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2-Quasi-λ-Nuclear Maps

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Abstract

In this paper we generalize the well-known result which says that the composition of quasi-nuclear maps is nuclear. More precisely, we define what we call a 2-quasi-λ-nuclear map between normed spaces, and we prove that the composition of a 2-quasi-λ-nuclear map with a quasi-λ-nuclear map is a pseudo-λ-nuclear map. Also, we prove that a quasi-λ-nuclear map is a 2-quasi-λ-nuclear map. For a nuclear G_∞-space, we prove that a linear map T between normed spaces is 2-quasi-λ-nuclear if and only if it is quasi-λ-nuclear.

1. Basic Concepts

For two sequences of scalars $x = (x_n)$ and $y = (y_n)$ we write $x_n = O(y_n)$ if there is a $\rho > 0$ such that $x_n \leq \rho y_n$ for all $n \in \mathbf{N}$.

A set A of sequences of non-negative real numbers is called a **Köthe set**, if it satisfies the following conditions:

1. For each pair of elements $a, b \in A$ there is $c \in A$ with $a_n = O(c_n)$ and $b_n = O(c_n)$.

2. For every integer $r \in N$ there exists $a \in A$ with $a_r > 0$.

The space of all sequences $x = (x_n)$ such that

$$p_a(x) := \sum_n |x_n|\, a_n < +\infty$$

for all $a \in A$, is called the **Köthe space**, $\lambda(A)$, generated by A[3].

A Köthe set P will be called a **power set of infinite type** if it satisfies the following conditions:

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1. For each $a \in P$, $1 \leq a_n \leq a_{n+1}$ for all n.
2. For each $a \in P$, there exists $b \in P$ such that $a_n^2 = O(b_n)$.

A Köthe space of the form $\lambda(P)$ where P is a power set of infinite type is called a G_∞**-space** or a **smooth sequence space of infinite type**[8].

Let $\alpha = (\alpha_n)$ be an unbounded non-decreasing sequence of positive real numbers. Then $P_\infty = \{(k^{\alpha_n}) : k \in \mathbf{N}\}$ is a countable Köthe set. The corresponding Köthe space $\Lambda_\infty(\alpha) = \lambda(P_\infty)$ is called the **power series of infinite type**.

The space s of **rapidly decreasing sequences** is a G_∞-space which is generated by $A = \{(n^k) : k = 1, 2, 3, ...\}$.

Let E and F be two arbitrary normed spaces. A linear map T from E into F is called a **nuclear** map if there are sequences (a_n), (y_n) in E' and F respectively, with

$$\sum_n ||a_n|| \, ||y_n|| < +\infty \text{ such that } T(x) = \sum_n \langle x, a_n\rangle y_n,$$

and a **quasi-nuclear** map if there is a sequence (a_n) in E' with $\sum_n ||a_n|| < +\infty$ such that $||T(x)|| \leq \sum_n |\langle x, a_n\rangle|$ (see [4, P. 49, P. 56]).

In the rest of this paper, letter λ stands for a fixed sequence space contained in ℓ_1.

A linear map T of a normed space E into a normed space F is called a **pseudo-λ-nuclear map** if there exist a sequence (α_n) in λ and a bounded sequences (a_n) and (y_n) in E' and F respectively such that $Tx = \sum_n \alpha_n \langle x, a_n\rangle y_n$, for all x in E, and a **quasi-λ-nuclear map** if there exist a sequence (α_n) in λ and a bounded sequence (a_n) in E' such that $||Tx|| \leq \sum_n |\alpha_n| \, |\langle x, a_n\rangle|$, for all x in E([1][6]).

A linear map T from a normed space E into a normed space F is called a **2-quasi-nuclear map**, if there is a sequence (a_n) in E' with

$$\sum_n ||a_n||^2 < +\infty \quad \text{such that} \quad ||Tx|| \leq \left(\sum_n |\langle x, a_n\rangle|^2\right)^{1/2} [5].$$

One of our goals in the present paper is to generalize the following Theorem:

Theorem 1.1 [4] *The composition of quasi nuclear maps is nuclear.*

To proceed in our work and to achieve our goals, we introduce the following definition:

Definition 1.1 *A bounded linear map T of a normed space E into a normed space F is called a* **2-quasi-λ-nuclear map** *if there exist a sequence (α_n) in λ and a bounded* *sequence* (a_n) *in* E' *such that*

$$||Tx|| \leq \left(\sum_n |\alpha_n| \, |\langle x, a_n \rangle|^2\right)^{1/2},$$

for all x *in* E.

2. Main Results

Let $\mathcal{N}(E,F)$, $\mathcal{QN}_1(E,F)$, $\mathcal{QN}_2(E,F)$, $\mathcal{P}\lambda\mathcal{N}(E,F)$, $\mathcal{Q}\lambda\mathcal{N}_1(E,F)$, and $\mathcal{Q}\lambda\mathcal{N}_2(E,F)$ denote the collection of all nuclear, quasi-nuclear, 2-quasi-nuclear, pseudo-λ-nuclear, quasi-λ-nuclear, and 2-quasi-λ-nuclear maps, respectively, between normed spaces E and F. It is an easy matter to see the following propositions.

Proposition 2.1 *If* $T \in \mathcal{P}\lambda\mathcal{N}(E,F)$, *then* $T \in \mathcal{N}(E,F)$.

Proposition 2.2 *If* $T \in \mathcal{Q}\lambda\mathcal{N}_1(E,F)$, *then* $T \in \mathcal{QN}_1(E,F)$.

Proposition 2.3 *If* $T \in \mathcal{Q}\lambda\mathcal{N}_2(E,F)$, *then* $T \in \mathcal{QN}_2(E,F)$.

Let $\mathbf{B}(E,F)$ denotes the collection of all bounded linear map between normed spaces E and F. Then we have the following proposition.

Proposition 2.4 *Let* E, F *and* G *be normed spaces. Let* T *and* S *be linear maps from* E *into* F *and from* F *into* G *respectively. Then*

1. *If* $T \in \mathbf{B}(E,F)$ *and* $S \in \mathcal{P}\lambda\mathcal{N}(F,G)$, *then* $ST \in \mathcal{P}\lambda\mathcal{N}(E,G)$.
2. *If* $T \in \mathcal{P}\lambda\mathcal{N}(E,F)$ *and* $S \in \mathbf{B}(F,G)$, *then* $ST \in \mathcal{P}\lambda\mathcal{N}(E,G)$.
3. *If* $T \in \mathbf{B}(E,F)$ *and* $S \in \mathcal{Q}\lambda\mathcal{N}_1(F,G)$, *then* $ST \in \mathcal{Q}\lambda\mathcal{N}_1(E,G)$.
4. *If* $T \in \mathcal{Q}\lambda\mathcal{N}_1(E,F)$ *and* $S \in \mathbf{B}(F,G)$, *then* $ST \in \mathcal{Q}\lambda\mathcal{N}_1(E,G)$.
5. *If* $T \in \mathbf{B}(E,F)$ *and* $S \in \mathcal{Q}\lambda\mathcal{N}_2(F,G)$, *then* $ST \in \mathcal{Q}\lambda\mathcal{N}_2(E,G)$.
6. *If* $T \in \mathcal{Q}\lambda\mathcal{N}_2(E,F)$ *and* $S \in \mathbf{B}(F,G)$, *then* $ST \in \mathcal{Q}\lambda\mathcal{N}_2(E,G)$.

Our next result indicates the relationship between quasi-λ-nuclear and 2-quasi-λ-nuclear maps.

Theorem 2.1 *Each quasi-*λ*-nuclear map is 2-quasi-*λ*-nuclear.*

Proof. Let $T : E \to F$ be a quasi-λ-nuclear map between normed spaces E and F. Then there exist a sequence (α_n) in λ and a bounded sequence (a_n) in E' such that $||Tx|| \leq \sum_n |\alpha_n| \, |\langle x, a_n \rangle|$, for all x in E. By Hölder's inequality, we have

$$||Tx|| \leq \left(\sum\nolimits_n |\alpha_n|\right)^{1/2} \left(\sum\nolimits_n |\alpha_n| \, |\langle x, a_n \rangle|^2\right)^{1/2}.$$

Hence,

$$||Tx|| \leq \left(\sum\nolimits_n |\alpha_n| \, |\langle x, \sqrt{\beta}\, a_n \rangle|^2\right)^{1/2},$$

where $\beta = \sum_n |\alpha_n|$. Since $(\alpha_n) \in \lambda$ and $(\sqrt{\beta}\, a_n)$ is a bounded sequence in E', T is a 2-quasi-λ-nuclear map. □

The relationship between pseudo-ℓ_1-nuclear and nuclear maps is given by the following proposition.

Proposition 2.5 *A linear map T from a normed space E into a normed space F is nuclear if and only if it is pseudo-ℓ_1-nuclear.*

The following proposition gives the relationship between quasi-nuclear and quasi-ℓ_1-nuclear maps.

Proposition 2.6 *A linear map T from a normed space E into a normed space F is quasi-nuclear if and only if it is quasi-ℓ_1-nuclear.*

The next proposition indicates the relationship between 2-quasi-nuclear and 2-quasi-ℓ_1-nuclear maps.

Proposition 2.7 *A linear map T from a normed space E into a normed space F is a 2-quasi-nuclear map if and only if it is a 2-quasi-ℓ_1-nuclear map.*

The following result is direct consequence of Proposition 2.6, Theorem 2.1 and Proposition 2.7.

Corollary 2.1 *Each quasi-nuclear map is a 2-quasi-nuclear map.*

The following well-known results are essential for proving Theorem 2.2.

Lemma 2.1 *(see Pietsch [4, P. 57]) Each normed space F can be considered as a linear subspace of a Banach space $\ell_\infty(I)$.*

Lemma 2.2 *[4, P. 57] If G_0 is a linear subspace of the normed space G, then each continuous linear map S_0 of G_0 into a Banach space $\ell_\infty(I)$ can be extended to a continuous linear map S from G into $\ell_\infty(I)$ with $||S|| = ||S_0||$.*

Theorem 2.2 *Each quasi-λ-nuclear map $T : E \to F$ between normed spaces E and F is also pseudo-λ-nuclear if it is regarded as a map from E into a Banach space $\ell_\infty(I)$ in which F is embedded.*

Proof. Since T is a quasi-λ-nuclear map, there exist a sequence (α_n) in λ and a bounded sequence (a_n) in E' such that $||Tx|| \leq \sum_n |\alpha_n| \, |\langle x, a_n\rangle|$, for all x in E. Let $G_0 = \{(\langle x, \alpha_n a_n\rangle) \colon x \in E\}$. Then G_0 forms a subspace of ℓ_1. Define a map $S_0 : G_0 \to F$ by $S_0(\langle x, \alpha_n a_n\rangle) = Tx$. Then S_0 is a continuous linear map from G_0 into F with $||S_0|| \leq 1$ because

$$\begin{aligned} ||S_0(\langle x, \alpha_n a_n\rangle)|| &= ||Tx|| \\ &\leq \sum\nolimits_n |\alpha_n| \, |\langle x, a_n\rangle| \\ &= \sum\nolimits_n |\langle x, \alpha_n a_n\rangle| \\ &= ||(\langle x, \alpha_n a_n\rangle)||_1. \end{aligned}$$

So by Lemmas 2.1 and 2.2, the map S_0 can be extended to a continuous linear map S from ℓ_1 into the Banach space $\ell_\infty(I)$ with $||S|| \leq 1$. Let $y_n = Se_n$, where (e_n) is the standard basis of ℓ_1. Then

$$||y_n||_\infty = ||Se_n||_\infty \leq ||S|| \, ||e_n||_1 \leq 1.$$

Also, the map S has the representation $S(\zeta_n) = \sum_n \zeta_n y_n$, for $(\zeta_n) \in \ell_1$. Since $Tx = S(\langle x, \alpha_n a_n\rangle)$, the map T has the form

$$Tx = \sum\nolimits_n \langle x, \alpha_n a_n\rangle y_n = \sum\nolimits_n \alpha_n \langle x, a_n\rangle y_n.$$

Since $(\alpha_n) \in \lambda$ and (a_n) is a bounded sequence in E', and (y_n) is a bounded sequence in $\ell_\infty(I)$, we get the pseudo-λ-nuclearity of the map T as a map from E into $\ell_\infty(I)$. □

The following known result is a crucial in proving our next result.

Theorem 2.3 *[5] If T is a bounded linear map from a normed space E into a Banach space F, then the following conditions are equivalent:*

1. *T is a 2-quasi-nuclear map.*

2. *T factors through the diagonal map $D_\mu : \ell_\infty \to \ell_2$ for some $\mu \in \ell_2$, that is, there are two bounded linear maps S_1 from E into ℓ_∞ and S_2 from ℓ_2 into F such that $T = S_2 D_\mu S_1$.*

Theorem 2.4 *If $T : E \to F$ is a quasi-λ-nuclear map between normed spaces and if S is a 2-quasi-λ -nuclear map from F into a Banach space G, then ST is a pseudo-λ-nuclear map.*

Proof. Since S is a 2-quasi-λ-nuclear map, by Theorem 2.3 and Proposition 2.3, S can be factored through a diagonal map $D_\mu : \ell_\infty \to \ell_2$ for some $\mu \in \ell_2$, that is, there are two bounded linear maps S_1 from F into ℓ_∞ and S_2 from ℓ_2 into G such that $S = S_2 D_\mu S_1$. Then by using Proposition 2.4 and Theorem 2.2, we get the pseudo-λ-nuclearity of ST. □

The following result follows from Theorem 2.1 and Theorem 2.4.

Corollary 2.2 *The Composition of quasi-λ-nuclear maps is a pseudo-λ-nuclear map.*

Remark. In case $\lambda = \ell_1$, we get the well-known result which says that the composition of quasi-nuclear maps is a nuclear map [4].

In order to prove our last main result, we introduce the following definition to facilitate our subsequent arguments.

Definition 2.1 *If A is a set of sequences, then we define the set $B(A)$ by $B(A) = \{x \colon x\,a \in \ell_\infty \ \forall \ a \in A\}$.*

Remark. It is easy matter to see that if A is a Köthe set, then $\lambda(A) \subseteq B(A)$. However, we have the following result.

Proposition 2.8 *If $\lambda(P)$ is a nuclear Köthe space, then $B(P) = \lambda(P)$.*

Proof. Let $x \in B(P)$ be given. For $a \in P$, by Grothendieck-Pietsch criterion for nuclearity, we choose $b \in P$ such that $(a_n/b_n) \in \ell_1$. Since $x \in B(P)$ and $b \in P$, there is $\alpha > 0$ such that $|x_n|\, b_n \leq \alpha$ for each n. Let

$$N_a = \{n \colon a_n \neq 0\} \quad \text{and} \quad N_b = \{n \colon b_n \neq 0\}.$$

Then $N_a \subseteq N_b$. Thus,

$$\begin{aligned}\sum_n |x_n|\, a_n &= \sum_{n\in N_a} |x_n|\, a_n \\ &\leq \sum_{n\in N_b} |x_n|\, a_n \\ &= \sum_{n\in N_b} |x_n|\, b_n\, \frac{a_n}{b_n} \\ &\leq \alpha \sum_{n\in N_b} \frac{a_n}{b_n} < +\infty.\end{aligned}$$

Therefore $x \in \lambda(P)$. □

Theorem 2.5 *Suppose that $\lambda = \lambda(P_0)$ is a nuclear G_∞-space. A bounded linear map between normed spaces is a quasi-λ-nuclear map if and only if it is a 2-quasi-λ-nuclear map.*

Proof. The "if" part condition follows from Theorem 2.1. To prove the "only if" part, let $T : E \to F$ be a 2-quasi-λ-nuclear map between normed spaces E and F. Then there exist a sequence (α_n) in λ and a bounded sequence (a_n) in E' such that $||Tx|| \leq \left(\sum_n |\alpha_n|\, |\langle x, a_n\rangle|^2\right)^{1/2}$. Then we have $||Tx|| \leq \sum_n \sqrt{|\alpha_n|}\, |\langle x, a_n\rangle|$. To finish our proof it is enough to show that $(\sqrt{|\alpha_n|}) \in \lambda$. For $a \in P_0$, find $\beta > 0$ and $b \in P_0$ such that $a_n^2 \leq \beta^2 b_n \ \forall n$. So by Proposition 2.8, $\sup_n \sqrt{|\alpha_n|} a_n \leq \sup_n \sqrt{|\alpha_n| b_n} < \infty$ since $(\alpha_n) \in \lambda$. □

3. Examples

In this section, we give some examples to show that the converse of our main previous results are not true in general. The following proposition will be of great use in our next example.

Proposition 3.1 *[6] A diagonal map $D = (d_n)$ with $d_n \geq 0$, is a pseudo-$\Lambda_\infty(\alpha)$-nuclear map from ℓ_1 into ℓ_1 if and only if the d_n's which are different from zero can be rearranged into a sequence in $\Lambda_\infty(\alpha)$.*

Now we give an example of a 2-quasi-nuclear map which is not a 2-quasi-λ-nuclear map.

Example 3.1 *Define a map $D : \ell_1 \to \ell_2$ by $Dx = (x_n/2^n)$. Then D is a 2-quasi-nuclear map which is not a 2-quasi-$\Lambda_\infty(n)$-nuclear map.*

Proof. To show that D is a 2-quasi-nuclear map, let $a_n = e_n/2^n$. Then

$$||Dx||_2^2 = \left|\left|\left(\frac{x_n}{2^n}\right)\right|\right|_2^2 = \sum_n \left|\frac{x_n}{2^n}\right|^2 = \sum_n |\langle x, a_n\rangle|^2 .$$

Since (a_n) is a sequence in ℓ_∞ with $\sum_n ||a_n||_\infty^2 < +\infty$, D is a 2-quasi-nuclear map. To show that D is not a 2-quasi-$\Lambda_\infty(n)$-nuclear map, define a map $A : \ell_2 \to \ell_1$ by putting $Ax = (x_n/2^n)$. Then A is quasi-nuclear. Therefore A is 2-quasi-nuclear. By Theorem 2.3, A can be factored through D_μ for some $\mu \in \ell_2$, that is, there are bounded maps $S_2 : \ell_2 \to \ell_\infty$, $D_\mu : \ell_\infty \to \ell_2$, and $S_1 : \ell_2 \to \ell_1$ such that $A = S_1 D_\mu S_2$. If we assume that D is 2-quasi-$\Lambda_\infty(n)$-nuclear, then by Theorem 2.5, D is quasi-$\Lambda_\infty(n)$-nuclear. Then by Proposition 2.4, S_2D is quasi-$\Lambda_\infty(n)$-nuclear. Therefore by Theorem 2.2, S_2D is pseudo-$\Lambda_\infty(n)$-nuclear. Thus by Proposition 2.4, AD is pseudo-$\Lambda_\infty(n)$ -nuclear. Since $AD : \ell_1 \to \ell_1$ is given by $ADx = (x_n/4^n)$ and AD is pseudo-$\Lambda_\infty(n)$-nuclear, by Proposition 3.1 we have, $(1/4^n) \in \Lambda_\infty(n)$, which is a contradiction. So A is not 2-quasi-$\Lambda_\infty(n)$-nuclear. □

Now, we give an example of a 2-quasi-λ-nuclear map which is not quasi-λ-nuclear. To achieve that we need the following definitions and results. For two normed spaces E and F and for integers $r \geq 0$, $\mathcal{A}_r(E, F)$ denotes the collection of all finite rank linear maps A from E into F whose range is at most r-dimensional.

Definition 3.1 *[4; P. 120] Let T be a linear map from a normed space E into a normed space F. The r-**th approximation number** $\alpha_r(T)$ of T is defined to be* $\inf\{||T - A|| : A \in \mathcal{A}_r(E, F)\}$.

Definition 3.2 *[4; P. 144] Let B be an arbitrary bounded subset in a normed space E with closed unit ball U. The infimum of all $\delta > 0$ for which there is a linear subspace F of E with dimension at most n such that $B \subset \delta U + F$ is called the n-**th diameter** of B and is denoted by $d_n(B)$.*

It is clear that $d_0(B) \geq d_1(B) \geq \cdots \geq 0$.

Definition 3.3 *[see 7] Let $T : E \to F$ be a bounded linear map between normed spaces E and F with closed unit balls U and V respectively. The n-th diameter of T, denoted by $d_n(T)$, is defined to be $d_n(T(U))$.*

Lemma 3.1 *[2, P. 23] Suppose that T is a linear map from a normed space E into a normed space F. Then $d_n(T) \le \alpha_n(T) \le \sqrt{n}\, d_n(T)$.*

Lemma 3.2 *[2, P. 23] Suppose that T is a compact map from a Banach space X into a Banach space F. Then $\alpha_n(T) = \alpha_n(T')$, where T' is the dual map of T.*

Lemma 3.3 *[2, P. 23] Suppose that T is a compact map from a Hilbert space H_1 into a Hilbert space H_2. Then $\alpha_n(T) = d_n(T)$.*

To this end, we have furnished the necessary background to give an example of a 2-quasi-λ-nuclear map which is not quasi-λ-nuclear.

Example 3.2 *Let $P = \left\{\left(n^{\ln(kn)}\right) : k = 1, 2, \ldots\right\}$. Define the map D on ℓ_2 by $Dx = (\sqrt{\alpha_n}\, x_n)$ where $\alpha_n = \frac{1}{n^2\, n^{\ln(n^2)}}$. Then we have the following assertions:*

1. *$\lambda(P)$ is a nuclear Köthe space which is subset of ℓ_1.*
2. *There are no $\rho > 0$ and $m \in N$ such that the inequality*

$$n^{\ln(n^2)} \le \rho\, n^{\ln(m\,n)} \text{ holds for all } n \in \mathbf{N}.$$

3. *$(\alpha_n) \in \lambda(P)$ and $(\sqrt{\alpha_n}) \notin \lambda(P)$.*
4. *D is a 2-quasi-$\lambda(P)$-nuclear map.*
5. *D is not a quasi-$\lambda(P)$-nuclear map.*

Proof. The proofs of 1,2, and 3 are trivial. Since $(\alpha_n) \in \lambda(P)$ and (e_n) is a bounded sequence in ℓ_2, D is a 2-quasi-λ(P)-nuclear map. To prove 5, assume that D is a quasi-$\lambda(P)$-nuclear map. Then there exist a sequence $(\beta_n) \in \lambda(P)$ and a bounded sequence (a_n) in ℓ_2 such that

$$||Dx|| \le \sum\nolimits_n |\beta_n|\, |\langle x, a_n\rangle|.$$

Let $\gamma_n = \sum_{m=n}^{\infty} |\beta_m|$. We claim that, $\gamma = (\gamma_n) \in \lambda(P)$. For $k \in \mathbf{N}$, we have

$$\begin{aligned}
\sum\nolimits_n |\gamma_n|\, n^{\ln(kn)} &= \sum\nolimits_{n=1}^{\infty} \left(\sum\nolimits_{m=n}^{\infty} |\beta_m|\right) n^{\ln(kn)} \\
&= \sum\nolimits_{n=1}^{\infty} \left(|\beta_n| \sum\nolimits_{i=1}^{n} i^{\ln(ki)}\right) \\
&\le \sum\nolimits_n |\beta_n|\, n\, n^{\ln(kn)}.
\end{aligned}$$

If we choose $m \in \mathbf{N}$ so that $n \le n^{\ln m}$, then we have

$$\sum_n |\gamma_n| \, n^{\ln(kn)} \le \sum_n |\beta_n| \, n^{\ln(mkn)} < +\infty.$$

Therefore $\gamma = (\gamma_n) \in \lambda(P)$. Let

$$M_n = \{x \in \ell_2 \colon \langle x, a_i \rangle = 0, \; i = 1, 2, \ldots, n\}.$$

If $x \in M_n$, then

$$||Dx|| \le \sum_{m=n}^{\infty} |\beta_m| \, |\langle x, a_m \rangle| \le \gamma_n \sup_n ||a_n|| \, ||x||.$$

Hence, $D(U \cap M_n) \subseteq \gamma_n U$ where U is the unit ball of ℓ_2. Therefore

$$D'(U^\circ) \subseteq \gamma_n U^\circ + M_n^\perp,$$

which gives $d_n(D') \le \gamma_n$. By Lemma 3.2, Lemma 3.3 and Theorem 8.3.2 [4, P. 130], we have $\alpha_n(T) = \sqrt{\alpha_n} \le \gamma_n$. Since $(\sqrt{\alpha_n}) \notin \lambda(P)$, we have $(\gamma_n) \notin \lambda(P)$, which is a contradiction. Therefore D is not a quasi-$\lambda(P)$-nuclear map. □

Problem. Is Theorem 2.5 still valid for any G_∞-space $\lambda(P)$ which is not nuclear?

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