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Flat Marcinkiewicz Integral Operators

Ahmad Al-Salman and Hussain Al-Qassem

Abstract

In this paper, we study Marcinkiewicz integral operators with rough kernels supported by surfaces of revolutions. We prove that our operators are bounded on L^p under certain convexity assumptions on our surfaces and under very weak conditions on the kernel.

Key Words: Marcinkiewicz Integral, rough kernel, flat curves, Fourier transform.

1. Introduction

Let $\mathbf{S}^{n-1}$ be the unit sphere in $\mathbf{R}^n$ $(n \geq 2)$ equipped with the normalized Lebesgue measure $d\sigma$ and $\Omega \in L^1\left(\mathbf{S}^{n-1}\right)$ be a homogeneous function of degree zero that satisfies

$$\int_{\mathbf{S}^{n-1}} \Omega(x)\ d\sigma(x) = 0. \tag{1.1}$$

Let $\Gamma : \mathbf{R}^n \to \mathbf{R}^d$, $d \geq n+1$ be a mapping such that the surface $\Gamma(\mathbf{R}^n)$ is smooth in $\mathbf{R}^d$. The Marcinkiewicz integral operator $\mu_{\Omega,\Gamma}$ associated to Γ and Ω is defined by

$$\mu_{\Omega,\Gamma} f(x) = (\int_{-\infty}^{\infty} \left| \int_{|y|\leq 2^t} f(x-\Gamma(y))\, |y|^{-n+1}\, \Omega(y) dy \right|^2 2^{-2t} dt)^{\frac{1}{2}}. \tag{1.2}$$

The problem regarding the operator $\mu_{\Omega,\Gamma}$ is that under what conditions on Γ and Ω, the operator $\mu_{\Omega,\Gamma}$ maps $L^p(\mathbf{R}^d)$ into $L^p(\mathbf{R}^d)$ for some $1 < p < \infty$. It is well known that if

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$d = n+1$, $\Gamma(y) = (y, 0)$, and $\Omega \in Lip_\alpha(\mathbf{S}^{n-1})$, $(0 < \alpha \leq 1)$, E. M. Stein has proved that $\mu_{\Omega,\Gamma}$ is bounded on L^p for all $1 < p \leq 2$. Subsequently, A. Benedek, A. Calderón, and R. Panzone proved the L^p boundedness of $\mu_{\Omega,\Gamma}$, $\Gamma(y) = (y, 0)$, for all $1 < p < \infty$ provided that $\Omega \in C^1(\mathbf{S}^{n-1})$ ([3]). Recently, there has been a notable progress in obtaining L^p boundedness results of the operator $\mu_{\Omega,\Gamma}$ under the assumption that $\frac{\partial^\alpha \Gamma}{\partial y^\alpha}(0) \neq 0$ for some multi-index $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are non negative integers (see [1], [5], among others). Our main focus in this paper is investigating the L^p boundedness of $\mu_{\Omega,\Gamma}$ if $\frac{\partial^\alpha \Gamma}{\partial y^\alpha}(0) = 0$ for all multi-indices α, i.e., when Γ has infinite order of contact with its tangent plane at the origin. In this paper, we shall assume that Γ is a surface of revolution obtained by rotating a one-dimensional curve around one of the coordinate axes. More specifically, we let $\Gamma(y) = (y, \phi(|y|))$, where ϕ is a real valued function defined on $\mathbf{R}^+$. Here we allow ϕ to be flat at the origin. In what follows we shall simply denote $\mu_{\Omega,\Gamma}$ by μ_ϕ. We should point out here that the study of integral operators with kernels supported by surfaces of revolutions has a long history (see [2], [6], [9], [11], among others).

Our main result in this paper is the following:

Theorem 1.1. *Suppose that $\phi : \mathbf{R}^+ \to \mathbf{R}$ is an increasing convex function. If $\Omega \in L(Log^+ L)(\mathbf{S}^{n-1})$* and satisfies (1.1), *then μ_ϕ is bounded on $L^p(\mathbf{R}^{n+1})$ for $1 < p < \infty$.*

Here, $L(Log^+ L)(\mathbf{S}^{n-1})$ is the space of all $L^1(\mathbf{S}^{n-1})$ functions Ω that satisfies

$$\int_{\mathbf{S}^{n-1}} |\Omega(y')| \, Log^+(|\Omega(y')|) d\sigma(y') < \infty.$$

It is worth pointing out that $L(Log^+ L)(\mathbf{S}^{n-1})$ contains the space $L^q(\mathbf{S}^{n-1})$ (for any $q > 1$) properly and the condition $\Omega \in L(Log^+ L)(\mathbf{S}^{n-1})$ is known to be the most desirable size condition for the L^p boundedness of the related Calderón-Zygmund singular integral operator ([4]).

We shall obtain Theorem 1.1 as a consequence of a more general result in which we allow our kernels to be rough in the radial direction. To be more specific, for $1 < \gamma < \infty$, let Δ_γ be the set of all measurable functions $h : \mathbf{R}^+ \to \mathbf{R}$ which satisfy

$$\|h\|_{\Delta_\gamma} = \sup_{R>0} (R^{-1} \int_0^R |h(t)|^\gamma \, dt)^{\frac{1}{\gamma}} < \infty \tag{1.3}$$

and let $\Delta_\infty = L^\infty(\mathbf{R}^+)$. For $h \in \Delta_\gamma$ for some $\gamma > 1$, let $\mu_{\phi,h}$ be the operator defined by (1.2) with $\Gamma(y) = (y, \phi(|y|))$ and Ω replaced by Ωh. Then we have the following theorem.

Theorem 1.2. *Suppose that* $\phi : \mathbf{R}^+ \to \mathbf{R}$ *is an increasing convex function* and $h \in \Delta_\gamma$ *for some* $\gamma > 1$. *If* $\Omega \in L(Log^+ L)\left(\mathbf{S}^{n-1}\right)$ and satisfies (1.1), *then* $\mu_{\phi,h}$ *is bounded on* $L^p(\mathbf{R}^{n+1})$ *for* $|1/p - 1/2| < \min\left\{1/2, 1/\gamma^{'}\right\}$.

It is easy to see that if $h \in \Delta_\gamma$ for some $\gamma \geq 2$, then $\mu_{\phi,h}$ in Theorem 1.2 is bounded on $L^p(\mathbf{R}^{n+1})$ for all $1 < p < \infty$. Hence, Theorem 1.1 can be deduced from Theorem 1.2 by taking $h = 1$.

Throughout this paper, the letter C is a positive constant that may vary at each occurrence but it is independent of the essential variables.

2. Preparation

Suppose that $\Omega \in L^1\left(\mathbf{S}^{n-1}\right)$ is a homogeneous function of degree zero that satisfies (1.1). For a suitable function $\phi : \mathbf{R}^+ \to \mathbf{R}$ and a measurable function $h : \mathbf{R}^+ \to \mathbf{R}$, consider the family of measures $\{\sigma_{\beta,\phi,h} : \beta \in \mathbf{R}^+\}$ defined on $\mathbf{R}^{n+1}$ by

$$\int_{\mathbf{R}^{n+1}} f d\sigma_{\beta,\phi,h} = \beta^{-1} \int_{|y|<\beta} f(y, \phi(|y|))\, |y|^{-n+1}\, \Omega(y')h(|y|)dy. \tag{2.1}$$

Also, let $\sigma^*_{\phi,h}$ be the maximal function defined by

$$\sigma^*_{\phi,h} f(x, x_{n+1}) = \sup_{\beta>0} \left|\sigma_{\beta,\phi,h} * f(x, x_{n+1})\right|. \tag{2.2}$$

Lemma 2.1. *Suppose that* $h \in \Delta_\gamma$ *for some* $\gamma > 1$ *and* $\Omega \in L^2(\mathbf{S}^{n-1})$ with $\|\Omega\|_{L^1} \leq 1$. Then for $\theta = \min\left\{(3\gamma^{'})^{-1}, (12)^{-1}\right\}$, we have

$$|\hat{\sigma}_{\beta,\phi,h}(\xi, \tau)| \leq 2\, \|h\|_\gamma\, \|\Omega\|_{L^2}\, |\beta\xi|^{-\theta}. \tag{2.3}$$

Proof. Using polar coordinates, Hölder's inequality, and noticing that $|\hat{\sigma}_{\beta,\phi,h}(\xi, \tau)| \leq \|h\|_{\Delta_\gamma}$, we get

$$|\hat{\sigma}_{\beta,\phi,h}(\xi, \tau)| \leq \|h\|_{\Delta_\gamma} \max\{\|\Omega\|_{L^2}^{1-\frac{2}{\gamma'}} (F(\beta, \xi))^{\frac{2}{\gamma'}}, F(\beta, \xi)\}, \tag{2.4}$$

where

$$F(\beta,\xi)=(\int_0^1\left|\int_{\mathbf{S}^{n-1}}e^{-i\beta r\xi\cdot y'}\Omega(y')d\sigma(y')\right|^2dr)^{\frac{1}{2}}.$$

Now it is easy to see that

$$(F(\beta,\xi))^2\le\int_{\mathbf{S}^{n-1}}\int_{\mathbf{S}^{n-1}}|\Omega(z')|\,|\Omega(y')|\left|\int_0^1 e^{-i\beta r\xi\cdot(y'-z')}dr\right|d\sigma(y')d\sigma(z'). \tag{2.5}$$

By combining the estimate $\left|\int_0^1 e^{-i\beta r\xi\cdot(y'-z')}dr\right|\le|\beta\xi|^{-1}\left|\xi'\cdot(y'-z')\right|^{-1}$ with the trivial estimate $\left|\int_0^1 e^{-i\beta r\xi\cdot(y'-z')}dr\right|\le 1$, we get

$$\left|\int_0^1 e^{-i\beta r\xi\cdot(y'-z')}dr\right|\le|\beta\xi|^{-\frac{1}{6}}\left|\xi'\cdot(y'-z')\right|^{-\frac{1}{6}}. \tag{2.6}$$

Thus by (2.5), (2.6), and Hölder's inequality, we have

$$F(\beta,\xi)\le C\left\|\Omega\right\|_{L^2}|\beta\xi|^{-\frac{1}{12}}. \tag{2.7}$$

Hence by (2.4), (2.7), and the trivial estimate $|\hat{\sigma}_{\beta,\phi,h}(\xi,\tau)|\le\|h\|_\gamma$, we get (2.3).

Now we prove the following result concerning the maximal function $\sigma^*_{\phi,h}$:

Theorem 2.2. *Suppose that* $\phi:\mathbf{R}^+\to\mathbf{R}$ *is an increasing convex function. If* $\|\Omega\|_{L^1(\mathbf{S}^{n-1})}\le 1$ *and* $\|\Omega\|_{L^2(\mathbf{S}^{n-1})}\le 2^{3(w+1)}$ *for some* $w\ge 0$, *then for* $1<p<\infty$ *we have*

$$\left\|\sigma^*_{\phi,1}f\right\|_p\le(w+1)C\left\|f\right\|_p \tag{2.8}$$

Proof. Without loss of generality we may assume that $\Omega\ge 0$. Let $\{\sigma_{2^t,\phi,1}:t\in\mathbf{R}\}$ be a family of measures defined as in (2.1). Choose $\theta\in\mathcal{S}(\mathbf{R}^n)$ such that $\hat{\theta}(\xi)=1$ for $|\xi|\le\frac{1}{2}$, and $\hat{\theta}(\xi)=0$ for $|\xi|\ge 1$. Let $\theta_r(x)=r^{-n}\theta(\frac{x}{r})$ for $r\ge 0$. Define the families of measures $\{\tau_t:t\in\mathbf{R}\}$ and $\{\lambda_t:t\in\mathbf{R}\}$ on $\mathbf{R}^{n+1}$ by

$$\hat{\tau}_t(\xi,\eta)=\hat{\sigma}_{2^t,\phi,1}(\xi,\eta)-\hat{\theta}_{2^t}(\xi)\hat{\sigma}_{2^t,\phi,1}(0,\eta). \tag{2.9}$$

By Lemma 2.1 and the estimate $\|\tau_t\| \leq C$, we have

$$|\hat{\tau}_t(\xi,\eta)| \leq C 2^{3(w+1)} \left|2^t \xi\right|^{-\frac{1}{12}}; \tag{2.10}$$

from which, when combined with the trivial estimate $\|\tau_t\| \leq C$, we get

$$|\hat{\tau}_t(\xi,\eta)| \leq C \left|2^t \xi\right|^{-\frac{1}{12(w+1)}}. \tag{2.11}$$

On the other hand, by the estimate $\|\tau_t\| \leq C$ and noticing that $|\hat{\tau}_t(\xi,\eta)| \leq C\,|2^t \xi|$, we get

$$|\hat{\tau}_t(\xi,\eta)| \leq C \left|2^t \xi\right|^{\frac{1}{12(w+1)}}. \tag{2.12}$$

Let μ_ϕ be the maximal function defined on $\mathbf{R}^{n+1}$ by

$$\mu_\phi(f)(x,x_{n+1}) = \sup_{t\in\mathbf{R}} \left| 2^{-t} \int_0^{2^t} f(x, x_{n+1} - \phi(t))dt \right|.$$

By the convexity assumption on ϕ, we have

$$\|\mu_\phi(f)\|_p \leq C \ \|f\|_p \tag{2.13}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $1 < p < \infty$.

We now choose a collection of $\mathcal{C}^\infty$ functions $\{\psi_{w,t}\}_{t\in\mathbf{R}}$ on $(0,\infty)$ such that:

$$\begin{aligned} supp(\psi_{w,t}) &\subseteq [2^{-(w+1)t-(w+1)}, 2^{-(w+1)t+(w+1)}],\ 0 \leq \psi_{w,t} \leq 1, \\ \left|\frac{d^s\psi_{w,t}}{du^s}(u)\right| &\leq \frac{C}{u^s},\ \text{and} \sum_{j\in\mathbf{Z}} \psi_{w,j+t}(u) = 1. \end{aligned} \tag{2.14}$$

Let $\varphi_{w,t}$ be such that $\hat{\varphi}_{w,t}\ (\xi) = \psi_{w,t}(|\xi)|)$. For $j \in \mathbf{Z}$, define the operators

$$\mathbf{J}_{w,j}(f)(x) = (\int_{-\infty}^{\infty} \left|\tau_{(w+1)t} * \varphi_{w,j+t}\ * f(x)\right|^2 dt)^{\frac{1}{2}}; \tag{2.15}$$

$$\mathbf{g}_{w,j}(f)(x) = (\int_{-\infty}^{\infty} |\varphi_{w,j+t}\ * f(x)|^2 dt)^{\frac{1}{2}}. \tag{2.16}$$

By a well known argument (see [11], pages 26-28), it is easy to prove that

$$\|\mathbf{g}_{w,j}(f)\|_p \leq C \|f\|_p \tag{2.17}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $1 < p < \infty$. Let τ^* be the maximal function corresponding to the family $\{\tau_t : t \in \mathbf{R}\}$. Then it is easy to see that

$$\sigma^*_{\phi,1}(f) \leq 2\sqrt{w+1}\sum_{j\in\mathbf{Z}}\mathbf{J}_{w,j}(f) + ((M_{\mathbf{R}^n} \otimes id_{\mathbf{R}}) \circ \mu_\phi(f)); \tag{2.18}$$

$$\tau^*(f) \leq 2\sqrt{w+1}\sum_{j\in\mathbf{Z}}\mathbf{J}_{w,j}(f) + 2((M_{\mathbf{R}^n} \otimes id_{\mathbf{R}}) \circ \mu_\phi(f)), \tag{2.19}$$

where $M_{\mathbf{R}^n}$ is the Hardy Littlewood maximal function defined on $\mathbf{R}^n$.

Now by the trivial estimate $\|\tau_t\| \leq C$, (2.11)-(2.12), and Plancherel's theorem, it is easy to see that

$$\|\mathbf{J}_{w,j}(f)\|_2 \leq C2^{-|j|}\|f\|_2 \tag{2.20}$$

for all $j \in \mathbf{Z}$ and $f \in L^2(\mathbf{R}^{n+1})$. Thus, by (2.13), (2.18)-(2.20) and the L^p boundedness of $M_{\mathbf{R}^n}$, we obtain

$$\|\tau^* f\|_2 \leq 2\sqrt{w+1}C\|f\|_2\,, \tag{2.21}$$

holds for $f \in L^2(\mathbf{R}^{n+1})$. Now by a similar argument as in the proof of the lemma on page 544 in ([7]), we have

$$\|\mathbf{J}_{w,j}(f)\|_{p_0} \leq (w+1)^{\frac{1}{4}}C\|f\|_{p_0} \tag{2.22}$$

for $f \in L^{p_0}(\mathbf{R}^{n+1})$ and $\left|\frac{1}{p_0} - \frac{1}{2}\right| = \frac{1}{2q}$, with $q = 2$. Therefore, by (2.19)-(2.20) and (2.22) we get

$$\|\tau^* f\|_p \leq (w+1)^{\frac{1}{2}+\frac{1}{4}}C\|f\|_p \tag{2.23}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $\frac{4}{3} < p < 4$.

Now repeat the same argument employed in the proof of the inequalities (2.22)-(2.23) using $q = \frac{4}{3} + \epsilon$ ($\epsilon \to 0^+$) this time, we get

$$\|\tau^* f\|_p \leq (w+1)^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}C\|f\|_p \tag{2.24}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $\frac{7}{8} < p < 8$. By successive application of the above argument, we get

$$\|\tau^* f\|_p \leq (w+1)C\|f\|_p \tag{2.25}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $1 < p < \infty$. Thus by an argument similar to the one that led to (2.22), we have

$$\|\mathbf{J}_{w,j}(f)\|_p \leq \sqrt{w+1}C\|f\|_p \tag{2.26}$$

for $f \in L^p(\mathbf{R}^{n+1})$ and $1 < p < \infty$. Hence by (2.13), (2.18), (2.20), and (2.26), we obtain (2.8). This ends the proof of our theorem. □

Now by an application of Hölder's inequality, we immediately obtain the following corollary.

Corollary 2.3. *Suppose that* $\phi : \mathbf{R}^+ \to \mathbf{R}$ *is an increasing convex function and* $h \in \Delta_\gamma$ *for some* $\gamma > 1$. *If* $\|\Omega\|_{L^1(\mathbf{S}^{n-1})} \leq 1$ *and* $\|\Omega\|_{L^2(\mathbf{S}^{n-1})} \leq 2^{3(w+1)}$ *for some* $w \geq 0$, *then for* $\gamma' < p \leq \infty$ *and* $f \in L^p(\mathbf{R}^{n+1})$ *we have*

$$\left\|\sigma^*_{\phi,h} f\right\|_p \leq (w+1)C\|f\|_p . \tag{2.27}$$

3. Proof of main results

We shall prove Theorem 1.2 as a consequence of the following theorem:

Theorem 3.1. *Suppose that* $\phi : \mathbf{R}^+ \to \mathbf{R}$ *is an increasing convex function* and $h \in \Delta_\gamma$ *for some* $\gamma > 1$. *If* $\Omega \in L^1(\mathbf{S}^{n-1})$ is a homogeneous function of degree zero that satisfies (1.1) with $\|\Omega\|_{L^1(\mathbf{S}^{n-1})} \leq 1$ *and* $\|\Omega\|_{L^2(\mathbf{S}^{n-1})} \leq 2^{3(w+1)}$ *for some* $w \geq 0$, *then for* $|1/p - 1/2| < \min\left\{1/2, 1/\gamma'\right\}$ *and* $f \in L^p(\mathbf{R}^{n+1})$ we have

$$\|\mu_{\phi,h}(f)\|_p \leq C(w+1)\|f\|_p .$$

Proof. Since $\Delta_\gamma \subseteq \Delta_2$ for all $\gamma \geq 2$, we may assume that $1 < \gamma \leq 2$. Let $\{\sigma_{2^t,\phi,h} : t \in \mathbf{R}\}$ be the family of measures defined as in (2.1). Then $\mu_{\phi,h}$ can be written as

$$\mu_{\phi,h}(f)(x, x_{n+1}) = (\int_{-\infty}^{\infty} |\sigma_{2^t,\phi,h} * f(x, x_{n+1})|^2 \, dt)^{\frac{1}{2}}. \tag{3.1}$$

Let $\{\psi_{w,t}\}_{t\in\mathbf{R}}$ be as in (2.14) and let $\mathbf{J}_{w,j}$ be the operator defined on $\mathbf{R}^{n+1}$ as in (2.15) with $\tau_{(w+1)t}$ replaced $\sigma_{2^{(w+1)t},\phi,h}$. Then it is easy to see that

$$\mu_{\phi,h}(f)(x, x_{n+1}) \leq \sqrt{w+1} \sum_{j\in\mathbf{Z}} \mathbf{J}_{w,j}(f)(x, x_{n+1}). \tag{3.2}$$

Thus, by Lemma 2.1 and the trivial estimate $\left\|\sigma_{2^{(w+1)t},\phi,h}\right\| \leq C$, we have

$$\left|\hat{\sigma}_{2^{(w+1)t},\phi,h}(\xi,\tau)\right| \leq C2^{3(w+1)}\left|2^{(w+1)t}\xi\right|^{-\frac{1}{3\gamma'}}. \tag{3.3}$$

On the other hand, by definition of $\sigma_{2^{(w+1)t},\phi,h}$ and the cancelation property of Ω, we obtain

$$\left|\hat{\sigma}_{2^{(w+1)t},\phi,h}(\xi,\tau)\right| \leq C\left|2^{(w+1)t}\xi\right|. \tag{3.4}$$

Therefore, by (3.3)-(3.4), we get

$$\left|\hat{\sigma}_{2^{(w+1)t},\phi,h}(\xi,\tau)\right| \leq C\min\{\left|2^{(w+1)t}\xi\right|^{-\frac{1}{3\gamma'(w+1)}}, \left|2^{(w+1)t}\xi\right|^{\frac{1}{w+1}}\}. \tag{3.5}$$

Now by (3.3), (3.5), and Plancherel's theorem, we have

$$\left\|\mathbf{J}_{w,j}(f)\right\|_2 \leq 2^{-|j|}C\left\|f\right\|_2 \tag{3.6}$$

for $f \in L^2(\mathbf{R}^{n+1})$ and $j \in \mathbf{Z}$. By Corollary 2.3 and a similar argument as in the proof of Theorem 7.5 in ([8]), we have

$$\left\|\mathbf{J}_{w,j}(f)\right\|_p \leq \sqrt{w+1}C\left\|f\right\|_p \tag{3.7}$$

for all p satisfying $|1/p-1/2| < 1/\gamma'$ and $f \in L^p(\mathbf{R}^{n+1})$.

By interpolating between (3.6) and (3.7), we get that

$$\left\|\mathbf{J}_{w,j}(f)\right\|_p \leq 2^{-\theta_p|j|}\sqrt{w+1}C\left\|f\right\|_p \tag{3.8}$$

for all p satisfying $|1/p-1/2| < 1/\gamma'$, $f \in L^p(\mathbf{R}^{n+1})$, and for some constant $\theta_p > 0$ independent of j and w. Hence by (3.2) and (3.8), the proof is complete.

Now we turn to the proof of Theorem 1.2:

Proof of Theorem 1.2. Let $\phi : \mathbf{R}^+ \to \mathbf{R}$ be an increasing convex function, $\Omega \in L(\log^+ L)(\mathbf{S}^{n-1})$ with $\|\Omega\|_{L^1(\mathbf{S}^{n-1})} = 1$, and $h \in \Delta_\gamma$ *for some* $\gamma > 1$. Since $\Delta_\gamma \subseteq \Delta_2$ for $\gamma \geq 2$, we may assume that $1 < \gamma \leq 2$. Now by a similar argument as in [2], there exist $D \subseteq \mathbf{N}\cup\{0\}$, a sequence of functions $\{b_w : w \in D\} \subset L^1(\mathbf{S}^{n-1})$, and a sequence of positive real numbers $\{C_w : w \in D\}$ such that

$$\begin{aligned} \|b_w\|_{L^2} &\leq C2^{3(w+1)}, \qquad \|b_w\|_{L^1} \leq 2; && (3.9) \\ \sum_{w\in D} wC_w &\leq 1 + \|\Omega\|_{L(\log^+ L)}; && (3.10) \\ \int_{\mathbf{S}^{n-1}} b_w \, d\sigma &= 0; && (3.11) \\ \Omega &= \sum_{w\in D} C_w b_w. && (3.12) \end{aligned}$$

For $w \in D$, let $\mu_{\phi,h,w}$ be the operator defined as in (1.6) with Ω replaced by b_w. Therefore, by (3.9)-(3.12) and Theorem 3.1, we obtain that

$$\|\mu_{\phi,h} f\|_p \leq C\{\sum_{w\in D} wb_w\} \|f\|_p \leq C\{1 + \|\Omega\|_{L(\log^+ L)}\} \|f\|_p$$

holds for all p satisfying $|1/p - 1/2| < 1/\gamma^{'}$ and $f \in L^p(\mathbf{R}^{n+1})$. This ends the proof of Theorem 1.2.

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Ahmad Al-SALMAN,
Hussain Al-QASSEM
Department of Math
Yarmouk University
Irbid-Jordan
alsalman@yu.edu.jo
husseink@yu.edu.jo

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