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Rayleigh Number in a Stability Problem for a Micropolar Fluid

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Abstract

Approximate numerical evaluations of the Rayleigh number are obtained for a stability problem of thermal convection in a heat-conducting micropolar fluid layer between two rigid boundaries [7]. The influences of all the physical parameters on the values of the Rayleigh number are studied. Also, approximate neutral curves and neutral surfaces are represented in various parameters spaces.

Key Words: Fourier series methods, micropolar fluid, thermal convection, Rayleigh number.

1. Introduction

The general theory of fluid microcontinua is attributed to AC Eringen. His work was concerned with a nonlinear theory of microelastic solids, but his treatment of motion, balance of moments, conservation of energy, and entropy production is applicable to all continuous media consisting of microelements, e.g. micropolar fluids.

In this paper, we are concerned with the onset of thermal convection in a heat conducting micropolar fluid situated in a horizontal unbounded layer between two rigid walls. In a particular case this stability problem [12] was solved theoretically in [3] using the Chandrasekhar - Galerkin method. In this method, the unknown functions are expanded upon complete sets of functions which satisfy all the boundary conditions. For the case $Q = 0$, $\overline{\delta} = 0$, where Q and $\overline{\delta}$ are two physical parameters, the Budianski

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-DiPrima (B-D) method was proposed and a variational formulation of the problem was presented too leading us to the same secular equation as the direct B-D method. Herein, the problem is treated analytically and numerically by the B- D method, also for the cases $Q \neq 0$ and/or $\overline{\delta} \neq 0$. We calculate the values of the Rayleigh number for various values of the micropolar parameters.

In [6] a simple general direct method for solving two-point eigenvalue problems is applied. The multipicity of the characteristic roots is discussed yielding the characteristic bifurcation manifolds of the parameter space corresponding to potential false secular manifolds. The exact eigensolutions describing the thermal convection wer deduced. The secular equation is obtained only for the hydrodynamic case. For the micropolar case the detection of false neutral manifolds is still an open problem. Datta and Sastry [2] studied the Bénard problem of thermal instability of fluids between two horizontal planes heated from below in order to exhibits the micro-rotation effects. In [12], Rama Rao conducted the study of the onset of thermal conducting micropolar layer between rigid boundaries in the absence of a magnetic field using the finite difference method. His numerical results are presented for comparison in this paper.

As stated in [3], assuming that the exchange of stability principle holds [12], the linear stability against normal mode perturbations is governed by the two-point problem

$$\begin{cases} (1+R)\Big[(D^2-a^2)^2 - QD^2\Big]W + R(D^2-a^2)Z - R_a \cdot a^2\Theta = 0, \\ \Big[A(D^2-a^2) - 2R\Big]Z - R(D^2-a^2)W = 0, \\ (D^2-a^2)\Theta + W - \overline{\delta}Z = 0, \end{cases} \tag{1}$$

$$W = DW = Z = \Theta = 0 \text{ at } z = \pm 0.5. \tag{2}$$

The micropolar parameters are $R = \dfrac{k}{\mu}$, $A = \dfrac{\gamma}{\mu d^2}$, $\overline{\delta} = \dfrac{\delta}{\varrho_0 c_v d^2}$, Q is the intensity of the magnetic field, R_a stands for the Rayleigh number and $a > 0$ is the wave number. The numbers μ, k, α, β, γ and δ are material constants. The functions W, Θ, Z : $[-0.5, 0.5] \to \mathbb{R}$ characterize the amplitude of the perturbation of the vertical component of the velocity, temperature and the vertical component of the spin vorticity, respectively.

The problem (1)-(2) generates a linear differential operator

$$L : \mathcal{D}(L) \subset (L^2(-0.5; 0.5))^4 \to (L^2(-0.5; 0.5))^4,$$

$$\mathcal{D}(L) = \{(W, Z, \Theta) \in (C^\infty[-0.5; 0.5])^4 | W = DW = Z = \Theta = 0 \text{ at } z = \pm 0.5\}.$$

The problem is to determine in the parameter space (R_a, R, A, $\overline{\delta}$, Q, a) the neutral hypersurface which separates the domain of stability from the linear instability domain. This means that we will search for the smallest eigenvalue R_{amin}, when the parameters R, A, $\overline{\delta}$, Q, a are kept constant. Here, the equation of the neutral hypersurface, $SE_g = 0$, is obtained by the Budianski-DiPrima method.

Each of the unknown functions in problem (1)-(2) can be uniquely written as a sum of an odd function and an even function, e.g. $W = W_o + W_e$, $Z = Z_o + Z_e$, $\Theta = \Theta_0 + \Theta_e$. In this way, taking into account that an even function is equal to an odd one only if they are both null, the problem splits in the following two two-point problems

$$\begin{cases} (1+R)\Big[(D^2-a^2)^2 - QD^2\Big]W_e + R(D^2-a^2)Z_e - Ra\cdot a^2\Theta_e = 0, \\ \Big[A(D^2-a^2) - 2R\Big]Z_e - R(D^2-a^2)W_e = 0, \\ (D^2-a^2)\Theta_e + W_e - \overline{\delta}Z_e = 0, \end{cases} \tag{1_e}$$

$$W_e = DW_o = Z_e = \Theta_e = 0 \text{ at } z = \pm 0.5. \tag{2_e}$$

and

$$\begin{cases} (1+R)\Big[(D^2-a^2)^2 - QD^2\Big]W_o + R(D^2-a^2)Z_o - Ra\cdot a^2\Theta_o = 0, \\ \Big[A(D^2-a^2) - 2R\Big]Z_o - R(D^2-a^2)W_o = 0, \\ (D^2-a^2)\Theta_o + W_o - \overline{\delta}Z_o = 0, \end{cases} \tag{1_o}$$

$$W_o = DW_e = Z_o = \Theta_o = 0 \text{ at } z = \pm 0.5. \tag{2_o}$$

The new problems will have even and odd solutions respectively. In this case $SE_g = SE_e \cdot SE_o$, where $SE_e = 0$ and $SE_o = 0$ are the equations of the neutral hypersurface corresponding to even and odd solutions respectively. So, instead of solving the eigenvalue problem (1)-(2), we solve the problems $(1)_e$-$(2)_e$ and $(1)_o$-$(2)_o$ in the class of even and odd functions separately. The smallest eigenvalue R_{amin} will be a solution of $SE_e = 0$ or $SE_o = 0$ and, as a consequence, the corresponding eigenfunctions W, Z, Θ will be even or odd.

An important observation is that, if for a particular choice of the parameters, the smallest eigenvalue correspond to the even solution, then the situation remains the same for any other values of those parameters.

Obviously, solving $SE_e = 0$ or $SE_o = 0$ instead of $SE_g = 0$ simplifies the computations, i.e. instead of evaluating a n-th order determinant, we will evaluate a $\frac{n}{2}$-th order determinant [6].

2. The Budiansky-DiPrima method

The B-D method is based on the Fourier expansion of all unknown functions upon total sets of functions which do not satisfy all boundary conditions of the problem. The remained boundary conditions lead to some constraints for the Fourier coefficients.

Even problem (1_e)-(2_e). In this case, the unknown functions W, Θ, Z are expanded upon the total set $\{E_{2n-1}\}_{n\in\mathbb{N}}$, where $E_{2n-1}(z) = \sqrt{2}\cos(2n-1)\pi z,\ n \in \mathbb{N}$, namely

$$\begin{cases} W = \sum\limits_{n=1}^{\infty} \sqrt{2}W_{2n-1}\cos(2n-1)\pi z, \\ Z = \sum\limits_{n=1}^{\infty} \sqrt{2}Z_{2n-1}\cos(2n-1)\pi z, \\ \Theta = \sum\limits_{n=1}^{\infty} \sqrt{2}\Theta_{2n-1}\cos(2n-1)\pi z. \end{cases} \tag{3}$$

The series expansions of the derivatives occurring in (1) are obtained by the backward integration technique [6]. Substitute these expressions in (1), impose the condition that the obtained equations be orthogonal to E_{2m-1}, $m = 1, 2, ...$ to get the system

$$\begin{cases} (1+R)[A_n^2 + Q(2n-1)^2\pi^2]W_{2n-1} - RA_nZ_{2n-1} - Ra\cdot a^2\Theta_{2n-1} = \\ = 2\sqrt{2}(-1)^n(1+R)(2n-1)\pi\alpha \\ \\ RA_nW_{2n-1} - (AA_n + 2R)Z_{2n-1} = 0, \\ \\ W_{2n-1} - \overline{\delta}Z_{2n-1} - A_n\Theta_{2n-1} = 0, \end{cases} \tag{4}$$

where $A_n = (2n-1)^2\pi^2 + a^2$.

The single boundary condition which is not automatically satisfied by the chosen series is $DW = 0$ at $z = \pm 0.5$ and it introduces the constraint

$$\sum_{n=1}^{\infty}(-1)^n\sqrt{2}(2n-1)\pi W_{2n-1} = 0. \tag{5}$$

The secular equation is obtained by solving the system (4) and substituting the obtained expression for W_{2n-1} in (5). In the next section, it is written for the cases: ($Q = 0, \overline{\delta} = 0$), ($Q = 0, \overline{\delta} \neq 0$), ($Q \neq 0, \overline{\delta} = 0$) and ($Q \neq 0, \overline{\delta} \neq 0$).

Odd problem (1_o)-(2_o). The unknown odd functions are expanded upon the total set $\{F_{2n-1}\}_{n\in\mathbb{N}}$, $F_{2n-1}(z)=\sqrt{2}\sin(2n-1)\pi z$. Then (1_o)-(2_o) becomes

$$\begin{cases} (1+R)[A_n^2+Q(A_n-a^2)]W_{2n-1}-RA_nZ_{2n-1}-Ra\cdot a^2\Theta_{2n-1}= \\ =2\sqrt{2}(-1)^n[\alpha(1+R)+\beta R], \\ RA_nW_{2n-1}-[AA_n+2R]Z_{2n-1}=2\sqrt{2}A(-1)^n\beta, \\ W_{2n-1}-\overline{\delta}Z_{2n-1}-A_n\Theta_{2n-1}=2\sqrt{2}(-1)^n\gamma, \end{cases} \tag{6}$$

where $\alpha=D^3W(0.5)$, $\beta=DZ(0.5)$, $\gamma=D\Theta(0.5)$ are arbitrary constants. The following constraints

$$\begin{cases} \Gamma_1\equiv\sum\limits_{n=1}^{\infty}\sqrt{2}(-1)^{n+1}W_{2n-1}=0, \\ \Gamma_2\equiv\sum\limits_{n=1}^{\infty}\sqrt{2}(-1)^{n+1}Z_{2n-1}=0, \\ \Gamma_3\equiv\sum\limits_{n=1}^{\infty}\sqrt{2}(-1)^{n+1}\Theta_{2n-1}=0 \end{cases} \tag{7}$$

must be satisfied.

Solving the system (6) and introducing the solution $(W_{2n-1}, Z_{2n-1}, \Theta_{2n-1})$ in (7) we obtain the following system in α, β, γ

$$\begin{cases} \alpha(1+R)B^{1,1}+\beta[RB^{1,1}+A\overline{\delta}R_a\cdot a^2B^{0,0}-RAB^{0,2}]-R_a\cdot a^2B^{1,0}\gamma=0, \\ -\alpha(1+R)RB^{0,2}+\beta[(1+R)A(B^{0,3}+QB^{0,2}-Qa^2B^{0,1})-ARa\cdot a^2B^{0,0}- \\ -R^2B^{0,2}]+RR_a\cdot a^2B^{0,1}\gamma=0, \\ \alpha(1+R)(R\overline{\delta}B^{0,1}-B^{1,0})+\beta[\overline{\delta}R^2B^{0,1}+RAB^{0,1}-RB^{1,0}-(1+R)\overline{\delta}A\cdot \\ \cdot(B^{0,2}+QB^{0,1}-Qa^2B^{0,0})]+\gamma[(1+R)(B^{1,2}+QB^{1,1}-Qa^2B^{1,0})-R^2B^{0,2}]=0, \end{cases}$$

where

$$\Delta^{-1}=(1+R)A_nD_n[A_n^2+Q(A_n-a^2)]+R\overline{\delta}A_nR_a\cdot a^2-R_a\cdot a^2D_n-R^2A_n^3$$

is the Cramer determinant of the system (6), $D_n=AA_n+2R$ and $B^{p,q}=\sum\limits_{n=1}^{\infty}(D_n)^p(A_n)^q\Delta$.

The equation of the neutral surface follows by imposing to the determinant of the homogeneous system in α, β, γ to vanish, i.e.

$$\begin{vmatrix} B^{1,1} & M1 & -R_a \cdot a^2 B^{1,0} \\ -RB^{0,2} & M2 & RR_a \cdot a^2 B^{0,1} \\ R\overline{\delta}B^{0,1} - B^{1,0} & M3 & (1+R)(B^{1,2} + QB^{1,1} - Qa^2B^{1,0}) - R^2B^{0,2} \end{vmatrix} = 0, \qquad (8)$$

where

$$\begin{cases} M1 = RB^{1,1} + A\overline{\delta}R_a \cdot a^2 B^{0,0} - RAB^{0,2}, \\ M2 = (1+R)A(B^{0,3} + QB^{0,2} - Qa^2B^{0,1}) - AR_a \cdot a^2 B^{0,0} - R^2B^{0,2}, \\ M3 = \overline{\delta}R^2B^{0,1} + RAB^{0,1} - RB^{1,0} - (1+R)\overline{\delta}A(B^{0,2} + QB^{0,1} - Qa^2B^{0,0}). \end{cases}$$

3. Numerical results

According to Chandrasekhar [1], the lowest "state" is even in the case of two rigid boundaries. Here, we performe numerical evaluations for the odd solution too.

In this case, the computations are more complicated due to the infinite series that occur in the expression of $B^{p,q}$. In order to compare our results to the classical ones, we considered $A = 0.0001$, $R = 0.0001$, $\overline{\delta} = Q = 0$. The sum was truncated for $n \leq 11$ so, for instance, for $a = 3.117$ and $a = 5.365$, we obtained

$n = 3$	$R_a = 27864.2192$
$n = 5$	$R_a = 26407.7455$
$n = 7$	$R_a = 25931.0603$
$n = 11$	$R_a = 25554.0021$

$a = 3.117$

$n = 3$	$R_a = 19155.9650$
$n = 5$	$R_a = 18335.2184$
$n = 7$	$R_a = 18081.1626$
$n = 11$	$R_a = 17888.5193$

$a = 5.365$

In [1], the exact characteristic values for the first odd mode of instability is $R_a = 24982$ for $a = 3.117$, and the critical value of R_a is $R_{a_{critic}} = 17610.39$ for $a = 5.365$, which are in good agreement with our computations.

For the even part, we obtained the following secular equations defining the manifolds that separate the stability domain from the instability domain in the parameter space.

Case $Q = 0$, $\overline{\delta} = 0$. In this case the secular equation reads

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2 A_n D_n}{A_n^3 \Big[D_n(1+R) - R^2\Big] - R_a \cdot a^2 D_n} = 0, \tag{9}$$

which converges like the numerical series $\sum_{i=1}^{n} \frac{1}{n^2}$. Numerical computations performed in (9) leads to Table 1 and Figure 1. The results show that as the parameter R increased, the Rayleigh number increases rapidly.

Also, the growth of the wave number a implies a steep growth of the Rayleigh number. The variations in A does not imply significant changes in the values of the Rayleigh number.

In Chandrasekhar [1], by a variational method, the problem is treated for $A = R = \overline{\delta} = Q = 0$ and it is obtain a critical value for the Rayleigh number $R = 1715.08$ for $a = 3.117$. These results are the same as ours.

Table 1. Rayleigh numbers for the case $Q = 0$, $\overline{\delta} = 0$.

A	R	a	R_a
0.001	0.001	3.117	1850.624086
0.001	0.5	3.117	2302.200180
0.001	0.5	5.00	3213.376953
0.002	0.5	5.00	3250.429895
0.001	1.00	5.00	3860.174964
0.002	1.00	5.00	3911.264998
0.001	2.00	6.70	9015.560744
0.001	2.00	6.80	9349.859491
0.001	2.00	14.00	93743.02414
0.001	2.00	9.50	24618.15747
0.001	4.00	9.50	36480.43048
0.001	6.00	9.50	48513.80933
0.001	8.00	9.50	60537.15202

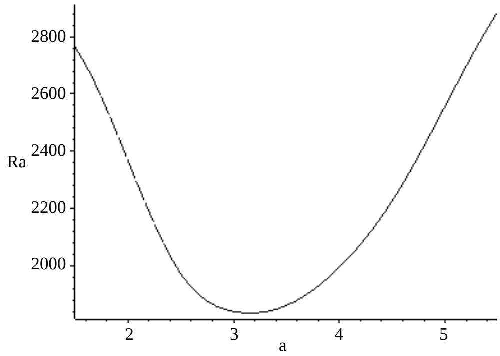

Figure 1. Neutral Rayleigh numbers at $Q = 0,\ \overline{\delta} = 0,\ A = 0.001$.

The case $Q = 0$, $\overline{\delta} \neq 0$. In this case, the secular equation, the secular equation has the form of the following (convergent) series

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2 A_n D_n}{A_n^3 \left[D_n(1+R) - R^2\right] + Ra \cdot a^2(\delta R A_n - D_n)} = 0. \tag{10}$$

This case was treated in [12] too. The (approximate) values of the Rayleigh number are deduced from the secular equation (10) by means of a large number of terms in (10). The stability curves are presented in Figure 2 for various values of the micropolar parameters A and $\overline{\delta}$.

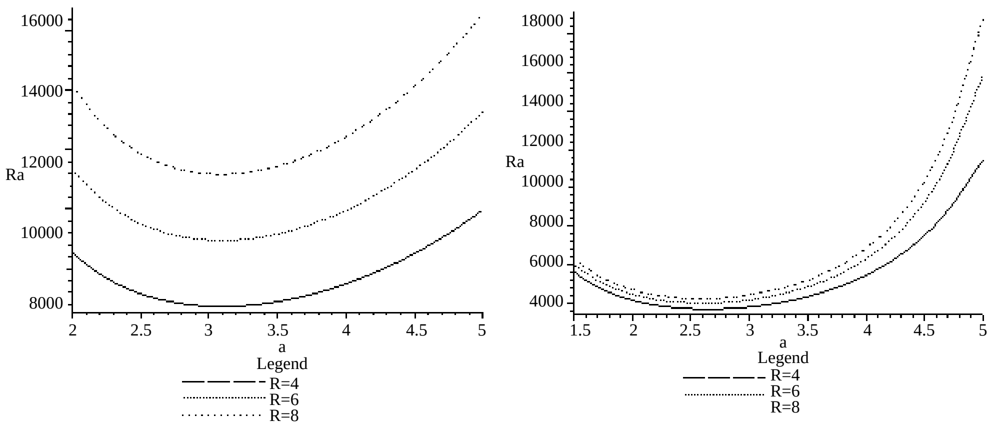

Figure 2. Neutral curve in the space parameters (a, R_a) in the case $Q = 0,\ \overline{\delta} \neq 0$.

We chosed the same values for the micropolar parameters and for the wave number as in [12]. Table 2 shows a very good agreement between the numerical results from [12] and those obtained by us. The graphical representations of the neutral surfaces in the space parameters (a, R, R_a) (Figure 3) show that the critical values of the Rayleigh number is obtained, in this case, for a value of the wave number around $a = 3.117$.

Table 2. Rayleigh numbers for the case $Q = 0$, $\overline{\delta} \neq 0$.

A	R	$\overline{\delta}$	a	R_a − obtained in [12]	R_a − obtained by us
0.001	2	0.1	6.90	−5204.902	−5218.031767
0.001	4	0.1	6.80	−7723.6289	−7778.615586
0.001	6	0.1	6.85	−10259.840	−10271.62894
0.001	8	0.1	6.85	−12785.852	−12836.01081
0.001	2	0.05	9.00	−16641.5305	−16674.77845
0.001	2	0.02	14.00	−97263.438	−97384.66469
0.005	2	0.1	6.75	−5863.3438	−5909.801271
0.01	2	0.1	6.70	−6723.4570	−6763.317816
0.05	2	0.1	6.70	−17143.020	−17344.22361

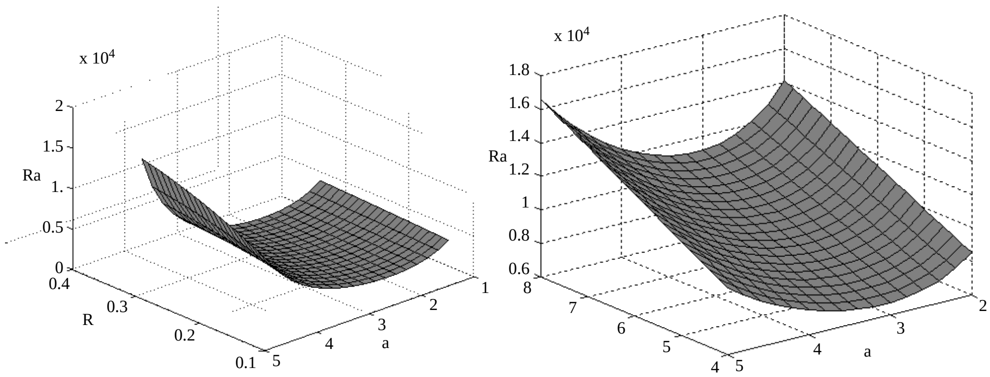

Figure 3. Neutral surface in the space parameters (a,R,R_a) in the case $Q = 0$, $\overline{\delta} \neq 0$.

In [12] it was treated the hydrodynamic case. In addition we performed numerical computations for the Rayleigh number when the intensity of the hydromagnetic field is different from zero. There are presented for the following two situations.

The case $Q \neq 0$, $\overline{\delta} = 0$. In this case the secular equation has the following form

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2 A_n D_n}{A_n D_n (1+R)[A_n^2 + Q(2n-1)^2\pi^2] - R_a a^2 D_n - R^2 A_n^3} = 0, \tag{11}$$

and it is a convergent series too.

In Figure 4 for comparison we presented the neutral curves obtained for different values of the micropolar parameters A, R. The graphical representation shows that the growth of the intensity of the magnetic field produces a delay in the onset of instability. Also, for the wave number a between $[2, 6]$ we obtain an approximate minimum value for the Rayleigh number. In Figure 5, the neutral surface in the space parameters (Q, a, Ra) is presented. Some of the numerical evaluations performed in order to show the influence of the parameter Q are presented in Table 3.

Table 3. Critical values of the Rayleigh number in the case $Q \neq 0$, $\delta = 0$.

A	R	Q	a	R_a
0.001	2	1	6.90	9936.883064
0.001	2	$\sqrt{5}$	6.70	9242.094792
0.001	2	$\sqrt{1000}$	6.75	10682.83266
0.001	2	$\sqrt{5000}$	3.20	8808.891133
0.001	2	$\sqrt{5000}$	6.75	12371.99000
0.001	4	$\sqrt{10}$	6.70	13952.95872
0.002	4	$\sqrt{20}$	6.70	14365.32681
0.001	4	$\sqrt{100}$	6.80	14925.52555
0.002	2	$\sqrt{500}$	6.00	8297.337337
0.002	2	$\sqrt{500}$	6.50	9655.476425
0.001	6	$\sqrt{500}$	6.80	21059.23273
0.003	2	$\sqrt{50}$	3.20	4409.302773
0.003	2	$\sqrt{50}$	4.20	4770.599495
0.003	2	$\sqrt{50}$	6.75	9923.446849
0.003	2	$\sqrt{75}$	5.20	6108.457424

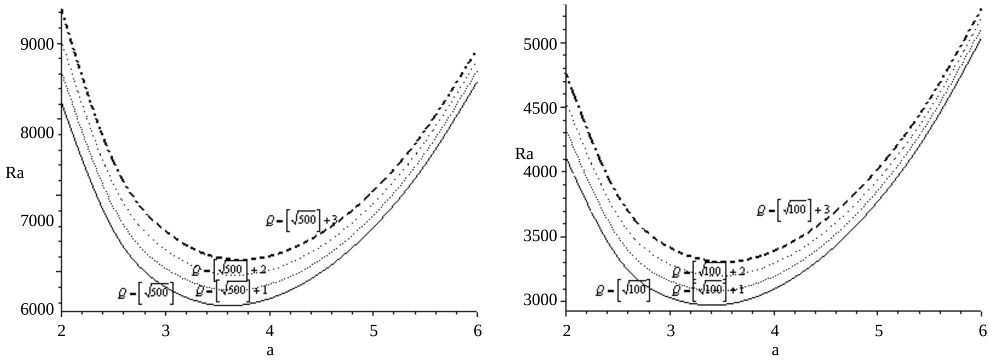

Figure 4. Neutral curve in the parameter space (a, R_a).

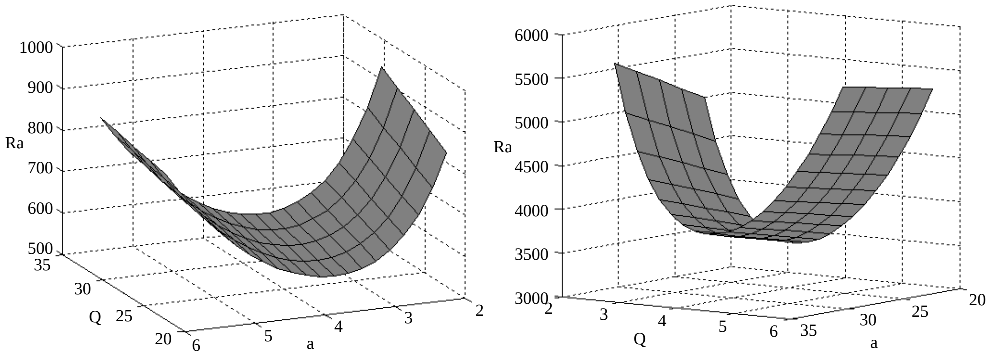

Figure 5. Neutral surface in the parameter space (Q, a, R_a).

The case $Q \neq 0$, $\delta \neq 0$. In this case, the secular equation has the form of the series

$$\sum_{n=1}^{\infty} \frac{(2n-1)^2 A_n D_n}{A_n D_n (1+R)[A_n^2 + Q(2n-1)^2\pi^2] - R^2 A_n^3 + R_a a^2(\delta R A_n - D_n)} = 0, \qquad (12)$$

which converges as the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2}$. The hydrodynamic case, when the micropolar parameter $\overline{\delta}$ is also different from zero, is the more general case. Numerical results for various values of the critical Rayleigh number are presented in Table 4. The neutral curves from Figure 6 are obtained by keeping three of the micropolar parameters constant.

Table 4. Critical values of the Rayleigh number in the case $Q \neq 0$, $\overline{\delta} \neq 0$.

A	R	a	δ	Q	R_a
0.001	2	3.00	0.1	20	236.5144011
0.002	2	3.00	0.05	$\sqrt{5}$	322.4400120
0.002	4	3.00	0.1	10	-3451.853699
0.001	8	5.00	0.1	$\sqrt{50.0}$	330716.4729
0.001	2	6.90	0.1	$\sqrt{5}$	-77.60407492
0.001	2	6.80	0.1	1	-66.31270945
0.001	4	6.75	0.05	$\sqrt{10}$	11314.80899
0.001	6	6.75	0.05	$\sqrt{10}$	23163.41752
0.001	6	6.75	0.05	$\sqrt{5}$	23201.19899
0.002	2	6.70	0.1	$\sqrt{5}$	-685071.9618
0.002	4	6.70	0.1	$\sqrt{10}$	693771.3059

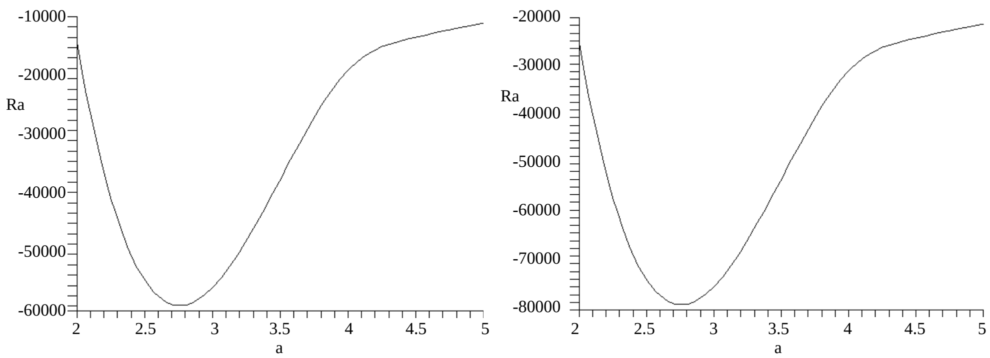

Figure 6. Neutral curve in the parameter space (a, R_a).

Figures 7,8 represent the graphics for the neutral surfaces obtained by keeping the parameters (A, R), respectively (A, Q) constant. The large number of graphical representations and numerical evaluations presented in the paper is requested by the large number of physical parameters. The influence of each of this parameter on the values of the Rayleigh number is given.

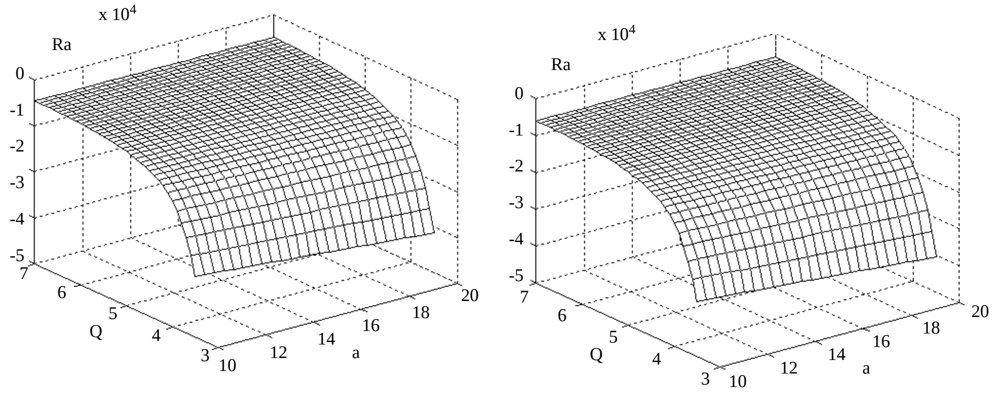

Figure 7. Neutral surface for the case $Q \neq 0$, $\overline{\delta} \neq 0$ in the parameter space (a,Q,R$_a$).

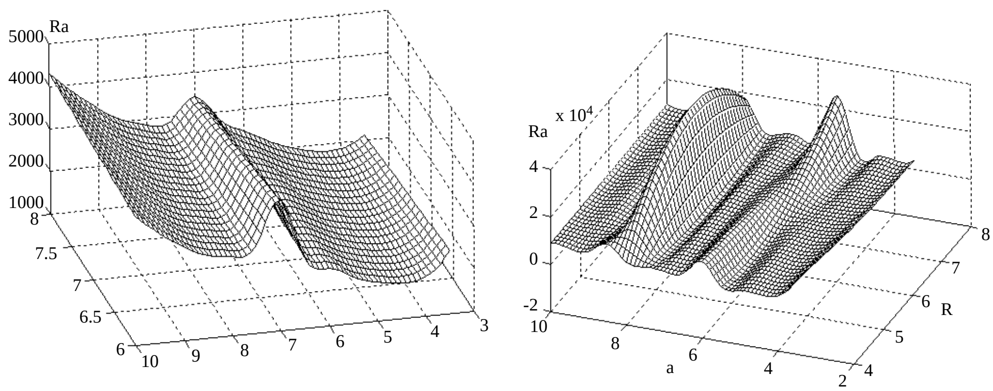

Figure 8. Neutral surface in the parameter space (a,R,R$_a$).

4. Conclusions

In this paper we determined numerically approximate for the Rayleigh number on the neutral curve and on the neutral surfaces. Both the hydrodynamic case ($Q = 0$) and the presence of a magnetic field in treated. In each case the secular equation was found and discussed for various values of the physical parameters.

The evaluations showed that, in the hydrodynamic case, when the micropolar parameter $\overline{\delta}$ is not null the viscosity parameter k has a stabilizing influence on the flow. This agrees with the results obtain by Datta and Sastry in [2]. For $\overline{\delta} = Q = 0$ the completed neutral curves and neutral surfaces show the following influence of the micropolar parameter R: the domain of stability enlarges as R increases. When A and $\overline{\delta}$ increases, large values of the wave number seems to have a stabilizing effect on the fluid.

The B-D method was chosen in order to avoid very difficult standard numerical computations. For the hydrodynamic case the obtained results have a good accuracy compared with the results obtained in [1] and [12]. This is an important characteristic if we take into consideration that the problem does not easily lead us to an exact solution. In addition, we obtained numerical results for the case $Q \neq 0$, not to be found in [7] and [12].

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