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CR-Submanifolds of an S-manifold

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Abstract

The study of CR-submanifolds of a Kaehler manifold was initiated by Bejancu [1]. Since then many papers have appeared on CR-submanifolds. The purpose of this paper is to studied the CR-submanifolds of an S-manifold. In particular, we studied the integrability of the distributions D and $D^{\perp}$ of a CR-submanifold of an S-manifold.

Key words and phrases: CR-submanifolds, S-manifold, CR-submanifold of an S-manifold.

0. Introduction

Many authors have studied the geometry of submanifolds of Kaehler, Sasakian and trans Sasakian manifolds. The main ones can be found in [8]. For manifolds with an f-structure f, D. E. Blair has introduced the S-manifold as the analogue of the Kaehler structure in the almost complex case and of the quasi-Sasakian structure in the almost contact case [3].

The purpose of this paper is to study the integrability of the distributions of a CR-submanifold of an S-manifold. In sections 1 and 2 we review basic formulas and definitions for submanifolds in Riemannian manifolds and in S-manifold respectively, which we shall use later. In section 3 we study CR-submanifold of an S-manifold and discuss the integrability of the distributions D and $D^{\perp}$.

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1. Preliminaries

Let N be a Riemannian manifold of dimension n and M an m-dimensional submanifold of N. Let g be the metric tensor field on N as well as the induced metric on M. We denote by $\overline{\nabla}$ the covariant differentiation in N and by ∇ the covariant differentiation in M determined by the induced metric. Let TN (resp. TM) be the Lie algebra of vector fields in N (resp. in M) and $T^{\perp}M$ the set of all vector fields normal to M. The Gauss and Weingarten formulas are respectively given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{1.1}$$

$$\overline{\nabla}_X N = -A_N X + \nabla^{\perp}_X N \tag{1.2}$$

for $X, Y \in TM$ and $N \in T^{\perp}M$, where $\nabla^{\perp}$ is the connection in the normal bundle, h is the second fundamental form of M and A_N the Weingarten endomorphism associated with N. Then A_N and h are related by the relation

$$g(A_N X, Y) = g(h(X, Y), N). \tag{1.3}$$

2. CR-submanifold of S-manifold

Let (N, g) be a Riemannian manifold with $\dim(N) = 2m + s$. It is said to be an S-manifold if there exist on N a f-structure f ([4]) of rank $2n$ and s global vector fields $\xi_1, \xi_2, \cdots, \xi_s$ (structure vector fields) such that ([7])

- (i) If $\eta_1, \eta_2, \cdots, \eta_s$ are the dual 1-forms of $\xi_1, \xi_2, \cdots, \xi_s$, then

$$f\xi_\alpha = 0, \tag{2.4}$$

$$\eta_\alpha \circ f = 0, \tag{2.5}$$

$$f^2 = -I + \sum \eta_\alpha \otimes \xi_\alpha, \tag{2.6}$$

$$g(X, Y) = g(fX, fY) + \Phi(X, Y), \tag{2.7}$$

from any $X, Y \in TN, \alpha = 1, 2, \cdots, s$, where

$$\Phi(X, Y) = \sum \eta_\alpha(X)\eta_\alpha(Y).$$

- (ii) The f-structure f is normal, that is

$$[f, f] + 2\sum d\eta_\alpha \otimes \xi_\alpha = 0, \tag{2.8}$$

where $[f, f]$ is the Nijenhuis torsion of f.

- (iii)

$$\eta_1 \wedge \eta_2 \wedge \cdots \eta_s \wedge (d\eta_\alpha)^n \neq 0, \tag{2.9}$$

and

$$d\eta_1 = d\eta_2 = \cdots = d\eta_s = F, \tag{2.10}$$

for any α, where F is the fundamental 2-form defined by

$$F(X, Y) = g(X, fY), \qquad X, Y \in TN.$$

In the case $s = 1$, an S-manifold is a Sasakian manifold.

For the Riemannian connection $\overline{\nabla}$ of g on an S-manifold N, we have

$$\overline{\nabla}_X \xi_\alpha = -fX, \quad X \in TN,\ \alpha = 1, 2, \cdots, s. \tag{2.11}$$

$$(\overline{\nabla}_X f)Y = \sum\{g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2X\}, \quad X, Y \in TN. \tag{2.12}$$

Now, let M be an m-dimensional submanifold immersed in N. M is said to be an invariant submanifold if $\xi_\alpha \in TM$ for any α and $fX \in TM$ for any $X \in TM$. On the other hand, it is said to be an anti-invariant submanifold if $fX \in T^\perp M$ for any $X \in TM$.

Now assume that the structure vector fields $\xi_1, \xi_2, \cdots \xi_s$ are tangent to M (and so, $\dim(M) \geq s$). Then M is called a CR-submanifold of N if there exist two differentiable distributions D and $D^\perp$ on M satisfying:

- (i) $TM = D \oplus D^\perp$ (direct sum);
- (ii) The distribution D is invariant under f, that is $fD_x = D_x$ for any $x \in M$;
- (iii) The distribution $D^\perp$ is anti-invariant under f, that is, $fD_x^\perp \subseteq T_x^\perp M$ for any $x \in M$.

We denote by $2p+s$ and q the real dimensions of D_x and $D_x^{\perp}$ respectively, for any $x \in M$. Then if $p=0$ we have an anti-invariant submanifold tangent to $\xi_1, \xi_2, \cdots, \xi_s$, and if $q=0$, we have an invariant submanifold. A CR-submanifold is said to be D-totallygeodesic if $h(X,Y)=0$ for any $X, Y \in D$ and it is said to be $D^{\perp}$-totallygeodesic if $h(Z,W)=0$ for any $Z, W \in D^{\perp}$. Now denote by P and Q the projection morphisms of TM on D and $D^{\perp}$, respectively, we call D(resp.$D^{\perp}$) the horizontal (resp.vertical) distribution. Then for any $X \in TM$, we have

$$X = PX + QX,$$

where PX and QX belong to the distribution D and $D^{\perp}$, respectively. Also for a vector field N normal to M, we put

$$fN = tN + nN,$$

where tN (resp. nN) denotes the vertical (resp. normal) component of fN. The pair $(D, D^{\perp})$ is called ξ_α-horizontal (resp. ξ_α-vertical) if $\xi_\alpha x \in D_x$ (resp. $\xi_\alpha x \in D_x^{\perp}$) for each $x \in M$.

3. The distributions D and $D^{\perp}$

Lemma 1 *Let M be a CR-submanifold of an S-manifold N, then we have*

$$P\nabla_X fPY - PA_{fQY}X - fP\nabla_X Y = \sum[g(X,Y)P\xi_\alpha - \eta_\alpha(Y)PX], \tag{3.13}$$

$$Q\nabla_X fPY - QA_{fQY}X - th(X,Y) = \sum[g(X,Y)Q\xi_\alpha - \eta_\alpha(Y)QX], \tag{3.14}$$

$$h(X, fPY) - fQ\nabla_X Y + \nabla_X^{\perp} fQY = nh(X,Y), \quad \forall\, X, Y \in TM. \tag{3.15}$$

Proof. Let N be an S-manifold and M be a CR-submanifold of N then from (2.9) for $X, Y \in TM$, we have

$$(\overline{\nabla}_X f)Y = \sum[g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2X],$$

$$\begin{aligned}
\overline{\nabla}_X fY - f\overline{\nabla}_X Y &= \sum[g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2X] \\
&= \sum\{g(X,Y)\xi_\alpha - \eta_\alpha(X)\eta_\alpha(Y)\xi_\alpha - \eta_\alpha(Y)X + \eta_\alpha(Y)\eta_\alpha(X)\xi_\alpha\} \\
&= \sum\{g(X,Y)\xi_\alpha - \eta_\alpha(Y)X\},
\end{aligned}$$

therefore

$$\overline{\nabla}_X(fPY + fQY) - f\overline{\nabla}_X Y = \sum\{g(X,Y)\xi_\alpha - \eta_\alpha(Y)X\},$$

$$\overline{\nabla}_X fPY + \overline{\nabla}_X fQY - f\overline{\nabla}_X Y = \sum\{g(X,Y)\xi_\alpha - \eta_\alpha(Y)X\}.$$

Now using Gauss and Weingarten formulas, we have

$$\begin{aligned} h(X, fPY) + \nabla_X fPY - A_{fQY}X + \nabla^{\perp}_X fQY - f\nabla_X Y - fh(X,Y) \\ = \sum\{g(X,Y)\xi_\alpha - \eta_\alpha(Y)X\}, \end{aligned}$$

or

$$\begin{aligned} h(X, fPY) + P\nabla_X fPY + Q\nabla_X fPY - PA_{fQY}X - QA_{fQY}X + \nabla^{\perp}_X fQY \\ - fP\nabla_X Y - fQ\nabla_X Y - th(X,Y) - nh(X,Y) \\ = \sum\{g(X,Y)(P\xi_\alpha + Q\xi_\alpha) - \eta_\alpha(Y)(PX + QX)\}. \end{aligned}$$

Now comparing the horizontal, vertical and normal parts, we obtain (3.13), (3.14) and (3.15). □

Lemma 2 *If M is ξ_α-horizontal CR-submanifold of an S-manifold N, then*

$$-A_{fW}Z - fP\nabla_Z W - th(Z,W) = \sum g(Z,W)\xi_\alpha, \tag{3.16}$$

$$\nabla^{\perp}_Z fW = fQ\nabla_Z W + nh(Z,W) \tag{3.17}$$

for all $Z, W \in D^{\perp}$.

Proof. Let N be an S-manifold, and M be a CR-submanifold of N, then from (2.9) we have

$$(\overline{\nabla}_X f)Y = \sum\{g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2X\}, \quad \forall X, Y \in TM;$$

therefore

$$(\overline{\nabla}_Z f)W = \sum\{g(fZ, fW)\xi_\alpha + \eta_\alpha(W)f^2Z\}, \quad \forall Z, W \in D^{\perp};$$

and since $\xi_\alpha \in D$, we have

$$\begin{aligned}(\overline{\nabla}_Z f)W &= \sum\{g(fZ, fW)\xi_\alpha\} \\ &= \sum\{g(Z,W)\xi_\alpha - \eta_\alpha(Z)\eta_\alpha(W)\xi_\alpha\} \\ &= \sum g(Z,W)\xi_\alpha;\end{aligned}$$

therefore

$$\overline{\nabla}_Z fW - f\overline{\nabla}_Z W = \sum g(Z,W)\xi_\alpha.$$

Now using Gauss and Wiengarten formulas, we have

$$-A_{fW}Z + \nabla^{\perp}_Z fW - f\nabla_Z W - fh(Z,W) = \sum g(Z,W)\xi_\alpha$$

$$-A_{fW}Z + \nabla^{\perp}_Z fW - fP\nabla_Z W - fQ\nabla_Z W - th(Z,W) - nh(Z,W) = \sum g(Z,W)\xi_\alpha.$$

Now comparing tangent and normal parts, we obtain

$$-A_{fW}Z - fP\nabla_Z W = \sum g(Z,W)\xi_\alpha + th(Z,W),$$

$$\nabla^{\perp}_Z fW - fQ\nabla_Z W = nh(Z,W) \quad \forall Z, W \in D^{\perp}$$

which completes the proof. □

Lemma 3 *If M is ξ_α-vertical CR-submanifold of an S-manifold N, then*

$$\nabla_X fY - fP\nabla_X Y = \sum g(X,Y)\xi_\alpha + th(X,Y), \tag{3.18}$$

$$h(X, fY) = fQ\nabla_X Y + nh(X,Y), \quad \textit{for all} \quad X, Y \in D. \tag{3.19}$$

Proof. From (2.9) we have

$$(\overline{\nabla}_X f)Y = \sum\{g(fX, fY)\xi_\alpha + \eta_\alpha(Y)f^2X\},$$

since $\xi_\alpha \in D^\perp$, then for all $X, Y \in D$ we have

$$\begin{aligned}(\overline{\nabla}_X f)Y &= \sum g(fX, fY)\xi_\alpha \\ &= \sum \{g(X,Y)\xi_\alpha - \eta_\alpha(X)\eta_\alpha(Y)\xi_\alpha\} \\ &= \sum g(X,Y)\xi_\alpha.\end{aligned}$$

Therefore

$$\overline{\nabla}_X fY - f\overline{\nabla}_X Y = \sum g(X,Y)\xi_\alpha.$$

Now using Gauss formula, we obtain for all $X, Y \in D$

$$\nabla_X fY + h(X, fY) - f\nabla_X Y - fh(X,Y) = \sum g(X,Y)\xi_\alpha$$

$$\begin{aligned}\nabla_X fY + h(X, fY) - fP\nabla_X Y - fQ\nabla_X Y - th(X,Y) - nh(X,Y) \\ = \sum g(X,Y)\xi_\alpha.\end{aligned}$$

Now comparing tangent and normal parts, we get

$$\nabla_X fY - fP\nabla_X Y = \sum g(X,Y)\xi_\alpha + th(X,Y),$$

$$h(X, fY) = fQ\nabla_X Y + nh(X,Y).$$

which completes the proof. □

Remark 4 *Let M be a CR-submanifold of an S-manifold N. Then we have*

$$\nabla_X \xi_\alpha = -fPX, \qquad \forall X \in TM \tag{3.20}$$

$$h(X, \xi_\alpha) = -fQX \qquad \forall X \in TM \tag{3.21}$$

$$\nabla_X \xi_\alpha = 0 \qquad \forall X \in D^\perp \tag{3.22}$$

$$h(X, \xi_\alpha) = 0 \qquad \forall X \in D \tag{3.23}$$

$$h(\xi_\alpha, \xi_\alpha) = 0 \tag{3.24}$$

$$A_V \xi_\alpha \in D^\perp \qquad \forall V \in T^\perp M. \tag{3.25}$$

$$\eta_\alpha(A_V X) = 0, \qquad \forall X \in D.$$

Proof. By Gauss formula in equation (2.8), we easily obtain

$$\overline{\nabla}_X \xi_\alpha = -fX \Rightarrow \nabla_X \xi_\alpha + h(X, \xi_\alpha) = -fX,$$

which gives

$$\nabla_X \xi_\alpha + h(X, \xi_\alpha) = -fPX - fQX.$$

Now comparing tangent and normal parts, we get

$$\nabla_X \xi_\alpha = -fPX \quad \text{and} \quad h(X, \xi_\alpha) = -fQX.$$

Hence

$$h(X, \xi_\alpha) = 0 \qquad \text{for all} \quad X \in D,$$

and

$$h(\xi_\alpha, \xi_\alpha) = 0 \qquad (f\xi_\alpha = 0)$$

$$\nabla_X \xi_\alpha = -fPX \Rightarrow \nabla_X \xi_\alpha = 0 \qquad \forall\ X \in D^\perp.$$

Let $X \in D$, then we have

$$g(A_V \xi_\alpha, X) = g(h(X, \xi_\alpha), V) = g(0, V) = 0.$$

Using (3.23) in the above equation, we get

$$g(A_V \xi_\alpha, X) = 0, \quad \forall X \in D \quad \text{which leads to} \quad A_V \xi_\alpha \in D^\perp.$$

Also,

$$g(A_V \xi_\alpha, X) = 0, \quad \forall X \in D, \Rightarrow g(A_V X, \xi_\alpha) = 0, \Rightarrow \eta_\alpha(A_V X) = 0.$$

□

Remark 5 *Let M be a CR-submanifold of an S-manifold N, if M is ξ_α-horizontal, then the distribution D is integrable* ⇔

$$h(X, fY) = h(Y, fX) \qquad \forall\ X, Y \in D. \tag{3.26}$$

Proof. From Equation (3.3) we have

$$h(X, fY) - fQ\nabla_X Y = nh(X, Y) \quad \forall X, Y \in D. \tag{3.27}$$

Now interchanging X and Y, we have

$$h(Y, fX) - fQ\nabla_Y X = nh(Y, X) \quad \forall\ X, Y \in D. \tag{3.28}$$

Subtracting (3.27) and (3.28), we obtain

$$h(X, fY) - h(Y, fX) = fQ[X, Y].$$

Hence $Q[X, Y] = 0$, iff

$$h(X, fY) = h(Y, fX) \qquad \forall\ X, Y \in D.$$

□

Remark 6 *Let M be a CR-submanifold of an S-manifold N, then M is a foliate if D is involutive.*

Remark 7 *Let M be a CR-submanifold of an S-manifold N, if M is a foliate ξ_α-horizontal, then*

$$h(fX, fY) = -h(X, Y), \qquad \forall\ X, Y \in D. \tag{3.29}$$

Proof. Since every involutive is integrable, then by (3.26) we have

$$h(X, fY) = h(fX, Y),$$

then

$$\begin{aligned} h(fX, fY) &= h(f^2X, Y) = h(-X + \sum \eta_\alpha(X)\xi_\alpha, Y) \\ &= h(-X, Y) + h(\sum \eta_\alpha(X)\xi_\alpha, Y) \\ &= -h(X, Y) \qquad \text{(by equation 3.24).} \end{aligned}$$

□

Remark 8 *Let M be a CR-submanifold of an S-manifold N, then M is mixed totally geodesic if and only if one of the following satisfied:*

$$A_V X \in D \qquad (\forall X \in D,\ V \in T^{\perp}M), \tag{3.30}$$

$$A_V X \in D^{\perp} \qquad (\forall\ X \in D^{\perp},\ \ V \in T^{\perp}M). \tag{3.31}$$

Proof. Consider $A_V X$, let $X \in D, V \in T^{\perp}M$ and $Y \in D^{\perp}$, then

$$\begin{aligned} g(A_V X, Y) &= g(h(X,Y), V) \\ &= 0 \ \Leftrightarrow \ A_V X \in D. \end{aligned}$$

Hence

$$\begin{aligned} g(h(X,Y),V) = 0 \quad &\Leftrightarrow \quad h(X,Y) = 0 \\ &\Leftrightarrow \quad A_V X \in D \qquad \forall X \in D, V \in T^{\perp}M. \end{aligned}$$

In a similar way is deduced relation. (3.31). □

Remark 9 *The horizontal (resp. vertical) distribution on D (resp. $D^{\perp}$) is said to be parallel [1] with respect to the connection ∇ on M if $\nabla_X Y \in D$ (resp. $\nabla_Z W \in D^{\perp}$) for any $X, Y \in D$ (resp. $Z, W \in D^{\perp}$).*

Remark 10 *Let M be a ξ_α-horizontal CR-submanifold of an S-manifold N, then the horizontal distribution D is parallel if and only if*

$$h(X, fY) = h(fY, X) = fh(X, Y). \tag{3.32}$$

Proof. Since every parallel is involutive then the first equality follows immediately. Now since D is parallel, we have

$$\nabla_X fY \in D, \quad \forall X, Y \in D,$$

Then from (3.14) we have

$$th(X,Y) = 0 \qquad \forall X, Y \in D \text{ if } \xi_\alpha \in D, \tag{3.33}$$

and from (3.3) if $\xi_\alpha \in D$ then D is parallel

$$\Leftrightarrow \quad h(X, fY) = nh(X, Y).$$

But, we have

$$fh(X, Y) = th(X, Y) + nh(X, Y),$$

and from (3.21) we have $fh(X, Y) = nh(X, Y)$, which completes the proof. □

Remark 11 *Let M be a CR-submanifold of an S-manifold N, if M is ξ_α-vertical, then the distribution $D^\perp$ is integrable* $\Leftrightarrow$

$$A_{fX}Y - A_{fY}X = \sum[\eta_\alpha(X)Y - \eta_\alpha(Y)X], \qquad \forall X, Y \in D^\perp \tag{3.34}$$

Proof. If $X, Y \in D^\perp$, then (3.1) and (3.2) become

$$-PA_{fY}X - fP\nabla_X Y = 0, \tag{3.35}$$

$$-QA_{fY}X - th(X, Y) = \sum[g(X, Y)\xi_\alpha - \eta_\alpha(Y)X]. \tag{3.36}$$

Now adding (3.23) and (3.24), we have

$$-A_{fY}X - fP\nabla_X Y - th(X, Y) = \sum[g(X, Y)\xi_\alpha - \eta_\alpha(Y)X]. \tag{3.37}$$

Now interchanging X and Y, we have

$$-A_{fX}Y - fP\nabla_Y X - th(Y, X) = \sum[g(X, Y)\xi_\alpha - \eta_\alpha(X)Y]. \tag{3.38}$$

Subtracting the equations(3.25) and (3.26), we obtain

$$-A_{fY}X + A_{fX}Y - fP[X, Y] = \sum[-\eta_\alpha(Y)X + \eta_\alpha(X)Y].$$

Hence $P[X, Y] = 0$, $\Leftrightarrow$

$$A_{fX}Y - A_{fY}X = \sum[\eta_\alpha(X)Y - \eta_\alpha(Y)X].$$

Therefore $D^\perp$ is integrable $\Leftrightarrow$ (3.22) holds. □

Corollary 12 *If M is a ξ_α-horizontal CR-submanifold of an S-manifold N then $D^\perp$ is integrable if and only if*

$$A_{fY}X = A_{fX}Y \quad \forall\ X, Y \in D^\perp. \tag{3.39}$$

Proof. The proof can be obtained directily from Lemma (3). □

Remark 13 *Let M be a ξ_α-horizontal CR-submanifold of an S-manifold N then $D^\perp$ is parallel if and only if*

$$-A_{fW}Z = \sum g(Z, W)\xi_\alpha + th(Z, W) \quad \forall\, Z, W \in D^\perp. \tag{3.40}$$

Proof. From (3.4) we have,

$$-A_{fW}Z - fP\nabla_Z W = \sum g(Z, W)\xi_\alpha + th(Z, W) \quad \forall Z, W \in D^\perp,$$

hence

$$\nabla_Z W \in D^\perp,$$

$$\Leftrightarrow P\nabla_Z W = 0.$$

Using this we get

$$-A_{fW}Z = \sum g(Z, W)\xi_\alpha + th(Z, W) \quad \forall Z, W \in D^\perp.$$

□

Remark 14 *Let M be a ξ_α-vertical CR-submanifold of an S-manifold N, then the distribution $D^\perp$ is parallel if and only if*

$$A_{fW}Z \in D^\perp \qquad \forall\ Z, W \in D^\perp. \tag{3.41}$$

Proof. Using the Gauss and Weingarten formulas for $Z, W \in D^\perp$, we have

$$-A_{fW}Z + \nabla^\perp_Z fW - f\nabla_Z W - fh(Z, W) = \sum\{g(Z, W)\xi_\alpha - \eta_\alpha(W)Z\}.$$

Now take inner product with $Y \in D$, we have

$$-g(A_{fW}Z, Y) + g(\nabla^{\perp}_{Z} fW, Y) - g(f\nabla_Z W, Y) - g(fh(Z, W), Y)$$
$$= \sum\{g(Z, W)g(\xi_\alpha, Y) - \eta_\alpha(W)g(Z, Y)\}.$$

Hence since $\xi_\alpha \in D^\perp$ then we have

$$-g(A_{fW}Z, Y) = g(f\nabla_Z W, Y) = -g(\nabla_Z W, fY),$$

implies that

$$g(A_{fW}Z, Y) = 0 \Leftrightarrow A_{fW}Z \in D^\perp.$$

Therefore

$$\nabla_Z W \in D^\perp \;\Leftrightarrow\; A_{fW}Z \in D^\perp \quad \forall\; Z, W \in D^\perp.$$

□

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