

Nonlinear Quantum Wave Equation of Radiation Electron and Dissipative Systems

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Abstract

As well known, an electron will produce radiative reaction force when the electron emits electromagnetism radiation. The electron radiative effect had not been considered in Schrödinger wave equation. In this paper, we give the nonlinear quantum wave equations for the radiative electron and some dissipative systems.

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1. Introduction

Linearity of the Schrödinger equation and the validity of the superposition principle have been essential ingredients of quantum theory since its earliest days. Practically all physical phenomena behave nonlinearly when examined over a sufficiently large range of the dynamical parameters that determine their evolution.

A number of earlier works that have attempted to extend quantum theory in a nonlinear way, there are: The work by Kibble and Randjbar-Daemi is close to ours in that they consider how nonlinear modifications of quantum field theory can be made compatible with Lorentz or more generally coordinate invariance [1, 2]. Besides considering a coupling of quantum fields to classical gravity according to general relativity, which induces an intrinsic nonlinearity [2, 3], these authors study mean-field type nonlinearities, where parameters of the model are state dependent through their assumed dependence on expectations of certain operators. Work by Bialynicki-Birula and Mycielski introduces a logarithmic nonlinearity into the nonrelativistic Schrödinger equation, with which many of the features of standard quantum mechanics are left intact [4]. A number of different nonrelativistic models of this kind have been systematically studied by Weinberg, offering also an assessment of the observational limits on such modifications of the Schrödinger equation [5]. Independently, Doebner and Goldin and collaborators have also studied nonlinear modifications of the nonrelativistic Schrödinger equation [6]. This was originally motivated by attempts to incorporate dissipative effects. Later, however, they have shown that classes of nonlinear Schrödinger equations, including many of those considered earlier, can be obtained through nonlinear transformations of the linear quantum mechanical equation. The nonlinear quantum mechanics has a practical importance in different fields, like condensed matter, quantum optics and atomic and molecular physics; even quantum gravity may involve nonlinear quantum mechanics [7-10]. Another important example is in the modern field of quantum computing [11-14].

As well known, an electron will produce radiative reaction force when the electron emits electromagnetic radiation. The electron radiative effect is not considered in Schrödinger wave equation and Dirac quantum theory. In this paper, we study the quantum effect of lower and higher energy electron radiative reaction, and give the nonlinear quantum wave equation to describe the nonrelativistic and relativistic radiation electron.

2. Nonlinear quantum wave equation for nonrelativistic radiation electron

The radiation reaction force of low energy electron can argument based on conservation of energy for a nonrelativistic electron. An electron of mass m and charge e acted on by an external force \vec{F}_{ext} moves according to the Newton equation of motion: $m\dot{\vec{v}} = \vec{F}_{ext}$. To account for this radiative energy loss and its effect on the motion of the electron we should modify the Newton equation by adding a radiative force \vec{F}_{rad} , and the Newton equation is $m\dot{\vec{v}} = \vec{F}_{ext} + \vec{F}_{rad}$.

Since the electron is accelerated, it emits radiation at a rate given by Larmor power formula [15]

$$p = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} |\dot{\vec{v}}|^2, \quad (v \ll c) \quad (1)$$

The electron radioactive energy in unit time is

$$\frac{dw}{dt} = \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} |\dot{\vec{v}}|^2, \quad (2)$$

and the radioactive energy in the time interval $(0, t)$ is

$$W = \frac{1}{4\pi\epsilon_0} \int_0^t \frac{2e^2}{3c^3} |\dot{\vec{v}}|^2 dt. \quad (3)$$

By conservation of energy, we have

$$\frac{d}{dt}(T + V) = -\frac{d}{dt}W, \quad (4)$$

or

$$E = T + V + W = \text{constant}, \quad (5)$$

we directly promote all classical variables E , T and W to quantum operators

$$\hat{E} \rightarrow i\hbar \frac{\partial}{\partial t}, \quad \hat{p} \rightarrow -i\hbar \nabla, \quad (6)$$

Eq. (5) becomes

$$i\hbar \frac{\partial}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 + V + \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} \int_0^t \dot{\vec{v}}^2 dt', \quad (7)$$

and act the wave function $\psi(\vec{r}, t)$ on the right side of Eq. (7), we have

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V \psi(\vec{r}, t) + \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} \int_0^t \dot{\vec{v}}^2 dt' \cdot \psi(\vec{r}, t), \quad (8)$$

The complex time dependent wave function $\psi(\vec{r}, t)$ is expressed in terms of a Lagrange action $S(\vec{r}, t)$ [8]

$$\psi(\vec{r}, t) = \sqrt{\rho(\vec{r}, t)} e^{iS(\vec{r}, t)/\hbar}, \quad (9)$$

and

$$\frac{\psi(\vec{r}, t)}{\psi^*(\vec{r}, t)} = e^{2iS(\vec{r}, t)/\hbar}. \quad (10)$$

Using the relation from the Hamilton-Jacobi theory

$$\vec{p} = \nabla S(\vec{r}, t) = m\vec{v}, \quad (11)$$

or

$$\vec{v} = \frac{\nabla S(\vec{r}, t)}{m}. \quad (12)$$

From Eqs. (10)-(12), we can express the velocity and acceleration in terms of $\psi(\vec{r}, t)$ and $\psi^*(\vec{r}, t)$ as follows

$$\vec{v} = -i \frac{\hbar}{2m} \nabla \ln \frac{\psi(\vec{r}, t)}{\psi^*(\vec{r}, t)}, \quad (13)$$

and

$$\dot{\vec{v}} = -i \frac{\hbar}{2m} \frac{\partial}{\partial t} \nabla \ln \frac{\psi(\vec{r}, t)}{\psi^*(\vec{r}, t)}, \quad (14)$$

and then Eq. (8) becomes

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V\psi(\vec{r}, t) - \frac{1}{4\pi\epsilon_0} \frac{2e^2}{3c^3} \frac{\hbar^2}{4m^2} \int_0^t \left(\frac{\partial}{\partial t'} \nabla \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')} \right)^2 dt' \cdot \psi(\vec{r}, t). \quad (15)$$

The Eq. (15) is radioactive electron nonlinear quantum wave function for the low energy radioactive electron.

3. Nonlinear quantum wave equation for the dissipative systems

The heavy ion scattering experiments give strong indication for a reaction called deep inelastic process [8]. The heavy ion lose their entire available kinetic energy during the collision and are then repelled just by their mutual Coulomb interaction energy. Thus nuclear friction seems to play an important role. Moreover, there is also evidence that the fission process is damped during the descent from saddle to scission. Classical as well as microscopic calculations in these phenomena which include frictional effects have

already been made [9, 10, 11]. Quantal friction, however, is still an open problem. Other applications are, for instance, the motion of Stokes' ball in a viscous medium, Brownian motion, or an electric oscillator composed of inductance, capacitor, and resistor. In the following, we should give the quantum wave equation for these dissipative systems.

1. The dissipative system for the dissipative force $\vec{F}_n = -k\vec{v}$.

For a dissipative system, there is a conserved quantity, which is the sum of a particle's mechanical energy and the work doing by the dissipative force acting on the particle. It is

$$E = T + V - \int \vec{F}_n \cdot d\vec{r} = \text{constant}, \quad (16)$$

where T and V are the particle's kinetic energy and potential energy, respectively, the integrate $\int \vec{F}_n \cdot d\vec{r}$ is the work doing by the dissipative force acting on the particle, and E is the system's total energy.

Substituting $\vec{F} = -k\vec{v}$ into Eq. (16), we have

$$E = T + V + k \int_0^t \vec{v}^2 dt', \quad (17)$$

substituting Eqs. (6), (13) into (17), we obtain the operator equation

$$i\hbar \frac{\partial}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 + V + k \int_0^t -\frac{\hbar^2}{4m^2} (\nabla \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')})^2 dt', \quad (18)$$

and act the wave function $\psi(\vec{r}, t)$ on the right side of Eq. (18), we have

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V \psi(\vec{r}, t) - \frac{\hbar^2 k}{4m^2} \int_0^t (\nabla \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')})^2 dt' \cdot \psi(\vec{r}, t), \quad (19)$$

The Eq. (19) is the nonlinear quantum wave equation corresponding to the dissipative force $\vec{F} = -k\vec{v}$.

2. The dissipative system for the dissipative force $\vec{F}_n = -k\vec{v}^2 \frac{\vec{v}}{|\vec{v}|}$.

Substituting $\vec{F}_n = -k\vec{v}^2 \frac{\vec{v}}{|\vec{v}|}$ into Eq. (16), we have

$$E = T + V + k \int_0^t v^3 dt'. \quad (20)$$

Similarly, we can obtain the nonlinear quantum wave equation corresponding to the dissipative force $\vec{F} = -k\vec{v}^2 \frac{\vec{v}}{|\vec{v}|}$. It is

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V \psi(\vec{r}, t) + ik \frac{\hbar^3}{8m^3} \int_0^t (\nabla \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')})^3 dt' \cdot \psi(\vec{r}, t). \quad (21)$$

3. The dissipative system for the dissipative force $\vec{F} = -k\dot{\vec{v}}$.

Substituting $\vec{F} = -k\dot{\vec{v}}$ into Eq. (16), we have

$$E = T + V + k \int_0^t \dot{\vec{v}} \cdot \vec{v} dt'. \quad (22)$$

We can obtain the nonlinear quantum wave equation corresponding to the dissipative force $\vec{F} = -k\dot{\vec{v}}$. It is

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(\vec{r}, t) + V \psi(\vec{r}, t) - \frac{\hbar^2}{4m^2} k \int_0^t \nabla \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')} \cdot \nabla \frac{\partial}{\partial t'} \ln \frac{\psi(\vec{r}, t')}{\psi^*(\vec{r}, t')} dt' \cdot \psi(\vec{r}, t). \quad (23)$$

4. Conclusion

The electron in an atom produces radiative reaction. We should consider the quantum effect of the radiation when we calculate the atom energy spectrum and wave functions. The nonlinear term in Eq. (15) is the radiation quantum correction. For the dissipative systems, we can research their quantum property by the new nonlinear quantum wave equations.

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