Turk J Math
29 (2005) , 65 – 74.

On the Nilpotency Class of Lie Rings With Fixed-Point-Free Automorphisms

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Abstract

Let L be a solvable Lie ring with derived length s. Assume that L admits an automorphism ϕ of prime order $p \geq 11$ such that $C_L(\phi) = 0$. It is proved that the class of L is less than $\frac{(p-2)^{s+1}}{(p-3)^2}$.

Key words and phrases: automorphisms, Lie rings

1. Introduction

An automorphism ϕ of a Lie ring L is called fixed-point-free if $C_L(\phi) = 0$. Here, as usual, $C_L(\phi)$ denotes the set $\{x \in L; x^\phi = x\}$. In [1] Higman showed that there exists a function $h(p)$ depending only on p such that if L is any Lie ring admitting a fixed-point-free automorphism ϕ of prime order p, then L is nilpotent and the nilpotency class of L is at most $h(p)$. He also showed that the class of a nilpotent group with a fixed-point-free automorphism of order p is at most $h(p)$. The minimal function satisfying the above condition is now called the Higman function. It is well-known elementary results that $h(2) = 1$ and $h(3) = 2$. Higman proved that $h(p) \geq \frac{p^2-1}{4}$ for $p > 2$ and that $h(5) = 6$. Scimemi showed that $h(7) = 12$. The question about exact values of $h(p)$ for $p \geq 11$ still is open.

1991 *Mathematics Subject Classification:* 17B70.
The author was supported by CNPq-Brazil

An upper bound for $h(p)$ was given by Kreknin and Kostrikin in [3]. They showed that

$$h(p) \leq \frac{(p-1)^{2^{p-1}-1}-1}{p-2}.$$

One of the crucial steps in the result of Kreknin and Kostrikin is the following theorem.

Theorem 1.1 *Let L be a solvable Lie ring with derived length s. Assume that L admits an automorphism ϕ of prime order p such that $C_L(\phi)=0$. Then L is nilpotent and the class of L is at most $\frac{(p-1)^s-1}{p-2}$.*

In [5] Meixner strengthened Theorem 1.1 by showing that under the above hypothesis L is of class at most $(p-1)^{s-1}$. Our goal in the present paper is to obtain a further improvement of Theorem 1.1. Since the precise values of $h(p)$ with $p \leq 7$ are known, we consider the case $p \geq 11$. It will be shown (see Theorem 3.7) that in this case the class of L is less than $\frac{(p-2)^{s+1}}{(p-3)^2}$. Combining this with Kreknin's theorem [4], that says that the derived length of a Lie ring with a fixed-point-free automorphism of finite order is bounded by a function of the order of the automorphism, it is immediate that

$$h(p) \leq \frac{(p-2)^{2^{p-1}}-1}{(p-3)^2}.$$

A yet better bound for $h(p)$ can be obtained by analysing the proof of Kreknin's theorem, but this is beyond the purpose of the paper.

2. Some elementary lemmas

Given elements $l_1, l_2, \ldots, l_m$ of a Lie ring L, we denote by $[l_1, l_2, \ldots, l_m]$ the element $[\ldots[[l_1, l_2], \ldots, l_{m-1}], l_m]$. Let $L_{i_1}, \ldots, L_{i_m}$ be some not necessarily distinct subsets of L. We denote by $[L_{i_1}, \ldots, L_{i_m}]$ the subgroup of the additive group of L generated by all elements of the form $[l_1, l_2, \ldots, l_m]$, where each l_j belongs to L_{i_j}. The symbols $L^{(k)}$ and $\gamma_k(L)$ denote the kth term of the derived series of L and the kth term of the lower central series of L. As usual, we write L' in place of $L^{(1)}$. The centralizer $C_L(R)$ of a subset R in L is defined by $C_L(R)=\{x \in L; [R,x]=0\}$.

Throughout the paper $\mathbb{Z}_p$ denotes the additively written cyclic group of prime order $p \geq 11$. Let $t, i_1, \ldots, i_k$ be not necessarily distinct non-zero elements in $\mathbb{Z}_p$. We say that

$i_1, \ldots, i_k$ produce t if there exists a subset S of $\{1, 2, \ldots, k\}$ such that $t = \sum_{j \in S} i_j$. We assume here that the sum of the empty set of elements of $\mathbb{Z}_p$ is zero. Our first lemma is taken from [3]. For the reader's convenience we include the proof.

Lemma 2.1 *Let $i_1, \ldots, i_k$ be not necessarily distinct non-zero elements of $\mathbb{Z}_p$ (p a prime). Then either they produce at least $k+1$ pairwise distinct elements of $\mathbb{Z}_p$ or else they produce all elements of $\mathbb{Z}_p$.*

Proof. Let M_s denote the set of all elements produced by $i_1, \ldots, i_s$. We prove the lemma by induction on k. If $k = 1$ then M_k consists of two elements, namely 0 and i_1. Thus in the case $k = 1$ the lemma is true. Assume that $k \geq 2$. Note that $M_k = M_{k-1} \cup M_{k-1} + i_k$. By induction, M_{k-1} either consists of at least k elements or $M_{k-1} = \mathbb{Z}_p$. Clearly the lemma fails to be true if and only if $|M_{k-1}| = k < p$ and $M_{k-1} + i_k = M_{k-1}$. The condition $M_{k-1} + i_k = M_{k-1}$ implies that M_{k-1} contains the subgroup of $\mathbb{Z}_p$ generated by i_k. Therefore $M_{k-1} = \mathbb{Z}_p$, a contradiction against the assumption that $|M_{k-1}| = k < p$. $\square$

Lemma 2.2 *Let $i_1, \ldots, i_k$ be non-zero elements of $\mathbb{Z}_p$ which produce exactly $k + 1 < p$ pairwise distinct elements of $\mathbb{Z}_p$. Then for any $j \in \{1, \ldots, k\}$ we have either $i_j = i_1$ or $i_j = -i_1$.*

Proof. We use induction on k. Let $k = 2$. If $i_2 \neq \pm i_1$, then the elements $0, i_1, i_2, i_1 + i_2$ are pairwise distinct. Hence i_1, i_2 produce at least $k + 2 = 4$ elements. Therefore, if $k \leq 2$, the lemma holds.

Assume that $k \geq 3$. As in the proof of Lemma 2.1 let M_s denote the set of all elements produced by $i_1, \ldots, i_s$. We have $M_k = M_{k-1} \cup M_{k-1} + i_k$.

By Lemma 2.1 M_{k-1} contains at least k elements and, certainly, our hypothesis implies that M_{k-1} contains at most $k + 1$ elemens. If $|M_{k-1}| = k + 1$ then $M_{k-1} = M_k$ and therefore M_{k-1} contains the subgroup of $\mathbb{Z}_p$ generated by i_k. In this case $|M_{k-1}| = p$ which, contradicts the assumption that $k + 1 < p$. Thus, $|M_{k-1}| = k$ and we are in a position to apply the induction hypothesis. So for each $j \in \{1, \ldots, k - 1\}$ we have either $i_j = i_1$ or $i_j = -i_1$.

We certainly can reverse the rôles of i_k and of i_{k-1}. Since $k \geq 3$, we obtain that either $i_k = i_1$ or $i_k = -i_1$. The lemma follows. $\square$

Lemma 2.3 *Let $t, i_1, \ldots, i_{p-1}$ be not necessarily distinct non-zero elements of $\mathbb{Z}_p$ such that $i_1, \ldots, i_{p-2}$ do not produce $-t$. Then for any $j = 1, \ldots, p-2$, satisfying the condition that $i_j + i_{p-1} \neq 0$, the elements $i_1, \ldots, i_{j-1}, i_j + i_{p-1}, i_{j+1}, \ldots, i_{p-2}$ produce $-t$.*

Proof. Since the ordering of $i_1, \ldots, i_{p-2}$ plays no rôle, it suffices to prove the lemma only for $j = p-2$. We have to show that $i_1, \ldots, i_{p-3}, i_{p-2} + i_{p-1}$ produce $-t$. Let M_s denote the set of all elements produced by $i_1, \ldots, i_{s-1}, i_s$. Then

$$M_{p-2} = M_{p-3} \cup M_{p-3} + i_{p-2}.$$

Also, if M^* is the set of all elements produced by $i_1, \ldots, i_{p-3}, i_{p-2} + i_{p-1}$, then

$$M^* = M_{p-3} \cup M_{p-3} + i_{p-2} + i_{p-1}.$$

By Lemma 2.1 M_{p-3} contains at least $p-2$ elements. Since

$$-t \notin M_{p-2} = M_{p-3} \cup M_{p-3} + i_{p-2},$$

we conclude that $-t - i_{p-2} \notin M_{p-3}$. Hence, $M_{p-3} = \mathbb{Z}_p \backslash \{-t, -t - i_{p-2}\}$. Now the assumption that $i_{p-2} + i_{p-1} \neq 0$ implies that $-t - i_{p-2} \in M_{p-3}$. We see that

$$-t \in M_{p-3} + i_{p-2} + i_{p-1} \subseteq M^*.$$

The proof is complete. □

3. $\mathbb{Z}_p$-graded Lie rings

Recall that, for an additively written abelian group A, a Lie ring L is said to be A-graded if the additive group of L is presented as a sum $L = \sum_{i \in A} L_i$ of subgroups L_i indexed by elements of A in such a way that $[L_i, L_j] \leq L_{i+j}$ for all $i, j \in A$. Given an A-graded Lie ring $L = \sum_{i \in A} L_i$, an ideal N of L is called homogeneous if $N = \sum_{i \in A} N_i$, where $N_i = N \cap L_i$. It is easy to see that the members of the derived series and the members of the lower central series of an A-graded Lie ring are homogeneous.

Lemma 3.1 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring with $L_0 = 0$. Let $t, i_1, \ldots, i_k$ be not necessarily distinct non-zero elements in $\mathbb{Z}_p$ such that $i_1, \ldots, i_k$ produce $-t$. Assume $R_{i_1}, \ldots, R_{i_k}$ are subsets of $L_{i_1}, \ldots, L_{i_k}$ respectively, and denote by R the subring generated by the R_{i_j}. Let $M = L_t \cap C_L(R')$. Then $[M, R_{i_1}, \ldots, R_{i_k}] = 0$.*

Proof. Let us choose arbitrary elements $m \in M$ and $l_j \in R_{i_j}$; $j = 1, \ldots, k$. Since for any $x \in C_L(R')$ and any $a, b \in R$ we have $[x, a, b] = [x, b, a]$, it follows that

$$[m, l_1, l_2, \ldots, l_j, l_{j+1}, \ldots, l_k] = [m, l_1, l_2, \ldots, l_{j+1}, l_j, \ldots, l_k].$$

Thus the above commutator does not change under any permutation of the the l_j. By the hypothesis there exist several indices $i_{a_1}, i_{a_2}, \ldots, i_{a_r}$, among the $i_1, \ldots, i_k$, whose sum equals $-t$. Then

$$[m, l_1, \ldots, l_k] = [\underline{m, l_{a_1}, \ldots, l_{a_r}}, l_{b_1}, \ldots, l_{b_{r_1}}], \quad (r + r_1 = k).$$

The underlined subcommutator lies in $L_0 = 0$ so the commutator $[m, l_1, \ldots, l_k]$ is zero as well. Since $[M, R_{i_1}, \ldots, R_{i_k}]$ is generated by commutators of the form $[m, l_1, \ldots, l_k]$, the lemma follows. $\Box$

Lemma 3.2 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring such that $L_0 = 0$. Let $r \in \mathbb{Z}_p$ and $H = \langle L_r, L_{-r} \rangle$. Then $[L, \underbrace{H, \ldots, H}_{p-1}] = 0$.*

Proof. Since $L = \sum L_i$, it is sufficient to show that for any $t \in \mathbb{Z}_p$ and any $i_1, \ldots, i_{p-1} = \pm r$ we have

$$[L_t, L_{i_1} \ldots L_{i_{p-1}}] = 0.$$

By Lemma 2.1 $i_1, \ldots, i_{p-1}$ produce p elements. In particular, they produce $-t$. Let J be some minimal subset of $\{1, \ldots, p-1\}$ such that $-t = \sum\limits_{j \in J} i_j$. We claim that either $i_j = r$ for any $j \in J$ or $i_j = -r$ for any $j \in J$. Indeed, suppose $i_{j_1} = r$ and $i_{j_2} = -r$. Then $i_{j_1} + i_{j_2} = 0$. Put $J_1 = J \backslash \{j_1, j_2\}$. Obviously we have $-t = \sum\limits_{j \in J_1} i_j$. This contradicts the minimality of J.

Thus without any loss of generality we may assume that $i_j = r$ for any $j \in J$. So if $s = |J|$, then $-t = s \cdot r$. Since $[L_{-r}, L_r] \leq L_0 = 0$, it is clear that $[L, L_{-r}, L_r] = [L, L_r, L_{-r}]$.

We now easily derive that $[L_t, L_{i_1}, \ldots, L_{i_{p-1}}] = [L_t, \underbrace{L_r, \ldots, L_r}_{s}, L_{k_1}, \ldots, L_{k_{p-1-s}}]$, where $\{k_1, \ldots, k_{p-1-s}\} = \{i_1, \ldots, i_{p-1}\} \backslash J$. Since $[L_t, \underbrace{L_r, \ldots, L_r}_{s}] \leq L_0 = 0$, it follows that $[L_t, L_{i_1} \ldots L_{i_{p-1}}] = 0$. □

The next theorem was first proved by Meixner [5]. Our proof is quite different, though.

Theorem 3.3 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring such that $L_0 = 0$. Assume L is metabelian. Then L is nilpotent of class at most $p - 1$.*

Proof. It suffices to show that for any $i_1, \ldots, i_p \in \mathbb{Z}_p \backslash \{0\}$ and any $l_k \in L_{i_k}$, $k = 1, \ldots, p$, we have $[l_1, l_2, \ldots, l_p] = 0$. If $i_3, \ldots, i_p$ produce $-i_1 - i_2$ then, since $[l_1, l_2] \in L_{i_1+i_2}$, Lemma 3.1 tells that $[l_1, l_2, \ldots, l_p] = 0$. Suppose that $i_3, \ldots, i_p$ do not produce $-i_1 - i_2$. In this case by Lemma 2.2 there exists $r \in \mathbb{Z}_p$ such that $i_j = \pm r$ for any $j = 3, \ldots, p$. Set $H = \langle L_r, L_{-r} \rangle$. If one of i_1, i_2 equals $\pm r$ then obviously

$$[l_1, \ldots, l_p] \in [L, \underbrace{H, \ldots, H}_{p-1}]$$

and it follows from Lemma 3.2 that $[l_1, l_2, \ldots, l_p] = 0$.

Thus, assume that none of i_1, i_2 equals $\pm r$ and write

$$[l_1, \ldots, l_p] = [l_1, l_3, l_2, l_4, \ldots, l_p] - [l_2, l_3, l_1, l_4, \ldots, l_p].$$

Since $i_2 \neq \pm r$, Lemmas 2.1 and 2.2 yield that $i_2, i_4, \ldots, i_p$ produce any element in $\mathbb{Z}_p$. In particular they produce $-(i_1 + i_3)$. Note that $[l_1, l_3] \in L' \cap L_{i_1+i_3}$. As L' is abelian, Lemma 3.1 implies that $[l_1, l_3, l_2, l_4, \ldots, l_p] = 0$. Similarly, one can derive that $[l_2, l_3, l_1, l_4, \ldots, l_p] = 0$. This shows that $[l_1, l_2, \ldots, l_p] = 0$ for any $l_k \in L_{i_k}$; $k = 1, \ldots, p$. The lemma follows. □

Lemma 3.4 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring with $L_0 = 0$. Let M and N be homogeneous ideals of L such that $M \leq C_L(N')$. Then*

$$[M, \underbrace{N, \ldots, N}_{p-2}] \leq Z(L).$$

Proof. For any $i \in \mathbb{Z}_p$ we let $M_i = M \cap L_i$ and $N_i = N \cap L_i$. Choose arbitrary elements

$$m \in M_t,\ n_k \in N_{i_k};\ k = 1, \ldots, p-2,\ \ l \in L_{i_{p-1}}.$$

Since $M = \sum M_i$ and $N = \sum N_i$, it suffices to show that

$$[m, n_1, \ldots, n_{p-2}, l] = 0.$$

If $i_1, \ldots, i_{p-2}$ produce $-t$, then $[m, n_1, \ldots, n_{p-2}, l] = 0$ by Lemma 3.1. Assume that $i_1, \ldots, i_{p-2}$ do not produce $-t$. We have

$$[m, n_1, \ldots, n_{p-2}, l] = [m, n_1, \ldots, n_{p-3}, l, n_{p-2}] + [m, n_1, \ldots, n_{p-3}, [n_{p-2}, l]].$$

Note that $[n_{p-2}, l] \in N_{i_{p-2}+i_{p-1}}$. Lemmas 3.1 and 2.3 now tell us that if $i_{p-2} + i_{p-1} \neq 0$ then

$$[m, n_1, \ldots, n_{p-3}, [n_{p-2}, l]] = 0.$$

Obviously this is true also when $i_{p-2} + i_{p-1} = 0$ because, in this case, $[n_{p-2}, l] = 0$. Therefore we can conclude that

$$[m, n_1, \ldots, n_{p-2}, l] = [m, n_1, \ldots, n_{p-3}, l, n_{p-2}].$$

The above argument allows us to transfer l in the commutator $[m, n_1, \ldots, n_{p-3}, l, n_{p-2}]$ further on the left, so we obtain

$$[m, n_1, \ldots, n_{p-2}, l] = [m, l, n_1, \ldots, n_{p-2}].$$

Note that $[m, l] \in M_{t+i_{p-1}}$ and, since $i_{p-1} \neq 0$, it follows that $t + i_{p-1} \neq t$. Taking into account that $i_1, \ldots, i_{p-2}$ do not produce $-t$ and using Lemma 2.1 we conclude that $i_1, \ldots, i_{p-2}$ produce any element of $\mathbb{Z}_p$ except $-t$. In particular, they produce $-t - i_{p-1}$. Now the equality $[m, n_1, \ldots, n_{p-2}, l] = 0$ follows from Lemma 3.1. The lemma is established. □

Let for the rest of the paper $\sigma(s)$ stand for the number

$$\frac{(p-2)^s - 1}{p-3}.$$

We denote by $Z_i(L)$ the ith term of the upper central series of L.

Lemma 3.5 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring with $L_0 = 0$. Suppose that M is a homogeneous ideal of L such that $L/C_L(M)$ is solvable of derived length s. Then $M \leq Z_{\sigma(s+1)}(L)$.*

Proof. We will use induction on s, the case $s \leq 1$ being obvious from Lemma 3.4. Assume that $s \geq 2$. Let $N = L^{(s-1)}$. Then N is a homogeneous ideal of L and $M \leq C_L(N')$. By Lemma 3.4,

$$[M, \underbrace{N, \ldots, N}_{p-2}] \leq Z(L).$$

Let $\bar{L} = L/Z(L)$, and let $\bar{X}$ denote the image in $\bar{L}$ of a subset X of L. Set $K = [M, \underbrace{N, \ldots, N}_{p-3}]$. Then $\bar{L}/C_{\bar{L}}(\bar{K})$ is of derived length at most $s-1$ and so, by the induction hypothesis,

$$\bar{K} \leq Z_{\sigma(s)}(\bar{L}).$$

Considering now $\bar{L}/Z_{\sigma(s)}(\bar{L})$ and repeating the argument, we obtain

$$[\bar{M}, \underbrace{\bar{N}, \ldots, \bar{N}}_{p-4}] \leq Z_{2\sigma(s)}(\bar{L})$$

and, more generally,

$$[\bar{M}, \underbrace{\bar{N}, \ldots, \bar{N}}_{p-i}] \leq Z_{(i-2)\sigma(s)}(\bar{L}).$$

It becomes clear that $\bar{M} \leq Z_{(p-2)\sigma(s)}(\bar{L})$. Therefore $M \leq Z_{(p-2)\sigma(s)+1}(L)$. Remark that $(p-2)\sigma(s) + 1 = \sigma(s+1)$. The lemma follows. □

Theorem 3.6 *Let $L = \sum L_i$ be a $\mathbb{Z}_p$-graded Lie ring such that $L_0 = 0$. If L is of derived length s then L is nilpotent and the class of L is at most $1+(p-2)\sum_{i=0}^{s-1}\sigma(i)$. In particular, the class of L is less than $\frac{(p-2)^{s+1}}{(p-3)^2}$.*

Proof. Assume that $s \geq 2$ and use induction on s. Let N be the metabelian term of the derived series of L and let $M = N'$. The induction hypothesis will be that L/M

is nilpotent and the class of L/M is at most $1+(p-2)\sum_{i=0}^{s-2}\sigma(i)$. Theorem 3.3 tells us that $M \le Z_{p-2}(N)$. Set $M_i = [M, \underbrace{N, \dots, N}_{p-2-i}]$. Then $M_i \le Z_i(N)$ and $M_{p-2} = M$. Since $M_1 \le Z(N)$, it follows that the derived length of $L/C_L(M_1)$ is at most $s-2$. Now Lemma 3.5 yields $M_1 \le Z_{\sigma(s-1)}(L)$. Considering $L/Z_{\sigma(s-1)}(L)$ and applying Lemma 3.5 again we obtain

$$M_2 \le Z_{2\sigma(s-1)}.$$

Eventually, we derive

$$M = M_{p-2} \le Z_{(p-2)\sigma(s-1)}(L).$$

Therefore, L is of class at most

$$1+(p-2)\sum_{i=0}^{s-2}\sigma(i)+(p-2)\sigma(s-1) = 1+(p-2)\sum_{i=0}^{s-1}\sigma(i).$$

A direct calculation shows that

$$1+(p-2)\sum_{i=0}^{s-1}\sigma(i) = 1-\frac{(p-2)}{(p-3)^2}-\frac{(p-2)(s-1)}{(p-3)}+\frac{(p-2)^{s+1}}{(p-3)^2},$$

which is less than $\frac{(p-2)^{s+1}}{(p-3)^2}$. □

Theorem 3.7 *Let L be a solvable Lie ring with derived length s. Assume that L admits an automorphism ϕ of prime order p such that $C_L(\phi)=0$. Then L is nilpotent and the class of L is less than $\frac{(p-2)^{s+1}}{(p-3)^2}$.*

Proof. Let ω be a primitive nth root of unity, and let us set $K = L \otimes \mathbb{Z}[\omega]$. Then, in a natural way, ϕ can be regarded as an automorphism of K with the property that $C_K(\phi)=0$. Put $K_i = \{l \in K | l^\phi = \omega^i l\}$ and $R = \sum_{i\in\mathbb{Z}_p} K_i$. Then $[K_i, K_j] \le K_{i+j}$ for any $i, j \in \mathbb{Z}_p$. Thus, the ring R (viewed as an algebra over $\mathbb{Z}[\omega]$) becomes $\mathbb{Z}_p$-graded. Since $C_K(\phi)=0$, it is clear that $K_0=0$. Furthermore, the derived length of K equals that of L so that R is solvable and has derived length at most s. Set $f = \frac{(p-2)^{s+1}}{(p-3)^2}$. Now Theorem 3.6 tells us that R is nilpotent with class at most f.

On the other hand, the subring R contains pK [2, Lemma 4.1.1] so it follows that $\gamma_{f+1}(pK) = 0$. However $\gamma_{f+1}(pK)$ is the same as $p^{f+1}\gamma_{f+1}(K)$. It follows that the additive group of $\gamma_{f+1}(K)$ is a p-group. Since $\gamma_{f+1}(K)$ admits a fixed-point-free automorphism of order p, we conclude that $\gamma_{f+1}(K) = 0$, as required. $\square$

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Received 03.10.2003