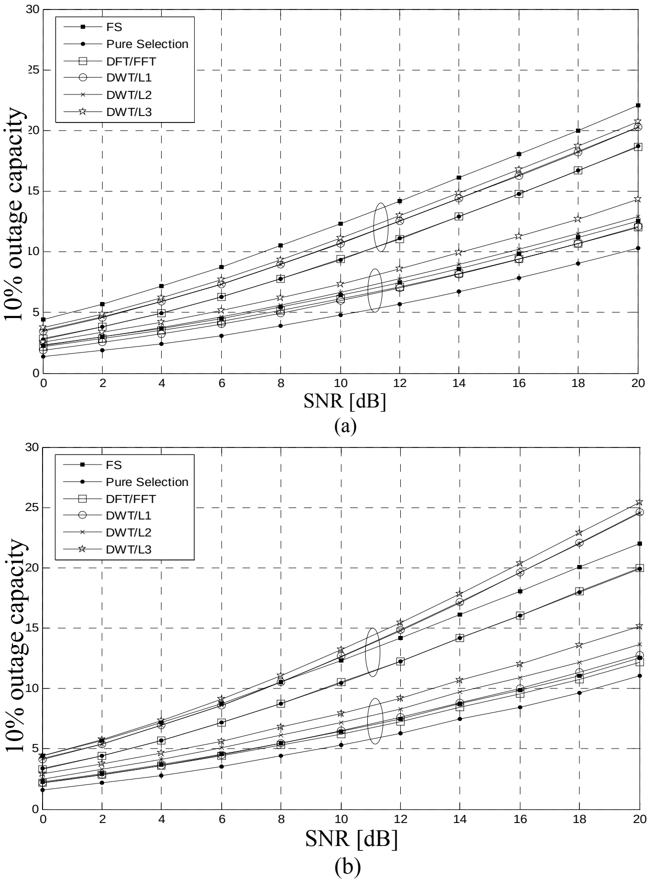
(a)

(b)

Fig. 2 10% Outage capacity vs. SNR for (8×3; 4) MIMO system:(a) without DWT-based and (b) with DWT-based

TABLE I
PERFORMANCE EVALUATION FOR OUR PROPOSED SYSTEM

Performance Evaluation			FS	Optimal	NBS
Without DWT	i.i.d b/s/Hz		22	19.95	19.06
	$\sigma_r = 2^o$ b/s/Hz		12.48	10.98	9.19
	%Degardation		43%	44.9%	51.7%
DWT-based	i.i.d	b/s/Hz	28.52	25.38	23.03
		Improv.%	29.6%	27.2%	20.8%
	$\sigma_r = 2^o$	b/s/Hz	16.83	15.25	12.39
		Improv.%	34.9%	38.9%	34.8%
	%Degardation		40.9%	39.9%	46%
Degardation decreasement			2.1%	5%	5.7%



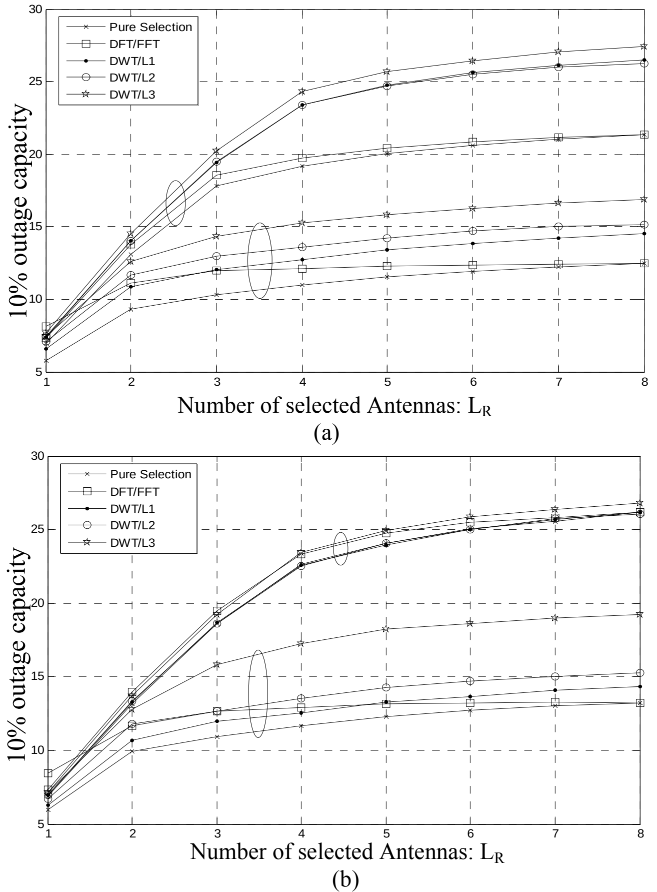
(a)

(b)

Fig. 3 10% Outage capacity vs. SNR for (a) (8×3; 3) and (b) (8×3; 4) [The upper panel (i.i.d) and the lower panel ($\sigma_r = 2^o$)]

Fig. 3 explains the influence of the wavelet decomposition on the channel matrix dimension. For comparison purposes; FS without DWT, optimal algorithm without DWT (pure selection) and optimal algorithm with DFT/FFT are plotted. Virtually, the dimension of **H** is increased by 1, thus observed system, virtually, will be ($M_R \times M_T + 1$), hence the Molisch condition ($L_R \geq M_T$) [4] must be followed. Explaining that as follows; DWT operation must be applied on the channel matrix **H** which the number of rows and columns is even. Otherwise, DWT will operate with added zero padding row or column. Notice that, there is an increase in the performance, exceeding the FS-without DWT, due to the increase of the decomposition level. However, our proposed DWT-based antenna selection outperforms DFT/FFT-based antenna selection.

Fig. 4 shows the 10% outage capacity of (8×3) and (8×4) MIMO systems, respectively, verses L_R [1: 8], reaching to FS (L_R = 8), at 20dB SNR. Fig. 4 considers the systems in correlated channel, (σ_r = 20°) and (σ_r = 2°). Notice that, in Fig. 4 (a), the performance for both pure optimal and optimal-DFT/FFT increases linearly with L_R until ($L_R = M_T$). Now, the effect of our proposed technique will appear clearly. The performance of optimal-DWT, for different level of decomposition, increases linearly with L_R until (L_R = rank ($\mathbf{\Psi H}$) = M_T + 1) where the number of transmit-antennas increased virtually by one because of the DWT decompoition. Hence, L_R must be, in design, equals M_T + 1 and this is only the negligible paid penalty, compared with optimal-DFT/FFT ($L_R = M_T$), to achieve the high performance. To make a fair comparison between DFT/FFT-based and our DWT-based, the number of elements at Tx is increased, M_T = 4, see Fig. 4 (b). When M_T is increased to be 4, DWT-based did not change the dimension of the decomposed channel matrix but the decomposed channel matrix coefficients, especially DWT/L3, are increased. Naturally the performance of optimal-DFT/FFT is improved.



(a)

(b)

Fig. 4 10% Outage capacity vs. L_R for : (a) 8×3 MIMO system and (b) 8×4 MIMO system, correlation cases: upper panel (σ_r =20°) and lower panel (σ_r =2°)

Fig. 5 shows the performance improvement with the angle spread variation at 20dB SNR with two values of L_R = (4 ,8). Notice that, at the pin-hole channel, $\boldsymbol{\sigma_r}$ = 0 (**R** collapse to a rank-1 matrix), there is no any improvement for DFT/FFT-based performance when L_R is increased from 4 to 8. On the opposite side, our proposed technique improves the pure optimal algorithm and incrementally changes its performance when L_R is increased from 4 to 8. That's mean the performance evaluation is reaching %73 in this hard environment. At FS case, L_R = 8, the performance for both DFT/FFT-based and pure optimal is the same for all angle spread range.

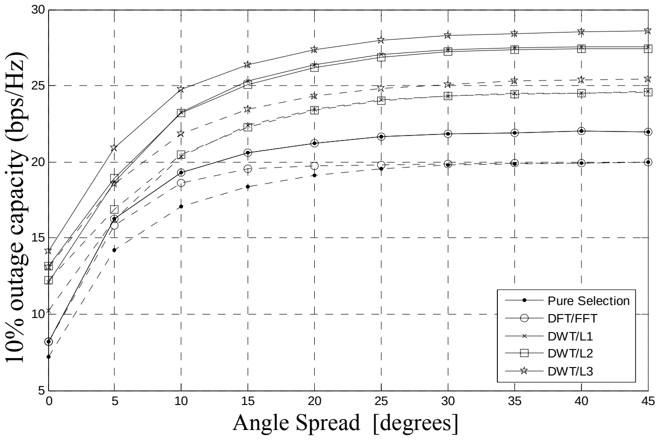


Fig. 5 10% Outage capacity vs. angle spread for MIMO system (8×3) with different L_R values: dot line (L_R = 4) and solid line (L_R = 8)

The main result is shown in Fig. 5 that DWT-based with L_R = 4 outperforms DFT/FFT-based with FS for all wide range of angular spread. Also, we can see the performance for all observed schemes is improving the pure optimal algorithm especially at the high correlation region, low values of the angle spread, and then they are saturated after certain value of angle spread. It should be noted, however, that for high values of angle spread, the observed schemes reach the capacity of i.i.d. channel and receive correlation matrix, **R**, approaches identity.

Now our proposed scheme will be applied on the slight, preferable and applicable antenna selection algorithm. The algorithm is NBS, which suggested to be used in 4G of cellular systems, because its lower complexity and simplicity while it did not need any estimation process in receiver [14] and it is a near-optimal at low SNR. When the proposed is applied on the optimal and NBS algorithms, optimal algorithm has %5≈ (5bps/Hz) evaluation in its performance, whereas the performance evaluation of NBS reaches %5.7≈ (3.5bps/Hz), see Table I. In the following Figs. 6 and 7, DWT/L3-based is compared with DFT/FFT-based for both optimal and NBS algorithms. In such figures, optimal algorithm is denoted by the black markers and NBS algorithm is mentioned by white markers; circle for pure selection, square for DFT/FFT-based and star for DWT-based. Also, FS, without DWT-based, is plotted for comparison purposes.

Fig. 6 shows %10 outage capacity verses SNR for (8×3; 4) MIMO system. As expected, in i.i.d. case, the performance of the FFT-based optimal algorithm and the pure optimal algorithm is the same for the wide range of SNR. Under this performance curve, also, the performance of the FFT-based

NBS algorithm and the pure NBS algorithm is the same for all range of SNR. On the opposite, DWT-based provides improvements for both optimal and NBS algorithms, which they are exceeding FS curve. On the other side, if the correlation case is considered, the pure optimal and NBS algorithms behaviors are, also, clearly degraded due to the high correlation. The advantage of using the bases DFT/FFT and DWT are clearly obtained. Notice that, DWT-based NBS algorithm is exceeding FS and outperforming FFT-based optimal algorithm for all SNR values. Table II ensures this result and summarizes the evaluation performance of such schemes behavior at 20dB SNR. The evaluation is related to the pure optimal and NBS algorithms for two cases of the channel (i.i.d and correlation). There is a significant advantage for DWT-based compared to DFT/FFT-based. The improvements in the performance of DWT- optimal are about %27 (5.4 bps/Hz) and %38.9 (4.27 bps/Hz) for i.i.d. and correlated channels, respectively. The evaluation for DWT-NBS is about %20.8 (3.85 bps/Hz) and %34.8 (3.15 bps/Hz) for i.i.d. and correlated channels, respectively.

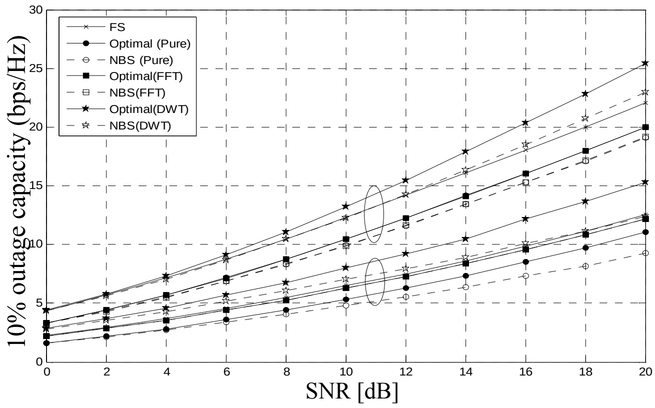


Fig. 6 10% Outage capacity vs. SNR for (8×3; 4) MIMO system with Optimal and NBS for i.i.d case (upper panel) and correlated case (lower panel)

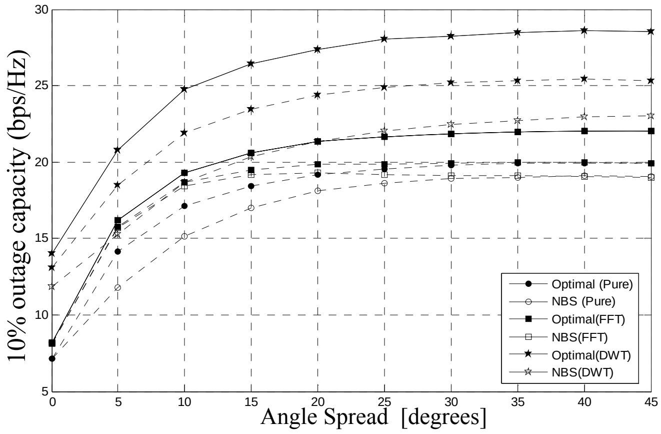


Fig. 7 10% Outage capacity vs. σ_r for (8×3) MIMO system with Optimal and NBS for two different values of L_R: $L_R = 4$ (dotted line)

TABLE II
PERFORMANCE EVALUATION FOR OUR PROPOSED SYSTEM COMPARED TO FFT-BASED FOR (8×3; 4) WITH OPTIMAL AND NBS ALGORITHMS (AT SNR=20DB)

Performance Evaluation			Optimal	NBS
FFT-based	i.i.d	b/s/Hz	19.98	19.18
		Improv.%	0.00	0.00
	$\sigma_r = 2^o$	b/s/Hz	12.19	12.19
		Improv.%	11%	31.9%
	%Degardation		39%	36.4%
DWT/ L3 - based	i.i.d	b/s/Hz	25.38	23.03
		Improv.%	27.2%	20.8%
	$\sigma_r = 2^o$	b/s/Hz	15.25	12.39
		Improv.%	38.9%	34.8%
	%Degardation		39.9%	46%
Degardation decreasement	i.i.d		27.2%	20.8%
	$\sigma_r = 2^o$		27.9%	3.36%

Finally, Fig. 7 shows the variation of capacity verses the angle spread at Rx, $\boldsymbol{\sigma_r}$, with two observation values of L_R (4 and 8) at reasonable SNR=20dB. Notice that, the performance of DWT-based Optimal and NBS algorithms with $L_R = 4$ outperform the performance of FFT-based Optimal, with FS case, for all wide range of angular spread except very short zone.

Industry trends to the smallest packaging of devices. NBS did not need any estimation in receiver and it provides a significant performance as antenna selection technique to overcome the problem of MIMO systems complexity under correlated channels.

For our simple example of (8×3; 4) MIMO system, NBS reaches 23b/s/Hz in i.i.d. channel while it has about 12b/s/Hz and 22b/s/Hz for high and low correlated channel, respectively. Further, MIMO technique is applied on mobile system with wide bandwidth, i.e. the bandwidth of WCDMA is nearly 10MHz, to approach the capacities about 120Mbps to 220Mbps. While WLAN, especially indoor communications, has the bandwidth of 40MHz then the rate is reaching order of 800Mbps.

V. CONCLUSION

The performance loss due to antenna subset selections in MIMO systems, especially in the highly correlated channels, is very important issue. The paper addresses this problem and provides a proposed system, which is the embedded DWT operation in RF chains. DWT-based is a minor hardware overhead to the complexity of the system w.r.t. the out coming evaluation in the performance, optimal algorithm has %5 evaluation in its performance, whereas the performance evaluation of NBS reaches %5.7. DWT-based provides more improvements, lower hardware and computational complexity than DFT/FFT-based.

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