Ion-trap simulation of the quantum phase transition in an exactly solvable model of spins coupled to bosons

Gian Luca Giorgi,[1,*] Simone Paganelli,[2,†] and Fernando Galve[1]

[1] *Instituto de Física Interdisciplinar y Sistemas Complejos IFISC (CSIC-UIB), Campus Universitat Illes Balears, E-07122 Palma de Mallorca, Spain*

[2] *Grup de Física Teòrica, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*

It is known that arrays of trapped ions can be used to efficiently simulate a variety of many-body quantum systems. Here, we show how it is possible to build a model representing a spin chain interacting with bosons which is exactly solvable. The exact spectrum of the model at zero temperature and the ground state properties are studied. We show that a quantum phase transition occurs when the coupling between spins and bosons reaches a critical value, which corresponds to a level crossing in the energy spectrum. Once the critical point is reached, the number of bosonic excitations in the ground state, which can be assumed as an order parameter, starts to be different from zero. The population of the bosonic mode is accompanied by a macroscopic magnetization of the spins. This double effect could represent an useful resource for the phase transition detection since a measure on the phonon can give information about the phase of the spin system. A finite temperature phase diagram is also given in the adiabatic regime.

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I. INTRODUCTION

Quantum spin chains play a fundamental role in the study of many-body systems and quantum phase transitions. These phenomena take place at zero temperature, i.e. in a purely quantum regime, and are driven by the variation of an internal parameter causing a critical change in the ground state due to level crossing in the energy spectrum [1]. During last years, a renewed interest in quantum phase transitions has been developed to understand the behavior of entanglement near the critical point [2–5]. On the other side, the study of interactions between spins and light is considered as the starting point to introduce the framework of open systems, where the appearance of dissipation and decoherence can be understood [6]. A prototypical example is the so called spin-boson model [7], consisting of a single spin interacting with a multi-mode electromagnetic radiation, modeled as distribution of quantum harmonic oscillators. Here, we will discuss the case where a series of spins (arranged in an isotropic XY chain) interact with one or more bosonic modes.

Despite the importance of spin systems, they are not always directly accessible experimentally, and efficient simulation methods are needed to study their properties. It has been shown that spin chains can be efficiently simulated using internal and motional degrees of freedom of trapped particles [8–12]. In particular, arrays of laser-cooled trapped ions seem to be very promising from an experimental point of view. Ions can be trapped with high spatial accuracy, and their internal states can be manipulated with high precision by means of the interaction with electromagnetic fields. A first experimental evidence of coupling between two-level systems, consisting in the transition from paramagnetic into ferromagnetic order in a two-spin quantum Ising model has been recently reported [13]. Furthermore, ion traps offer the possibility of engineering spin-boson coupling [11, 14]. Since spin chains are also important for practical applications, like quantum communication protocols [15], their simulation would allow to estimate the decoherence effects which derive from the interaction with the environment [16].

As said before, we propose a model which allows to observe the phase transition in an isotropic XY spin chain coupled with external boson modes. The model we will introduce in the following is exactly solvable. The phase transition manifests itself in a non-analytical variation of the amplitude of the bosonic field. In correspondence of this change, the chain acquires a finite magnetization. The physical implementation of this model can be done within the framework of trapped ions discussed before.

The paper is organized as follows. In Sec. II we first introduce the general argument of simulation of spin chains with trapped ions. Then, we show how it is possible to simulate a particular model, consisting of a chain of spins in contact with a bosonic environment. The critical properties at zero temperature of this model are discussed in Sec. III, while in Sec. IV we study the phase diagram at finite temperature in the case of a single slowly oscillating phonon. Finally, in Sec. V we present our conclusions.

II. ARRAYS OF TRAPPED IONS AND INTERACTION BETWEEN SPINS AND BOSONS

As shown in Refs. [8, 9], Coulomb chains in linear Paul traps can simulate spin-spin interactions. An effective spin-spin Hamiltonian emerges as the result of the

*Electronic address: gianluca@ifisc.uib-csic.es

†Electronic address: paganelli@ifae.es

interaction of the internal states of ions with the phononic degrees of freedom generated by Coulomb repulsion. The total Hamiltonian includes a phonon bath (H_ν), a force produced by a set of lasers along the directions $\alpha = x, y, z$ (H_f), and an effective magnetic field that can be generated by forcing transitions between the internal ion states (H_m). In units $\hbar = 1$, we have (see [8])

$$\begin{aligned} H_\nu &= \sum_{\alpha,n} \varepsilon_{\alpha,n} a^\dagger_{\alpha,n} a_{\alpha,n}, \\ H_f &= -2 \sum_{\alpha,l} F_\alpha q_{\alpha,l} (1 + \sigma^\alpha_l), \\ H_m &= \sum_{\alpha,l} B^\alpha \sigma^\alpha_l, \end{aligned} \tag{1}$$

where $a_{\alpha,n}$ is the annihilation operator of the n vibrational mode on the α direction of the phonon bath due to the Coulomb repulsion, F_α are the laser forces, $q_{\alpha,l}$ is the displacement, with respect to its equilibrium position, of the ion l in direction α, and B^α is the effective magnetic field. Expressing the coordinates in terms of collective modes through the matrices M^α,

$$H_f = \sum_{\alpha,l,n} F_\alpha \frac{M^\alpha_{l,n}}{\sqrt{2m\varepsilon_{\alpha,n}}} (a^\dagger_{\alpha,n} + a_{\alpha,n})(1 + \sigma^\alpha_l). \tag{2}$$

A spin-spin Hamiltonian is obtained after the application of a suitable canonical transformation $U = e^S$, introduced to eliminate H_f: if

$$S = \sum_{\alpha,l,n} F_\alpha \frac{M^\alpha_{l,n}}{\sqrt{2m\varepsilon^3_{\alpha,n}}} (a^\dagger_{\alpha,n} - a_{\alpha,n})(1 + \sigma^\alpha_l), \tag{3}$$

the total system Hamiltonian turn out to be

$$\tilde{H} = U H U^\dagger = H_\nu + \frac{1}{2} \sum_{\alpha,l,j} J^\alpha_{l,j} \sigma^\alpha_l \sigma^\alpha_j + \sum_{\alpha,l} B'^\alpha \sigma^\alpha_l + H_E, \tag{4}$$

where $B'^\alpha = B^\alpha - F^2_\alpha/(m\varepsilon^2_\alpha)$, the coupling parameters J are function of F, M, ε, and H_E is a residual term that can be neglected at low temperatures or by using highly anisotropic traps [8]. A particular case of Eq. (4) is represented by the isotropic XY chain in the presence of a transverse field, numerically studied in in Ref. [9].

Here, we show that modifying the unitary transformation allows to obtain other kinds of phase transitions, involving for instance the interaction between spins and bosons. A possible way to do that consists in applying different transformations along the three axes. In fact, by limiting the transformation given before to the directions x and y, through

$$S_{x,y} = \sum_{\alpha=x,y} \sum_{l,n} F_\alpha \frac{M^\varepsilon_{l,n}}{\sqrt{2m\omega^3_{\alpha,n}}} (a^\dagger_{\alpha,n} - a_{\alpha,n})(1 + \sigma^\alpha_l), \tag{5}$$

we obtain

$$\begin{aligned} e^{-S_{x,y}} H e^{S_{x,y}} &= \sum_{l,n} F_z \frac{M^z_{l,n}}{\sqrt{2m\varepsilon_{z,n}}} (a^\dagger_{z,n} + a_{z,n})(1 + \sigma^z_l) \\ &+ H_\nu + \frac{1}{2} \sum_{\alpha=x,y} \sum_{l,j} J^\alpha_{l,j} \sigma^\alpha_l \sigma^\alpha_j \\ &+ \sum_l B^z \sigma^z_l + \sum_{\alpha=x,y} \sum_l B'^\alpha \sigma^\alpha_l + H'_E. \end{aligned} \tag{6}$$

Here, the correction H'_E can be neglected in the same limit of H_E. In the case of a spatially homogeneous array, we can drop the dependence from the ion position both in M and in J. Moreover, if the Coulomb interaction can be considered as a perturbation with respect to the trapping potential (stiff limit), the trapping frequencies can be tuned to obtain relevant spin-spin coupling only between nearest-neighbors. Along the transverse direction, a boson displacement can be applied to eliminate the term $\sum_n F_z M^z_n/(\sqrt{2m\varepsilon_{z,n}})(a^\dagger_{z,n} + a_{z,n})$. As a result,

$$\begin{aligned} \tilde{H} &= \sum_{\alpha=x,y} \frac{J^\alpha}{2} \sum_l \sigma^\alpha_l \sigma^\alpha_{l+1} + \sum_n \frac{g^z_n}{\sqrt{N}} (a^\dagger_{z,n} + a_{z,n}) \sum_l \sigma^z_l \\ &+ (B^z - 2 \sum_n \frac{g^{z2}_n}{\omega_n}) \sum_l \sigma^z_l + \sum_{\alpha=x,y} \sum_l B'^\alpha \sigma^\alpha_l \\ &+ H_\nu - N \sum_n \frac{g^{z2}_n}{\omega_n}, \end{aligned} \tag{7}$$

where $g^z_n = F_z M^z_n \sqrt{(N/2m\varepsilon_{z,n})}$. The last term is a constant that barely translates all the energies, and the fields B' can be set to zero. By fixing the external magnetic field $B^z = 2\sum_n (g^{z2}_n/\omega_n)$, we arrive to

$$\begin{aligned} \tilde{H} &= \frac{1}{2} \sum_l (J^x \sigma^x_l \sigma^x_{l+1} + J^y \sigma^y_l \sigma^y_{l+1}) \\ &+ \sum_n \frac{g^z_n}{\sqrt{N}} (a^\dagger_{z,n} + a_{z,n}) \sum_l \sigma^z_l + H_\nu - \frac{N B^z}{2}. \end{aligned} \tag{8}$$

It is worth noting that, in this resulting model, the spins are effective variables which, because of the unitary transformation, are not trivially related to the original internal degrees of freedom of the ions. Nevertheless, the bosonic variables do refer to the real phonon modes along the z direction, so their measurement appears to be the natural way to get information about the overall system.

III. CRITICAL PROPERTIES

To describe the critical properties of this model, let us first discuss the simpler single-mode version. By assuming $J^x = J^y = J$, the Hamiltonian is

$$H = J \sum_{l=1}^{N} [\sigma^+_l \sigma^-_{l+1} + h.c.] + \omega a^\dagger a + \frac{g}{\sqrt{N}} (a^\dagger + a) \sum_{l=1}^{N} \sigma^z_l, \tag{9}$$

where N is the total number of spin, and where periodic boundary conditions are imposed. As it is useful to work with adimensional quantities, we use J as energy unit and define the rescaled phonon frequency $\gamma = \omega/J$ as well as the bare coupling constant $\lambda = g^2/(J\omega)$. With this notation the Hamiltonians reads

$$H = \sum_{l=1}^{N} [\sigma_l^+ \sigma_{l+1}^- + h.c.] + \gamma a^\dagger a + \sqrt{\frac{\lambda\gamma}{N}}(a^\dagger + a) \sum_{l=1}^{N} \sigma_l^z. \tag{10}$$

To diagonalize it, first we introduce the Jordan-Wigner transformation [17], defined through $\sigma_l^z = 1 - 2c_l^\dagger c_l$, $\sigma_l^+ = \prod_{j<l} \left(1 - 2c_j^\dagger c_j\right) c_l$, and $\sigma_l^- = \prod_{j<l} \left(1 - 2c_j^\dagger c_j\right) c_l^\dagger$, mapping spins into spinless fermions. Then, the Fourier transform

$$c_k = \frac{1}{\sqrt{N}} \sum_{l=1}^{N} e^{i2\pi kl/N} c_l \tag{11}$$

allows us to write, in the thermodynamic limit,

$$H = \sum_k \epsilon_k c_k^\dagger c_k + \gamma a^\dagger a + \sqrt{\frac{\lambda\gamma}{N}}(a^\dagger + a) \sum_k (1 - 2c_k^\dagger c_k), \tag{12}$$

with $\epsilon_k = -2\cos(2\pi k/N)$. Fermions can be decoupled from the bosonic degree of freedom by applying the displacement operator $D = \exp[\hat{\alpha}(a^\dagger - a)]$, being $\hat{\alpha} = -\sqrt{\lambda/(\gamma N)} \sum_k (1 - 2c_k^\dagger c_k)$ an operator in the fermionic space. Since $[\hat{\alpha}, \sum_k \epsilon_k c_k^\dagger c_k] = 0$, fermions are not transformed into polarons because of the application of D. Moreover, it is immediately seen that $[b, b^\dagger] = 1$. The final Hamiltonian is

$$DHD^\dagger = \sum_k \epsilon_k c_k^\dagger c_k + \gamma b^\dagger b - \frac{\lambda}{N} [\sum_k (1 - 2c_k^\dagger c_k)]^2. \tag{13}$$

At a first sight, it is similar to that of the XX chain in a transverse field [18], whose Hamiltonian in the Jordan-Wigner space would be $\sum_k \epsilon_k c_k^\dagger c_k - h \sum_k (1 - 2c_k^\dagger c_k)$. While the latter model has the $U(1)$ symmetry, since it is invariant under rotations around the z axis, the Hamiltonian (9) has an additional symmetry, being also invariant under the action of the operator $\mathcal{S} = \prod_l \sigma_l^x \otimes \exp[i\pi a^\dagger a]$. The $U(1)$ symmetry implies, in the language of fermions, that eigenstates of H have a fixed number of particles. The symmetry $\mathcal{S}$ could be broken, for instance, by adding a displacing bosonic field $a + a^\dagger$, and then by sending it to zero. The average value of $a + a^\dagger$ can be assumed as an order parameter to study the breakdown of $\mathcal{S}$.

Since we are interested in the implementation in an array of ion traps, finite-size effects should be taken into account. A detailed analysis of XYZ models on finite cycles has been made in several papers [18–20]. In our model, after the Jordan-Wigner transformation, the Hamiltonian becomes

$$\begin{aligned} H &= \sum_{l=1}^{N-1} (c_{l+1} c_l^\dagger + c_l c_{l+1}^\dagger) - \mathcal{P}(c_1^\dagger c_N + c_N^\dagger c_1) \\ &+ \gamma a^\dagger a + \sqrt{\frac{\lambda\gamma}{N}}(a^\dagger + a) \sum_k (1 - 2c_k^\dagger c_k), \end{aligned} \tag{14}$$

where N is assumed to be an even number and where $\mathcal{P} = \prod_{l=1}^{N} (1 - 2c_l^\dagger c_l)$. Its possible eigenvalues are ± 1. Notice that $\mathcal{P}$ is a measure of the parity of the number of particles of each state. Since $[H, \mathcal{P}] = 0$, all eigenstates of H have definite parity, and we can proceed to a separate diagonalization of H in the two subspaces corresponding to $\mathcal{P} = \pm 1$. Then, if we introduce the two new Hamiltonians $H^\pm = H(\mathcal{P} = \pm 1)$, the complete set of eigenvectors of H will be given by the odd eigenstates of H^- and the even eigenstates of H^+. The choice of $\mathcal{P} = -1$ or $\mathcal{P} = +1$ amounts to have, respectively, periodic or antiperiodic boundary conditions. This implies that H^+ and H^- are diagonalized through two Fourier transforms which differ from each other because of the set of allowed values of k. If half-integer values ($k = 1/2, 3/2, \ldots, N - 1/2$) will be used to diagonalize H^+, in the case of H^- we have $k = 1, 2, \ldots, N$. Now, the displacement operator D can be introduced, and a structure formally identical to that given in Eq. 13 can be done both for H^+ and H^-, with the proper choice of k.

The ground state of the bosonic part can be identified as the vacuum state of the operator b, and it corresponds, in the original representation, to a coherent state of amplitude equal to the average value of the operator $\hat{\alpha}$ calculated on the fermionic ground state, whose structure is now enlightened. Since both H^+ and H^- are exactly solvable, their spectra can be calculated by measuring the energy of all possible configurations, obtained adding the desired number of fermions. The energy of a state with m particles is

$$E_{\mathcal{M}} = -\frac{\lambda(N - 2m)^2}{N} - 2 \sum_{k \in \mathcal{M}} \cos\frac{2\pi k}{N}. \tag{15}$$

where $\mathcal{M}$ is a string representing the occupation of different modes. The sum is extended only to the values of k where a particle is present. For large values of λ, the first term will be larger than the sum, unless $N - 2m$ is very small. The ground state corresponds to the sequences which maximize $(N - 2m)^2$, i.e. to $m = 0$ or $m = N$, and it is twofold degenerate. In fact, both the states $|\Phi^+\rangle = |0\rangle \otimes |\alpha\rangle$ and $|\Phi^-\rangle = \prod_k c_k^\dagger |0\rangle \otimes |-\alpha\rangle$ have energy equal to $E_0 = -N\lambda$. Here the state $|\Phi^+\rangle$ is, in the original fermion-boson representation (before the application of D), the tensor product of the fermionic vacuum and of a coherent bosonic state of amplitude $\alpha = -g\sqrt{N}/\omega$, while in $|\Phi^-\rangle$ the fermionic system is fully occupied, and the boson coherent state has amplitude $-\alpha$. As λ decreases, the two terms of Eq. (15) start to compete with each other. Let us call λ_m the value such

that $E_0 = \min\{E_{\mathcal{M}}\}$. Since by symmetry $\lambda_m = \lambda_{N-m}$, we will limit to consider $m = 1, 2, \ldots, N/2$. It can be shown that, if $m < m'$, then $\lambda_m < \lambda_{m'}$. In other words, the first ground state level crossing takes place between $|\Phi^+\rangle$ (or $|\Phi^-\rangle$) and a state with $N/2$ particles. This transition occurs at the value of $\lambda_{N/2} = \lambda_c$ determined by

$$N\lambda_c = 4\sum_{k=1}^{N/4} \cos\frac{(2k-1)\pi}{N}. \tag{16}$$

In the thermodynamic limit, performed replacing the sum with an integral, we obtain $\lambda_c = 2/\pi$. As a consequence of the half filling, $\sum_k (1 - 2c_k^\dagger c_k) = 0$, and the boson part is left in its vacuum. This state can be written as $|\Phi^{HF}\rangle = \prod_{|k|<N/4} c_k^\dagger |0\rangle \otimes |0\rangle$.

By further decreasing λ, no other transitions are observed. Then, the state $|\Phi^{HF}\rangle$ is the Hamiltonian ground state for every $\lambda < \lambda_c$. It is worth noting that both $|\Phi^{HF}\rangle$ and $|\Phi^+\rangle$ (or $|\Phi^-\rangle$) are eigenstates of H^+. In any case, eigenstates of H^- are excitations. In Fig. 1, we plot the lowest energy levels for a chain of 8 spins. The transition $|\Phi^\pm\rangle \to |\Phi^{HF}\rangle$ is observed.

While $|\Phi^{HF}\rangle$ is eigenstate of the symmetry operator $\mathcal{S}$, this is not true in the case of $|\Phi^+\rangle$ and $|\Phi^-\rangle$. In fact, $\mathcal{S}|\Phi^\pm\rangle = |\Phi^\mp\rangle$. The possibility of obtaining a coherent emission of light on the bosonic mode is inherently linked to the breakdown of the Hamiltonian symmetry, since the only coherent state which is also eigenstate of $\exp[i\pi a^\dagger a]$ is the vacuum. While symmetry breaking takes place independently on the system size (the number of spins), only in the thermodynamic limit a true phase transition takes place, since the Hilbert spaces that could be built up starting from $|\Phi^+\rangle$ and $|\Phi^-\rangle$ become unitarily nonequivalent.

In the presence of multi-mode radiation, Hamiltonian (8) can be recovered modifying Hamiltonian (9) in the following way

$$\begin{aligned} H_{mm} &= \sum_{l=1}^{N}[\sigma_l^+\sigma_{l+1}^- + h.c.] + \sum_n \gamma_n a_n^\dagger a_n \\ &+ \sum_n \sqrt{\frac{\lambda_n\gamma_n}{N}}(a_n^\dagger + a_n)\sum_{l=1}^{N}\sigma_l^z, \end{aligned} \tag{17}$$

and it is subject to the symmetry $\mathcal{S}_{mm} = \prod_l \sigma_l^x \otimes \exp[i\pi\sum_n a_n^\dagger a_n]$. The diagonalization is performed through the application of the Jordan-Wigner transformation and of the operator $\prod_n D_n$, with $D_n = \exp[\hat{\alpha_n}(a_n^\dagger - a_n)]$, and $\hat{\alpha_n} = -\sqrt{\lambda_n/(\gamma_n N)}\sum_k(1 - 2c_k^\dagger c_k)$. As a result, we have

$$\tilde{H}_{mm} = \sum_k \epsilon_k c_k^\dagger c_k + \sum_n \omega_n b_n^\dagger b_n - \frac{\Lambda}{N}[\sum_k (1 - 2c_k^\dagger c_k)]^2, \tag{18}$$

where $\Lambda = [\sum_n (g_n^2/J\omega_n)]$ is related to the spectral density of the bath. In Ref. [14], the authors showed how baths characterized by different spectral densities could be simulated. Being $\tilde{H}_{mm}$ formally identical to $DHD^\dagger$, we expect a phase transition for $\Lambda_c = 2/\pi$. For $\Lambda > \Lambda_c$, the two degenerate ground states are $|\Phi^+_{mm}\rangle = |0\rangle \otimes \prod_n |\alpha_n\rangle$ and $|\Phi^-_{mm}\rangle = \prod_k c_k^\dagger |0\rangle \otimes \prod_n |-\alpha_n\rangle$, while for $\Lambda < \Lambda_c$ the non degenerate half filled ground state is $|\Phi^{HF}_{mm}\rangle = \prod_{|k|<N/4} c_k^\dagger |0\rangle \otimes \prod_n |0\rangle$.

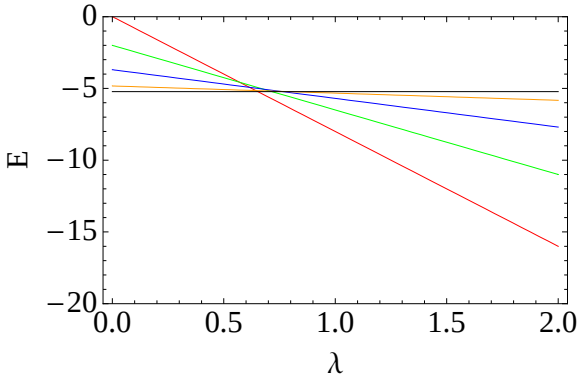

FIG. 1: (Color online) Ground state energy of H as a function of λ for a chain of 8 spins, with $\omega = J = 1$. For any given number of particles, only the lower level is plotted. The red line represents the energy of $|\Phi^+\rangle$ and $|\Phi^-\rangle$, while the black one corresponds to $|\Phi^{HF}\rangle$. As we can see, there is only one transition, occurring at $\lambda_c \simeq 0.65$. All energies derived from numbers of fermions different from $0, N/2, N$ are excitations. Specifically, the energy of the one-particle state is drawn in green, the two-particle state is in blue, and the orange line corresponds to the three-particle state. Due to the particle-hole symmetry, states with m and $N - m$ particles have the same energy.

IV. TEMPERATURE EFFECTS IN THE ADIABATIC LIMIT

In this section we analyze the phase transition at finite temperature in the so-called adiabatic limit ($\gamma \ll 1$) with a single coupled mode. This limit corresponds to the case of a very slowly oscillating phonon and it is asymptotically exact at high temperature. The adiabatic Hamiltonian is obtained by neglecting the kinetic energy of the phonon and treating the coordinate as a parameter

$$H_{AD} = \sum_{l=1}^{N}[\sigma_l^+\sigma_{l+1}^- + h.c.] + \frac{\nu^2}{4\lambda} + \frac{\nu}{\sqrt{N}}\sum_{l=1}^{N}\sigma_l^z, \tag{19}$$

where we introduced the adimensional coordinate $\nu = \frac{g}{J}\sqrt{2\omega m}x$. The Hamiltonian is a isotropic XY model in an external field, parametrically dependent on ν, it can be diagonalized in the same way as described previously and reduced in two blocks corresponding to odd and even pseudo-fermion occupation number

$$H_\pm = \sum_k \epsilon_k^{AD}(\nu) c_k^\dagger c_k + \frac{\nu^2}{4\lambda} + \frac{\nu}{\sqrt{N}}, \tag{20}$$

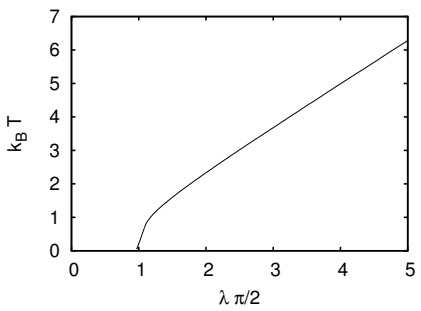

FIG. 2: Phase diagram in the adiabatic approximation. Data are obtained with 10 spins and numerical evaluation of the adiabatic potential. The left region corresponds to a single-well adiabatic potential, the right region to a double-well potential.

with $\epsilon_k^{AD}(\nu) = -2\left(\frac{\nu}{\sqrt{N}} + \cos\frac{2\pi}{N}k\right)$. The energy of a configuration with a filling m is

$$E_{\mathcal{M}} = -2\left(\frac{\nu m}{\sqrt{N}} + \sum_{k\in\mathcal{M}} \cos\frac{2\pi}{N}k\right). \tag{21}$$

The partition function, defined by

$$Z \propto \int d\nu Z_0(\nu,\beta) \exp -\beta(\frac{\nu^2}{4\lambda} + \frac{\nu}{\sqrt{N}}), \tag{22}$$

with

$$Z_0(\nu,\beta) = \mathrm{tr}\left\{e^{-\beta\sum_k \epsilon_k^{AD}(\nu) c_k^\dagger c_k}\right\}, \tag{23}$$

can be written as $Z \propto \int d\nu \exp -\beta V_{AD}(\nu,\beta)$ so defining an adiabatic potential

$$V_{AD}(\nu,\beta) = \frac{\nu^2}{4\lambda} + \frac{\nu}{\sqrt{N}} + -\frac{1}{\beta}\ln Z_0(\nu,\beta). \tag{24}$$

Above the critical temperature, $V_{AD}(\nu,\beta)$ is minimized by $\nu = 0$ while, at the critical point, the potential becomes double well allowing for a displacement of the phonon. The phase diagram obtained is plotted in Fig. (2).

V. CONCLUSIONS

To resume, we have shown that a system of trapped ions can be mapped into an isotropic XY chain interacting with phonons, following the scheme of Ref. [8]. Modifying the original canonical transformations needed to write a spin-spin Hamiltonian, the vibrational degrees of freedom along a fixed direction are coupled with the effective spins the other degrees of freedom are mapped into. The resulting model is exactly solvable and exhibits a quantum phase transition, due to the breaking of a symmetry which takes into account both the spin and the boson degrees of freedom. In this phase, phonons are in a coherent state with finite amplitude. Since the phonons refer to the real ionic vibration and not to an effective quantity, the phase transition detection should be relatively easy. The experimental realization of the building block of such simulations (Ref. [13]), encourages this kind of investigation.

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