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Rickart-type Annihilator Conditions on Formal Power Series

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Abstract

Let R be an α-rigid ring and $R_0[[x;\alpha]]$ be the nearring of a formal skew power series in which addition and substitution are used as operations. It is shown that R is Rickart and any countable family of idempotents of R has a join in $I(R)$ if and only if $R_0[[x;\alpha]] \in \mathcal{R}_{r1}$ if and only if $R_0[[x;\alpha]] \in \mathcal{R}_{\ell 1}$ if and only if $R_0[[x;\alpha]] \in q\mathcal{R}_{r2}$. An example to show that, α-rigid condition on R is not superfluous, is provided.

Key Words: Annihilator conditions; Nearrings; Skew power series; Baer rings; α-rigid rings; Rickart rings.

1. Introduction

Throughout this paper all rings are associative with unity and all nearrings are left nearrings. We use R and N to denote a ring and a nearring respectively. Recall from [15] that ring R is *Baer* if R has unity and the right annihilator of every nonempty subset of R is generated, as a right ideal, by an idempotent. Kaplansky [15] shows that the definition of Baer ring is left-right symmetric. A generalization of Baer rings is a Rickart ring. A ring R is called *right* (resp. *left*) *Rickart* if the right (resp. left) annihilator of an element of R is generated by an idempotent. R is called a *Rickart* ring if it is both right and left Rickart ring. Note that for a reduced ring R, the concept of right Rickart and left Rickart are equivalent. The class of Baer rings includes all right Noetherian Rickart rings and all von Neumann regular rings. In 1974, Armendariz obtained the following

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result [2, Theorem B]: Let R be a reduced ring. Then $R[x]$ is a Baer ring if and only if R is a Baer ring. Recall a ring or a nearring is said to be *reduced* if it has no nonzero nilpotent elements. A generalization of Armendariz's result for several types of polynomials extensions over Baer rings, are obtained by various authors, [2], [4], [12] and [22]. According to Krempa [17], an endomorphism α of a ring R is said to be *rigid* if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$. Note that any rigid endomorphism of a ring is a monomorphism and α-rigid rings are reduced, by Hong et al. [12]. Properties of α-rigid rings has been studied in [10], [12–13], [17] and [21].

Birkenmeier and Huang in [5], have defined the *Rickart-type annihilator conditions* in the class of nearring as follows (for a nonempty $S \subseteq N$, let $r_N(S) = \{a \in N \mid Sa = 0\}$ and $\ell_N(S) = \{a \in N \mid aS = 0\}$). For every singleton subset S of N:

(1) $N \in \mathcal{R}_{r1}$ if $r_N(S) = eN$ for some idempotent $e \in N$;

(2) $N \in \mathcal{R}_{r2}$ if $r_N(S) = r_N(e)$ for some idempotent $e \in N$;

(3) $N \in \mathcal{R}_{\ell 1}$ if $\ell_N(S) = Ne$ for some idempotent $e \in N$;

(4) $N \in \mathcal{R}_{\ell 2}$ if $\ell_N(S) = \ell_N(e)$ for some idempotent $e \in N$.

When S is a principally generated ideal, the *quasi-Rickart* annihilator conditions in the class of nearrings are also defined and denoted similarly except replacing $\mathcal{R}$ by $q\mathcal{R}$. If N is a ring with unity then $N \in \mathcal{R}_{r1}$ (resp. $\mathcal{R}_{\ell 1}$) is equivalent to N being a right (resp. left) Rickart ring. In [3, p. 28], the $\mathcal{R}_{r2}$ condition is considered for rings with involution. They studied Baer-type annihilator conditions in the class of nearrings. In particular, they studied Baer-type annihilator conditions on the nearring of polynomials $R[x]$ (with the operations of addition and substitution) and formal power series and obtaining the following results: Let R be a reduced ring. (1) If R is Baer, then $R_0[x]$ (resp. $R_0[[x]]$) satisfies all the Baer-type annihilator conditions. (2) If $R_0[x]$ (resp. $R_0[[x]]$) satisfies any one of the Baer-type annihilator conditions, then R is Baer.

In [8] Fraser and Nicholson studied the formal power series extensions over a reduced Rickart ring. Also in [19], Liu studied the formal power series extensions over a principally quasi-Baer ring. Thus, it is natural to ask: *What can be said about various Rickart-typ annihilator conditions for skew formal power series under addition and substitution, where α is a endomorphism of R ?* We use $(x)f$ to denote the formal power series $\sum_{i=1}^{\infty} f_i x^i$, where $f_i \in R$ for each i, and $(x)f \circ (x)g$ indicates the substitution of $(x)f$ into $(x)g$. Let $(x)f = \sum_{i=1}^{\infty} a_i x^i$ and $(x)g = \sum_{j=1}^{\infty} b_j x^j$. Then

$$(x)f \circ (x)g = ((x)f)g = b_1(x)f + b_2((x)f)^2 + b_3((x)f)^3 + \cdots$$

$$= b_1a_1x + (b_1a_2 + b_2a_1\alpha(a_1))x^2 +$$
$$+ (b_1a_3 + b_2a_1\alpha(a_2) + b_2a_2\alpha^2(a_1) + b_3a_1\alpha(a_1)\alpha^2(a_1))x^3 \cdots .$$

We denote the collection of all power series with positive orders using the operations of addition and substitution by $R_0[[x;\alpha]]$, unless specifically indicated otherwise (i.e., $R_0[[x;\alpha]]$ denotes $(R_0[[x;\alpha]],+,\circ)$). Observe that the system $(R_0[[x;\alpha]],+,\circ)$ is a 0-symmetric left nearring, since, the operation "$\circ$", left distributes but does not right distribute over addition. Thus $(R_0[[x;\alpha]],+,\circ)$ forms a left nearring but not a ring. We denote the set of all idempotents of R by $I(R)$.

In this paper for an α-rigid ring R, we show that R is Rickart and any countable family of idempotents of R has a join in $I(R)$ if and only if $R_0[[x;\alpha]] \in \mathcal{R}_{r1}$ if and only if $R_0[[x;\alpha]] \in \mathcal{R}_{\ell 1}$ if and only if $R_0[[x;\alpha]] \in q\mathcal{R}_{r2}$. An example to show that α-rigid condition on R is not superfluous is provided.

As a consequence, for a reduced ring R, we show that R is Rickart and any countable family of idempotents of R has a join in $I(R)$ if and only if $R_0[[x]] \in \mathcal{R}_{r1}$ if and only if $R_0[[x]] \in \mathcal{R}_{\ell 1}$ if and only if $R_0[[x]] \in q\mathcal{R}_{r2}$.

2. Nearrings of skew formal power series

Definition 2.1 *(Krempa [17]) Let α be an endomorphism of R. α is called a rigid endomorphism if $a\alpha(a) = 0$ implies $a = 0$ for $a \in R$. A ring R is called to be α-rigid if there exists a rigid endomorphism α of R.*

Clearly, any rigid endomorphism is a monomorphism. Note that α-rigid rings are reduced rings. In fact, if R is an α-rigid ring and $a^2 = 0$ for $a \in R$, then $a\alpha(a)\alpha(a\alpha(a)) = 0$. Thus $a\alpha(a) = 0$ and so $a = 0$. Therefore R is reduced. But there exists an endomorphism of a reduced ring which is not a rigid endomorphism (see [12, Example 8]). However, if α is an inner automorphism (i.e., there exists an invertible element $u \in R$ such that $\alpha(r) = u^{-1}ru$ for any $r \in R$) of a reduced ring R, then R is α-rigid.

Lemma 2.2 *(Hong et al. [12]) Let R be an α-rigid ring and $a, b \in R$. Then we have the following:*

1. *If $ab = 0$ then $a\alpha^n(b) = \alpha^n(a)b = 0$ for each positive integer n.*
2. *If $a\alpha^k(b) = 0 = \alpha^k(a)b$ for some positive integer k, then $ab = 0$.*

3. If $e^2 = e \in R$, then $\alpha(e) = e$.

A nearring N is said to have the *insertion of factors property* (IFP) if for all $a, b, n \in N$, $ab = 0$ implies $anb = 0$.

In the sequel, $R_0[[x;\alpha]]$ denotes the nearring of skew formal power series $(R_0[[x;\alpha]], +, \circ)$ with positive orders and the operations of addition and substitution.

Proposition 2.3 *Let R be an α-rigid ring and $R_0[[x;\alpha]]$ be the 0-symmetric left nearring of skew power series with coefficients in R. Then $R_0[[x;\alpha]]$ is reduced.*

Proof. Assume to the contrary that $R_0[[x;\alpha]]$ is not reduced. Then there exists $(x)f = \sum_{i=n}^{\infty} a_i x^i \in R_0[[x;\alpha]]$ such that $(x)f \circ (x)f = 0$, $n \geq 1$, $a_n \neq 0$. Then $a_n^2 \alpha^n(a_n) \cdots \alpha^{n(n-1)}(a_n) = 0$ and hence $a_n = 0$, by Lemma 2.2, which is a contradiction. Therefore $R_0[[x;\alpha]]$ is reduced. □

Lemma 2.4 *Let R be an α-rigid ring and $(x)f = \sum_{i=1}^{\infty} a_i x^i$, $(x)g = \sum_{i=1}^{\infty} b_i x^i \in R_0[[x;\alpha]]$. Then $(x)f \circ (x)g = 0$ if and only if $b_i a_j = 0 = a_j b_i$ for all $i, j \geq 1$.*

Proof. Since $(x)f \circ (x)g = 0$, we have

$$b_1(x)f + b_2((x)f)^2 + b_3((x)f)^3 + \cdots = 0. \tag{1}$$

Then $b_1 a_1 = a_1 b_1 = 0$, since the coefficient of x is $b_1 a_1$ and R is reduced. Multiplying a_1 to eq. (1) from the left-hand side, then we have

$$a_1 b_2((x)f)^2 + a_1 b_3((x)f)^3 + \cdots = 0. \tag{2}$$

Hence $a_1 b_2 a_1 \alpha(a_1) = 0$, since it is coefficient of x^2 in eq. (2) and by Lemma 2.2, $a_1 b_2 = 0$. Inductively, we have $a_1 b_i = b_i a_1 = 0$ for all $i \geq 1$. Hence eq. (1) becomes $(\sum_{i=2}^{\infty} a_i x^i) \circ (\sum_{i=1}^{\infty} b_i x^i) = 0$, since R satisfies the IFP property. Continuing this process, we can prove $a_j b_i = 0$ for all $i, j \geq 1$.

Since R satisfies the IFP property, the converse follows from Lemma 2.2. □

Lemma 2.5 *Let R be a ring and α be an endomorphism of R. If $(x)E = \sum_{i=1}^{\infty} e_i x^i \in R_0[[x;\alpha]]$ is an idempotent, then $e_1^2 = e_1$. If R is α-rigid, then $(x)E = e_1 x$.*

Proof. Clearly, $e_1^2 = e_1$. Since $(x)E \circ (x)E = (x)E$, we have $e_1(x)E + e_2((x)E)^2 + \cdots = (x)E$ and that $e_1e_1(x)E + e_1e_2((x)E)^2 + \cdots = e_1(x)E$. Hence $(x)E \circ (e_1(x)E - e_1x) = (x)E \circ (e_1e_2x^2 + e_1e_3x^3 + \cdots) = 0$. Thus $e_1e_i = e_ie_1 = 0$ for all $i \geq 2$, by Lemma 2.4. Again, $(x)E \circ (e_2(x)E - e_2x) = (x)E \circ (-e_2x + e_2e_2x^2 + \cdots) = 0$. Thus $e_2^3 = 0$, by Lemma 2.4 and that $e_2 = 0$. Repeating the same procedure yields $(x)E \circ (-e_ix + e_ie_2x^2 + \cdots) = 0$ for all $i \geq 2$. Our conclusion then follows from Lemma 2.4. $\square$

Let $I(R)$ be the set of all idempotents of R.

Definition 2.6 *Let $\{e_1, e_2, \cdots\}$ be a countable family of idempotents of R. We say $\{e_1, e_2, \cdots\}$ has a join in $I(R)$ if there exists an $e \in I(R)$ such that*

1. *$e_i(1-e) = 0$, for all i and*
2. *if $f \in I(R)$ is such that $e_i(1-f) = 0$, for all i, then $e(1-f) = 0$.*

Theorem 2.7 *Let R be an α-rigid ring. Then the following conditions are equivalent:*

1. *R is Rickart and any countable family of idempotents of R has a join in $I(R)$;*
2. *$S = R_0[[x;\alpha]] \in \mathcal{R}_{r1}$;*
3. *$(R[[x;\alpha]], +, .)$ is a Rickart ring;*
4. *$S = R_0[[x;\alpha]] \in \mathcal{R}_{\ell 1}$;*
5. *$S = R_0[[x;\alpha]] \in q\mathcal{R}_{r2}$.*

Proof. 2.⇒1. Let $R_0[[x;\alpha]] \in \mathcal{R}_{r1}$ and $a \in R$. Then $r_S(ax) = ex \circ S$ for some idempotent $e \in R$, by Lemma 2.5. Hence for each $r \in R$, $erax = ax \circ (ex \circ rx) = 0$, since e is a central element. Thus $eR = Re \subseteq \ell_R(a)$. Now let $r \in \ell_R(a)$. Then $ax \circ rx = rax = 0$ and that $rx = ex \circ rx$. Hence $r = re \in Re$. Consequently $\ell_R(a) = Re$. Therefore R is a Rickart ring. Suppose that $\{e_1, e_2, \cdots\}$ is a countable family of idempotents of R. Set $(x)\phi = \sum_{i=1}^{\infty} e_ix^i \in S$. Since $S \in \mathcal{R}_{r1}$, $r_S((x)\phi) = ex \circ S$ for some idempotent $e \in R$. Let $g = 1 - e$. Then $e_i(1-g) = e_ie = 0$ for each i, since e is a central idempotent. Suppose that h is an idempotent of R such that $e_i(1-h) = 0$ for each i. Then $(1-h)e_i = 0$ and that $(x)\phi \circ (1-h)x = 0$. Thus $(1-h)x \in r_S((x)\phi) = ex \circ S$, and so $(1-h)x = ex \circ (1-h)x$. Hence $1 - h = (1-h)e$ and that $g(1-h) = (1-e)(1-h) = 0$. Therefore g is a join of the set $\{e_1, e_2, \cdots\}$.

1.$\Rightarrow$2. Let $(x)f = \sum_{i=1}^{\infty} a_i x^i \in S$. Then $\ell_R(a_i) = Re_i$ for each i, where $e_i^2 = e_i \in R$. Suppose that h be a join of the set $\{1 - e_i | i = 1, 2, \cdots\}$. Then $(1 - h)(1 - e_i) = (1-e_i)(1-h) = 0$ for each i. Hence $(1-h) = (1-h)e_i$ and that $(x)f \circ (1-h)x = 0$. Thus $(1-h)x \circ S \subseteq r_S((x)f)$. Let $(x)g = \sum_{j=1}^{\infty} b_j x^j \in r_S((x)f)$. Then $a_i b_j = b_j a_i = 0$ for each i, j, by Lemma 2.4. Hence $b_j = b_j e_i$ for each i, j and so $b_j(1-e_i) = (1-e_i)b_j = 0$. Since R is a Rickart ring, $\ell_R(b_j) = Rf_j$ for some idempotent $f_j \in R$. Thus $(1-e_i) = (1-e_i)f_j$ and so $(1-e_i)(1-f_j) = 0$ for each i, j. Since h is a join of the set $\{1-e_i | i = 1, 2, \cdots\}$, so $h(1-f_j) = 0$ for all j. Hence $b_j = b_j - f_j b_j = (1-f_j)b_j = (1-h)(1-f_j)b_j \in (1-h)R$. Thus $(x)g = \sum_{j=1}^{\infty}(1-h)(1-f_j)b_j x^j = (1-h)x \circ \sum_{j=1}^{\infty}(1-f_j)b_j x^j \in (1-h)x \circ S$, since $(1-h)$ is a central idempotent and $\alpha(1-h) = (1-h)$. Consequently $r_S((x)f) = (1-h)x \circ S$. Therefore $R_0[[x;\alpha]] \in \mathcal{R}_{r1}$.

The equivalence of 1. and 3. follows from [10, Theorem 2.5] and proof of 1.$\Leftrightarrow$4. is similar to that of 1.$\Leftrightarrow$2.

1.$\Rightarrow$5. Let $(x)f = \sum_{i=1}^{\infty} a_i x^i \in R_0[[x;\alpha]]$ and I the ideal of $R_0[[x;\alpha]]$ generated by $(x)f$. Since R is Rickart, $r_R(a_i) = \ell_R(a_i) = e_i R$, for some central idempotent $e_i \in R$. Suppose that h is a join of the set $\{1 - e_i | i = 1, \cdots\}$. Then $(1-e_i)(1-h) = 0$, hence $a_i r(1-h) = a_i r e_i (1-h) = 0$ for each $i \geq 1$ and $r \in R$. Hence $(1-h)x \in r_S(I)$. But $r_S(hx) = (1-h)x \circ S$. Thus $r_S(hx) \subseteq r_S(I)$. Let $(x)g = \sum_{j=1}^{\infty} b_j x^j \in r_S(I)$. Then $(x)f \circ (x)g = 0$ and that $a_i b_j = b_j a_i = 0$ for each i, j, by Lemma 2.4. Hence $b_j = e_i b_j$ for each i, j. Since R is Rickart, $r_R(b_j) = f_j R$ for each j, where f_j is a central idempotent of R. Thus $(1-e_i) \in r_R(b_j) = f_j R$ and so $(1-e_i) = f_j(1-e_i)$ for each i, j. Since h is a join of $\{1 - e_i | i = 1, \cdots\}$, $h(1-f_j) = 0$ for each j. Hence $b_j = b_j - b_j f_j = (1-f_j)b_j = (1-h)(1-f_j)b_j \in (1-h)R$. So $(x)g \in (1-h)x \circ S$. Consequently $r_S(I) = r_S(hx)$. Therefore $S \in q\mathcal{R}_{r2}$.

5.$\Rightarrow$1. Let $a \in R$ and I be an ideal of S generated by ax. Since $S \in q\mathcal{R}_{r2}$, $r_S(I) = r_S(ex)$ for some idempotent $e \in R$, by Lemma 2.5. We show $r_R(a) = r_R(e) = (1-e)R$. Since $ex \circ (1-e)x = 0$ so $ax \circ (1-e)x = 0$ and that $(1-e)a = 0$. Hence $r_R(e) \subseteq r_R(a)$. Suppose that $t \in r_R(a)$. Then $tx \in r_S(I)$, since R is α-rigid. Hence $ex \circ tx = 0$. Consequently $r_R(a) = r_R(e) = (1-e)R$. Therefore R is Rickart. Suppose that $\{f_1, f_2, \cdots\}$ is a countable family of idempotents of R. Set $(x)\phi = \sum_{i=1}^{\infty} f_i x^i$ and J be an ideal of S generated by $(x)\phi$. Then $r_S(I) = r_S(ex)$ for some idempotent $e \in R$, since $S \in q\mathcal{R}_{r2}$. We show that e is a join of $\{f_1, f_2, \cdots\}$. Since $r_S(ex) = (1-e)x \circ S$, so $(x)\phi \circ (1-e)x = \sum_{i=1}^{\infty}(1-e)f_i x^i = 0$. Then $f_i(1-e) = (1-e)f_i = 0$ for each i. Suppose that h is an idempotent of R such that $f_i(1-h)=0$ for each i. Hence $(1-h)x \in r_S(I)=r_S(ex)$, by Lemma 2.2. Thus $ex \circ (1-h)x=0$ and that $(1-h)e=e(1-h)=0$. Consequently e is a join of $\{f_1, f_2, \cdots\}$. □

The following examples show that there exists a Rickart ring R such that nearring $R_0[[x;\alpha]] \notin \mathcal{R}_{r1}$ by using Theorem 2.7.

Example 2.8 *(19, Example 6). Let W be an infinite set and B the Boolean ring of all subsets of W. Set R is the subring of B consisting of all finite and cofinite subsets of W. Then R is a commutative reduced Rickart ring, by [20, Example 3.7]. But R fails to have the property that all countable family of idempotents have a join in R. Let $\alpha : R \to R$ be the identity map. Thus by Theorem 2.7, nearring $R_0[[x;\alpha]] \notin \mathcal{R}_{r1}$.*

Example 2.9 *For a given field F, let*

$$R=\{(a_n)_{n=1}^{\infty} \in \Pi_{n=1}^{\infty} F_n \mid a_n \textit{ is eventually constant}\}$$

which is a subring of $\Pi_{n=1}^{\infty} F_n$, where $F_n=F$ for $n=1,2,\ldots$. Then the ring R is a commutative von Neumann regular ring and hence it is Rickart. Let α be the identity map on R. Then R is α-rigid. Thus by Theorem 2.7, nearring $R_0[[x;\alpha]] \notin \mathcal{R}_{r1}$.

Corollary 2.10 *Let R be a reduced ring. Then the following conditions are equivalent:*

1. *R is Rickart and any countable family of idempotents of R has a join in $I(R)$;*
2. *$S=R_0[[x]] \in \mathcal{R}_{r1}$;*
3. *$(R_0[[x]],+,.)$ is a Rickart ring;*
4. *$S=R_0[[x]] \in \mathcal{R}_{\ell 1}$;*
5. *$S=R_0[[x]] \in q\mathcal{R}_{r2}$.*

The following example shows that the condition "R is α-rigid" in Theorem 2.7 is not superfluous.

Example 2.11 *Let F be a field and consider the polynomial ring $R=F[y]$ over F. Then R is a commutative domain and so R is Rickart. Since 0 and 1 are the only idempotents of R, so each countable family of idempotents of R has a join. Let $\alpha : R \to R$ be* *an endomorphism defined by* $\alpha(f(y)) = f(0)$. *Then the nearring ring* $R_0[[x;\alpha]]$ *is not reduced. In fact, for* $0 \neq yx^2 \in R_0[[x;\alpha]]$, *we have* $yx^2 \circ yx^2 = y^2\alpha^2(y)x^4 = 0$. *So* $R_0[[x;\alpha]]$ *is not reduced. We claim that the only idempotents of* $R_0[[x;\alpha]]$ *are* 0 *and* x. *Let* $e = a_1(y)x + a_2(y)x^2 + \cdots \in R_0[[x;\alpha]]$ *be a nonzero idempotent. Then* $(a_1(y)x + a_2(y)x^2 + \cdots) \circ (a_1(y)x + a_2(y)x^2 + \cdots) = (a_1(y)x + a_2(y)x^2 + \cdots)$ *and that* $a_1(y)(a_1(y)x + a_2(y)x^2 + \cdots) + a_2(y)(a_1(y)x + a_2(y)x^2 + \cdots)^2 + \cdots = (a_1(y)x + a_2(y)x^2 + \cdots)$. *So* $a_1^2(y) = a_1(y)$. *Since* R *is a domain,* $a_1(y) = 0$ *or* $a_1(y) = 1$. *Assume that* $a_1(y) = 1$. *Since* $(a_1(y)a_2(y) + a_2(y)a_1(y)\alpha(a_1(y)))x^2 = a_2(y)x^2$, *hence* $a_2(y) + a_2(y) = a_2(y)$ *and that* $a_2(y) = 0$. *Since* $(a_1(y)a_3(y) + a_3(y)a_1(y)\alpha(a_1(y))\alpha^2(a_1(y)))x^3 = a_3(y)x^3$ *and* $\alpha(a_1(y)) = a_1(y) = 1$, *hence* $a_3(y) = 0$. *Continuing this process, we have* $e = a_1(y)x = x$. *If* $a_1(y) = 0$, *then it is clear that* $e = 0$. *Therefore the only idempotents of* $R_0[[x;\alpha]]$ *are* 0 *and* x. *Now we show that* $R_0[[x;\alpha]] \notin \mathcal{R}_{r1}$. *Note that* $r_{R_0[[x;\alpha]]}(yx) \neq R_0[[x;\alpha]]$, *since nearrring* $R_0[[x;\alpha]]$ *has unity. Thus* $r_{R_0[[x;\alpha]]}(yx) \neq r_{R_0[[x;\alpha]]}(0) = x \circ R_0[[x;\alpha]]$. *Since* $yx \circ x^2 = y\alpha(y)x^2 = 0$, *hence* $r_{R_0[[x;\alpha]]}(yx) \neq r_{R_0[[x;\alpha]]}(x) = 0 \circ R_0[[x;\alpha]]$. *Therefore* $R_0[[x;\alpha]] \notin \mathcal{R}_{r1}$.

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References

[1] Anderson, D.D. and Camillo, V.: Armendariz rings and Gaussian rings. Comm. Algebra 26(7), 2265-2275 (1998).

[2] Armendariz, E.P.: A note on extensions of Baer and p.p.-rings, J. Austral. Math. Soc. 18, 470-473 (1974).

[3] Berberian, S.K.: Baer $*$-rings; Springer-Verlag: Berlin, 1968.

[4] Birkenmeier, G.F., Kim, J.Y. and Park, J.K.: Polynomial extensions of Baer and quasi-Baer rings, J. Pure Appl. Algebra 159, 24-42 (2001).

[5] Birkenmeier, G.F. and Huang, F.K.: Annihilator conditions on polynomials, Comm. Algebra 29(5), 2097-2112 (2001).

[6] Birkenmeier, G.F. and Huang, F.K.: Annihilator conditions on polynomials II, Monatsh. Math. (2003).

[7] Birkenmeier, G.F. and Huang, F.K.: Annihilator conditions on formal power series, Algebra Colloq. 9(1), 29-37 (2002).

[8] Fraser, J.A., Nicholson, W.K.: Reduced PP-rings. Math. Japonica 34(5), 715-725 (1989).

[9] Hashemi, E. and Moussavi, A.: Polynomial extensions of quasi-Baer rings, Acta Math. Hungar. 107(3), 207-224 (2005).

[10] Hashemi, E. and Moussavi, A.: Skew power series extensions of α-rigid p.p.-rings, Bull. Korean Math. Soc. 41(4), 657-665 (2004).

[11] Hirano, Y.: On annihilator ideals of a polynomial ring over a noncommutative ring, J. Pure Appl. Algebra 168, 45-52 (2002).

[12] Hong, C.Y., Kim, N.K. and Kwak, T.K.: Ore extensions of Baer and p.p.-rings, J. Pure Appl. Algebra 151 (2000) 215-226.

[13] Hong, C.Y., Kim, N.K. and Kwak, T.K.: On skew Armendariz rings, Comm. Algebra 31(1), 103-122 (2003).

[14] Huh, C., Lee, Y. and Smoktunowicz, A.: Armendariz rings and semicommutative rings, Comm. Algebra 30(2), 751-761 (2002).

[15] Kaplansky, I.: Rings of Operators, Benjamin, New York, 1965.

[16] Kim, N.K. and Lee, Y.: Armendariz rings and reduced rings, J. Algebra 223, 477-488 (2000).

[17] Krempa, J.: Some examples of reduced rings, Algebra Colloq. 3(4), 289-300 (1996).

[18] Lee, T.K. and Wong, T.L.: On Armendariz rings, Houston J. Math. 29(3), 583-593 (2003).

[19] Liu, Z.: A note on principally quasi-Baer rings, Comm. Algebra 30(8), 3885-3890 (2002).

[20] Liu, Z. and Ahsan, J.: PP-rings of generalized power series, Acta Math. Sinica English Series 16, 573-578 (2000).

[21] Matczuk, J.: A characterization of α-rigid rings, Comm. Algebra 32(1), 4333-4337 (2004).

[22] Moussavi, A. and Hashemi, E.: On (α, δ)-skew Armendariz Rings, J. Korean Math. Soc. 42(2), 353-363 (2005).

[23] Rege, M.B., Chhawchharia, S.: Armendariz rings, Proc. Japan Acad. Ser. A Math. Sci. 73, 14-17 (1997).

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