Effective Mass Dirac-Morse Problem with any κ-value

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Abstract

The Dirac-Morse problem are investigated within the framework of an approximation to the term proportional to $1/r^2$ in the view of the position-dependent mass formalism. The energy eigenvalues and corresponding wave functions are obtained by using the parametric generalization of the Nikiforov-Uvarov method for any κ-value. It is also studied the approximate energy eigenvalues, and corresponding wave functions in the case of the constant-mass for pseudospin, and spin cases, respectively.

Keywords: generalized Morse potential, Dirac equation, Position-Dependent Mass, Nikiforov-Uvarov Method, Spin Symmetry, Pseudospin Symmetry

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The investigation of the solutions for quantum mechanical systems having certain potentials in the case of position-dependent mass (PDM) [1, 2] has been received great attentions. Many authors have studied the solutions of different potentials for spatially-dependent mass, such as hypergeometric type potentials [3], Coulomb potential [4], PT-symmetric kink-like, and inversely linear plus linear potentials [5]. It is well known that the theory based on the effective-mass Schrödinger equation is a useful ground for investigation of some physical systems, such as semiconductor heterostructures [6], the impurities in crystals [7-9], and electric properties of quantum wells, and quantum dots [10]. In the present work, we tend to solve the Dirac-Morse problem within the PDM formalism.

The pseudospin symmetry is an interesting result appearing in Dirac equation of a particle moving in an external scalar, and vector potentials in the case of it when the sum of the potentials is nearly zero. It was observed that the single particle states have a quasidegeneracy labeled with the quantum numbers $\tilde{\ell}$, and $\tilde{s}$, which are called the pseudo-orbital angular momentum, and pseudospin angular momentum quantum numbers, respectively [11-16]. The concept of pseudospin symmetry has received great attentions in nuclear theory because of being a ground to investigate deformation, and superdeformation in nuclei [17, 18], and to build an effective shell-model coupling scheme [19, 20]. The symmetry appears in that case, when the magnitude of scalar potential is nearly equal to the magnitude of vector potential with opposite sign [14, 21-25] and the Dirac equation has the pseudospin symmetry, when the sum of the vector, and scalar potentials is a constant, i.e., $\Sigma(r) = V_v(r) + V_s(r) = const.$ or $d\Sigma(r)/dr = 0$ [16]. The spin symmetry is another important symmetry occurring in Dirac theory in the presence of external scalar, and vector potentials. The spin symmetry appears in the Dirac equation, when the difference of scalar, and vector potentials is a constant, i.e., $\Delta(r) = V_v(r) - V_s(r) = const.$ [14, 16].

Recently, the pseudospin and/or spin symmetry have been studied by many authors for some potentials, such as Morse potential [26-28], Woods-Saxon potential [29], Coulomb [30], and harmonic potentials [31-33], Eckart potential [34-36], Pöschl-Teller potential[37, 38], Hulthén potential [39], and Kratzer potential [40]. In Ref. [41], the bound-state solutions of Dirac equation are studied for generalized Hulthén potential with spin-orbit quantum number κ in the position-dependent mass background. In this letter, we tend to show that the new scheme of the Nikiforov-Uvarov (NU) method could be used to find the energy spectra, and the corresponding eigenspinors within the framework of an approximation to

the term proportional to $1/r^2$ for arbitray spin-orbit quantum number κ, i.e. $\kappa \neq 0$, when the mass depends on position. The NU method is a powerful tool to solve of a second order differential equation by turning it into a hypergeometric type equation [42].

Dirac equation for a spin-$\frac{1}{2}$ particle with mass m moving in scalar $V_s(r)$, and vector potential $V_v(r)$ can be written as (in $\hbar = c = 1$ unit)

$$[\alpha \,.\, \mathbf{P} + \beta(m + V_s(r))]\, \Psi_{n\kappa}(r) = [E - V_v(r)]\, \Psi_{n\kappa}(r)\,. \tag{1}$$

where E is the relativistic energy of the particle, $\mathbf{P}$ is three-momentum, α and β are 4×4 Dirac matrices, which have the forms of $\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$, respectively, [43]. Here, σ is a three-vector whose components are Pauli matrices and I denotes the 2×2 unit matrix. $\mathbf{J}$ denotes the total angular momentum , and $\hat{K} = -\beta(\sigma.\mathbf{L} + 1)$ corresponds to the spin-orbit operator of the Dirac particle in a spherically symmetric potential, where $\mathbf{L}$ is the orbital angular momentum operator of the particle. The eigenvalues of the spin-orbit operator $\hat{K}$ are given as $\kappa = \pm(j+1/2)$, where $\kappa = -(j+1/2) < 0$ correspond to the aligned spin $j = \ell + 1/2$, and $\kappa = (j + 1/2) > 0$ correspond to the unaligned spin $j = \ell - 1/2$. The total angular momentum quantum number of the particle is described as $j = \tilde{\ell} + \tilde{s}$,where $\tilde{\ell} = \ell + 1$ is the pseudo-orbital angular momentum quantum number, and $\tilde{s} = 1/2$ is the pseudospin angular momentum quantum number. For a given $\kappa = \pm 1, \pm 2, \ldots$, the relation between the spin-orbit quantum number κ , and "two" orbital angular momentum quantum numbers are given by $\kappa(\kappa + 1) = \ell(\ell + 1)$, and $\kappa(\kappa - 1) = \tilde{\ell}(\tilde{\ell} + 1)$.

The Dirac spinor in spherically symmetric potential can be written in terms of upper and lower components as

$$\Psi_{n\kappa}(r) = \frac{1}{r} \begin{pmatrix} \chi_{n\kappa}\,(r) Y^{\ell}_{jm}(\theta, \phi) \\ i\phi_{n\kappa}\,(r) Y^{\tilde{\ell}}_{jm}(\theta, \phi) \end{pmatrix}, \tag{2}$$

where $Y^{\ell}_{jm}(\theta, \phi)$, and $Y^{\tilde{\ell}}_{jm}(\theta, \phi)$ are the spherical harmonics, and $\chi_{n\kappa}\,(r)/r$, and $\phi_{n\kappa}\,(r)/r$ are radial part of the upper and lower components. Substituting Eq. (2) into Eq. (1) enable us to write the Dirac equation as a set of two couple differential equations in terms of $\chi_{n\kappa}\,(r)$ and $\phi_{n\kappa}\,(r)$. By eliminating $\chi_{n\kappa}\,(r)$ or $\phi_{n\kappa}\,(r)$ in these coupled equations, we obtain

$$\Big\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} + \frac{1}{M_\Delta(r)} \Big(\frac{dm(r)}{dr} - \frac{d\Delta(r)}{dr} \Big) \Big(\frac{d}{dr} + \frac{\kappa}{r} \Big) \Big\} \chi_{n\kappa}(r) = M_\Delta(r) M_\Sigma(r) \chi_{n\kappa}(r)\,, \tag{3}$$

$$\Big\{\frac{d^2}{dr^2}-\frac{\kappa(\kappa-1)}{r^2}-\frac{1}{M_\Sigma(r)}\Big(\frac{dm(r)}{dr}+\frac{d\Sigma(r)}{dr}\Big)\Big(\frac{d}{dr}-\frac{\kappa}{r}\Big)\Big\}\phi_{n\kappa}(r)=M_\Delta(r)M_\Sigma(r)\phi_{n\kappa}(r)\,, \quad (4)$$

where $M_\Delta(r)=m+E_{n\kappa}-\Delta(r)\,,\ M_\Sigma(r)=m-E_{n\kappa}+\Sigma(r)$, and $\Delta(r)=V_v\,(r)-V_s\,(r)$, $\Sigma(r)=V_v\,(r)+V_s\,(r)$.

In the NU-method, the Schrödinger equation is transformed by using an appropriate coordinate transformation

$$\sigma^2(s)\Psi''(s)+\sigma(s)\tilde{\tau}(s)\Psi'(s)+\tilde{\sigma}(s)\Psi(s)=0\,, \quad (5)$$

where $\sigma(s)$, $\tilde{\sigma}(s)$ are polynomials, at most second degree, and $\tilde{\tau}(s)$ is a first degree polynomial. The polynomial $\pi(s)$, and the parameter k are required in the method

$$\pi(s)=\frac{1}{2}\left[\sigma'(s)-\tilde{\tau}(s)\right]\pm\sqrt{\frac{1}{4}\left[\sigma'(s)-\tilde{\tau}(s)\right]^2-\tilde{\sigma}(s)+k\sigma(s)}, \quad (6)$$

$$\lambda=k+\pi'(s), \quad (7)$$

where λ is a constant. The function under the square root in the polynomial in $\pi(s)$ in Eq. (6) must be square of a polynomial in order that $\pi(s)$ be a first degree polynomial. Replacing k into Eq. (6), we define

$$\tau(s)=\tilde{\tau}(s)+2\pi(s). \quad (8)$$

where the derivative of $\tau(s)$ should be negative [42]. Eq. (5) has a particular solution with degree n, if λ in Eq. (7) satisfies

$$\lambda=\lambda_n=-n\tau'-\frac{[n(n-1)\sigma'']}{2},\quad n=0,1,2,\ldots \quad (9)$$

To obtain the solution of Eq. (5) it is assumed that the solution is a product of two independent parts as $\Psi(s)=\phi(s)\ y(s)$, where $y(s)$ can be written as

$$y_n(s)\sim\frac{1}{\rho(s)}\frac{d^n}{ds^n}\left[\sigma^n(s)\ \rho(s)\right], \quad (10)$$

where the function $\rho(s)$ is the weight function, and should satisfy the condition

$$\left[\sigma(s)\ \rho(s)\right]'=\tau(s)\ \rho(s)\,, \quad (11)$$

and the other factor is defined as

$$\frac{1}{\phi(s)}\frac{d\phi(s)}{ds}=\frac{\pi(s)}{\sigma(s)}. \quad (12)$$

In order to clarify the parametric generalization of the NU method, let us take the following general form of a Schrödinger-like equation written for any potential,

$$\left\{\frac{d^2}{ds^2}+\frac{\alpha_1-\alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds}+\frac{-\xi_1 s^2+\xi_2 s-\xi_3}{[s(1-\alpha_3 s)]^2}\right\}\Psi(s)=0. \tag{13}$$

When Eq. (13) is compared with Eq. (5), we obtain

$$\tilde{\tau}(s)=\alpha_1-\alpha_2 s\ ;\ \sigma(s)=s(1-\alpha_3 s)\ ;\ \tilde{\sigma}(s)=-\xi_1 s^2+\xi_2 s-\xi_3\,. \tag{14}$$

Substituting these into Eq. (6)

$$\pi(s)=\alpha_4+\alpha_5 s\pm\sqrt{(\alpha_6-k\alpha_3)s^2+(\alpha_7+k)s+\alpha_8}\,, \tag{15}$$

where the parameter set are

$$\begin{aligned}&\alpha_4=\tfrac{1}{2}\,(1-\alpha_1)\,,\quad \alpha_5=\tfrac{1}{2}\,(\alpha_2-2\alpha_3)\,,\ \alpha_6=\alpha_5^2+\xi_1\\ &\alpha_7=2\alpha_4\alpha_5-\xi_2\,,\ \alpha_8=\alpha_4^2+\xi_3\,.\end{aligned} \tag{16}$$

In NU-method, the function under the square root in Eq. (15) must be the square of a polynomial [42], which gives the following roots of the parameter k

$$k_{1,2}=-(\alpha_7+2\alpha_3\alpha_8)\pm 2\sqrt{\alpha_8\alpha_9}\,, \tag{17}$$

where $\alpha_9=\alpha_3\alpha_7+\alpha_3^2\alpha_8+\alpha_6$. We obtain the polynomials $\pi(s)$ and $\tau(s)$ for $k=-(\alpha_7+2\alpha_3\alpha_8)-2\sqrt{\alpha_8\alpha_9}$, respectively

$$\pi(s)=\alpha_4+\alpha_5 s-[(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\,)s-\sqrt{\alpha_8}\,]\,, \tag{18}$$

$$\tau(s)=\alpha_1+2\alpha_4-(\alpha_2-2\alpha_5)s-2\,[(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\,)s-\sqrt{\alpha_8}\,]\,. \tag{19}$$

Thus, we impose the following for satisfying the condition that the derivative of the function $\tau(s)$ should be negative in the method

$$\begin{aligned}\tau'(s)\ &=\ -(\alpha_2-2\alpha_5)-2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\,)\\ &=\ -2\alpha_3-2(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\,)\quad <0.\end{aligned} \tag{20}$$

From Eqs. (7), (8), (19), and (20), and equating Eq. (7) with the condition that λ should satisfy given by Eq. (9), we find the eigenvalue equation

$$\begin{aligned}\alpha_2 n-(2n+1)\alpha_5\ &+\ (2n+1)(\sqrt{\alpha_9}+\alpha_3\sqrt{\alpha_8}\,)+n(n-1)\alpha_3\\ &+\ \alpha_7+2\alpha_3\alpha_8+2\sqrt{\alpha_8\alpha_9}=0.\end{aligned} \tag{21}$$

We obtain from Eq. (11) the polynomial $\rho(s)$ as $\rho(s) = s^{\alpha_{10}-1}(1-\alpha_3 s)^{\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1}$ and substituting it into Eq. (10) gives

$$y_n(s) = P_n^{(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1)}(1-2\alpha_3 s)\,, \tag{22}$$

where $\alpha_{10} = \alpha_1 + 2\alpha_4 + 2\sqrt{\alpha_8}$, $\alpha_{11} = \alpha_2 - 2\alpha_5 + 2(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8})$ and $P_n^{(\alpha,\beta)}(1-2\alpha_3 s)$ are the Jacobi polynomials. From Eq. (12), one obtaines

$$\phi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}}\,, \tag{23}$$

then the general solution $\Psi(s) = \phi(s)y(s)$ becomes

$$\Psi(s) = s^{\alpha_{12}}(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} P_n^{(\alpha_{10}-1,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1)}(1-2\alpha_3 s). \tag{24}$$

where $\alpha_{12} = \alpha_4 + \sqrt{\alpha_8}$ and $\alpha_{13} = \alpha_5 - (\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8}\,)$. Let us study the case where the parameter $\alpha_3 = 0$. In this type of problems, the eigenfunctions become

$$\Psi(s) = s^{\alpha_{12}}\, e^{\alpha_{13}s}\, L_n^{\alpha_{10}-1}(\alpha_{11}s)\,, \tag{25}$$

when the limits $lim_{\alpha_3\to 0}\, P_n^{(\alpha_{10}-1\,,\frac{\alpha_{11}}{\alpha_3}-\alpha_{10}-1)}(1-\alpha_3 s) = L_n^{\alpha_{10}-1}(\alpha_{11}s)$ and $lim_{\alpha_3\to 0}\,(1-\alpha_3 s)^{-\alpha_{12}-\frac{\alpha_{13}}{\alpha_3}} = e^{\alpha_{13}s}$ are satisfied and the corresponding energy spectrum is

$$\begin{aligned}\alpha_2 n - 2\alpha_5 n + (2n+1)(\sqrt{\alpha_9}\; -\; \alpha_3\sqrt{\alpha_8}\,) + n(n-1)\alpha_3 + \alpha_7 \\ +\; 2\alpha_3\alpha_8 - 2\sqrt{\alpha_8\alpha_9}\, + \alpha_5 = 0\,.\end{aligned} \tag{26}$$

The generalized Morse potential is given by [44]

$$V_M(x) = De^{-2\beta x} - 2De^{-\beta x}\,, \tag{27}$$

where $x = (r/r_0) - 1\,, \beta = \alpha r_0\,, D$ is the dissociation energy, r_0 is the equilibrium distance, and α is the potential width. The term proportional to $1/r^2$ in Eq. (4) can be expanded about $x = 0$ [45]

$$V_M(x) = \frac{\kappa(\kappa-1)}{r^2} = \frac{a_0}{(1+x)^2} = a_0(1-2x+3x^2+\ldots)\,;\; a_0 = \frac{\kappa(\kappa-1)}{r_0^2}\,, \tag{28}$$

Instead, we now replace $V_M(x)$ by the potential [45]

$$\tilde{V}_M(x) = a_0(a_1 + a_2e^{-\beta x} + a_3e^{-2\beta x})\,, \tag{29}$$

Expanding the potential $\tilde{V}_M(x)$ around $x = 0$, and combining equal powers with Eq. (28), one can find the arbitrary constants in the new form of the potential as

$$a_1 = 1 - \frac{3}{\beta} + \frac{3}{\beta^2}\,;\;\; a_2 = \frac{4}{\beta} - \frac{6}{\beta^2}\,;\;\; a_3 = -\frac{1}{\beta} + \frac{3}{\beta^2}\,. \tag{30}$$

Eq. (4) can not be solved analytically because of the last term in the equation, we prefer to use a mathematical identity such as $dm(r)/dr = -d\Sigma(r)/dr$ to eliminate this term. We obtain the mass function from the identity as

$$m(x) = m_0 + m_1 e^{-\beta x} + m_2 e^{-2\beta x}\,, \tag{31}$$

where m_0 corresponds to the integral constant, and the parameters m_1, and m_2 are $2D$, and $-D$, respectively. The parameter m_0 will denote the rest mass of the Dirac particle. By using the potential form given by Eq. (29) replaced by Eq. (28), inserting the mass function in Eq. (31), setting the "difference" potential $\Delta(r)$ to generalized Morse potential in Eq. (27) and using the new variable $s = e^{-\beta x}$, we have

$$\Big\{ \frac{d^2}{ds^2} + \frac{1}{s}\frac{d}{ds} + \frac{1}{s^2}\Big[-\delta^2(a_0a_1 + m_0^2 - E^2) - \delta^2[a_0a_2 + (m_0 - E)(m_1 + 2D)]s - \delta^2[a_0a_3 + (m_0 - E)(m_2 - D]s^2\Big]\Big\}\phi_{n\kappa}(s) = 0\,. \tag{32}$$

Comparing Eq. (32) with Eq. (13) gives the parameter set

$$\begin{aligned}
&\alpha_1 = 1\,, && -\xi_1 = -\delta^2[a_0a_3 + (m_0 - E)(m_2 - D] \\
&\alpha_2 = 0\,, && \xi_2 = -\delta^2[a_0a_2 + (m_0 - E)(m_1 + 2D)] \\
&\alpha_3 = 0\,, && -\xi_3 = -\delta^2(a_0a_1 + m_0^2 - E^2) \\
&\alpha_4 = 0\,, && \alpha_5 = 0 \\
&\alpha_6 = \xi_1\,, && \alpha_7 = -\xi_2 \\
&\alpha_8 = \xi_3\,, && \alpha_9 = \xi_1 \\
&\alpha_{10} = 1 + 2\sqrt{\xi_3}\,, && \alpha_{11} = 2\sqrt{\xi_1} \\
&\alpha_{12} = \sqrt{\xi_3}\,, && \alpha_{13} = -\sqrt{\xi_1}
\end{aligned} \tag{33}$$

where $\delta = 1/\alpha$. We write the energy eigenvalue equation of the generalized Morse potential by using Eq. (26)

$$2\delta\sqrt{a_0a_1 + m_0^2 - E^2} - \delta\,\frac{a_0a_2 + (m_0 - E)(m_1 + 2D)}{\sqrt{a_0a_3 + (m_0 - E)(m_2 - D)}} = 2n + 1\,. \tag{34}$$

Since the negative energy eigenstates exist in the case of the pseudospin symmetry [14, 15, 16], so we choose the negative energy solutions in Eq. (46). In Table I, we give some numerical values of the negative bound state energies obtained from Eq. (46) for CO molecule in atomic units, where we use the input parameter set as $D = 11.2256$ eV, $r_0 = 1.1283$ $\mathring{A}$, $m_0 = 6.8606719$ amu, and $a = 2.59441$ [46], and summarize our results for different $\tilde{\ell}$, and n values. The corresponding lower spinor component can be written by using Eq. (25)

$$\phi(s) = s^{w_1}\, e^{-\,w_2 s} L_n^{2w_1}(2w_2 s)\,, \tag{35}$$

where $w_1 = \delta\sqrt{a_0 a_1 + m_0^2 - E^2}$, and $w_2 = \delta\sqrt{a_0 a_3 + (m_0 - E)(m_2 - D)}$.

Let us study the two special limits, pseudospin and spin symmetry cases, respectively, in the case of the constant mass.

1. *Pseudospin Case*

The Dirac equation has the exact pseudospin symmetry if the "sum" potential could satisfy the condition that $d\Sigma(r)/dr = 0$, i.e. $\Sigma(r) = A(const.)$ [14]. The parameters in our formalism become $m_1 = m_2 = 0$. Setting the "difference" potential $\Delta(r)$ to the generalized Morse potential in Eq. (27), using Eq. (29) for the term proportional to $1/r^2$, and using the new variable $s = e^{-\beta x}$, we have from Eq. (4)

$$\Big\{ \frac{d^2}{ds^2} + \frac{1}{s}\frac{d}{ds} + \frac{1}{s^2}\Big[-\delta^2[a_0 a_1 + M(m_0 + E)] - \delta^2(2MD + a_0 a_2)s + \delta^2(MD - a_0 a_3)s^2\Big]\Big\}\phi(s) = 0\,. \tag{36}$$

where $M = m_0 + A - E$. By following the same procedure, the energy eigenvalue equation for the exact pseudospin symmetry in the case of constant mass is written

$$2\sqrt{a_0 a_1 + M(m_0 + E)} = \frac{a_0 a_2 + 2DM}{\sqrt{a_0 a_3 - DM}} + \alpha(2n + 1)\,. \tag{37}$$

and the corresponding wave functions read as

$$\phi^{m_1 = m_2 = 0}(s) = s^{w'_1}\, e^{-\,w'_2 s} L_n^{2w'_1}(2w'_2 s)\,, \tag{38}$$

where $w'_1 = \delta\sqrt{a_0 a_1 + M(m_0 + E)}$, and $w'_2 = \delta\sqrt{a_0 a_3 - DM}$. We must consideration the negative bound states solutions in Eq. (37) because there exist only the negative eigenvalues in the exact pseudospin symmetry [14, 15, 16].

2. Spin Case

The spin symmetry appears in the Dirac equation if the condition is satisfied that $\Delta(r) = V_v(r) - V_s(r) = A(const.)$. In this case, we have from Eq. (3)

$$\Big\{\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} - (m_0 + E - A)(m_0 - E - \Sigma(r))\Big\}\chi(r) = 0\,, \tag{39}$$

where we set the "sum" potential as generalized Morse potential given in Eq. (27), and use approximation for the term proportional to $1/r^2$ in Eq. (29) [45]

$$\tilde{V}_M(x) = b_0(b_1 + b_2 e^{-\beta x} + b_3 e^{-2\beta x})\,, \tag{40}$$

where $b_0 = \kappa(\kappa+1)/r_0^2$, and the parameters $b_i (i = 1, 2, 3)$ are given in Eq. (30). Using the variable $s = e^{-\beta x}$, and inserting Eq. (40) into Eq. (39), we obtain

$$\begin{aligned}\Big\{\frac{d^2}{ds^2} + \frac{1}{s}\frac{d}{ds} &+ \frac{1}{s^2}\Big[-\delta^2[b_0 b_1 + M'(m_0 - E)] + \delta^2(2DM' - b_0 b_2)s \\ &- \delta^2(b_0 b_3 + DM')s^2\Big]\Big\}\chi(s) = 0\,.\end{aligned} \tag{41}$$

where $M' = m_0 + E - A$. We write the energy eigenvalue equation, and corresponding wave equations in the spin symmetry limit, respectively,

$$\frac{\delta[2DM' - b_0 b_2]}{\sqrt{b_0 b_3 + DM'}} + 2\delta\sqrt{b_0 b_1 + M'(m_0 - E)} = 2n + 1\,, \tag{42}$$

and

$$\chi^{m_1=m_2=0}(s) = s^{w_1''}\, e^{-w_2'' s} L_n^{2w_1''}(2w_2'' s)\,, \tag{43}$$

where $w_1'' = \delta\sqrt{b_0 b_1 + M'(m_0 - E)}$, and $w_2'' = \delta\sqrt{b_0 b_3 + DM'}$. We must take into account the positive energy solutions in Eq. (42) in the case of the exact spin symmetry [14, 15, 16].

In Summary, we have approximately solved the effective mass Dirac equation for the generalized Morse potential for arbitrary spin-orbit quantum number κ in the position-dependent mass background. We have found the eigenvalue equation, and corresponding two-component spinors in terms of Legendre polynomials by using the parametric NU-method within the framework of an approximation to the term proportional to $1/r^2$. We have also obtained the energy eigenvalue equations, and corresponding wave functions for exact pseudospin, and spin symmetry limits in the case of constant mass. We have observed that our analytical results in the case of the pseudospin symmetry are good agreement with the ones obtained in the literature.

[1] Von Roos O 1983 Phys. Rev. B **27** 7547

[2] Levy-Leblond J M 1995 Phys. Rev. A **52** 1845

[3] Jia C S, Wang P Q, Liu J Y and He S 2008 Int. J. Theor. Phys. **47** 2513

[4] Alhaidari A D 2004 Phys. Lett. A **322** 72

[5] Jia C S and Dutra A S 2008 Ann. Phys. **323** 566

[6] Gora T and Williams F 1969 Phys. Rev. **177** 11979

[7] Luttinger J M and Kuhn W 1955 Phys. Rev. **97** 869

[8] Wanner G H 1957 Phys. Rev. **52** 191

[9] Slater J C 1949 Phys. Rev. **52** 1592

[10] Serra L and Lipparini E 1997 Europhys. Lett. **40** 667

[11] Hecht K T and Adler A 1969 Nucl. Phys. A **137** 139.

[12] Arima A, Harvey M and Shimizu K 1969 Phys. Lett. B **30** 517

[13] Blokhin A L, Bahri C and Draayer J P 1986 Phys. Rev. Lett. **74**(21) 4199.

[14] Ginocchio J N 1997 Phys. Rev. Lett. **78**(3) 436

[15] Ginocchio J N and Leviatan A 2001 Phys. Rev. Lett. **87**(7) 072502

[16] Ginocchio J N 2005 Phys. Rep. **414** 165

[17] Dudek J, Nazarewicz W, Szymanski Z and Le Ander G 1987 Phys. Rev. Lett. **59** 1405

[18] Bohr A, Hamamoto I and Mottelson B R 1982 Phys. Scr. **26** 267

[19] Troltenier D, Bahri C and Draayer J P 1995 Nucl. Phys. A **586** 53

[20] Ginocchio J N 1999 Phys. Rep. **315** 231

[21] Leviatan A and Ginocchio J N 2001 Phys. Lett. B **518** 214

[22] Von Neumann-Cosel P and Ginocchio J N 2000 Phys. Rev C **62** 014308

[23] Ginocchio J N 2001 Nucl. Phys. A **59** 41c

[24] Bell J S and Ruegg H 1975 Nucl. Phys. B **98** 151

[25] Ginocchio J N 2002 Phys. Rev C **66** 064312

[26] Bayrak O and Boztosun I 2007 J. Phys. A: Math. Theor. **40** 11119

[27] Berkdemir C 2006 Nucl. Phys. A **770** 32

[28] Qiang W C, Zhou R S and Gao Y 2007 J. Phys. A: Math. Theor. **40** 1677

[29] Gou J Y and Sheng Z Q 2005 Phys. Lett. A **338** 90

[30] Alhaidari A D, Bahlouli H and Al-Hasan A 2006 Phys. Lett. A **349** 87

[31] Lisboa R, Malheiro M, Castro A S, Alberto P and Fiolhais M 2004 Phys. Rev C **69** 024319

[32] Ginocchio J N 2005 Phys. Rev. Lett. **95** 252501

[33] Castro A S, Alberto P, Lisboa R and Fiolhais M 2006 Phys. Rev C **73** 054309

[34] Jia C S, Guo P and Peng X L 2006 J. Phys. A: Math. Gen. **39** 7737

[35] Zhang L H, Li X P and Jia C S 2008 Phys. Lett. A **372** 2201

[36] Soylu A, Bayrak O and Boztosun I 2008 J. Phys. A: Math. Theor. **41** 065308

[37] Xu Y, He S and Jia C S 2008 J. Phys. A: Math. Theor. **41** 255302

[38] Jia C S, Guo P, Diao Y F, Yi L Z and Xie X J 2007 Eur. Phys. J. A **34** 41

[39] Soylu A, Bayrak O and Boztosun I 2007 J. Math. Phys. **48** 082302

[40] Berkdemir C and Sever R 2008 J. Phys. A **41** 045302

[41] Arda A, Tezcan C and Sever R, in progress.

[42] Nikiforov A F and Uvarov V B 1988 Special Functions of Mathematical Physics (Birkhauser, Basel).

[43] Greiner G 1981 Relativistic Quantum Mechanics (Springer Verlag).

[44] Morse P M 1929 Phys. Rev. **34** 57

[45] Pekeris C L 1934 Phys. Rev. **45** 98

[46] Nasser I, Abdelmonem M S, Bahlouli H and Alhaidari A D 2007 J. Phys. B **40.21** 4245

TABLE I: Energy eigenvalues for the CO molecule for different values of $\tilde{\ell}$ and (n, κ) in the case of position dependent mass.

$\tilde{\ell}$	n	κ	state	$E < 0$
1	1	-1	$1s_{1/2}$	6.15913020
2	1	-2	$1p_{3/2}$	6.52968379
3	1	-3	$1d_{5/2}$	6.89146288
4	1	-4	$1f_{7/2}$	7.24974882