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# EFFECTS OF CHEMICAL REACTION, HEAT AND MASS TRANSFER ON NON-LINEAR LAMINAR BOUNDARY-LAYER FLOW OVER A WEDGE WITH SUCTION OR INJECTION

R. Kandasamy
Department of Mathematics, Institute of Road and Transport Technology
Erode-638 316, India
kandan_kkk@yahoo.co.in

S. P. Anjali Devi
Bharathiar University
Coimbatore-641 046, India.

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**Abstract.** An approximate numerical solution for the steady laminar boundary-layer flow over a wall of the wedge with suction or injection in the presence of species concentration and mass diffusion has been obtained by solving the governing equations using R.K. Gill method. The fluid is assumed to be a viscous and incompressible fluid. Numerical calculations up to third level of truncation are carried out for different values of dimensionless parameters and an analysis of the results obtained shows that the flow field is influenced appreciably by the chemical reaction (consumption and generation reactant) and suction or injection at the wall of the wedge.

*Keywords*: chemical reaction (consumption and generation reactant), suction or injection at the wall of the wedge, Boussinesq's approximation, steady laminar boundary-layer flow and mass diffusive

## 1. Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc.

Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. The flow of a fluid past a wedge is of fundamental importance since this type of flow constitutes a general and wide class of flows in which the free stream velocity is proportional to a power of the length coordinate measured from the stagnation point.

All industrial chemical processes are designed to transform cheaper raw materials to high value products (usually via chemical reaction). A 'reactor', in which such chemical transformations take place, has to carry out several functions like bringing reactants into intimate contact, providing an appropriate environment (temperature and concentration fields) for adequate time and allowing for removal of products. Fluid dynamics plays a pivotal role in establishing relationship between reactor hardware and reactor performance. For a specific chemistry catalyst, the reactor performance is a complex function of the underlying transport processes. The first step in any reaction engineering analysis is formulating a mathematical framework to describe the rate (and mechanisms) by which one chemical species is converted into another in the absence of any transport limitations (chemical kinetics). Once the intrinsic kinetics is available, the production rate and composition of the products can be related, in principle, to reactor volume, reactor configuration and mode of operation by solving mass, momentum and energy balances over the reactor. This is the central task of a reaction and reactor engineering activity. Analysis of the transport processes and their interaction with chemical reactions can be quite difficult and is intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Recent advances in understanding the physics of flows and computational flow modelling (CFM) can make tremendous contributions in chemical engineering.

In these types of problems, the well-known Falkner-Skan transformation is used to reduce boundary-layer equations into ordinary differential equations for similar flows [1]. It can also be used for non-similar flows for convenience in numerical work because it reduces, even if it does not eliminate, dependence on the x-coordinate. The solutions of the Falkner-Skan equations are sometimes referred to as wedge flow solutions with only two of the wedge flows being common in practice [2]. The dimensionless parameter, $m$ plays an important role in such type of problems because it denotes the shape factor of the velocity profiles. It has been shown [3] that when $m < 0$ ( increasing pressure), the velocity profiles have a point of inflexion whereas when m $> 0$ (decreasing pressure ), there is no point of inflexion. This fact is of great importance in the analysis of the stability of laminar flows with a pressure gradient. Yih [4] presented an analysis of the forced convection boundary-layer flow over a wedge with uniform suction and blowing, whereas Watanabe [5] investigated the behavior of the boundary-layer over a wedge with suction and injection in forced flow. Recently, laminar boundary layer flow over a wedge with suction/injection has been discussed by Kafoussias and Nanousis [6] and Anjali Devi and Kandasamy [7] analyzed the effects of thermal stratification on laminar boundary layer flow over a wedge with suction and injection.

Since no attempt has been made to analyze non-linear boundary-layer flow with chemical reaction, heat and mass transfer over a wedge with suction or injection at the wall in the presence of a uniform transverse magnetic field, we have investigated it in this article. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using R.K.Gill method. Numerical calculations up to third level of truncation were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. Examination of such flow models reveal the influence of chemical reaction on velocity, temperature and concentration profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of chemical reaction (generation and consumption reactant) and suction or injection at the wall of the wedge.

## 2. Mathematical analysis

Two-dimensional laminar boundary-layer flow of a viscous and Boussinesq fluid over a wall of the wedge with suction or injection is analysed. As shown in Figure 1, the x-axis is parallel to the wedge and the y-axis is taken normal to it. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of

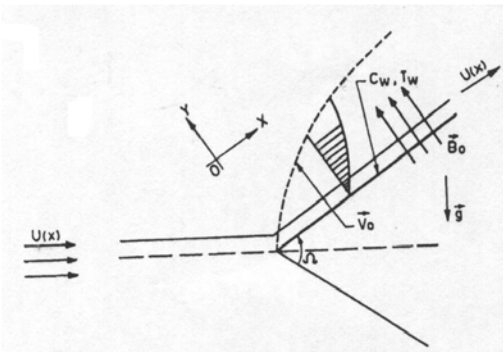


Figure 1. Flow analysis along the wall of the wedge

species far from the wall, $C_\alpha = 0$ is infinitesimally very small [8] and hence the Soret and Dufour effects are neglected. The chemical reactions take place in the flow and the physical properties $\mu$, $D$, $\rho$ and the rate of chemical reaction, $k_1$ are constant throughout the fluid. Under these conditions, the governing boundary layer equations of momentum, energy and diffusion for free convection flow under Boussinesq's approximation are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial^2 x} + U\frac{\partial U}{\partial X} + g\beta(T - T_\infty)\sin\frac{\Omega}{2} + g\beta^*(C - C_\infty)\sin\frac{\Omega}{2} \,, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \,, \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - k_1 C \,. \tag{4}$$

The boundary conditions are

$$\begin{array}{llll} u = 0\,, \quad v = v_o\,, & C = C_w\,, & T = T_w & \text{at } y = 0 \\ u = U(x)\,, & C = C_\alpha\,, & T = T_\alpha\,, & \text{at } y > \alpha \text{ by } y \to \alpha\,. \end{array} \tag{5}$$

As in [6], we introduce the following change of variables

$$\Psi(x,y) = \left(\frac{2U\nu x}{1+m}\right)^{1/2} f(x,\eta)\,, \tag{6}$$

$$\eta(x,y) = y\left[\frac{(1+m)U}{2\nu x}\right]^{1/2} \,. \tag{7}$$

Under this consideration, the potential flow velocity can be written [6] as

$$U(x) = cx^m\,, \qquad \beta_1 = \frac{2m}{1+m}\,, \tag{8}$$

where $c$ is a constant and $\beta_1$ is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \Omega/\Pi$ for a total angle $\Omega$ of the wedge.

The velocity components are given by

$$u = \frac{\partial \Psi}{\partial y}\,, \qquad v = -\frac{\partial \Psi}{\partial x} \tag{9}$$

It can be easily verified that the continuity equation (1) is identically satisfied. If we introduce the non-dimensional form of temperature and the concentration as

$$\theta = \frac{T - T_\alpha}{T_w - T_\alpha}\,, \tag{10}$$

$$\phi = \frac{C - C_\alpha}{C_w - C_\alpha}\,, \tag{11}$$

$$R_{e_x} = \frac{x}{\nu} \quad \text{(Reynolds number)}\,, \tag{12}$$

$$G_r = \nu g \beta \frac{T_w - T_\alpha}{U^3} \quad \text{(Grashof number)}\,, \tag{13}$$

$$G_c = \nu g \beta^* \frac{C_w - C_\alpha}{U^3} \quad \text{(Modified Grashof number)}\,, \tag{14}$$

$$P_r = \mu \frac{c_\rho}{k} \quad \text{(Prandtl number)}\,, \tag{15}$$

$$S_c = \frac{\nu}{D} \quad \text{(Schmidt number)}\,, \tag{16}$$

$$S = -v_0 \left( \frac{(1+m)x}{2\nu U} \right)^{1/2} \quad \text{(suction or injection parameter)}\,, \tag{17}$$

$$\gamma = \nu \frac{k_1}{U^2} \quad \text{(chemical reaction parameter)}\,. \tag{18}$$

Now equations (2) to (4) become

$$\frac{\partial^3 f}{\partial \eta^3} + f \frac{\partial^2 f}{\partial \eta^2} + \frac{2m}{1+m} \left[ 1 - \left( \frac{\partial^2 f}{\partial \eta^2} \right)^2 \right] + \frac{2m}{1+m} (G_c R_{e_x} \phi + G_r R_{e_x} \theta) \sin(\Omega/2) = = \frac{2x}{1+m} \left( \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial x \partial \eta} - \frac{\partial f}{\partial x} \frac{\partial^2 f}{\partial \eta^2} \right)\,, \tag{19}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + P_r f \frac{\partial \theta}{\partial \eta} = \frac{2P_r}{1+m} \theta \frac{\partial f}{\partial \eta} + P_r \frac{2x}{1+m} \left( \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial \eta} \right)\,, \tag{20}$$

$$\frac{\partial^2 \phi}{\partial \eta^2} + S_c f \frac{\partial \phi}{\partial \eta} - \frac{2S_c}{1+m} R_{e_x} \gamma \phi = \frac{2S_c}{1+m} \phi \frac{\partial f}{\partial \eta} + + 2x \frac{S_c}{1+m} \left( \frac{\partial f}{\partial \eta} \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial \eta} \frac{\partial f}{\partial x} \right)\,. \tag{21}$$

The boundary condition (5) can be written as

$$\eta = 0\,, \quad \frac{\partial f}{\partial \eta} = 0\,, \quad \frac{f}{2} \left( 1 + \frac{x}{U} \frac{\mathrm{d}U}{\mathrm{d}x} \right) + x \frac{\partial f}{\partial \xi} = -v_o \left( \frac{(1+m)x}{2vU} \right)^{1/2}\,, \quad \theta = 1\,, \quad \phi = 1$$
$$\eta > \alpha\,, \quad \frac{\partial f}{\partial \eta} = 1\,, \quad \theta = 0\,, \quad \phi = 0\,. \tag{22}$$

Equations (19) to (21) and the boundary condition (22) can be written as

$$\frac{\partial^3 f}{\partial \eta^3} + \left( f + \frac{1-m}{1+m} \xi \frac{\partial f}{\partial \xi} \right) \frac{\partial^2 f}{\partial \eta^2} - \frac{1-m}{1+m} \xi \frac{\partial^2 f}{\partial \xi \partial \eta} + + \frac{2m}{1+m} \left( 1 - \frac{\partial^2 f}{\partial \eta^2} \right) + \frac{2}{1+m} (G_c R_{e_x} \phi + G_r R_{e_x} \theta) \sin(\Omega/2) = 0\,, \tag{23}$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + P_r \left( f + \frac{1-m}{1+m} \xi \frac{\partial f}{\partial \xi} \right) \frac{\partial \theta}{\partial \eta} - \left( \frac{2P_r}{1+m} \theta \frac{\partial f}{\partial \eta} - \frac{1-m}{1+m} \xi \frac{\partial \theta}{\partial \xi} \right) \frac{\partial f}{\partial \eta} = 0\,, \tag{24}$$

$$\frac{\partial^2\phi}{\partial\eta^2}+S_cf\frac{\partial\phi}{\partial\eta}-\frac{2Sc}{1+m}R_{e_x}\gamma\Phi+S_c\frac{1-m}{1+m}\left(\frac{\partial\phi}{\partial\eta}\xi\frac{\partial f}{\partial\xi}-\frac{\partial f}{\partial\eta}\xi\frac{\partial\phi}{\partial\xi}\right)-$$
$$-\frac{2S_c}{1+m}\xi\frac{\partial f}{\partial\xi}=0\,, \quad (25)$$

$$\frac{\partial f}{\partial\eta}=0\,,\quad (1+m)\frac{f}{2}+\frac{1-m}{2}\xi\frac{\partial f}{\partial\xi}=S\,,\quad \theta=1\,,\quad \phi=1 \quad \text{at } \eta=0$$
$$\frac{\partial f}{\partial\eta}=1\,,\qquad \theta=0\,,\qquad \phi=0\,,\qquad \text{at } y>\alpha \text{ by } y\to\alpha\,, \quad (26)$$

where $S$ is the suction if $S>0$ and injection if $S<0$ and $\xi=kx^{(1-m)/2}$ is the dimensionless distance along the wedge ($\xi>0$). In this system of equations $f(\xi,\eta)$ is the dimensionless stream function; $\theta(\xi,\eta)$ is the dimensionless temperature; $\Phi(\xi,\eta)$ is the dimensionless concentration; $P_r$ is Prandtl number, $R_{e_x}$ is Reynolds number etc. which are defined by equations (9) to (18). The parameter $\xi$ indicates the dimensionless distance along the wedge ($\xi>0$). It is obvious that to retain the $x$-derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream-wise location through the $\xi$-derivatives, a locally autonomous solution, at any given stream-wise location cannot be obtained. In such a case, an implicit marching numerical solution scheme is usually applied proceeding the solution in the $\xi$-direction, i.e., calculating unknown profiles at $\xi_{i+1}$ when the same profiles at $\xi_i$ are known. The process starts at $\xi_i=0$ and the solution proceeds from $\xi_i$ to $\xi_{i+1}$ but such a procedure is time-consuming. However, when the terms involving $\partial f/\partial\xi$ , $\partial\theta/\partial\xi$ and $\partial f/\partial\xi$ and their $\eta$ derivatives are deleted, the resulting system of equations resembles, in effect, a system of ordinary differential equations for the functions $f$, $\theta$ and $\phi$ with $\xi$ as a parameter and the computational task is simplified. Furthermore a locally autonomous solution for any given $\xi$ can be obtained because the stream-wise coupling is severed. So, following the lines of [6], a recent numerical solution scheme is utilized for obtaining the solution of the problem. Now, due to the above factors, equations (23) to (25) are changed to

$$f'''+ff''+\frac{2m}{1+m}\left[1-(f')^2\right]+\frac{2}{1+m}(G_cR_{e_x}\phi+G_rR_{e_x}\theta)\sin(\Omega/2)=0\,, \quad (27)$$

$$\theta''+P_rf\theta'-\frac{2P_r}{1+m}f'\theta=0\,, \quad (28)$$

$$\phi''+S_cf\phi'-\frac{2S_c}{1+m}f'\phi-\frac{2S_c}{1+m}R_{e_x}\gamma\phi=0 \quad (29)$$

with boundary conditions

$$f(0)=\frac{2}{1+m}S\,,\quad f'(0)=0\,,\quad \theta(0)=1\,,\quad \phi(0)=1 \quad \text{at } \eta=0$$
$$f'(\alpha)=1\,,\quad \theta(\alpha)=0\,,\quad \phi(\alpha)=0\,,\quad \text{at } y>\alpha \text{ by } y\to\alpha\,. \quad (30)$$

Equations (27) to (29) with boundary conditions (30) are integrated using R .K Gill method. Chemical reaction, heat and mass transfer are studied for different values of suction/injection at the wall of the wedge and the strength of the applied magnetic field. In the following section, we discuss the results in detail.

## 3. **Results and discussion**

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations.

In the absence of mass transfer and magnetic effects, the results have been compared with that of the previous work [6] and it is found that they are in good agreement. The numerical results obtained are illustrated by means of Figures 2–7.

Effects due to the suction or injection with uniform chemical reaction at the wall of the wedge over the velocity, temperature and concentration are shown through Figures 2, 3 and 4.

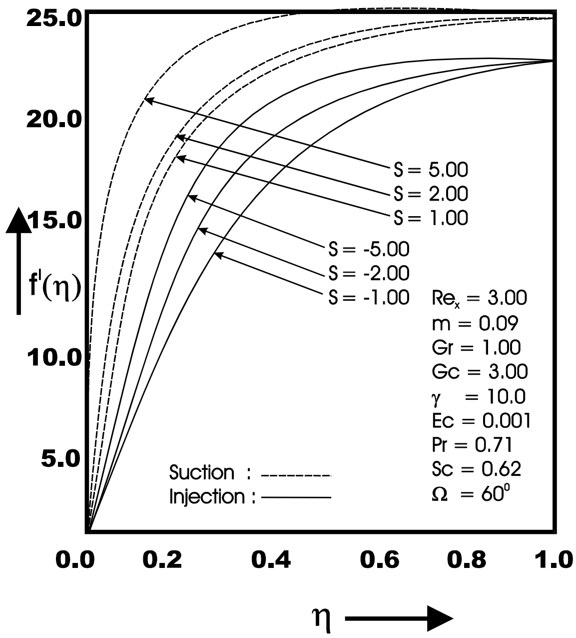


Figure 2. Velocity profiles for different values of suction/injection

Figure 2 depicts the dimensionless velocity profiles $f'(\eta)$ for different values of suction parameter ($S > 0$) and injection parameter ($S < 0$), respectively. It is observed that the velocity component of the fluid along the wall of the wedge increases with increase of suction and decreases with increase of injection at the wall of the wedge. On the contrary, the dimensionless temperature $\theta(\eta)$ and concentration $\phi(\eta)$ of the fluid reduce with increase of suction and increase with increase of injection and these are shown in Figures 3 and 4, respectively. So, the increase of suction accelerates the fluid motion and decreases the temperature distribution and concentration of the fluid along the wall of the wedge. On the other hand, the increase of injection decelerates the fluid motion and increases the temperature distribution and concentration of the

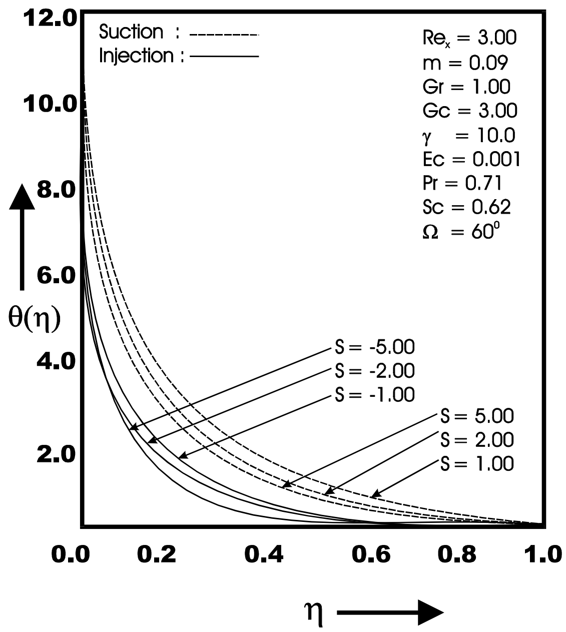


Figure 3. Temperature profiles for different values of suction/injection

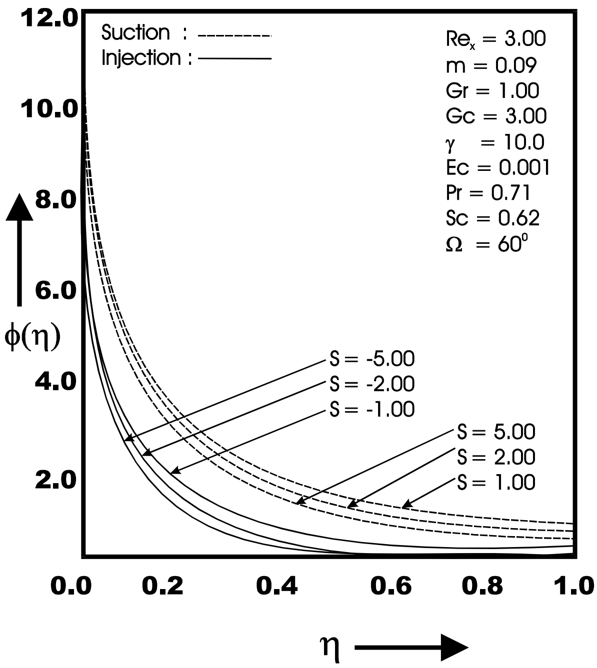


Figure 4. Concentration profiles for different values of suction/injection

fluid along the wall of the wedge. All this physical behavior is due to the combined effects of magnetic field, suction or injection and chemical reaction.

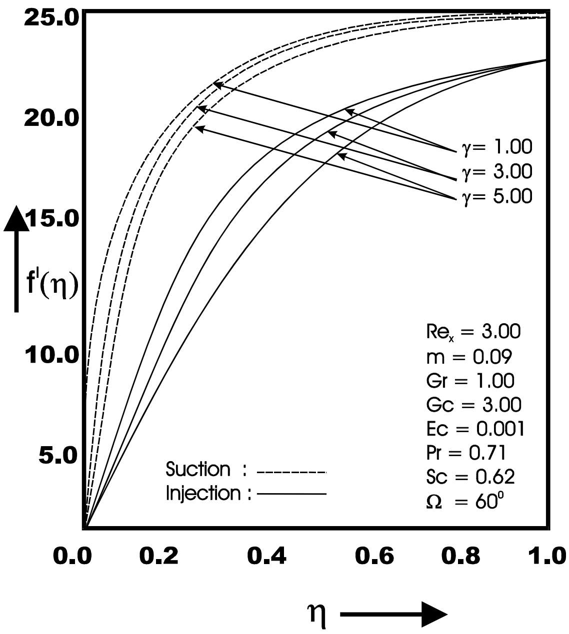


Figure 5. Influence of chemical reaction over the velocity profiles

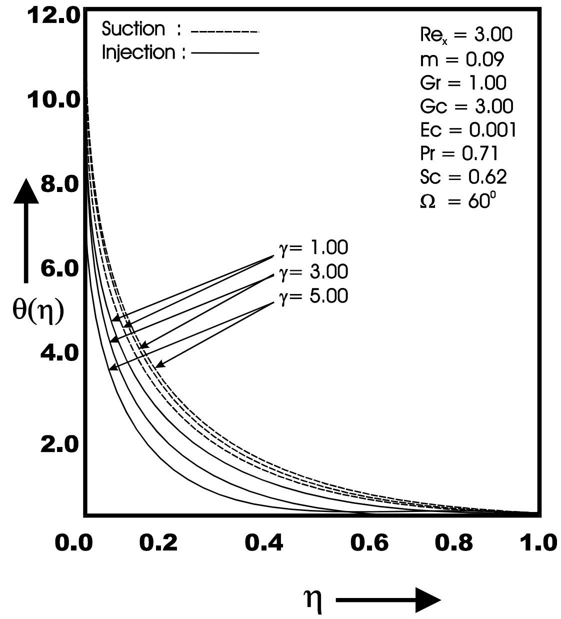


Figure 6. Effects of chemical reaction over the temperature profiles

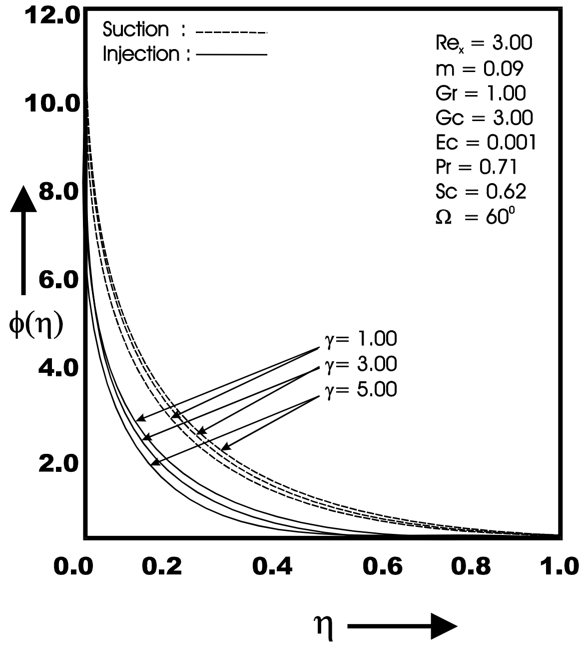


Figure 7. Effects of chemical reaction over the concentration profiles

The effects of chemical reaction over velocity, temperature and concentration of the fluid along the wall of the wedge are shown through Figures 5, 6 and 7.

## 4. Conclusion

We conclude the following from the above results and discussions:

- In the presence of uniform chemical reaction, the fluid flow along the wall of the wedge accelerates with increase of suction and decelerates with increase of injection. On the other hand, the temperature and concentration of the fluid reduce with increase of suction and increase with increase of injection of the fluid along the wall of the wedge. All these facts clearly depict the combined effects of chemical reaction and suction/injection.
- Due to the uniform magnetic field, in the case of suction, the increase of chemical reaction decelerates the fluid motion, temperature distribution and concentration of the fluid along the wall and for injection, it accelerates the fluid motion, temperature distribution and concentration of the fluid along the wall of the wedge, which affects the consumption reactions of the chemical reaction parameter.

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