Journal of Achievements in Materials and Manufacturing Engineering
VOLUME 25
ISSUE 1
November
2007

Analytic and experimental study for light alloy aluminium panels under compression

A. Fayza [a,*], A. Zghal [a], E. Bayraktar [b]
[a] Laboratory of Mechanics, Structures and Technological Development , High School of Sciences and Techniques, Tunisia. 5 Avenue Taha Hussein 1008 Montfleury, Tunis, Tunisia
[b] Supmeca/LISMMA-Paris, School of mechanical and manufacturing Engineering, EA 2336, St-Ouen, France
* Corresponding author: E-mail address: fayza-2002000@yahoo.com

Received 20.04.2007; published in revised form 01.11.2007

Materials

ABSTRACT

Purpose: Aluminium alloys have been indispensable for the progress of many technologies during the last decades. In particular, most stiffeners in aerospace structures are composed of aluminium panels often solicited with elastic and plastic bucking under particular boundary and loading conditions. The purpose of this paper is to enhance the difficulties encountered to predict the incipient elastic-plastic buckling for thin aluminium plates under combined compressive loads.
Design/methodology/approach: The used methodology was an analytic non linear approach, validated further with an experimental investigation. In fact, the instability of thin elastic-plastic rectangular panels made of 2024 T45 alloys is analyzed. General concept of the Von Kaman's equation with a set of trigonometric and harmonic functions was used in the analytic model. The computation of buckling loads concerns both elastic and plastic instability solutions. Developments in the plastic range were concerned with the (j2d) deformation and the (J2f) flow constitutive laws.
Findings: A methodology to develop this kind of analytic resolution is pointed out and has been illustrated for a set of variables. Several 2d and 3d plots, which can be used to predict incipient buckling strengths for plates with flat initial configurations, have been presented for the various load conditions.
Research limitations/implications: In the future it will be possible to apply the investigated analytic procedure to other particular cases.
Practical implications: Plots obtained with analytic solution can be used to predict incipient buckling strengths for plates with flat initial configurations are presented for the various tests. The interest of three dimensional representations is to indicate when plastic buckling occurs for a square plate under biaxial loading.
Originality/value: This paper presents a stable and low cost analytic solution to deal with instability phenomenon in elastic and plastic range for the design of light alloy aluminium plates. This approach is applied to assess the conditions for which plastic buckling can happen when particularly thin aluminium panels are used. This latter, can be implemented in finite element standard codes.
Keywords: Metallic alloys; Analytic solution; Aluminium alloys; Plasticity; Instability; Buckling

1. Introduction

In the literature a large number of studies have been undertaken on the unstable behaviour of rectangular plates, subject to elastic or plastic buckling [1-8]. In fact, both analytical and experimental studies dealing with 2024 T45 and 7075 T65 aluminium alloy plates problem are unusual. In this survey the analysis is focused on the elastic plastic buckling response of Kirchhoff rectangular plates made with 2024 T45 aluminium alloy. Important analytical investigations have been carried out for instability and buckling problems of thin panels. The difficulty in determining critical loads in plasticity is due to the interaction between the non linearity either in the geometry or in the material behaviour. In fact, a limited number of specific numerical standard codes based on Finite Elements techniques or Finite difference techniques are dealing with this problem.

The plastic buckling problem was examined earlier by Shanley and Hill [5], for solids structures compressed until the plastic range; which generally buckle under increasing loads (satisfying plastic loading condition). The quasi-static post buckling behaviour of continues elastic-plastic structures, was generally involved. The analytical work of Shanley [5] has employed the perturbation method to determine the post buckling behaviour and imperfection sensitivity of elastic plastic structures.

Till now, the assessment of the plastic buckling behaviour for elastic-plastic structures has been addressed both analytically and numerically. Although providing a valuable insight to the plastic buckling phenomenon, most contributions did not lead to an accurate evaluation of the critical load at a finite deflection, even for simple problems. In what follow, we will present here a new approach with an analytic solution for the elastic plastic instability problem of rectangular plates under compression conditions. An experimental methodology is also adopted and various applications are carried out with aluminium plates solicited statically under compressive buckling load. Aspect ratio, boundary conditions, material properties and constitutive laws in plastic buckling are main parameters for this analysis.

Nomenclature

a: Plate length, (mm)
b: Plate width, (mm)
h: Plate thickness, (mm)
$\hat{D}_f^{1111}, \hat{D}_f^{2222}, \hat{D}_f^{1212}, \hat{D}_f^{1122}$: Plate bending stiffness
E_1, E_2: Modulus of elasticity, (MPa)
G_{12} Shear modulus
ν_{12} Poisson's ratio
m Number of buckle half wave in the x direction.
n Number of buckle half wave in the y direction.
N_x, N_y, N_{xy} Membrane pre-buckling stresses resultants, (N/mm)
N_x^{cr} : Value of the membrane stress resultants at buckling.
K_{cr} : Non dimensional loading parameter and corresponding value at buckling.
λ : Proportional load factor.
w : Transversal displacement of the membrane plate .
C_1, C_2, C_3 : Non dimensionnel coefficients of rigidity.
E_s Sequent modules.
E_t : Tangent modulus.
ssss: Simply supported four edges.
sscc: Simply supported two loaded edges and clumped unloaded edges.

2. Formulation of the analytic method

To start with, we first outline the basic equations for boundary conditions describing the buckling of rectangular plates in the elastic and plastic range**,** Fig.1.

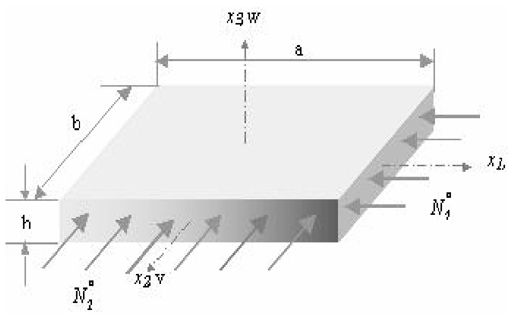

Fig. 1. Geometry of the plate

The formulation of the elementary theory of plates, uses the assumption that the normal stresses σ_{33} are negligible in the volume of the plate compared to the other components $\sigma_{33} = 0$. The plate's theory considers for a displacement field of a point M with co-ordinates (x_1, x_2, x_3) polynomial functions limited to 3 degrees.

It is noticed that with a 1st order model, the plane deformations vary linearly across the thickness of the plate. By neglecting the transverse shearing strains, we obtain two additional relations.

$$\begin{cases} \beta_1 = -\dfrac{\partial w}{\partial x_1} \\ \beta_2 = -\dfrac{\partial w}{\partial x_2} \end{cases} \tag{1}$$

Thus, the formulation for thin plate sections doesn't use more than three independent variables u, v and w.

2.1. Instability equation

Under axial compressive loads, the plate remains flat and the response curve is unique up to the critical load or the bifurcation buckling load. Beyond the buckling load, there are at least two possible response curves. One of these curves is the continuation of the pre-buckling curve, first part of the equilibrium path and called fundamental path, the other is called bifurcated path.

The equilibrium equation allows determining the initial configuration of the plate when it buckles. For symmetrical loading, this relation is:

$$M_{1,11} + M_{2,22} + 2M_{12,12} - N_1 \cdot \beta_{1,1} - N_2 \beta_{2,22} - N_{12}(\beta_{2,1} + \beta_{1,2}) = 0 \tag{2}$$

In this formulation, the constitutive law is written by equation (3), we consider a plane stress state and we neglect the influence of transverse shearing.

$$\{\tilde{\sigma}_{ij}\} = [D_{ijkl}]\{\tilde{\varepsilon}_{kl}\} \tag{3}$$

Generalized forces and moments are given by:

$$\{N\}=\int_{-h}^{h}\{\sigma\}dz \;;\qquad \{M\}=\int_{-h}^{h}z\{\sigma\}dz \quad\text{and}\quad \{M\}=\left[\hat{D}_f\right]\{\chi\}$$

$$\text{with } \{\chi\}=\begin{Bmatrix}-w_{,xx}\\ -w_{,yy}\\ -2w_{,xy}\end{Bmatrix} \text{ and } [\hat{D}_f]=h^3/12\;[D] \tag{4}$$

The critical buckling load can be determined as the minimum load verifying the differential equation (2). A solution w (x_1, x_2) that verify the equation (2) must take into account the boundary conditions of the considered plate. The parameters $\hat{D}_{ijkl}$ are the elastic plastic modules.

2.2. Constitutive laws

The rate form of the constitutive law is given by:

$$\dot{\sigma}_{ij}=\hat{D}_{ijkl}\dot{\varepsilon}_{kl} \tag{5}$$

With $\hat{D}_{ijkl}$ is the stiffness tensor and $\{\varepsilon\}=\begin{Bmatrix}-zw_{,11}\\ -zw_{,22}\\ -2zw_{,12}\end{Bmatrix}$ are the strains components, moments are then given by:

$$\begin{aligned}M_1&=-\frac{h^3}{12}\left[\hat{D}_{1111}w_{,11}+\hat{D}_{1122}w_{,22}\right]\\ M_2&=-\frac{h^3}{12}\left[\hat{D}_{2211}w_{,11}+\hat{D}_{2222}w_{,22}\right]\\ M_{12}&=-2\frac{h^3}{12}\left[\hat{D}_{1212}w_{,12}\right]\end{aligned} \tag{6}$$

The plane stress modulus are deduced from the three dimensional modulus $\hat{D}_{ijkl}$ for the general incremental constitutive law by;

$$\hat{D}_{ijkl}=D_{ijkl}-\frac{D_{ij33}D_{33kl}}{D_{3333}} \tag{7}$$

when we consider the two phenomenological constitutive laws (J2F and J2D) D_{ijkl} can be written as:

$$D_{ijkl}=D^e_{ijkl}-\alpha\frac{2G_J}{g_J}\sigma'_{ij}\,\sigma'_{kl} \tag{8}$$

where D^e_{ijkl} is the elastic stiffness tensor, which can be written as;

$$D^e_{ijkl}=2G_J\left[\frac{1}{2}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+\frac{\nu_J}{1-2\nu_J}\delta_{ij}\delta_{kl}\right] \tag{9}$$

where $\alpha = 0$ in elastic loading and 1 in plastic loading,

$$G_J=\frac{E_J}{2(1+\upsilon_J)} \text{ and } \sigma'_{ij}=\sigma_{ij}-\delta_{ij}\sigma_{kk} \text{ is the deviator-stress.} \tag{10}$$

$$g_J=\frac{2}{3}\bar{\sigma}^2\left(1+\frac{h_J}{2G_J}\right)\quad \bar{\sigma}_2=\frac{1}{2}\sigma'_{ij}\,\sigma'_{ij}\quad\text{and}\quad \frac{1}{h_J}=\frac{3}{2}\left(\frac{1}{E_t}-\frac{1}{E_J}\right) \tag{11}$$

- In J2F case $\nu_J = \nu$, $E_J = E$
- In J2D case $\nu_J = \nu_S$, $E_J = E_S$, with $\nu_S=\frac{1}{2}+\frac{E_S}{E}\left(\nu-\frac{1}{2}\right)$

The modulus E, Et, Es and the Poisson's ratio ν can be determined from a simple tension test. Moreover, it is worth noting that experimental tension curve adopted in this work can be simulated using a conventional power law in the form:

$$\frac{\bar{\varepsilon}}{\sigma_y}=\frac{1}{E}\left[\left(\frac{\bar{\sigma}}{\sigma_y}\right)^m\right] \text{ when } \bar{\sigma}\geq\sigma_y \tag{12}$$

The relation (3) can be rewritten as follows.

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}=\frac{E_1}{1-\nu_{12}\nu_{21}}\begin{bmatrix}1 & \nu_{12} & 0\\ \nu_{12} & \frac{E_2}{E_1} & 0\\ 0 & 0 & \frac{G_{12}}{E_2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix} \tag{13}$$

where $E_1 = E_t$, the matrix can be therefore described by the three non-dimensional parameters ν_{12}, $\frac{E_2}{E_1}$ and $\frac{G_{12}}{E_1}$. Explicit form of the different ratios is given in Table.1 [8].

Table 1.
Explicit form of the stiffness elements $\hat{D}_{ijkl}$

$\hat{D}_{1111}$	$\frac{E_1}{1-\nu_{12}\nu_{21}}$	$\hat{D}_{1122}$	$\frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}$
$\hat{D}_{2222}$	$\frac{E_2}{1-\nu_{12}\nu_{21}}$	$\hat{D}_{1212}$	$2\,G_{12}$

Table 2.
Mechanical properties of the aluminium alloys 2024T45and

Material	E_{11} Mpa	E_{22} Mpa	G_{12} Mpa	G_{12} Mpa	σ_y Mpa
2024T45	72500	72500	27600	27600	275

2.3. Analysis of boundary conditions

The explicit form of the critical loads can be expressed as follows. Let us consider the solution in the case of a simply supported edges labeled (ssss). The out of plane displacement w must satisfy the boundary conditions and w (x_1, x_2) takes the form:

$$w(x_1,x_2)=w_0\sin\left(\frac{m\pi x_1}{a}\right)\sin\left(\frac{n\pi x_2}{b}\right) \tag{14}$$

$$\begin{aligned}&\hat{D}_{2222}\,w_{,2222}+2(\hat{D}_{1122}+2\hat{D}_{1212})w_{,1122}+\hat{D}_{1111}\,w_{,1111}+\\ &N_1^{\circ}w_{,11}+\lambda N_1^{\circ}w_{,11}=0\end{aligned} \tag{15}$$

We assume that $N_2=\lambda N_1$

As it was discussed previously, the equation (15) allows us to deal with plastic solutions, while a convenient writing is possible, equation (16). The critical load corresponds to the minimum of the load verifying the equation below.

$$N_1 = \frac{\hat{D}_f^{1111}{}_1\left(\frac{m\pi}{a}\right)^4 + 2(\hat{D}_f^{1212} + 2\hat{D}_f^{1122})\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2}{\left(\frac{m\pi}{a}\right)^2 + \lambda\left(\frac{n\pi}{b}\right)^2}. + \frac{\hat{D}_f^{2222}\left(\frac{n\pi}{b}\right)^2}{\left(\frac{m\pi}{a}\right)^2 + \lambda\left(\frac{n\pi}{b}\right)^2} \tag{16}$$

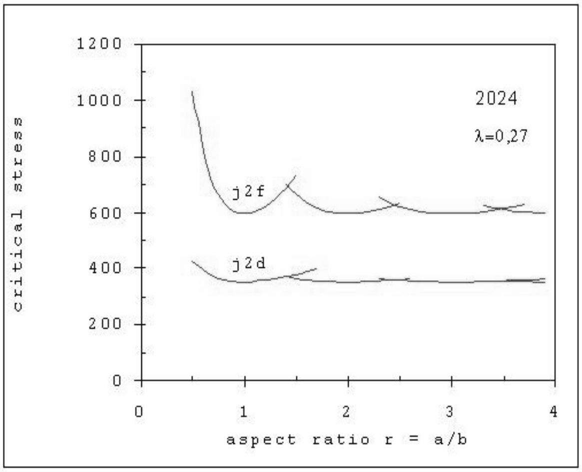

Fig. 2. Variation of the critical load versus the aspect ratio a/b for the 2024 alloy - J2F / J2D (λ =0.27)

In this last equation (15), all the coefficients are time independent, and may be assumed as constants. For a given proportional load factoŗ λ and for a given aspect ratio r, an algorithm of calculation was used for this analytic resolution approach.

Beyond the last study, we can underline the influence of the proportional load factor λ In fact, for particular values of the aspect ratio, the critical load can be in the plastic range.

3. Conclusions

This study has shown that with J2f law, critical loads are almost higher then those determined with J2d law, for aluminium 2024 T45. These results confirm also that the flow model, yields inconsistent predictions, completely disagree with other theories, and also with experimental results. The deformation theory gives always better results and there is a good qualitative agreement between J2d and the experimental results. A noticeable evolution was shown experimental critical loads depending on and λ and b for 2024. The interest of this 3D representation is to indicate the utmost plastic buckling, for a square plate under biaxial loading.

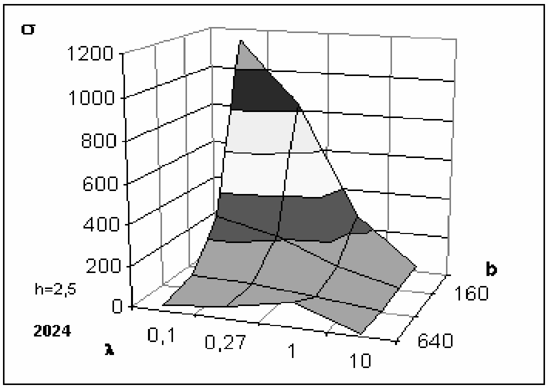

Fig. 3. Critical loads versus λ and b for 2024 aluminum

References

[1] K.D. Murphy, D. Ferreira, Thermal buckling of rectangular plates, International Journal of Solids and Structure 38 (2001) 3979-3994.

[2] G.H. Little, Efficient large deflection analysis of rectangular plates, International Journal of Solids and Structure 71 (1999) 333-352.

[3] K.K Shukla, Y Nath, Analytical solution for buckling and post buckling of angle ply laminated plates under thermo-mechanical loading, Applied mechanics department, iit Delphi, International Journal of Non Linear Mechanics (2000) 1097-1108.

[4] M. Azhari, S. Hshdar, M.A. Bradford, On the use of bunble functions in the local buckling analysis of plate structures by the split finite strip method, International Journal for Numerical Method in Engineering 48 (2000) 583-593.

[5] Shanley, Hill, Post buckling analysis of imperfect stiffened laminated cylindrical shells under combined external pressure and loading, International Journal of Mechanical Sciences 40 (1998) 339-355.

[6] Zghal, M. Potier Ferry, A. Nadari, Buckling analysis based on polycrystalline plasticity, Revue de Mécanique Appliquée et Théorique 1 (2000) 41-61.

[7] J. Henrik Sâllstrôm, Accurate calculation of elastic buckling loads for space frames built up of uniform beams with open thin walled cross section, International Journal for Numerical Method in Engineering 39 (1996) 2319-2333.

[8] Z.Q. Xiong, L.S. Xie, Study on the theory and application of the energy method for analyzing compressive instability in sheet forming, Journal of Materials Processing Technology 129 (2002) 255-260.