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A perturbation of m-order derivations on Banach algebras

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Abstract

Let $\mathcal{A}$ be a unital Banach algebra and let m, $1 \le m \le 4$, be an integer. If $f : \mathcal{A} \to \mathcal{A}$ is an approximate m-order derivation in the sense of Hyers-Ulam-Rassias, then $f : \mathcal{A} \to \mathcal{A}$ is an exact m-order derivation.

Key Words: m-order derivation, approximate m-order derivation, stability.

1. Introduction

The study of stability problems in the case of homomorphisms between metric groups originated from a famous talk given by S.M. Ulam [24] in 1940: *Under what condition does there exists a homomorphism near an approximate homomorphism?* In 1941, D.H. Hyers [8] answered affirmatively the question of Ulam for Banach spaces, which states that *if* $\delta > 0$ *is real number and* $f : \mathcal{X} \to \mathcal{Y}$ *is a mapping with* $\mathcal{X}$ *a normed space,* $\mathcal{Y}$ *a Banach space such that*

$$||f(x+y) - f(x) - f(y)|| \le \delta$$

for all $x, y \in \mathcal{X}$, *then there exists a unique additive mapping* $T : \mathcal{X} \to \mathcal{Y}$ *such that*

$$||f(x) - T(x)|| \le \delta$$

for all $x \in \mathcal{X}$. This stability phenomenon is called the *Hyers-Ulam stability* of the additive functional equation $f(x+y) = f(x) + f(y)$.

A generalized version of the theorem of Hyers for approximately additive mappings was given by T. Aoki [2] in 1950 and by Th.M. Rassias [17] in 1978 for linear mappings, respectively and the result is as follows:

If $f : \mathcal{X} \to \mathcal{Y}$ *is a mapping and there exist real numbers* $\theta \ge 0$ *and* $0 \le p < 1$ *such that*

$$\|f(x+y) - f(x) - f(y)\| \le \theta(\|x\|^p + \|y\|^p)$$

for all $x, y \in \mathcal{X}$, *then there exists a unique additive mapping* $T : \mathcal{X} \to \mathcal{Y}$ *such that*

$$\|f(x) - T(x)\| \le \frac{2\theta}{2 - 2^p}\|x\|^p$$

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for all $x \in \mathcal{X}$ *.*

On this fact, some authors say that the additive functional equation $f(x+y) = f(x) + f(y)$ has the Hyers-Ulam-Rassias stability property [5, 9, 11, 19, 20]. In 1991, Z. Gajda [6] answered the question for the case $p > 1$, which was raised by Th.M. Rassias [18]. Z. Gajda [6] gave an example to prove that it is not possible to prove a Th.M. Rassias's stability Theorem for the case when $p = 1$. Independently, a different new example was given by Th.M. Rassias and P. Semrl [21].

Let $\mathcal{A}$ be an algebra over the real or complex field $\mathbb{F}$. An additive map $d : \mathcal{A} \to \mathcal{A}$ is said to be a *ring derivation* if the functional equation $d(xy) = xd(y) + d(x)y$ holds for all $x, y \in \mathcal{A}$.

Recently, T. Miura *et al.* [15] examined the stability of ring derivations on Banach algebras:

Suppose that $\mathcal{A}$ *is a Banach algebra. Let* $p \geq 0$ *and* $\varepsilon \geq 0$ *be real numbers. If* $p \neq 1$ *and* $f : \mathcal{A} \to \mathcal{A}$ *is a mapping such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \varepsilon(\|x\|^p + \|y\|^p)$$

for all $x, y \in \mathcal{A}$*, and*

$$\|f(xy) - xf(y) - f(x)y\| \leq \varepsilon\|x\|^p\|y\|^p$$

for all $x, y \in \mathcal{A}$*, then there exists a unique ring derivation* $d : \mathcal{A} \to \mathcal{A}$ *such that*

$$\|f(x) - d(x)\| \leq \frac{2\varepsilon}{|2 - 2^p|}\|x\|^p$$

for all $x \in \mathcal{A}$*. In particular, if* $\mathcal{A}$ *is a Banach algebra without order, then* f *is an ring derivation.*

The stability result concerning derivations was first obtained by P. Šemrl [22] in operator algebras and various results for the stability of derivations have been obtained by many authors (for instances, [3, 4, 12, 13]).

Let $f : \mathcal{X} \to \mathcal{Y}$ be a mapping with $\mathcal{X}, \mathcal{Y}$ two vector spaces and let

$$D^m f(x,y) := \begin{cases} f(x+y) - f(x) - f(y), & \text{if } m = 1 \\ f(x+y) + f(x-y) - 2f(x) - 2f(y), & \text{if } m = 2 \\ f(2x+y) + f(2x-y) - 2f(x+y) & \text{if } m = 3 \\ \quad -2f(x-y) - 12f(x), & \\ f(2x+y) + f(2x-y) - 4f(x+y) - 4f(x-y) & \text{if } m = 4 \\ \quad -24f(x) + 6f(y), & \end{cases}$$

For each integer m, $1 \leq m \leq 4$, the functional equation $D^m f(x,y) = 0$ is said to be *additive, quadratic, cubic* [10] and *quartic* [14], respectively. For convenience' sake, a solution of the functional equation $D^m f(x,y) = 0$ will be called an m*-order mapping.*

In particular, the quadratic functional equation is used to characterize inner product spaces [1]. The Hyers-Ulam stability of quadratic functional equations was first proved by F. Skof [23]. S. Czerwik [5], K. W. Jun and H. M. Kim [10], obtained the Hyers-Ulam-Rassias stability result for the quadratic and cubic functional equation, respectively.

On the other hand, S.H. Lee *et. al.* [14] proved the Hyers-Ulam stability of the quartic functional equation. Using the Hyers' direct method in as the proof of [14, Theorem 3.1], we obtain the Hyers-Ulam-Rassias stability result for the quartic functional equation. Hence we have the following:

Proposition 1.1 *For each integer m, $1 \le m \le 4$, let $0 \le p \ne m$ and $\delta \ge 0$ be real numbers. If $f : \mathcal{X} \to \mathcal{Y}$ is a mapping with $\mathcal{X}$ a normed space, $\mathcal{Y}$ a Banach space such that*

$$\|D^m f(x,y)\| \le \delta(\|x\|^p + \|y\|^p),$$

for all $x, y \in \mathcal{X}$, then there exists a unique m-order mapping $T : \mathcal{X} \to \mathcal{Y}$ such that

$$\|f(x) - T(x)\| \le k\delta\|x\|^p$$

for all $x \in \mathcal{X}$, where: when $m = 1$, $k = \frac{2}{|2-2^p|}$ if $p \ne 1$, when $m = 2, 3$, $k = \frac{m}{|m^m - m^p|}$ if $p \ne m$ and when $m = 4$, $k = \frac{1}{2|2^4-2^p|}$ if $p \ne 4$.

We here introduce the following mapping:

An m-order mapping $\Delta : \mathcal{A} \to \mathcal{A}$ will be called an *m-order derivation* if the equality $\Delta(xy) = x^m\Delta(y) + \Delta(x)y^m$ is fulfilled for all $x, y \in \mathcal{A}$. As a simple example, let us consider the algebra of 2×2 matrices

$$\mathcal{A} = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : \; a, b \in \mathbb{C} \right\},$$

where $\mathbb{C}$ is a complex field. Then it is easy to see that the mapping $\Delta : \mathcal{A} \to \mathcal{A}$ defined by

$$\Delta\left(\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \right) = \begin{bmatrix} 0 & b^m \\ 0 & 0 \end{bmatrix}$$

is an m-order derivation, where m, $1 \le m \le 4$, is an integer.

It is natural to ask that there exists an approximate m-order derivation which is not an exact m-order derivation. The following example is a slight modification of an example due to [15].

Example 1.2 *Let X be a compact Hausdorff space and let $C(X)$ be the commutative Banach algebra of complex-valued continuous functions on X under pointwise operations and the supremum norm $\|\cdot\|_\infty$. We define $f : C(X) \to C(X)$ by*

$$f(a)(x) = \begin{cases} a(x)^m \log|a(x)| & \text{if } a(x) \ne 0, \\ 0 & \text{if } a(x) = 0 \end{cases}$$

for all $a \in C(X)$ and all $x \in X$, where m, $1 \le m \le 4$, is an integer. It is easy to see that

$$f(ab) = a^m f(b) + f(a) b^m$$

for all $a, b \in C(X)$.

Note that the following inequality holds for all $a \in C(X)$ with $a(x) \ne 0$:

$$|f(a)(x)| = |a(x)|^m \big|\log|a(x)|\big| \le \big(1 + |a(x)|\big)^{m+1} \le (1 + \|a\|_\infty)^{m+1}.$$

Hence we have $\|f(a)\|_\infty \le (1+\|a\|_\infty)^{m+1}$ *for all* $a \in C(X)$. *Using this inequality and the triangle inequality, we deduce that*

$$\|D^m f(a,b)\|_\infty \le M(a,b)$$

for all $a, b \in C(X)$, *where*

$$M(a,b) = \begin{cases} 3(1+\|a\|_\infty+\|b\|_\infty)^2 & \text{if } m=1, \\ 6(1+\|a\|_\infty+\|b\|_\infty)^3 & \text{if } m=2, \\ 18(1+2\|a\|_\infty+\|b\|_\infty)^4 & \text{if } m=3, \\ 40(1+2\|a\|_\infty+\|b\|_\infty)^5 & \text{if } m=4. \end{cases}$$

Hence we may regard f *as an approximate* m*-order derivation on* $C(X)$.

It will be of interest to investigate the stability problem of m-order derivations on Banach algebras as in the case of ring derivations. That is, the purpose of this paper is to prove the Hyers-Ulam-Rassias stability and the superstability of m-order derivations on Banach algebras.

2. Stability of m-order derivations

In this section, let $\mathbb{R}$ be the real field. $\mathbb{Q}$ and $\mathbb{N}$ will denote the set of the rational, the natural numbers, respectively and m, $1 \le m \le 4$, is an integer

Lemma 2.1 *Suppose that* $\mathcal{A}$ *is a Banach algebra. Let* $\delta, \varepsilon \ge 0$ *be real numbers and let* $p, q \ge 0$ *be real numbers with either* $p, q < m$ *or* $p, q > m$. *If* $f : \mathcal{A} \to \mathcal{A}$ *is a mapping such that*

$$\|D^m f(x,y)\| \le \delta(\|x\|^p + \|y\|^p) \tag{2.1}$$

for all $x, y \in \mathcal{A}$, *and*

$$\|f(xy) - x^m f(y) - f(x) y^m\| \le \varepsilon \|x\|^q \|y\|^q \tag{2.2}$$

for all $x, y \in \mathcal{A}$, *then there exists a unique* m*-order derivation* $\Delta : \mathcal{A} \to \mathcal{A}$ *such that*

$$\|f(x) - \Delta(x)\| \le k\delta\|x\|^p \tag{2.3}$$

for all $x \in \mathcal{A}$, *where: when* $m = 1$, $k = \frac{2}{|2-2^p|}$ *if* $p \ne 1$, *when* $m = 2, 3$, $k = \frac{m}{|m^m - m^p|}$ *if* $p \ne m$ *and when* $m = 4$, $k = \frac{1}{2|2^4-2^p|}$ *if* $p \ne 4$.

Proof. Assume that either $p, q < m$ or $p, q > m$. From Proposition 1.1, the inequality (2.1) guarantees that there exists a unique m-order mapping $\Delta : \mathcal{A} \to \mathcal{A}$ such that (2.3) holds for all $x \in \mathcal{A}$, where: when $m = 1$, $k = \frac{2}{|2-2^p|}$ if $p \ne 1$, when $m = 2, 3$, $k = \frac{m}{|m^m - m^p|}$ if $p \ne m$ and when $m = 4$, $k = \frac{1}{2|2^4-2^p|}$ if $p \ne 4$. We claim that

$$\Delta(xy) = x^m \Delta(y) + \Delta(x) y^m$$

for all $x, y \in \mathcal{A}$.

Set $\tau=1$ if $p,q<m$ and $\tau=-1$ if $p,q>m$. Since Δ is an m-order mapping, from [1, Proposition 1, p. 166], [10, Theorem 2.1] and [14, Theorem 2.1], we see that $\Delta(x)=2^{-\tau mn}\Delta(2^{\tau n}x)$ for all $x\in\mathcal{A}$ and all $n\in\mathbb{N}$. First, it follows from (2.3) that

$$\begin{aligned}\|2^{-\tau mn}f(2^{\tau n}x)-\Delta(x)\| &= 2^{-\tau mn}\|f(2^{\tau n}x)-\Delta(2^{\tau n}x)\|\\ &\le 2^{-\tau mn}k\delta\|2^{\tau n}x\|^p = 2^{\tau(p-m)n}k\delta\|x\|^p\end{aligned}$$

for all $x\in\mathcal{A}$ and all $n\in\mathbb{N}$. Since $\tau(p-m)<0$, we have

$$\|2^{-\tau mn}f(2^{\tau n}x)-\Delta(x)\|\to 0 \quad \text{as } n\to\infty. \tag{2.4}$$

Following the similar argument as the above, we obtain

$$\|2^{-2\tau mn}f(2^{2\tau n}xy)-\Delta(xy)\|\le 4^{\tau(p-m)n}k\delta\|xy\|^p$$

for all $x,y\in\mathcal{A}$ and all $n\in\mathbb{N}$, and so

$$\|2^{-2\tau mn}f(2^{2\tau n}xy)-\Delta(xy)\|\to 0 \quad \text{as } n\to\infty. \tag{2.5}$$

Since f satisfies (2.2), we get

$$\begin{aligned}&\|2^{-2\tau mn}f(2^{2\tau n}xy)-2^{-\tau mn}x^m f(2^{\tau n}y)-f(2^{\tau n}x)2^{-\tau mn}y^m\|\\ &= 2^{-2\tau mn}\|f((2^{\tau n}x)(2^{\tau n}y))-(2^{\tau n}x)^m f(2^{\tau n}y)-f(2^{\tau n}x)(2^{\tau n}y)^m\|\\ &\le 2^{-2\tau mn}\varepsilon\|2^{\tau n}x\|^q\|2^{\tau n}y\|^q = 4^{\tau(q-m)n}\varepsilon\|x\|^q\|y\|^q\end{aligned}$$

for all $x,y\in\mathcal{A}$ and all $n\in\mathbb{N}$. Reminding that $\tau(q-m)<0$, we obtain

$$\|2^{-2\tau mn}f(2^{2\tau n}xy)-2^{-\tau mn}x^m f(2^{\tau n}y)-f(2^{\tau n}x)2^{-\tau mn}y^m\|\to 0 \quad \text{as } n\to\infty. \tag{2.6}$$

Using (2.4), (2.5) and (2.6), we now see that

$$\begin{aligned}&\|\Delta(xy)-x^m\Delta(y)-\Delta(x)y^m\|\\ &\le \|\Delta(xy)-2^{-2\tau mn}f(2^{2\tau n}xy)\|\\ &\quad+\|2^{-2\tau mn}f(2^{2\tau n}xy)-2^{-\tau mn}x^m f(2^{\tau n}y)-2^{-\tau mn}f(2^{\tau n}x)y^m\|\\ &\quad+\|2^{-\tau mn}x^m f(2^{\tau n}y)-x^m\Delta(y)\|+\|2^{-\tau mn}f(2^{\tau n}x)y^m-\Delta(x)y^m\|\\ &\le \|\Delta(xy)-2^{-2\tau mn}f(2^{2\tau n}xy)\|\\ &\quad+\|2^{-2\tau mn}f(2^{2\tau n}xy)-2^{-\tau mn}x^m f(2^{\tau n}y)-2^{-\tau mn}f(2^{\tau n}x)y^m\|\\ &\quad+\|x^m\|\|2^{-\tau mn}f(2^{\tau n}y)-\Delta(y)\|+\|2^{-\tau mn}f(2^{\tau n}x)-\Delta(x)\|\|y^m\|\to 0 \quad \text{as } n\to\infty,\end{aligned}$$

which implies that $\Delta(xy)=x^m\Delta(y)+\Delta(x)y^m$ for all $x,y\in\mathcal{A}$. That is, Δ is an m-order derivation on $\mathcal{A}$, as claimed and the proof is complete. □

Lemma 2.2 *Suppose that $\mathcal{A}$ is a unital Banach algebra. Let $\delta, \varepsilon \geq 0$ be real numbers and let $p, q \geq 0$ be real numbers with either $p, q < m$ or $p, q > m$. If $f : \mathcal{A} \to \mathcal{A}$ is a mapping satisfying (2.1) and (2.2), then we have*

$$f(rx) = r^m f(x)$$

for all $x \in \mathcal{A}$ and all $r \in \mathbb{Q}$.

Proof. In the case when $r = 0$, it is trivial since $f(0) = 0$ by (2.1) or (2.2). Let e be a unit element of $\mathcal{A}$ and $r \in \mathbb{Q} \setminus \{0\}$ arbitrarily. Put $\tau = 1$ if $p, q < m$ and $\tau = -1$ if $p, q > m$. By Lemma 2.1, there exists a unique m-order derivation $\Delta : \mathcal{A} \to \mathcal{A}$ such that (2.3) is true. Recall that Δ is an m-order mapping, and hence it is easy to see that $\Delta(rx) = r^m \Delta(x)$ for all $x \in \mathcal{A}$ in view of [1, Proposition 1, p. 166], [10, Theorem 2.1] and [14, Theorem 2.1]. Then we get

$$\begin{aligned} &\|\Delta((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq r^m \|\Delta(2^{\tau n}ex) - f(2^{\tau n}ex)\| + r^m \|f(2^{\tau n}ex) - 2^{\tau mn} e f(x) - f(2^{\tau n}e)x^m\| \end{aligned} \tag{2.7}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. Now the inequalities (2.2), (2.3) and (2.7) yields that

$$\begin{aligned} &\|\Delta((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq r^m 2^{\tau np} k\delta \|x\|^p + r^m 2^{\tau nq} \varepsilon \|x\|^q \end{aligned} \tag{2.8}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$.

It follows from (2.3) and (2.8) that

$$\begin{aligned} &\|f((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq \|f((2^{\tau n}e)(rx)) - \Delta((2^{\tau n}e)(rx))\| \\ &\quad + \|\Delta((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq 2^{\tau np}(r^p + r^m) k\delta \|x\|^p + r^m 2^{\tau nq} \varepsilon \|x\|^q \end{aligned}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. That is, we have

$$\begin{aligned} &\|f((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq 2^{\tau np}(r^p + r^m) k\delta \|x\|^p + r^m 2^{\tau nq} \varepsilon \|x\|^q \end{aligned} \tag{2.9}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. From (2.2) and (2.9), we obtain

$$\begin{aligned} &\|2^{\tau mn}\{f(rx) - r^m f(x)\}\| \\ &= \|2^{\tau mn} e\{f(rx) - r^m f(x)\}\| \\ &\leq \|2^{\tau mn} e f(rx) + f(2^{\tau n}e) r^m x^m - f((2^{\tau n}e)(rx))\| \\ &\quad + \|f((2^{\tau n}e)(rx)) - r^m 2^{\tau mn} e f(x) - f(2^{\tau n}e) r^m x^m\| \\ &\leq \varepsilon \|2^{\tau n}e\|^q \|rx\|^q + 2^{\tau np}(r^p + r^m) k\delta \|x\|^p + r^m 2^{\tau nq} \varepsilon \|x\|^q \\ &= 2^{\tau np}(r^p + r^m) k\delta \|x\|^p + 2^{\tau nq}(r^q + r^m) \varepsilon \|x\|^q \end{aligned}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. This means that

$$\begin{aligned} &\|f(rx) - r^m f(x)\| \\ &\leq 2^{\tau(p-m)n}(r^p + r^m)k\delta\|x\|^p + 2^{\tau(q-m)n}(r^q + r^m)\varepsilon\|x\|^q \end{aligned} \tag{2.10}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. Since $\tau(p-m) < 0$ and $\tau(q-m) < 0$, if we take $n \to \infty$ in (2.10), then we arrive at

$$f(rx) = r^m f(x)$$

for all $x \in \mathcal{A}$. This completes the proof, since $r \in \mathbb{Q} \setminus \{0\}$ was arbitrary. □

Remark. In Lemma 2.2, if f is continuous, then it is easy to observe that $f(tx) = t^m f(x)$ for all $x \in \mathcal{A}$ and all $t \in \mathbb{R}$.

Now we are ready to prove our main result.

Theorem 2.3 *Suppose that $\mathcal{A}$ is a unital Banach algebra. Let $\delta, \varepsilon \geq 0$ be real numbers and let $p, q \geq 0$ be real numbers with either $p, q < m$ or $p, q > m$. If $f : \mathcal{A} \to \mathcal{A}$ is a mapping satisfying* (2.1) *and* (2.2)*, then $f : \mathcal{A} \to \mathcal{A}$ is an m-order derivation.*

Proof. Let Δ be a unique m-order derivation as in Lemma 2.2. Put $\tau = 1$ if $p, q < m$ and $\tau = -1$ if $p, q > m$. Since $f(2^{\tau n}x) = 2^{\tau m n} f(x)$ for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$ by Lemma 2.2, it follows from (2.3) that

$$\begin{aligned} \|f(x) - \Delta(x)\| &= \|2^{-\tau mn} f(2^{\tau n}x) - 2^{-\tau mn}\Delta(2^{\tau n}x)\| \\ &\leq 2^{-\tau mn} k\delta \|2^{\tau n}x\|^p \\ &= 2^{\tau(p-m)n} k\delta\|x\|^p \end{aligned}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. Namely,

$$\|f(x) - \Delta(x)\| \leq 2^{\tau(p-m)n} k\delta\|x\|^p \tag{2.11}$$

for all $x \in \mathcal{A}$ and all $n \in \mathbb{N}$. Since $\tau(p-m) < 0$, by letting $n \to \infty$ in (2.11), we conclude that $f(x) = \Delta(x)$ for all $x \in \mathcal{A}$ which implies that f is an m-order derivation. □

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