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Ideal Theory in Topological Algebras

Abdelhak Najmi

Abstract

Given a simplicial topologically non radical algebra A, we characterize its topological radical, $radA$. If furthermore A is advertive, then $radA$ coincides with the Jacobson radical $RadA$. On the other hand, it is shown that every two-sided invertive simplicial topological Gelfand-Mazur algebra has a functional spectrum and for every topologically nonradical simplicial Gelfand-Mazur *amits* the set $\mathcal{X}(A)$, of all continuous multiplicative linear functionals, is not empty.

Key Words: Left, right or two-sidedness, commutativity, almost commutativity, aits, alits, arits, amits, almits, armits, topological algebra, simplicial algebra, advertive or invertive algebra, radical, topological radical, Gelfand-Mazur algebra.

1. Notations and Preliminaries

Let A be a *topological algebra* over $\mathbb{C}$, the set of complex numbers, *with separately continuous multiplication* (in the sequel *topological algebra*). If for each $a, b \in A$ there exists $u, v \in A$ such that $ab = va = bu$ [6] (resp. $ab = va$ [7], $ab = bu$ [7]) then A is said to be *two-sided* or *bilateral* (resp. *left-sided* or *left-lateral*, *right-sided* or *right-lateral*) algebra. An *aits* (resp. *alits*, *almits*, *arits*, *armits*, *amits*) is by definition an algebra A for which every ideal (resp. left ideal, left maximal ideal, right ideal, right maximal ideal, (right or left) maximal ideal) is two-sided. Denote by $aits(A)$ (resp. $alits(A)$, $almits(A)$, $arits(A)$, $armits(A)$, $amits(A)$) an algebra A which is *aits* (resp. *alits*, *almits*, *arits*, *armits*, *amits*). Denote by $com(A)$ (resp. $bil(A)$, $ls(A)$, $rs(A)$) an algebra A which is commutative (resp. two-sided, left-sided, right-sided). It is shown in [6] and [7] that

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$$com(A) \Longrightarrow bil(A) \ \Longrightarrow \begin{cases} ls(A) \\ rs(A) \end{cases} .$$

Respectively, also we have $ls(A) \Longrightarrow alits(A) \Longrightarrow almits(A) \Longrightarrow amits(A)$.
We note that, by passage to the reverse algebra, the study of an *alits* or *arits*, an *almits* or *armits*, as well as of a *left* or a *right-sided* algebra reduces each type to the other one. Consequently, we have $rs(A) \Longrightarrow arits(A) \Longrightarrow armits(A) \Longrightarrow amits(A)$. Finally, one has the following reduction diagram

$$\begin{array}{ccccc}
 & & com(A) & & \\
 & & \downarrow & & \\
ls(A) & \leftarrow & bil(A) & \rightarrow & rs(A) \\
\downarrow & \swarrow & \downarrow & \searrow & \downarrow \\
alits(A) & \leftarrow & aits(A) & \rightarrow & arits(A) \\
\downarrow & \swarrow & & \searrow & \downarrow \\
almits(A) & & \downarrow & & armits(A) \\
 & \searrow & & \swarrow & \\
 & & amits(A) & &
\end{array}$$

where the symbol "$\rightarrow$" denotes "included in".

An element a of a topological algebra A is *right* (resp. *left*) *quasi-invertible* or *right* (resp. *left*) *advertible* if there exists an element $b \in A$ such that $a \circ b = ab - a - b = 0$ (resp. $b \circ a = 0$) and it is *quasi-invertible* or *advertible* if it is both left and right advertible; finally it is *topologically right* (resp. *left*) *quasi-invertible* or *right* (resp. *left*) *advertible* if there exists a net $(a_\lambda)_{\lambda\in\Lambda}$ such that $(a_\lambda \circ a)_{\lambda\in\Lambda}$ (resp. $(a \circ a_\lambda)_{\lambda\in\Lambda}$) converge to the zero element of A. In this context the terminology "*advertibly null net*" (A. Mallios), appropriately specialized, each time, concerning "sidedness", is also of use. In particular, when A has a unit element e then $a \in A$ is *topologically right* (resp. *left*) *invertible* if there exists a net $(a_\lambda)_{\lambda\in\Lambda}$ such that $(a_\lambda a)_{\lambda\in\Lambda}$ (resp. $(aa_\lambda)_{\lambda\in\Lambda}$) converge to e. We denote the set of all advertible (resp. invertible (when A has a unit), topologically advertible, topologically invertible) elements of A by $QinvA$ (resp. $InvA$, $TqinvA$, $TinvA$). Similarly we define the sets $RqinvA$ (resp. $RinvA$, $TrqinvA$, $TrinvA$) with r or R as the initial of "right". If A has a unite it is easy to see that $QinvA = e - InvA$ and $TqinvA = e - TinvA$. We will say that A is an *advertive* (resp. *invertive*) *algebra* if it is a topological algebra such that $TqinvA = QinvA$ (resp. $TinvA = InvA$). Recall that a B_0 *-algebra* is a topological algebra whose underlying topological vector space is a complete metrizable and locally convex space (hence, alias, a *Fréchet locally convex algebra*). When I is a two-sided ideal, we note by S the canonical map from A onto A/I. If I is a subset of A, then we denote by $cl_A(I)$ the closure of I in A. The set $\mathcal{X}(A)$ is the set of all nontrivial continuous characters (continuous multiplicative linear functionals) on A. For every $x \in A$, the spectrum of x is by definition

$$Sp_A(x) = \begin{cases} \left\{\lambda \in \mathbb{C}\setminus\{0\} : \frac{x}{\lambda} \notin QinvA\right\} \cup \{0\} & \text{if } A \text{ is not unital} \\ \{\lambda \in \mathbb{C} : x - \lambda e \notin InvA\} & \text{if } A \text{ is unital} \end{cases}$$

The spectral radius of A is by definition the function $\rho_A : x \longrightarrow \rho_A(x) = sup\,\{|\lambda| : \lambda \in Sp_A(x)\}$. Finally, $RadA$ will indicate the Jacobson radical of A.

We follow [3] for the definition of the topological radical ($radA$), topologically semi-simple algebra, topologically radical algebra, Gelfand-Mazur algebra (see also [10]), topologically nonradical Gelfand-Mazur algebra and simplicial algebra (normal in the sense of E. A. Michael ([11], p. 71)). We follow [12] for the definition of a *topologically spectral algebra* (i.e. for every $x \in A$, we have $Sp_A(x) = \{f(x) : f \in \mathcal{X}(A)\}$). We will say that A is *spectral* if there is a semi-norm P on A such that, for every $x \in A$, we have $\rho_A(x) \leq P(x)$. We denote by $M(A)$ (resp. $m(A)$) the set of all regular two-sided (resp. regular, two-sided and closed) ideals in A such that each one is maximal as left or as right; $i(A)$ the set of all two-sided regular and closed ideals of A. Also, if $\mathcal{L}(X)$ is the set of all continuous linear mappings on a topological space X endowed with the composition product $(a, b) \mapsto ab$, then consider $\mathcal{L}(X)^\circ$ as the reverse algebra of $\mathcal{L}(X)$ (algebra obtained by endowing the space $\mathcal{L}(X)$ with the reverse composition product $a.b = ba$; $a, b \in \mathcal{L}(X)$). Furthermore we endow $\mathcal{L}(X)$ (resp. $\mathcal{L}(X)^\circ$) with the topology of simple convergence.

2. Introduction

The identical map $i : \mathcal{L}(X) \mapsto \mathcal{L}(X)^\circ$ is an algebraic and topological isomorphism. We know that any morphism π of an algebra A into $\mathcal{L}(X)$ is called a representation of A on X and it define on X a left A-module multiplication if we put $ax = \pi(a)(x)$. Instead, contrary to ([3], p. 26), the right multiplication $xa = \pi(a)(x)$, doesn't defines on X a right A-module multiplication. But any *anti-morphism* π (that is a vector space morphism such that $\pi(ab) = \pi(b)\pi(a)$ for every $a, b \in A$) of an algebra A into $\mathcal{L}(X)$, called here a *reverse representation* of A on X (see e.g. proof of Proposition 13), defines on X a right A-module multiplication if we put $xa = \pi(a)(x)$. In [3], Abel has defined the topological radical $radA$, as the intersection of the kernels of all continuous irreducible representations of A on a linear Hausdorff space. He proved that $radA$ is the intersection of all closed maximal regular left (resp. right) ideals of A. We can define here $radA$, as the intersection of the kernels of all continuous irreducible reverse representations of A on a linear Hausdorff space X. Thus, with this new definition, we can prove by the same arguments as those of Theorem 1, p. 27, of [3] that $radA$ is the intersection of all closed maximal regular left (resp. right) ideals of A. So the two definitions coincide. In this context, see also, for instance, proof of Proposition 13.

First we deal with algebraic aspect of *alits* and *almits*. Many properties of one sided algebras are preserved by *alits* and *almits*, except the passage to the unitization which fails for the *alits* (Remark 5).

We give also some expressions of the topological radical in every simplicial topologically nonradical algebra. In [6] (resp. [7]), the authors proved that every Banach two-sided (resp. left-sided) algebra is almost commutative. So the set $\mathcal{X}(A)$ is not empty. Here, we get that the set $\mathcal{X}(A)$ is not empty for every topologically nonradical simplicial Gelfand-Mazur *amits*. On the other hand, it is shown that every two-sided invertive simplicial topological Gelfand-Mazur algebra is a topologically spectral algebra.

We describe the structure of an artinian, simplicial Gelfand-Mazur topologically nonradical topological *amits*. Furthermore, we prove that every simplicial Gelfand-Mazur topologically nonradical topological *amits* is almost commutative. Finally, we solve the *problem of the closed ideal* (whether a given topological algebra admits a proper and closed unilateral or bilateral ideal) for a *topological algebra* which is, in particular, *topologically nonradical* (cf. Lemma 21 below).

All algebras considered here will be complex. We will say that the algebra A is a *zero algebra* if $A^2 := \{xy : x, y \in A\} = \{0\}$. For every $x \in A$ put

$$\begin{aligned} Ann_l(x) &= \{y \in A : yx = 0\}, \\ Ann_r(x) &= \{y \in A : xy = 0\}, \end{aligned}$$

where L_x a supplementary of $Ann_l(x)$ and R_x a supplementary of $Ann_r(x)$.

Recall ([7]) that if A is a left (resp. right)-sided algebra, there exists a function f of two variables such that for every $x, y \in A : xy = f(x, y)x$ (resp. $xy = yf(x, y)$). Note that the function of left (resp. right)-sidedness is such a function with the fact that each partial function $f_x : t \longmapsto f(x, t)$ (resp. $f_x : t \longmapsto f(t, x)$) is into L_x (resp. R_x).

3. Algebraic Properties and Examples

Here are some examples of *alits*, *almits* and *amits*.

Example 1 Every left -sided algebra (and, also every commutative or two-sided algebra) is an *alits*.

Example 2 Let $A = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{C} \right\}$. The only left-sided (maximal) ideal of A is $RadA = \left\{ \begin{pmatrix} 0 & 0 \\ b & 0 \end{pmatrix} : b \in \mathbb{C} \right\}$. Of course it is two-sided. So A *is an alits* as well as *an almits* (then also *an amits*). We remark that A *is not left-sided algebra*. Because the equation $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} e & 0 \\ f & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ where the unknowns are e and f, give that $\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$: which is impossible.

Example 3 Let R be a zero algebra and θ an extra element to R. Let $A = R \oplus \mathbb{C}\theta$, with θ a right unit for A and $\theta r = 0$ for every $r \in R$. Then A is an associative algebra and every left ideal of A is a left ideal of R. So A *is an alits*. We can remark that $A^2 = A$ and A has infinitely many right units, namely $r + \theta$, for every r in R. Now let I be a proper ideal of R. Then $I \oplus \mathbb{C}\theta$ is a right ideal of A, which contains θ. So it is not a left ideal. Consequently A *is not an arits*. We can remark here that A *is not a left-sided algebra*. Indeed, let $r \in R$ with $r \neq 0$, then the equation $r(s + \theta) = (t + \lambda\theta)r$ where s is any element of R and t and λ are the unknowns, is equivalent to $r = 0$. Which is impossible. So A *is not left-sided*.

Example 4 Let $A = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in \mathbb{C} \right\}$. The *only left maximal ideal* of A is $M = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : a, b \in \mathbb{C} \right\}$, and it is *two-sided*. Then A *is an almits* (then also *an amits*). But, for example,

$$I = \left\{ \begin{pmatrix} 0 & a & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : a \in \mathbb{C} \right\},$$ is a *left ideal* of A which is *not two-sided*; so A *is not an alits*.

Contrary to the *aits* where the unitization is always a two-sided algebra (Proposition 9), we have the following Remark.

Remark 1 *If A is an alits*, then *the unitization A_1* of A (algebra obtained by junction of a unit e to A) *is not necessarily a left-sided algebra nor an alits.* For this, consider the algebra of Example 2. Then A_1 is not an *alits*. For it, all left ideals of A_1, different from $RadA$ and A, are of the form $T_d = RadA \oplus \mathbb{C}(u_d, -1)$ or $L_d = A_1(u_d, -1) = \mathbb{C}(u_d, -1)$, where $u_d = \begin{pmatrix} 1 & 0 \\ d & 0 \end{pmatrix}$, $d \in \mathbb{C}$. All ideals T_d are two-sided. But none of left ideals L_d is two-sided. Indeed, for every $d \in \mathbb{C}$ let $a, b \in \mathbb{C}$ such that $da \neq b$. Then, for example, $\left(\begin{pmatrix} 1 & 0 \\ d & 0 \end{pmatrix} - e \right) \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ da - b & 0 \end{pmatrix} \notin \mathbb{C}(u_d, -1)$. So A_1 *is not an alits.* Therefore A_1 *can't be a left-sided algebra.* Now we have the following proposition.

Proposition 1 *1. The unitization of a radical algebra is always an amits, an almits and an armits.*

2. The unitization of a radical alits is always an alits.

Proof. 1. Let R be a radical algebra. Every proper left ideal I of R gives a proper left ideal $I \oplus \{0\}$ of $R \oplus \mathbb{C}e$. Let $J = \{(j, \alpha) : (j, \alpha) \in R \oplus \mathbb{C}e\}$ be a proper left ideal of $R \oplus \mathbb{C}e$, different from the $I \oplus \{0\}$, for every proper left ideal I of R. There exists a $(j, \alpha) \in J$ with $\alpha \neq 0$. Put $i = -\frac{j}{\alpha}$. Then $(i, -1) \in J$. Since i is quasi-invertible in R, there exists $r \in R$ such that $ri - r - i = 0$. Then $(r, -1)(i, -1) = (0, 1) \in J$. So $J = R \oplus \mathbb{C}e$. Therefore every proper left ideal of $R \oplus \mathbb{C}e$ is of the form $I \oplus \{0\}$, with I a proper left ideal of R. Since R is the only maximal, left maximal and right maximal ideal of $R \oplus \mathbb{C}e$, the conclusion follows. 2. As in the proof of 1. every left ideal of $R \oplus \mathbb{C}e$ is a left ideal of R. So it is two-sided. Hence $R \oplus \mathbb{C}e$ is an *alits* □

Corollary 2 *Let R be a radical algebra and R_1 its unitization. Then R_1 is two-sided if, and only if, all ideals of R are two-sided.*

Proof. If R_1 is two-sided, then by Lemma I-19 [7], R is two- sided. So all its ideals are two-sided. Conversely, as in the proof of the last proposition, every proper left ideal of R_1 is of the form $I \oplus \{0\}$, with I a proper left ideal of R. So R_1 is an *alits*. The same study can be done for proper right ideals. So R_1 is an *arits*. By Proposition I-4, p. 18, of [6], the algebra R_1 is two-sided □

Lemma 3 *Let A be a non radical algebra; and I a left regular ideal of A with right unit element θ of A modulo I. Furthermore suppose that I is two-sided and A/I is left-sided. Then A/I is unitary and I is also right regular with left unit element θ of A modulo I.*

Proof. Let I be a left regular ideal of A with right unit element θ of A modulo I. Then $a\theta - a \in I$, for every $a \in A$. Consequently, A/I is right unitary with right unit $e = S(\theta)$. Since $B = A/I$ is left-sided, we have $xe = e'(x, e)x = x$ for every $x \in B$. Consider a fixed $x \in B$ and put $e'(x, e) = e'$. Then we have $e'x = x$; and so $e'xy = xy$, for every $y \in B$. So the algebra xB is unitary with unit $e = e'$. Consequently $a\theta - a \in I$ and $\theta a - a \in I$, for every $a \in A$. □

Corollary 4 *Let A be a non radical left-sided algebra; and I a left regular ideal of A with right unit element θ of A modulo I. Then A/I is unitary and I is also right regular with left unit element θ of A modulo I.*

Proof. It is sufficient to remark that A/I is left-sided. So we can apply the above lemma □

For the next proposition, recall the next lemma from [6].

Lemma 5 *Let A be a left-sided algebra and let f be the function of left-sidedness. Then, for every $x \in A$, each partial application $f_x : t \longmapsto f_x(t)$ is linear from L_x into L_x.*

Lemma 6 *Let A be a left-sided algebra. The following assertions are equivalent.*

1. *A is two-sided*

2. *For the function f of left-sidedness, each partial application*

$t \longmapsto f_x(t) = f(x,t)$, *from* $A \longrightarrow L_x$, *is onto for every* $x \in A$.

Proof. 1. $\Longrightarrow$ 2.. Let $y \in A$ be fixed and $x \in L_y$. Since A is right-sided, there exists $u \in A$ such that $xy = yu$. Let f be the function of left-sidedness of A. Then $yu = f_y(u)y$. So $xy = f_y(u)y$. Consequently $(x - f_y(u))\, y = 0$. So $x - f_y(u) \in Ann_l(y) \cap L_y$. But $Ann_l(y) \cap L_y = \{0\}$. Then for every $x \in L_y$, there exists $u \in L_y$ such that $x = f_y(u)$. 2. $\Longrightarrow$ 1.. Let $y \in A$ be fixed. Every $z \in A$ is written as $z = z_1 + z_2$, with $z_1 \in Ann_l(y)$ and $z_2 \in L_y$. There exists $x \in A$ such that $z_2 = f_y(x)$. Since $yx = f_y(x)y$, we have $zy = f_y(x)y = yx$. So, there exists $x \in A$ such that $zy = yx$. And so A right-sided. □

In the next proposition we can restrict ourselves to the study of a left-sided algebra, because, by passage to the reverse algebra, the study of a right-sided one can then be reduced to the previous case.

Proposition 7 *Let A be a left (resp. right)-sided algebra of finite dimension. Then A is two-sided.*

Proof. Let $x \in A$ be fixed, $y \in L_x$ and let f be the function of left-sidedness of A, then $xy = f_x(y)x$. We can remark here that $y \in Ann_r(x)$ if, and only if, $f_x(y) \in Ann_l(x)$ and, equivalently, $y \in R_x$ if, and only if, $f_x(y) \in L_x$. Let $z \in L_x$ such that $f_x(y-z) = 0$, then $(f_x(y-z))\, x = 0$. So $f_x(y-z) \in Ann_l(x) \cap L_x = \{0\}$ and so $y - z \in Ann_r(x) \cap R_x = \{0\}$. Therefore f_x is an injection from $L_x \longrightarrow L_x$. Hence it is also onto, because L_x is of finite dimension. All the more f_x is onto from A onto L_x. One concludes by the previous Lemma 6. □

Lemma 8 *Let A be a non radical aits. If I is a left (resp. right) regular ideal of A with right (resp. left) unit element θ of A modulo I. Then A/I is two-sided, unitary and I is also right (resp. left) regular with left (resp. right) unit element θ of A modulo I.*

Proof. We can restrict our selves to the case when I is left regular, because the other case can be returned to the first one. All ideals of $B = A/I$ are two- sided and B is right unitary with right unit $e = S(\theta)$. So $B := A/I$ is right-sided, because we have $Bx \subset BxB \subset xB$ for every $x \in B$. Since $B = A/I$ is right-sided, we have $ex = xe'(x,e)$ for every $x \in B$. Consider a fixed $x \in B$ and put $e'(x,e) = e'$. Then we have $ex = xe'$; and so $yex = yxe'$, for every $y \in B$. Then $yx = yxe'$, for every $y \in B$. So the algebra Bx is right unitary with units e and e'. Then $e = e'$. Hence $ex = xe = x$ for every $x \in B$. Consequently B is two-sided. So $a\theta - a \in I$ and $\theta a - a \in I$ for every $a \in A$. □

Contrary to the *alits* where the unitization is not always an *alits*, we have the following proposition.

Proposition 9 *1. Let A be an aits. Then its unitization A_1 is a two-sided algebra.*

2. Let A be an almits (resp amits). Then its unitization A_1 is of the same type.

Proof. 1. By Proposition 1, it is enough to consider a nonradical algebra. If A is an *aits*, then every proper left ideal I of A gives a proper left ideal $I \oplus \{0\}$ of $A_1 = A \oplus \mathbb{C}e$. So it is two-sided. Let $J = \{(j,\beta) : (j,\beta) \in A_1\}$ be a proper left ideal of A_1, different from all ideals $I \oplus \{0\}$, with I a proper left ideal of A. There exist a $(j,\beta) \in J$ with $\beta \neq 0$. Put $i = -\frac{j}{\beta}$. Then $(i,-1) \in J$. For every $(x,\alpha) \in A_1$ we have $(x,\alpha)(j,\beta) \in J$ and so $(xj - x, 0) \in J$. But $I_j^l = \{xj - x : x \in A\}$ is a left regular ideal of A with right unit element j of A modulo I_j^l and $I_j^l \oplus \{0\} \subset J$. By Lemma 8, $I_j^r \oplus \{0\} \subset J$ with $I_j^r = \{jx - x : x \in A\}$. So, for every $(x,\alpha) \in A_1$ we have $(j,\beta)(x,\alpha) \in J$. So all ideals of A_1 are two-sided. We conclude by Proposition I-4 of [7]. 2. If A is an *almits* (resp. *amits*), by Proposition 1, it is enough to suppose that A is not a radical algebra . Every left maximal (resp. left maximal or right maximal) ideal M_1 of A_1 which is included in A is two-sided. If M_1 is not included in A_1 let $(j,\beta) \in M_1$ with $\beta \neq 0$. Put $i = -\frac{j}{\beta}$. Then $(i,-1) \in M_1$. For every $(x,\alpha) \in A_1$ we have $(x,\alpha)(j,\beta) \in M_1$ (resp. $(j,\beta)(x,\alpha) \in M_1$, when M_1 is right maximal). So $(xi - x, 0) \in M_1$ (resp. $(ix - x, 0) \in M_1$, when M_1 is right maximal). Hence $xi - x \in I := A \cap M_1$ (resp. $ix - x \in I := A \cap M_1$, when M_1 is right maximal). But I is a regular left (resp. right) maximal ideal of A. Since it is two-sided and left (resp. right) regular with right (resp. left) unit element i of A modulo I, then A/I is a field. So $(ix - x, 0) \in M_1$ (resp. $(xi - x, 0) \in M_1$). Therefore

$(i,-1)(x,\alpha) \in M_1$ (resp. $(x,\alpha)(i,-1) \in M_1$). Hence $(j,\beta)(x,\alpha) = -\beta(i,-1)(x,\alpha) \in M_1$ (resp. $(x,\alpha)(j,\beta) \in M_1$). Consequently M_1 is two-sided □

Proposition 10 *1. Let A and B be two algebras and h a morphism algebra from A to B. If A is an alits, then $h(A)$ is a sub-alits of B. In particular, if I is a two-sided ideal of A, then the quotient algebra A/I is also an alits.*

2. Cartesian product is an alits if, and only if, every factor is an alits.

3. Every inductive limit of a family of alits is an alits.

Remark 2 By 1. of Proposition 10, if A is an *alits*, then the algebra $A/RadA$ is of the same type. But the converse is false. To see this, let $A = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in \mathbb{C} \right\}$, then $RadA = \left\{ \begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} : b \in \mathbb{C} \right\}$; and so $A/RadA$ is isomorphic to $\mathbb{C}$. Hence, the quotient algebra is commutative. But $I = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{C} \right\}$ is a left ideal which is not two-sided.

4. Topological Almits

We don't know if the completion of an *almits* (resp. *amits*) is necessarily an *almits* (resp. *amits*) or not. But to have a partial answer, let us recall at the following definition.

Definition 1. ([11]) A topological algebra A is *factor finite* if A/J is of finite dimension for every, closed, regular one sided maximal ideal J in A.

Proposition 11 *Let A be a topological algebra the completion $\widehat{A}$ of which is a topological algebra too. If A is a factor finite almits (resp. amits), then $\widehat{A}$ is an almits (resp. amits) too.*

Proof. Let J be a closed maximal left ideal of $\widehat{A}$. By Lemma B 13, p. 74, of [11], $\overline{A \cap J}^{\widehat{A}} = J$ and $A \cap J$ is a left maximal ideal of A. So, by assumption, it is two-sided.

Let $y \in \widehat{A}$, $j \in J$ and a net $(y_\alpha)_{\alpha \in \Lambda}$ of elements of A which converge to y. Then $jy = \lim_\alpha jy_\alpha \in \left\{iy = \lim_\alpha iy_\alpha : i \in J\right\} := \lim_\alpha Jy_\alpha = \lim_\alpha \overline{J \cap A}^{\widehat{A}} y_\alpha \subset \lim_\alpha \overline{(J \cap A)\, y_\alpha}^{\widehat{A}} \subset \lim_\alpha \overline{(J \cap A)}^{\widehat{A}} = \overline{(J \cap A)}^{\widehat{A}} = J$. Consequently $J\widehat{A} \subset J$. The case where J is a right maximal ideal is handled in a similar way. $\square$

To describe the topological radical of a topologically nonradical algebra, we need the following proposition.

Proposition 12 *Let A be a topologically non radical simplicial algebra. Then $radA \subset TqinvA$.*

Proof. Let $b \in A \backslash TlqinvA$ and let I be the closure of the left ideal $\{a - ab : a \in A\}$. Then I is closed regular left ideal of A and b is a right unit of A modulo I. Since A is a simlicial algebra, then there exists a closed regular maximal left ideal M of A such that $I \subset M$. Hence $b \notin M$. Consequently $b \notin radA$ ([3], Theorem 1, p. 27). So $radA \subset TlqinvA$. Similarly we can prove that $radA \subset TrqinvA$. Finally, we conclude that $radA \subset TqinvA$ $(= TlqinvA \cap TrqinvA)$. $\square$

We shall say that a closed (resp. closed left) (resp. closed right) ideal I of A is *topologically* (resp. *left*) (resp. *right*) *quasi-regular ideal* , if $I \subset TqinvA$ (resp. $I \subset TlqinvA$) (resp. $I \subset TrqinvA$).

Proposition 13 *Let A be a topologically non radical simplicial algebra. Then*

1.

$$\begin{aligned}
radA &= \{a \in A : \lambda a + ba \in TlqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\} \\
&= \{a \in A : \lambda a + ba \in TrqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\} \\
&= \{a \in A : \lambda a + ba \in TqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\} \\
&= \{a \in A : \lambda a + ab \in TlqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\} \\
&= \{a \in A : \lambda a + ab \in TrqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\} \\
&= \{a \in A : \lambda a + ab \in TqinvA, \forall \lambda \in \mathbb{C}, \forall b \in A\}
\end{aligned}$$

2. *$radA$ is a (closed) topologically quasi-regular ideal which includes all topologically left or right or quasi-regular ideals of A.*

Proof. 1. Let us prove the first equality. Let A be a simplicial algebra over $\mathbb{C}$ and let $a \in radA$. Since $radA$ is a two-sided ideal of A then $\lambda a + ba \in radA$ for each $\lambda \in \mathbb{C}$ and $b \in A$. Therefore, by the last proposition, $\lambda a + ba \in TlqinvA$ for each $\lambda \in \mathbb{C}$ and $b \in A$. Hence $radA \subset \{a \in A : \lambda a + ba \in TlqinvA, \forall\lambda \in \mathbb{C}, \forall b \in A\}$. To show the converse, it is enough in the proof of Theorem 3 ([3], p. 29) to replace " A-module" by " left A-module" and "$TqinvA$" by "$TlqinvA$". The first and second equalities are similarly proved. To show that

$$radA = \{a \in A : \lambda a + ab \in TlqinvA, \forall\lambda \in \mathbb{C}, \forall b \in A\},$$

we consider reverse representations in place of representations, right A-(sub) module in place of A-(sub)module and follow Mati Abel's proof of Theorem 3 of [3], p. 29. 2. By Proposition 12, the topological radical $radA$ is a topologically quasi-regular ideal of A. By using representations and reverse representations alternatively, we can use Abel's proof of Theorem 3 (loc. cit.), to show that all left, right and two-sided quasi-regular ideals are included in $radA$ □

Proposition 14 *Let A be an advertive simplicial topologically nonradical algebra. Then we have $radA = RadA$.*

Proof. It is known that

$$RadA = \{a \in A : \lambda a + ba \in QinvA, \forall\lambda \in \mathbb{C}, \forall b \in A\}.$$

Since A is an advertive algebra then $QinvA = TqinvA$. Therefore, by the last proposition, $radA = RadA$. □

Lemma 15 *If A is a unital (with unit e) and two-sided topological algebra then $TinvA = A\backslash(\cup\{I : I \in i(A)\}$.*

Proof. The inclusion $TinvA \subset A\backslash(\cup\{I : I \in i(A)\}$ is done by ([2], Lemma 1, p. 17). For the converse inclusion, let $a \in A\backslash(\cup\{I : I \in i(A)\}$. If $a \notin TinvA$ then $a \notin InvA$ and then Aa is an ideal for which $I = cl(Aa) \neq A$ (otherwise, there exists a net $(x_\alpha)_\alpha$

such that $\lim_{\alpha} x_\alpha a = e$; so a should be in $TinvA$; contradiction). Besides, I is obviously a vector subspace and also a two-sided ideal. For it, let $x \in A$ and $y \in I$. Then there exists a net $(z_\alpha)_\alpha$ such that $\lim_{\alpha} z_\alpha a = y$; so $xy = \lim_{\alpha}(xz_\alpha)a \in I$. Hence $AI \subset I$. In an other hand $cl(Aa) = cl(aA)$, so, if $z \in A$ and $y \in I$, there exist a net $(y_\alpha)_\alpha \subset A$ such that $y = \lim_{\alpha} ay_\alpha$. So $yz = \lim_{\alpha} a(y_\alpha z) \in I$. Hence $IA \subset I$. Consequently, I is a closed two-sided ideal which contains a. But it is not possible. So $a \in TinvA$. □

Recall that Abel ([3]) has proved that every unital two-sided topological algebra, which satisfies the condition

$$\bigcup_{M \in M(A)} M = \bigcup_{M \in m(A)} M \tag{4.1}$$

is an invertive algebra. Now by the preceding lemma and the same proof as that one given by Abel ([3]) in the commutative case we have the following.

Proposition 16 *Every two-sided invertive simplicial algebra satisfies condition (4.1).*

Corollary 17 *A unital simplicial two-sided topological algebra A is invertive if, and only if, A satisfies condition (4.1).*

Proposition 18 *Every two-sided invertive simplicial Gelfand-Mazur algebra is a topologically spectral algebra.*

Proof. Let $x \in A$ and $\lambda \in Sp_A(x)$. Then $x - \lambda e \notin InvA = TinvA$. By Lemma 15, there exists a closed (regular and two-sided) ideal I of A such that $x - \lambda e \in I$. But A is simplicial, so there exists an ideal $M \in m(A)$ such that $I \subset M$. As A is a Gelfand-Mazur algebra , the maximal ideal M define an $f \in \mathcal{X}(A)$ such that $M = Kerf$. Therefore $f(x) = \lambda$. Whence $Sp_A(x) = \{f(x) : f \in \mathcal{X}(A)\}$. □

The following result extends to our case (*non-commutativity* of the algebra concerned) a previous one of Mati Abel in [2: p.19, Proposition 7]. That is, one has the following proposition.

Proposition 19 *For every topologically nonradical simplicial Gelfand-Mazur almits or amits A the set $\mathcal{X}(A)$ is not empty.*

Proof. Since $A \backslash radA$ is not empty, by Proposition 13, there exists $a \in A \backslash radA$, $b \in A$ and $\lambda \in \mathbb{C}$ such that $c = \lambda a + ab \notin TlqinvA$. Hence $c \notin LqinvA$. Then $I = \{a - ac : a \in A\}$ is a regular left ideal with right unit element c of A modulo I and $J = cl_A(I) \neq A$. Hence J is of the same type as I and, in addition, it is closed. Since A is a simplicial algebra, then there exists a closed and maximal left ideal M of the same type as J such that $J \subset M$. But M is two-sided and the quotient A/M has a right unit and no proper left ideals. Therefore A/M is a division algebra. Since A is a Gelfand-Mazur algebra, A/M is isomorphic to $\mathbb{C}$; and thereby M defines an $f \in \mathcal{X}(A)$ such that $M = Kerf$. □

By Proposition 8 ([2]), in a topological algebra with non empty set $\mathcal{X}(A)$, if $a \in TqinvA$ (resp. if A is a unital algebra and $a \in TinvA$) then $f(a) \neq 1$ (resp. $f(a) \neq 0$) for each $f \in \mathcal{X}(A)$. So we have the following proposition which is in some way a reciprocal of the result just mentioned.

Proposition 20 *Let A be a topologically nonradical simplicial Gelfand-Mazur almits or amits. Let $a \in A$, then from $f(a) \neq 1$ (resp. $f(a) \neq 0$, when A is unital) for each $f \in \mathcal{X}(A)$ follows that $a \in TqinvA$ (resp. $a \in TinvA$).*

Proof. By the last proposition, the set $\mathcal{X}(A)$ is not empty. Let $a \in A$ and $f(a) \neq 1$ (resp. $f(a) \neq 0$, when A is unital) for each $f \in \mathcal{X}(A)$. If $a \notin TqinvA$ (resp. $a \notin TinvA$), then $a \notin QinvA$ (resp. $a \notin InvA$). Hence, $I = \{b - ba : b \in A\}$ is a regular left ideal with right unit element a of A modulo I (resp. $I = Aa$ is a left ideal of A) and $J = cl_A(I) \neq A$. Consequently, J is a regular and closed left ideal with right unit element a of A modulo J (resp. J is a closed left ideal of A). Since A is a simplicial topological algebra then there exists a regular, closed and maximal left ideal M, with right unit element a of A modulo M (resp. there exists a closed and maximal left ideal M), which contains J. But A is an *almits* (or *amits*), so M is two-sided. Hence, by the fact that A is a Gelfand-Mazur algebra, M defines an $f \in \mathcal{X}(A)$ such that $M = Kerf$. Consequently, $f(a) = 1$ (resp. $f(a) = 0$). But it is not possible. Hence, $a \in TqinvA$ (resp. $a \in TinvA$) □

The following lemma yields a positive response to the *problem of closed ideal* in a topological algebra.

Lemma 21 *A given topological algebra has a closed regular unilateral ideal if, and only if, it is topologically nonradical.*

Proof. Necessary condition. Consider, for example, if A admits a regular and closed left ideal I with right unit u of A modulo I. As A is simplicial, there exists a regular and closed maximal M containing I. Consequently A admits at least a continuous and irreducible representation. As, by definition, a topological algebra is topologically nonradical if it does not admits any continuous irreducible representation, then A is topologically nonradical. Conversely, if any element of A is topologically advertible, then A must be an ideal of topologically advertible elements. So A must be topologically radical a contradiction. Then, there exists an $x \in A$ such that x is not, for example, left topologically advertible. So x is not left advertible. Then $I_l = \{zx - z : z \in A\}$ is a regular left ideal of A with right unit element x of A modulo I_l such that $Cl_A(I_l)$ is a closed regular left ideal of A with right unit element x of A modulo $Cl_A(I_l)$. □

Proposition 22 *Every topologically nonradical artinian and simplicial topological Gelfand-Mazur amits A is almost commutative.*

Proof. The case of a radical algebra is trivial. If A is not radical, then the quotient algebra $A/RadA$ is artinian and semi-simple. So, by Theorem 27, p. 315, of [9], the quotient is isomorphic to a finite product of simple algebras, say $\prod_{i=1}^{n} A_i$. The quotient $A/RadA$ is an *amits*, so every A_i is an *amits* too. Because A_i is semi-simple, it can't be a proper zero-algebra. So $A_i = \{0\}$ or A_i is a field. On another hand, by Lemma 21 and the fact that A is a simplicial *amits*, $m(A) \neq \emptyset$. Now by a result of Mart Abel ([1], Corollary 1, p. 3) the topological algebra $A/RadA$ is a Gelfand-Mazur algebra. If A_i is different from $\{0\}$ it is isomorphic to $(\prod_{j=1}^{n} A_j)/(\prod_{j=1}^{i-1} A_i \times \{0\} \times \prod_{j=i+1}^{n} A_i)$. By the preceeding reference, A_i is a Gelfand-Mazur division algebra. So A_i is isomorphic to $\mathbb{C}$ (see [4], Theorem 1, p. 120). □

Lemma 23 *Let A be a topologically nonradical and simplicial topological algebra and $x \in A$. Then the following assertions are equivalent.*

1. *x is topologically advertible.*

2. *x is not a unit element of A modulo any regular and closed one-sided ideal of A.*

3. *x is not a unit element of A modulo any regular, maximal and closed one-sided ideal of A.*

Proof. 1. $\Rightarrow$ 2.. If x is a unit element of A modulo, for example, a closed right ideal I, one has $xy - y \in I$ for any $y \in A$. Since x is topologically advertible, there exists a generalized sequence $(z_\alpha) \subset A$ such that $xz_\alpha - z_\alpha \longrightarrow x$. Consequently $x \in I$. Since $y = y - xy + xy$ for every $y \in A$, one has $I = A$; a contradiction. 2. $\Rightarrow$ 1.. If x is not topologically advertible, as in the proof of the preceedent lemma, there exist a closed regular one-sided ideal I such that x is a unit element of A modulo I. 2. $\Rightarrow$ 3. is obvious. 3. $\Rightarrow$ 2.. Suppose that x is a unit element of A modulo a regular, closed one-sided ideal of A. Since the algebra A is simplicial, the element x is a unit element of A modulo a maximal, regular, closed one-sided ideal of A. □

The following corollary is an improvement on Lemma 15.

Corollary 24 *If A is a topologically nonradical aits (or simply, with the less restricting assumption: an algebra in which all regular ideals are two-sided) and simplicial topological algebra (which is neither necessarily unital, nor necessarily bilateral) then*

$$TqinvA = A \backslash \bigcup_{I \in i(A)} I = A \backslash \bigcup_{M \in m(A)} M$$

Proof. The equation $TqinvA = A \backslash \bigcup_{I \in i(A)} I$ is the interpretation of equivalence 1. $\Leftrightarrow$ 2. of Lemma 23. While equation $TqinvA = A \backslash \bigcup_{M \in m(A)} M$ is the interpretation of equivalence 1. $\Leftrightarrow$ 3. of Lemma 23. □

Lemma 25 *Let A be a topological algebra. Moreover, consider the following assertions:*

1. *$a \in QinvA$ if, and only if $f(a) \neq 1$ for every $f \in \mathcal{X}(A)$.*

2. *$a \in InvA$ if, and only if $f(a) \neq 0$ for every $f \in \mathcal{X}(A)$.*

3. RadA is closed and A is almost commutative.

Then 1. $\Rightarrow$ 3. *and* 2. $\Rightarrow$ 3..

Proof. 1. $\Rightarrow$ 3.. We know that $RadA \subset \bigcap_{f\in\mathcal{X}(A)} Ker(f)$. If $a \in \bigcap_{f\in\mathcal{X}(A)} Ker(f)$, then $f(a) \neq 1$ for every $f \in \mathcal{X}(A)$. So $a \in QinvA$. Since $\bigcap_{f\in\mathcal{X}(A)} Ker(f)$ is an ideal of quasi-invertible elements, then it is included in $RadA$. So $RadA = \bigcap_{f\in\mathcal{X}(A)} Ker(f)$ and it is closed. Now, since $xy - yx \in RadA$ for all $x, y \in A$, the quotient algebra $A/RadA$ is commutative. 2. $\Rightarrow$ 3.. One can come down to the previous case. □

As it is shown by the following examples, *the converse is false.*

Remark 2

1. Let $A = \mathbb{C}(X) \times \mathbb{C}$, where $\mathbb{C}(X)$ is the field of rational fractions which can be provided with a topology of a metrizable *l.c.a.* with continuous multiplication ([13], 3, p. 731). Then $A/RadA = A/\{0\} = A$ *is commutative.* But *the only non vanishing character of the unital algebra A is* $f : (x, \lambda) \mapsto \lambda$; and we have $f((0,1)) = 1 \neq 0$. Nevertheless $(0,1)$ is not invertible.

2. Let $A = \mathbb{C}[t]$ be the algebra of polynomial functions of one indeterminate, equipped with the following algebra norm $P(t) \longrightarrow \|P(t)\| = \left\|\sum_{i=0}^{n} a_i t^i\right\| = \sum_{i=0}^{n} |a_i|$. Obviously *A is almost commutative.* All characters of A are of the form f_z, $z \in \mathbb{C}$, with $f_z(P) = P(z)$. But we have $\mathcal{X}(A) = \{f_z : |z| \leq 1\}$. Here, for example, we have, $f_z(X-2) \neq 0$, for every $f_z \in \mathcal{X}(A)$. However, *the set of invertible elements is* $\mathbb{C}\backslash\{0\}$.

The following proposition generalizes Theorem 5 of Mati Abel [5] to the case of *amits algebras.*

Proposition 26 *1. Let A be a simplicial and topologically non radical Gelfand-Mazur amits. Then* *(a)* $\mathcal{X}(A)$ *is non empty.*

(b) $a \in QinvA$ *if, and only if* $f(a) \neq 1$ *for every* $f \in \mathcal{X}(A)$.

(c) If A has a unite then $a \in InvA$ *if, and only if* $f(a) \neq 0$ *for every* $f \in \mathcal{X}(A)$.

(d) $RadA = \bigcap_{f \in \mathcal{X}(A)} Ker(f)$. *So it is closed.*

(e) A is almost commutative.

(f) Let $x \in A$. *Then*

i. In the non unitary case, $Sp_A(x) = \{f(x) : f \in \mathcal{X}(A)\} \cup \{0\}$.

ii. In the unitary case,
$Sp_A(x) = \{f(x) : f \in \mathcal{X}(A)\} \cup \{0\}$, *if x is not invertible.*
$Sp_A(x) = \{f(x) : f \in \mathcal{X}(A)\}$, *if x is invertible.*

iii. The amits A is advertive. If in addition $\mathcal{X}(A)$ *is compact, then A is spectral.*

2. *Let A be an advertibly complete l.m.c. amits and suppose that* $A^2 = A$, *then it is a Gelfand-Mazur algebra and so it is advertive.*

Proof. 1.(a). By Lemma 21 and the fact that A is a simplicial *amits*, $m(A)$ is not empty. Let $M \in m(A)$ be, for example, left maximal. Since A/M is a unital algebra without proper left ideal, then A/M is a division algebra. Since M is closed and A/M is a Gelfand-Mazur algebra ([1]), it is isomorphic to $\mathbb{C}$, the field of complex numbers ([4]). Consequently, there exists $f \in \mathcal{X}(A)$ such that $M = Ker(f)$. 1.(b). The necessary condition is obvious. For the sufficient condition, suppose that a is not advertible. By Lemma 23, a is a right or left unit element of A modulo a regular, maximal and closed respectively left or right ideal M of A. Since A is an *amits*, M is two-sided. Now by Corollary 1, p. 3, of [1], the algebra A/M is a (topological) Gelfand-Mazur (division) algebra. So it is isomorphic to $\mathbb{C}$. Then, there exists a nontrivial continuous character f such that $M = Ker(f)$. Consequently, $f(a) = 1$; contradiction. 1.(c). Since $InvA = e - QinvA$, it is enough to use the previous assertion. 1.(d). As in the proof of Lemma 25 we have $RadA = \bigcap_{f \in \mathcal{X}(A)} Ker(f)$. Consequently it is closed. 1.(e). By Lemma 25, A is almost commutative. 1.(f). i. Let $\lambda \in Sp_A(x)$ and $\lambda \neq 0$, then $\lambda^{-1}x$ is not quasi-invertible. Or equivalently, by 1.(b) , $f(\lambda^{-1}x) = 1$ for certain $f \in \mathcal{X}(A)$. Hence $f(x) = \lambda$ for certain $f \in \mathcal{X}(A)$. 1.(f).ii. Let e be the unit element of A. If x is not invertible, then

$0 \in Sp_A(x)$. Let $\lambda \in Sp_A(x)$ and $\lambda \neq 0$, then $x - \lambda e$ is not invertible. Or equivalently, by 1.(c)., $f(x-\lambda e) = 0$ for certain $f \in \mathcal{X}(A)$. Hence $f(x) = \lambda$ for certain $f \in \mathcal{X}(A)$. 1.(f).iii. By 1.(f).i., 1.(f).ii. and Proposition 6, p. 19, of [2], A is an advertive algebra. Now from 1.f.i. or 1.f.ii., we have $\rho_A(x) = sup\{|f(x)| : f \in \mathcal{X}(A)\}$. Since $\mathcal{X}(A)$ is compact, then A is spectral. 2. If A is a radical (so also topologically radical) advertibly complete *l.m.c.a*, by Lemma 21, the fact that A is simplicial is trivial. Now, if A is a nonradical advertibly complete *l.m.c.a*, then by ([8], Corollary II-6), it is simplicial. Let us suppose now that $A^2 = A$ and let M be a two-sided ideal which is right maximal. Put $B = A/M$. If $B^2 = \{0\}$, then $A^2 = A \subset M$ (this includes the case where A is topologically radical, i.e. $A = radA$); which is impossible. Consequently, A is topologically non radical, B is a field and M is right and left regular. Besides, M is also left maximal. Indeed, if I is a right ideal which contains M and is strictly included in A, then $S(I)$ is an ideal of the field B, which is different from B. Hence $S(I) = \{0\}$. So $M = I$. Hence every two sided maximal ideal is regular and is maximal as left and as right. So if $M \in m(A)$, then A/M is isomorphic to $\mathbb{C}$. Consequently A is a Gelfand-Mazur algebra. Now, by 1.(f).i and 1.(f).ii, A is a topologically spectral algebra. By Proposition 6, p. 19, of [2], A is an advertive algebra. □

Remark 3 The part 2. of the last proposition is the reciprocal result in case of *l.m.c.a.* of Corollary 1 of M. Abel ([2], p. 16).

W. Zelazko ([14]) has given an example of a B_0-convex algebra with closed radical which is Q -algebra and which is not m-convex.

Corollary 27 *Let A be a simplicial Gelfand-Mazur B_0-convex algebra which is an amits and for which $RadA \neq A$. Then $B := A/RadA$ is a l.m.c.a..*

Proof. The quotient algebra B is a B_0-algebra. By the previous proposition, B is commutative, then, by Theorem B ([14]), the algebra B is m-convex. □

Proposition 28 *Let A be a topological amits and M a left maximal ideal of A. Then M is either the kernel of a nontrivial continuous character of A, or a hyperplane of A of codimension 1 containing A^2. In particular, this is the case when M is closed and not regular.*

Proof. Since A is an *amits*, then M is two-sided. Besides M is regular if, and only if, A/M is a field. Then if M is not regular, A/M can not be a field; and then it is a zero-algebra such that $dim(A/M) = 1$. If M is regular and closed, then it is easy to prove that M is the kernel of a nontrivial continuous character of A. □

Corollary 29 *Let A be a topological amits. Then the closed regular maximal left or right ideals of A of codimension* 1*, are exactly the kernels of nontrivial continuous characters of A, hence, two-sided, as well.*

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Abdelhak NAJMI
Ecole Normale Supérieure
Takaddoum B. P. 5118, 10105
Rabat (Maroc)
e-mail: najmiabdelhak@yahoo.fr

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