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NONLINEAR HYDROMAGNETIC FLOW, HEAT AND MASS TRANSFER OVER AN ACCELERATING VERTICAL SURFACE WITH INTERNAL HEAT GENERATION AND THERMAL STRATIFICATION EFFECTS

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Abstract. The study of heat and mass transfer on nonlinear hydromagnetic flow is of great practical importance to engineers because of its almost universal occurrence in many branches of science and engineering and hence a large amount of research work has been reported in this field. In particular, the study of heat and mass transfer with heat source and thermal stratification is of considerable importance in chemical and hydrometallurgical industries and this work deals with a problem of such study. Steady flow of an incompressible, viscous, electrically conducting and Boussinesq fluid over an accelerating vertical plate with heat source and thermal stratification effect is considered in the presence of an uniform transverse magnetic field. The problem is considered to be of MHD laminar boundary layer type. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using R.K.Gill method. The velocity, temperature and concentration of the fluid are shown graphically to observe the effects of parameters entering the problem. Finally a sufficient discussion of different results is presented

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1. **Introduction**

Mixed convection flow occurs frequently in nature. The temperature distribution varies from layer to layer and these types of flows have wide applications in industry, agriculture and oceanography. Further, they are especially used in dyeing-industries.

One of the most significant types of flow which has many practical applications in industrial manufacturing processes is the boundary layer behavior over a continuous moving solid surface. For example, heat treated material travelling on a conveyor belt possesses the characteristics of a continuous moving surface.

Heat and mass transfer for an electrically conducting fluid flow under the influence of a magnetic field is considered to be of significant importance due to its application in many engineering problems such as nuclear reactors and those dealing with liquid metals. MHD flows have been of much interest to the engineering community only since the introduction of liquid metal heat exchangers, whereas the thermal instability investigations are directly applicable to the problems in geophysics and astrophysics.

The effects of power-law surface temperature and power-law surface heat flux in the heat transfer characteristics of a continuous linear stretching surface were investigated by Chen and Char [1]. Processes involving the mass transfer effect have long been recognized as important, principally in chemical processing equipment. Crane [2], Vlegger [3] and Gupta and Gupta [4] analyzed the temperature distribution for the problem of a stretching surface. Georgantopoulos et. al [5]have studied the effects of free convection and mass transfer in a conducting liquid, when the fluid is subjected to a transverse magnetic field. Recently, Acharya et. al [6] have studied heat and mass transfer on an accelerating surface subjected to both power-law surface temperature and power-law heat flux variations with a temperature dependent heat source in the presence of suction and blowing.

2. **Nomenclature**

B_o^2	Magnetic field of strength	P_r	Prandtl number
C	Species concentration in the fluid	R_e	Reynolds number
C_∞	Species concentration in the fluid away from the surface	S_c	Schmidt number
C_w	Species concentration near the surface	T_w	Temperature of the wall
c_p	Specific heat at constant pressure	T_∞	Temperature far away from the wall
D	Chemical molecular diffusivity	u, v	Velocity components
g	Acceleration due to gravity	α	Thermal diffusivity
G_r	Grashof number	β	Coefficient of volumetric thermal expansion
G_c	Modified Grashof number	ν	Kinematic viscosity
K	Thermal conductivity	ρ	Density of the fluid

Heat and mass transfer on MHD laminar boundary-layer flow over an accelerating surface with internal heat source and thermal stratification effects is investigated in the present work. The fluid is assumed to be viscous, incompressible and electrically conducting with a magnetic field applied transversely to the direction of the flow. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using R.K.Gill method. This method has the following advantages over other available methods: (i) It utilizes less storage registers. (ii) It controls the growth of rounding errors and is usually stable. (iii) It is computationally economical. In the absence of chemical reaction and magnetic effects, the results are in excellent agreement with those in [6]. Numerical calculations for different values of dimensionless parameters entering the problem under consideration are carried out for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of the Schmidt number, thermal stratification and magnetic field on flow field.

3. **Mathematical analysis**

Consider a steady, viscous, incompressible and electrically conducting fluid flowing over an accelerating surface in the presence of a temperature dependent heat source. The problem is considered to be of laminar boundary layer type and two-dimensional. According to the coordinate system, the x-axis is parallel to the vertical surface and the y-axis is chosen normal to it. A transverse magnetic field of strength B_o is applied parallel to the y-axis. The fluid properties are assumed to be constant in a limited temperature range. The concentration of species far from the wall, C_∞ is infinitesimally very small [8] and hence the Soret and Dufour effects are neglected. The physical properties ρ , μ and D are constant throughout the fluid. In writing the following equations, it is assumed that the induced magnetic field, the external electric field and the electric field due to the polarization of charges are negligible. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion for free convection flow with Joule's dissipation (neglecting viscous dissipation) and under Boussinesq's approximation are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta\,(T-T_\infty)+g\beta^*\,(C-C_\infty)-\frac{\sigma B_o^2}{\rho}u\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\beta_1 u}{\rho c_p}\,(T_\infty-T)\,, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}\,. \tag{4}$$

The boundary conditions are

$$\begin{array}{llll} u=ax\,, \quad v=0\,, & C=C_\infty+A_o x^r\,, & T=T_\infty+A_1 x^r & \text{at } y=0\,; \\ u=0\,, & C=C_\infty\,, & T=T_\infty(x)=(1-n)T_o+nT_w(x) & \text{at } y\to 0\,; \end{array} \tag{5}$$

where n is constant, such that $0\le n<1$. The parameter n is defined as thermal stratification parameter and is equal to $m_1/(1+m_1)$ – see [7] – where m_1 is a constant. T_o is a constant reference temperature, say $T_\infty(0)$. The suffixes w and ∞ denote surface and ambient conditions, a is a constant and r is the temperature parameter.

A_o, A_1 and β are also constants, β is the volumetric coefficient of thermal expansion, β^* is the volumetric coefficient with concentration and σ is electrical conductivity.

We introduce the following new variables

$$\Psi(x,y) = (\upsilon a)^{1/2} f(\eta) \ , \qquad \eta(x,y) = y\sqrt{\frac{a}{\upsilon}} \, , \tag{6}$$

where $f(\eta)$ is the dimensionless stream function. If the velocity components are given by

$$u = \frac{\partial \Psi}{\partial y} \ , \qquad v = -\frac{\partial \Psi}{\partial x} \tag{7}$$

one can easily verify that the continuity equation (1) is identically satisfied and thus the concentration of species far from the wall is infinitesimally small. The following non-dimensional quantities are introduced

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \ , \tag{8}$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty} \ , \tag{9}$$

$$\delta = \frac{\beta_1}{\rho c_p} x \qquad \text{(longitudinal coordinate)} \ , \tag{10}$$

$$R_e = \frac{U}{\sqrt{a\upsilon}} \qquad \text{(Reynolds number)} \ , \tag{11}$$

$$G_r = \frac{\nu g \beta}{U^3} (T_w - T_\infty) \qquad \text{(Grashof number)} \ , \tag{12}$$

$$G_c = \frac{\nu g \beta^*}{U^3} (C_w - C_\infty) \qquad \text{(modified Grashof number)} \ , \tag{13}$$

$$P_r = \frac{\mu c_\rho}{k} \qquad \text{(Prandtl number)} \ , \tag{14}$$

$$S_c = \frac{\upsilon}{D} \qquad \text{(Schmidt number)} \ , \tag{15}$$

$$M^2 = \frac{\sigma B_o^2}{\rho c K^2} \qquad \text{(magnetic parameter)} \ . \tag{16}$$

Making use of equations (6)–(16) we obtain from equations (2)–(4) that

$$f''' + G_c R_e \phi + G_r R_e \theta + f f'' - (f')^2 - \frac{M^2}{R_e}(1 - f') = 0 \ , \tag{17}$$

$$\theta'' - P_r \left(\theta - \frac{n}{1-n}\right) f' - P_r \theta (r + \delta) f' + P_r f \theta' = 0 \ , \tag{18}$$

$$\phi'' - S_c f' \phi r + S_c f \phi' = 0 \ . \tag{19}$$

These equations are associated with the following boundary conditions

$$\begin{array}{llll} f(0) = 0 \ , & f'(0) = 1 \ , & \phi(0) = 1 \ , & \theta(0) = 1 \ , \\ f'(\infty) = 0 \ , & & \phi(\infty) = 0 \ , & \theta(\infty) = 0 \ . \end{array} \tag{20}$$

Equations (17) to (19) with boundary conditions (20) are integrated using the Runge-Kutta Gill method. Results of the problem are obtained for different values of the

Schmidt number, magnetic and thermal stratification parameters and they are discussed in detail in the following section.

4. Results and discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. In the absence of chemical reaction and magnetic effects, the results have been compared with those in a previous work [6] and it is found that they are in good agreement. The numerical results we have obtained are illustrated by means of Figures 1-12.

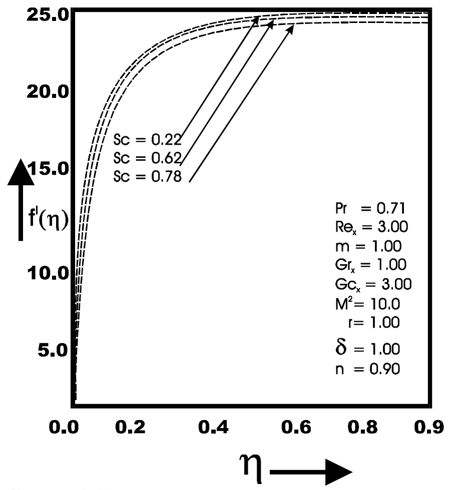

Figure 1. Effect of Schmidt number over the velocity profiles

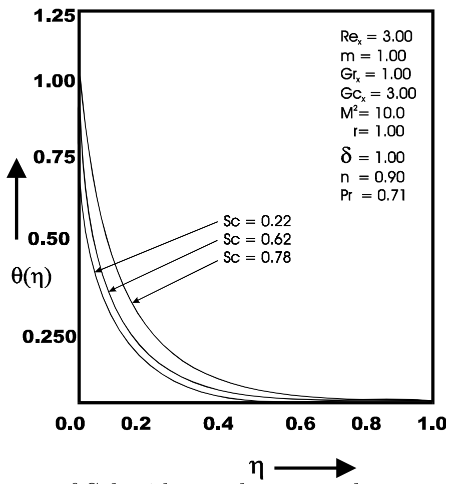

Figure 2. Influence of Schmidt number over the temperature profiles

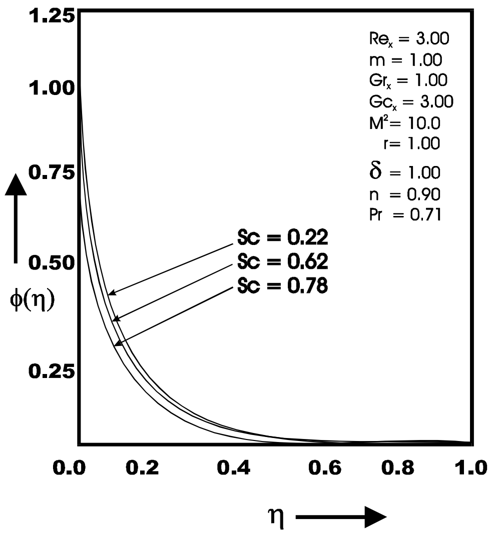

Figure 3. Effects of Schmidt number over the concentration profiles

In the presence of a uniform thermal stratification parameter with constant magnetic field, it is clear that the velocity and the concentration decrease and the temperature of the fluid increases with increase of the Schmidt number and these are displayed through the Figures 1, 2 and 3, respectively.

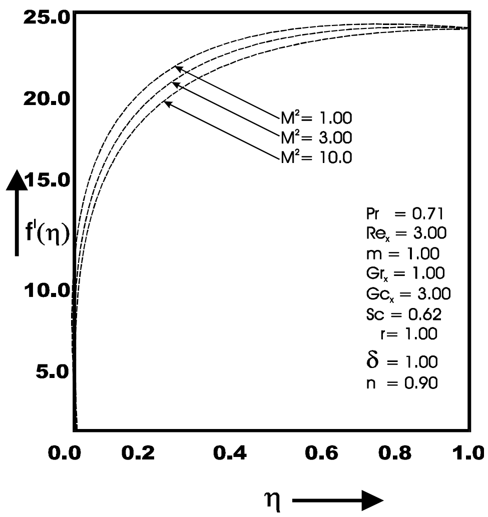

Figure 4. Influence of magnetic field over the velocity profiles

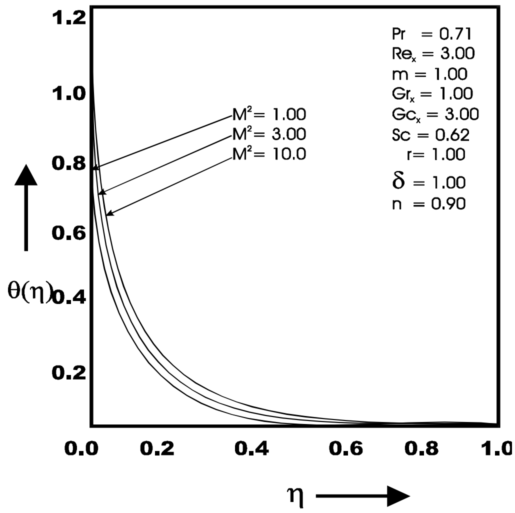

Figure 5. Effect of magnetic field over the temperature profiles

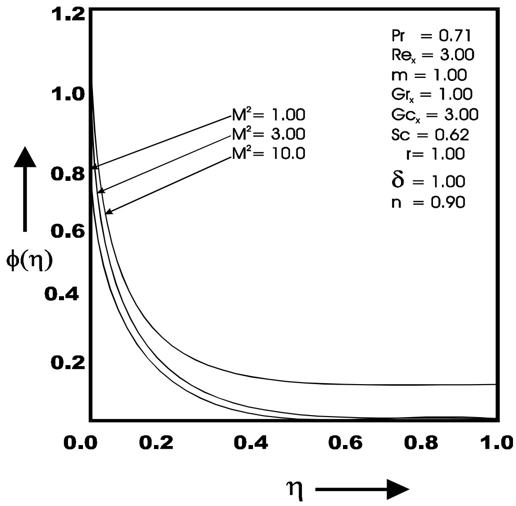

Figure 6. Influence of magnetic field over the concentration profiles

In the presence of a uniform Schmidt number and thermal stratification parameter, it is seen that the increase in the strength of a magnetic field, leads to a fall in the velocity of the fluid and a rise in the temperature and concentration of the fluid along the accelerating surface and are shown in Figures 4, 5 and 6, respectively.

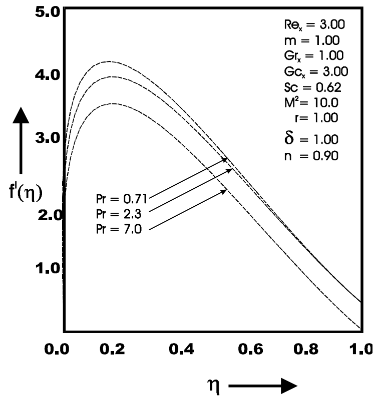

Figure 7. Effect of Prandtl number over the velocity profiles

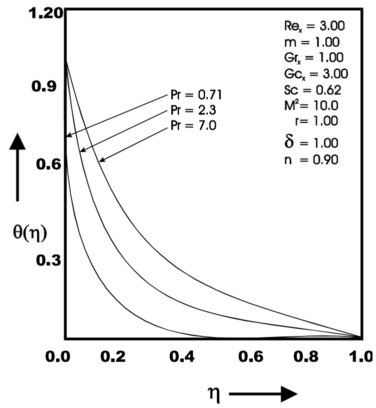

Figure 8. Influence of Prandtl number over the temperature profiles

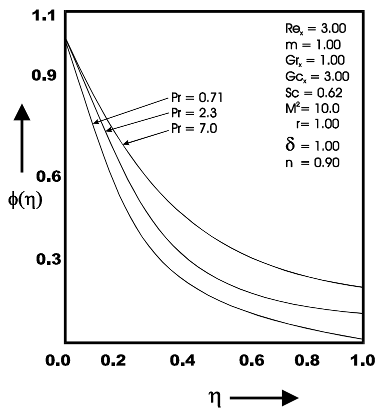

Figure 9. Effect of Prandtl number over the concentration profiles

Figure 7 depicts the dimensionless velocity profiles for different values of the Prandtl number with constant Schmidt number and magnetic field. It is observed that the velocity of the fluid decreases. The temperature and concentration of the fluid increase along the accelerating surface with increase of the Prandtl number and these are displayed through Figures 8 and 9.

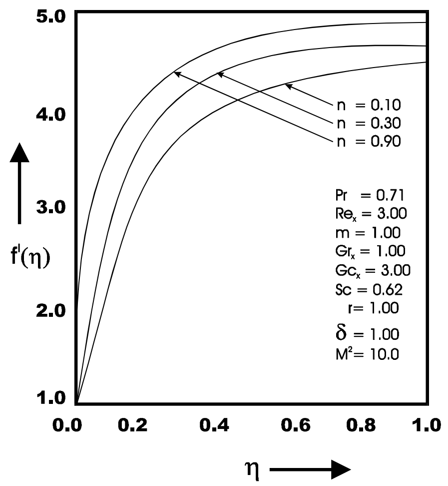

Figure 10. Effect of thermal stratification over the velocity profiles

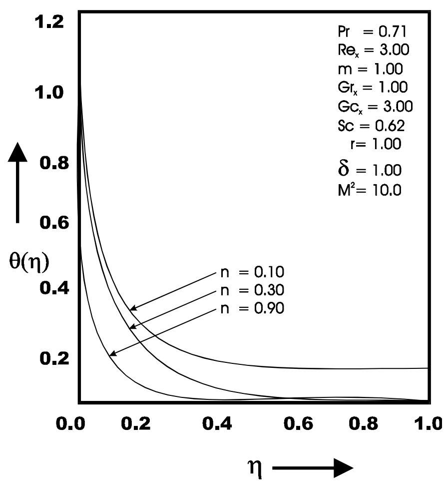

Figure 11. Influence of thermal stratification over the temperature profiles

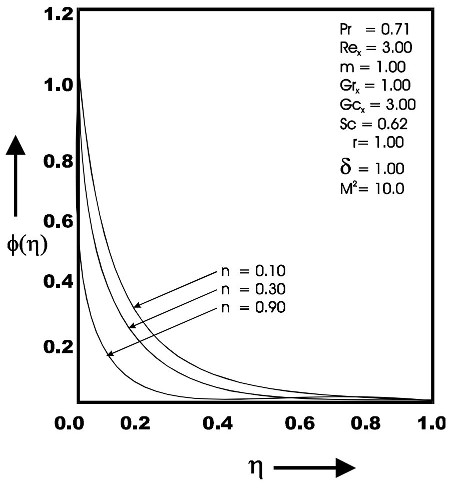

Figure 12. Effect of thermal stratification over the concentration profiles

In the case of the constant Schmidt number with a uniform magnetic field, it is observed that the increase of thermal stratification parameter accelerates the fluid motion and decelerates the temperature and concentration of the fluid along the accelerating surface and these are vivid through Figures 10, 11 and 12, respectively.

5. Conclusions

- In the presence of a uniform Schmidt number and magnetic field, the velocity decreases and the temperature and concentration of the fluid increase with an increase of the Prandtl number.
- Due to the constant thermal stratification parameter with a uniform magnetic field, the velocity and the concentration decrease and the temperature of the fluid increases with an increase of the Schmidt number.
- An increase in the strength of a magnetic field leads to a fall in the velocity and rise in the temperature and concentration of the fluid along the surface.
- The velocity increases and the temperature and concentration of the fluid decrease with increase of the thermal stratification parameter.
- In the presence of water vapour ($S_c = 0.62$) and the uniform magnetic field, it is noted that the velocity of the fluid increases and the temperature and concentration of the fluid decrease near the plate, respectively.
- In the presence of air ($P_r = 0.71$), it is interesting to note that the velocity of the fluid increases near the plate and thereafter decreases.
- The temperature and concentration of the fluid decrease at a very fast rate in the case of water ($P_r = 7.0$) in comparison to air ($P_r = 0.71$).
- In the case of a uniform thermal stratification parameter, it is noted that the velocity of the fluid increases and the temperature and concentration of the fluid decrease at a fast rate near the plate.

- In the presence of a uniform magnetic field and air ($P_r = 0.71$), the velocity of the fluid increases due to an increase in S_c showing that it decreases gradually as it is replaced by hydrogen ($S_c = 0.22$), by water vapor ($S_c = 0.62$) and ammonia ($S_c = 0.78$).
- The temperature and concentration decrease due to increase in Sc indicating that it increases as it is replaced by hydrogen ($S_c = 0.22$), by water vapor ($S_c = 0.62$) and ammonia ($S_c = 0.78$).

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