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Semi-Slant Submanifolds of a Nearly Kaehler Manifold

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Abstract

The aim in the present paper is to study some basic geometric properties of semi-slant submanifolds of a nearly Kaehler manifold.

Key words and phrases: Slant, Semi-slant, totally umbilical, totally geodesic.

1. Introduction

Slant submanifolds of a Kaehler manifold have been studied extensively in [2], [3], [4] [5] etc., whereas slant submanifolds of a nearly Kaehler manifold are yet to be explored to that extent. A nearly Kaehler structure on a manifold provides an interesting study with differential geometric point of view (c.f., [7], [8] etc.), consequently, the study of submanifolds of a nearly Kaehler manifold vis-a-vis that of a Kaehler manifold assumes significance in general. The study of differential geometry of semi-slant submanifolds, as a generalized version of CR-submanifolds of a Kaehlerian manifold was initiated by N. Papaghiuc [10]. Our aim, in the present note is to extend the study and explore some basic geometric aspects of semi-slant submanifolds of a nearly Kaehler manifold some of which have already been studied in the setting of a Kaehler manifold. The paper is organized in the following way.

In section 2, we recall some necessary details of almost Hermitian , nearly Kaehler manifolds and their submanifolds to provide some pre-requisites for the succeeding sections. In section 3, we have obtained some results regarding the integrability and the

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parallelism of the distributions on a semi-slant submanifold of an almost Hermitian manifold. In fact, this section intends to make a foundation for studying differential geometry of the semi-slant submanifolds of a nearly Kaehler manifold. Section 4 deals with the integrability of the distributions on the semi-slant submanifolds of a nearly Kaehler manifold and some important geometric aspects of the leaves of these distributions. In section 5, we have obtained some geometrically interesting results for totally umbilical semi-slant submanifolds of a nearly Kaehler manifold.

2. Preliminaries

In this section we give some terminology and notations used throughout this paper. We recall some necessary facts and formulas from the theory of almost Hermitian manifolds and their submanifolds.

Let $\bar{M}$ be an almost Hermitian manifold with almost complex structure J and Riemannian metric g. Further, let $T\bar{M}$ denote the tangent bundle of $\bar{M}$ and $\bar{\nabla}$, the operator of covariant differentiation with respect to g in $\bar{M}$. If the almost complex structure J satisfies

$$(\bar{\nabla}_U J) + (\bar{\nabla}_V J)U = 0 \tag{2.1}$$

for any $U, V \in T\bar{M}$, then the manifold $\bar{M}$ is called a nearly Kaehler manifold. Obviously every Kaehler manifold is nearly Kaehler. The geometric meaning of the nearly Kaehler condition is that geodesics are holomorphicaly planer curves. A curve γ on an almost Hermitian manifold is holomorphicaly planer if the holomorphic section determined by its tangent field is parallel along the curve. So far as non Kaehler nearly Kaehler manifolds are concerned, one of the most prominent example is that of S^6. On a 6-dimensional unit sphere S^6, one can construct an almost complex structure using the properties of the Cayley division algebra on R^7. We refer to [6] for this construction. It is known that this almost complex structure on S^6 is not integrable and that it is a nearly Kaehler structure on S^6.

For an almost complex structure J on a manifold $\bar{M}$, the Nijenhuis tensor field S is defined by

$$S(U, V) = [U, V] + J[JU, V] + J[U, JV] - [JU, JV] \tag{2.2}$$

for any $U, V \in T\bar{M}$. It is well known that vanishing of the tensor S is a necessary and sufficient condition for an almost complex manifold to be a complex manifold. The following is a useful relation:

$$S(JU, V) = S(U, JV) = -JS(U, V). \tag{2.3}$$

For an arbitrary submanifold M of a Riemannian manifold $\bar{M}$, Gauss and Weingarten formulae are respectively given by

$$\bar{\nabla}_U V = \nabla_U V + h(U, V) \tag{2.4}$$

and

$$\bar{\nabla}_U \xi = -A_\xi U + \nabla_U^\perp \xi \tag{2.5}$$

for all $U, V \in TM$, where ∇ is the induced Riemannian connection on M, ξ is a vector field normal to M, h is the second fundamental form of M, $\nabla^\perp$ is the normal connection in the normal bundle $T^\perp M$ and A_ξ is the shape operator of the second quadratic form. Moreover, we have

$$g(A_\xi U, V) = g(h(U, V), \xi), \tag{2.6}$$

where g denotes the Riemannian metric on $\bar{M}$ as well as the metric induced on M. The mean curvature vector H of M is given by

$$H = \frac{1}{n} \sum_{i=1}^{n} h(e_i, e_i), \tag{2.7}$$

where n is the dimension of M and $\{e_1, e_2, \cdots e_n\}$ is a local orthonormal frame of vector fields on M.

A distribution D on a submanifold M of an almost Hermitian manifold $\bar{M}$ is said to be a slant distribution if for each $U \in D_x$, the angle θ between JU and D_x is constant, i.e., independent of $x \in M$ and $U \in D_x$. A submanifold M of $\bar{M}$ is said to be a slant submanifold if the tangent bundle TM of M is slant.

A semi-slant submanifold M of $\bar{M}$ is a submanifold which admits two orthogonal complementary distributions D_1 and D_2 such that D_1 is holomorphic i.e., $JD_1 = D_1$ and D_2 is slant with slant angle $\theta \neq 0$.

3. Semi-Slant Submanifolds of an Almost Hermitian Manifold

Throughout, we assume that M is a semi-slant submanifold of an almost Hermitian manifold $\bar{M}$. For $U, V \in TM$ and $\xi \in T^\perp M$, we decompose JU and $J\xi$ into tangential and normal parts as

$$JU = PU + FU, \tag{3.1}$$

$$J\xi = t\xi + f\xi. \tag{3.2}$$

For the submanifold M, the tangent bundle TM and the normal bundle $T^{\perp}M$ of M are decomposed as

$$TM = D_1 \oplus D_2 \tag{3.3}$$

$$T^{\perp}M = FD_2 \oplus \mu \tag{3.4}$$

where μ is the orthogonal complementary distribution of FD_2 in $T^{\perp}M$ and is invariant subbundle of $T^{\perp}M$ under J. Moreover, in view of equation (3.3), we may write

$$U = T_1U + T_2U, \tag{3.5}$$

where T_1 and T_2 denote the projection operators onto D_1 and D_2, respectively. Following are some easy observations which we enlist for later use.

$$\left.\begin{array}{l} (a)\ \ PD_1 \subseteq D_1,\ \ (b)\ \ FD_1 = \{0\}, \\ \\ (c)\ \ PD_2 \subseteq D_2,\ \ (d)\ \ t(T^{\perp}M) = D_2. \end{array}\right\} \tag{3.6}$$

Moreover,

$$\left.\begin{array}{l} (e)\ \ P^2 + tF = -I,\ \ f^2 + Ft = -I \\ \\ (f)\ \ FP + fF = 0,\ \ tf + Pt = 0. \end{array}\right\} \tag{3.7}$$

The covariant differentiation of the morphisms P, F, t and f are defined respectively as

$$(\bar{\nabla}_U P)V = \nabla_U PV - P\nabla_U V \tag{3.8}$$

$$(\bar{\nabla}_U F)V = \nabla^{\perp}_U FV - F\nabla_U V \tag{3.9}$$

$$(\bar{\nabla}_U t)\xi = \nabla_U t\xi - t\nabla^{\perp}_U \xi \tag{3.10}$$

$$(\bar{\nabla}_U f)\xi = \nabla^{\perp}_U f\xi - f\nabla^{\perp}_U \xi \tag{3.11}$$

for any $U, V \in TM$ and $\xi \in T^{\perp}M$.

A distribution D on M is said to be an autoparallel distribution if $\nabla_U V \in D$ for each $U, V \in D$ and D is said to be parallel if $\nabla_U X \in D$ for each $U \in TM$ and $X \in D$.

If a distribution D on M is autoparallel, then it is clearly integrable and its leaves are totally geodesic in M. If D is parallel then the orthogonal complementary distribution $D^{\perp}$ is also parallel which implies that D is parallel if and only if $D^{\perp}$ is parallel. In this case, M is locally the Riemannian product of the leaves of D and $D^{\perp}$. If M is a CR-submanifold with parallel holomorphic and totally real distributions, then M is said to be a CR-product.

The above observations can be re-stated as the following lemma.

Lemma 3.1. *Let M be a semi-slant submanifold of an almost Hermitian manifold $\bar{M}$. Then M is locally the Riemannian product of the leaves of D_1 and D_2 if and only if*

$$\nabla_U X \in D_1 \quad or \quad \nabla_U Z \in D_2$$

for each $X \in D_1$, $Z \in D_2$ and $U \in TM$.

The following is an easy consequence of the above lemma and can be proved on taking account equation (3.8).

Corollary 3.1. *If a semi-slant submanifold of an almost Hermitian manifold $\bar{M}$ is a Riemannian product of the leaves of D_1 and D_2, then*

$$(\bar{\nabla}_U P)X \in D_1 \quad \text{or equivalently} \quad (\bar{\nabla}_U P)Z \in D_2$$

for each $U \in TM$, $X \in D_1$ and $Z \in D_2$.

Remark. The converse of Corollary 3.1 is true when M is a CR-submanifold. In this case we have a stronger result, i.e. the following corollary.

Corollary 3.2. *A CR-submanifold M in an almost Hermitian manifold $\bar{M}$ is a CR-product if and only if*

$$(\bar{\nabla}_U P)V \in D_1$$

for each $U, V \in TM$.

In terms of the normal valued 1-form F, we have the following characterization for M to be a Riemannian product in $\bar{M}$.

Corollary 3.3. *A semi-slant submanifold M of an almost Hermitian manifold $\bar{M}$ is a Riemannian product of the leaves of D_1 and D_2 if and only if*

$$(\bar{\nabla}_U F)X = 0$$

for each $U \in TM$ and $X \in D_1$.

The proof follows by putting $V = X$ in equation (3.9) and taking account of observation (3.6)(a).

Now, for $U, V \in TM$, let us denote the tangential and normal parts of $(\bar{\nabla}_U J)V$ by $\mathcal{P}_U V$ and $\mathcal{Q}_U V$, respectively. Then by an easy computation, we obtain the following formulae:

$$\mathcal{P}_U V = (\bar{\nabla}_U P)V - A_{FV}U - th(U, V), \tag{3.12}$$

$$\mathcal{Q}_U V = (\bar{\nabla}_U F)V + h(U, PV) - fh(U, V). \tag{3.13}$$

In the following, we obtain integrability conditions for the distributions D_1 and D_2 on a semi-slant submanifold of an almost Hermitian manifold.

Theorem 3.1. *The holomorphic distribution D_1 on a semi-slant submanifold of an almost Hermitian manifold is integrable if and only if*

$$\mathcal{Q}_X Y - \mathcal{Q}_Y X = h(X, PY) - h(Y, PX)$$

for each X, Y in D_1.

Proof. For $\xi \in T^{\perp}M$, we have

$$g(\bar{\nabla}_X JY - \bar{\nabla}_Y JX, \xi) = g(h(X, PY) - h(PX, Y), \xi)$$

or,

$$g(J(\bar{\nabla}_X Y - \bar{\nabla}_Y X) + \mathcal{Q}_X Y - \mathcal{Q}_Y X, \xi) = g(h(X, PY) - h(PX, Y), \xi)$$

or,

$$g(F[X, Y], \xi) = g(h(X, PY) - h(PX, Y) + \mathcal{Q}_Y X - \mathcal{Q}_X Y, \xi).$$

The assertion follows from the above relation. □

Theorem 3.2. *The slant distribution D_2 on a semi-slant submanifold of an almost Hermitian manifold is integrable if and only if*

$$T_1(\nabla_Z PW - \nabla_W PZ + A_{FZ}W - A_{FW}Z + \mathcal{P}_W Z - \mathcal{P}_Z W) = 0$$

for each Z, W in D_2.

Proof. For $X \in D_1$, we find that

$$g(P[Z, W], X) = g(\bar{\nabla}_Z JW - \bar{\nabla}_W JZ + \mathcal{P}_W Z - \mathcal{P}_Z W, X)$$

$$= g(\nabla_Z PW - \nabla_W PZ + A_{FZ}W - A_{FW}Z + \mathcal{P}_W Z - \mathcal{P}_Z W, X)$$

which proves the assertion. □

For the parallelism of the holomorphic and slant distributions on a semi-slant submanifold of an almost Hermitian manifold, we have the following proposition.

Proposition 3.1. *If D_1 is autoparallel then*

(a) $\mathcal{Q}_X Y - h(X, PY) = fh(X, Y)$,

(b) $\mathcal{P}_X Y + th(X, Y) \in D_1$

for each $X, Y \in D_1$.

Proof. (a) follows from the formula (3.13) on taking account of (3.6)(a), whereas (b) follows immediately from the formula (3.12). □

Proposition 3.2. *If D_2 is autoparallel then*

(a) $h(Z, PX) - \mathcal{Q}_Z X = fh(X, Z)$

(b) $\mathcal{P}_W Z + A_{FZ}W \in D_2$

for each $X \in D_1$ and $Z, W \in D_2$.

The proof is straightforward by virtue of formulae (3.12) and (3.13). □

4. Semi-Slant Submanifolds of a Nearly Kaehler Manifold

Throughout this section, we assume $\bar{M}$ to be a nearly Kaehler manifold and M, a semi-slant submanifold of $\bar{M}$. Thus, on $\bar{M}$ we have

$$(\bar{\nabla}_U J)V + (\bar{\nabla}_V J)U = 0.$$

Consequently for $U, V \in TM$

$$(a)\ \ \mathcal{P}_U V + \mathcal{P}_V U = 0 \quad \text{and} \quad (b)\ \ \mathcal{Q}_U V + \mathcal{Q}_V U = 0. \tag{4.1}$$

We begin the section by establishing the following characterization for the integrability of the holomorphic distribution on a semi-slant submanifold of a nearly Kaehler manifold.

Theorem 4.1. *For a semi-slant submanifold M of a nearly Kaehler manifold $\bar{M}$, the following statements are equivalent:*

(a) *The holomorphic distribution D_1 on M is integrable,*

(b) $h(X, PY) = h(PX, Y)$ *and* $\mathcal{Q}_X Y = 0$,

(c) $h(X, PY) + h(PX, Y) = 2(F\nabla_X Y + fh(X, Y))$

for each $X, Y \in D_1$.

Proof. From Lemma 3.1 and equation (4.1)(b), it follows that D_1 is integrable if and only if

$$2\mathcal{Q}_X Y = h(X, PY) + h(PX, Y). \tag{4.2}$$

It is known that the Nijenhuis tensor S of J on $\bar{M}$ satisfies

$$S(U, V) = 4J(\nabla_U J)V \tag{4.3}$$

for each $U, V \in T\bar{M}$. Moreover as $(\bar{\nabla}_U J)JV = -J(\bar{\nabla}_U J)V$, we get

$$S^{\perp}(X, Y) = -4\mathcal{Q}_X JY \tag{4.4}$$

for any $X, Y \in D_1$, where $S^{\perp}(X, Y)$ denotes the normal part of $S(X, Y)$. By equations (2.2) and (3.1) the left hand side of the above equation becomes $F([PX, Y)] + [X, PY])$ and therefore

$$F([PX, Y] + [X, PY]) = -4\mathcal{Q}_X JY. \tag{4.5}$$

The equivalence of (a) and (b) is established by virtue of equations (4.2) and (4.5).

Formula (3.13) in view of equation (4.1) can be modified to yield the following equation

$$F[X, Y] = 2(F\nabla_X Y + fh(X, Y)) - h(X, PY) - h(PX, Y), \tag{4.6}$$

from which it follows that D_1 is integrable if and only if

$$h(X, PY) + h(PX, Y) = 2(F\nabla_X Y + fh(X, Y)),$$

Which proves that (a) is equivalent to (c). Hence (a), (b) and (c) are equivalent. □

Corollary 4.1. *A totally umbilical semi-slant submanifold of a nearly Kaehler manifold is totally geodesic if D_1 is integrable.*

Corollary 4.2. *On a totally geodesic semi-slant submanifold of a nearly Kaehler manifold, the holomorphic distribution D_1 is integrable.*
With regards to the integrability of the slant distribution D_2 on a semi-slant submanifold of $\bar{M}$, we have the following theorem.

Theorem 4.2. *The slant distribution D_2 on a semi-slant submanifold of a nearly Kaehler manifold $\bar{M}$ is integrable if and only if*

$$T_1(\nabla_Z PW - \nabla_W PZ + A_{FZ}W - A_{FW}Z + 2\mathcal{P}_W Z) = 0 \tag{4.7}$$

for each $Z, W \in D_2$.
The assertion is an immediate consequence of Theorem 3.2 and equation (4.1)(a). □

Once the distributions on M are involutive then by Frobenius theorem M is foliated by the leaves of these distributions. In the following we have observed some geometric properties of the leaves of the holomorphic and slant distributions on M.

Proposition 4.1. *Let the holomorphic distribution D_1 on a semi-slant submanifold M of a nearly Kaehler manifold be integrable. Then the leaves of D_1 are totally geodesic in M if and only if*

$$h(X, PY) = fh(X, Y) \tag{4.8}$$

for each $X, Y \in D_1$.
The proof follows from Theorem 4.1 on using formula (3.13).

Proposition 4.2. *Let the slant distribution D_2 on M be integrable. Then the leaves of D_2 are totally geodesic in M if and only if*

$$T_1(A_{FZ}W + A_{FW}Z + 2th(Z, W)) = 0 \tag{4.9}$$

for each $Z, W \in D_2$.
The proof follows on using Theorem 4.2 and the autoparallelism of D_2. □

Theorem 4.3. *Let M be a semi-slant submanifold of a nearly Kaehler manifold $\bar{M}$. Then (P, g) is a nearly Kaehler structure on the holomorphic distribution D_1 if $th(X, Y) = 0$ for all $X, Y \in D_1$.*

Proof. In view of the observation (3.6)(a) and (3.7)(e)

$$P^2 X = -X \tag{4.10}$$

for each $X \in D_1$. Moreover, if $th(X, Y) = 0$, then by equations (3.12) and (4.1)(a), we have

$$(\nabla_X P)Y + (\nabla_Y P)X = 0. \tag{4.11}$$

The assertion is proved by virtue of (4.10) and (4.11). □

Theorem 4.4. *On a semi-slant submanifold of a nearly Kaehler manifold, if D_1 is autoparallel, then (P, g) is a nearly Kaehler structure on D_1.*

Proof. By equations (3.12), (4.1)(a), the observation (3.6) and the autoparallelism of D_1, it follows that

$$(\nabla_X P)Y + (\nabla_Y P)X = 0 \tag{4.12}$$

for each $X, Y \in D_1$. The above relation together with (4.10) proves the assertion. □

Remark. If D_1 is autoparallel, then by equation (3.12), (4.1)(a) and (3.6)(d),

$$th(X, Y) = 0$$

and therefore Theorem 4.3 can be treated as a corollary of Theorem 4.4.

In view of the above findings, we may state the following theorem.

Theorem 4.5. *Let M be a submanifold of a nearly Kaehler manifold $\bar{M}$. If M is complex, then it is a nearly Kaehler manifold and*

$$h(X, JY) = fh(X, Y)$$

for each $X, Y \in TM$.

Remark. If D_1 is autoparallel, then by equations (4.12) and (3.12), $th(X, Y) = 0$. In particular, if M is locally the Riemannian product of the leaves of D_1 and D_2 then $th(X, Y) = 0$, which is equivalent to the condition:

$$g(h(X, Y), FD_2) = 0$$

for each $X, Y \in D_1$.

The above condition is a clear extension of Chen′s characterization for the leaves of D_1 to be totally geodesic in a CR-submanifold of a Kaehler manifold (c.f., [1]).

5. Totally Umbilical Semi-Slant Submanifolds of a nearly Kaehler Manifold

Totally umbilical submanifolds are characterized by the property that $h(X,Y) = g(X,Y)H$, where H is the mean curvature normal vector field on M. Obviously, the condition $h(X,PX) = 0$ for each $X \in D_1$ is a more general condition on a semi-slant submnaifold than the umbilical condition. In the following we obtain a geometric property with this more general condition.

Proposition 5.1 *Let M be a semi-slant submanifold of a nearly Kaehler manifold $\bar{M}$ with $h(X,PX) = 0$, for each $X \in D_1$. If the holomorphic distribution D_1 is integrable then each leaf of D_1 is totally geodesic in M as well as in $\bar{M}$.*

Proof. By Theorem 4.1,

$$F\nabla_X Y = fh(X,Y). \tag{5.1}$$

Again, in view of the hypothesis and Theorem 4.1,

$$h(X,PY) = 0$$

i.e.,

$$h(X,Y) = 0 \tag{5.2}$$

for each $X, Y \in D_1$. Now taking account of equations (5.2) and (3.6)(b), equation (5.1) yields

$$\nabla_X Y \in D_1.$$

i.e., D_1 is autoparallel. In other words, the leaves of D_1 are totally geodesic in M, whereas equation (5.2) shows that they are totally geodesic in $\bar{M}$. Thus, proposition is proved completely. □

Note. An immediate consequence of Proposition 5.1 is Corollary 4.1. In other words, Proposition 5.1 re-affirms Corollary 4.1.
Infact, at this point, we are in a position to state the following theorem.

Theorem 5.1. *Let M be a totally umbilical semi-slant submanifold of a nearly Kaehler manifold $\bar{M}$. If the holomorphic distribution D_1 on M is integrable, then*

(a) *leaves of D_1 are totally geodesic in M and in $\bar{M}$ both,*

(b) *M is totally geodesic in $\bar{M}$,*

(c) $(\nabla_U P)V + (\nabla_V P)U = 0$

for each $U, V \in TM$.

Proof. Statements (a) and (b) are already proved in the Proposition 5.1 and the Corollary 4.1 respectively. For the statement (c), by formula (3.12) and (4.1)(a), we may write

$$(\nabla_U P)V + (\nabla_V P)U = A_{FU}V + A_{FV}U + 2th(U, V).$$

The right hand side of the above equation is zero as by part (b) M is totally geodesic in $\bar{M}$. This proves (c). □

Theorem 5.2. *Let* M *be a totally umbilical proper semi-slant submanifold of a nearly Kaehler manifold* $\bar{M}$. *If the mean curvature vector* H *lies in the invariant sub-bundle* μ *of the normal bundle* $T^{\perp}M$, *then* M *is totally geodesic in* $\bar{M}$.

Proof. By equation (5.1),

$$F\nabla_X X = \|X\|^2 fH$$

for each $X \in D_1$. As H is assumed to be in μ, $fH \in \mu$. On the other hand $FU \in FD_2$ for any $U \in TM$. In view of these observations, the above equation implies that

$$\nabla_X X \in D_1 \quad \text{and} \quad H \in FD_2.$$

The last observation together with the hypothesis verifies that $H = 0$, and therefore M is totally geodesic in $\bar{M}$. □

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