Journal of Computational and Applied Mechanics, Vol. 5., No. 1., (2004), pp. 89–102

QUASI–EVOLUTIONARY POLYOPTIMIZATION OF SPATIAL TRUSSES

Witold M. Paczkowski, Adrian Silicki
Department of Theory of Structure, Technical University of Szczecin
Szczecin, Poland
optim@ps.pl

Stefan Jendo
Institute of Fundamental Technological Research, Polish Academy of Sciences
Warsaw, Poland
sjendo@ippt.gov.pl

[Received: March 17, 2003]

Dedicated to Professor József FARKAS on the occasion of his seventy-fifth birthday

Abstract. In the paper the discrete quasi–evolutionary polyoptimization process is presented and illustrated with the optimization of an orthogonal double–layer spatial truss. The problem is solved in six cycles of evolution. During the analysis, values of the most important design variables connected with the structure of the object are obtained. As the result of the analysis, the cover that satisfies in the best way all considered requirements is obtained.

Mathematical Subject Classification: 65M06, 35L65, 35L67
Keywords: quasi–evolutionary optimization, polyoptimization, spatial trusses

1. Introduction

A modern large–scale object must satisfy many requirements, such as load bearing capacity and serviceability conditions and requirements of the investor and user. The structure has to be light, functional, friendly to the natural environment, and easy to maintain. All the requirements have influence on the economic aspect of the investment, like costs of erection, exploitation and utilization. Another important requirement is the attractive architecture form. The shape, dimensions and the general appearance of the structure should make an appropriate impression. In the first stage of the process of designing, architect's, user's, engineer's and investor's ideas of the structure come together [1, 2]. Those ideas are most frequently contradictory and the final concept of the structure must be a compromise that is sometimes very difficult to achieve. In such a situation optimization becomes an indispensable tool [3–6]. But in the case of complex problems some difficulties appear. Due to the necessity to consider many decision variables, the feasible range of solutions is quite numerous. Also, a considerable number of constraints must be taken into account. The choice of the best solution should be based on several criteria to make it satisfy all significant conditions and requirements in the best way. Solving that sort of problems only with methods based on a traditional approach to optimization may cause some difficulties. A quasi–evolutionary approach seems to be more effective.

2. Qasi–evolutionary polyoptimization

The quasi–evolutionary approach to solving optimization problems comes from observations of the outer world. In nature as well as in the history of human civilization one can notice continuous development, with trends to better and more complex forms. During the evolution, solutions that are difficult, poor or less functional are eliminated by stronger, lighter and more suitable ones for the purpose they are designed for. When new solutions are created, knowledge and experience from former attempts are applied. A similar approach may be employed when optimization problem is considered. It is useful, especially in the case of complex objects.

The quasi-evolutionary optimization process consists of few cycles, each of them being a standard numerical problem. The term 'quasi-evolutionary' is used here to avoid mistaking it for the traditional evolutionary optimization procedure in which genetic algorithms are required. In the quasi-evolutionary formulation, each solution gives information and experience that allow one to form better assumptions, select more effective methods of solution and help to exclude any insignificant elements of the solution. On the basis of the former results the problem is analyzed again in the next cycle. In this context, the quasi–evolutionary approach is similar to evolution in the world of nature, where only better adapted forms give start to a next generation [6].

3. Problem statement

The paper presents the process of a polyoptimal design of a cover of a sports hall. Dimensions of the space to cover are: width – 60 m, length – 105 m, height – 8 m. The only input requirement is that the cover should be realized as a two–layer spatial truss made of steel tubes. Values of all remaining variables describing the structure are to be obtained as results of the polyoptimization analysis. Since the object of the analysis is rather complex, solving the problem formulated in a traditional way may cause some difficulties. They can come from the fact that many decision variables must be considered, which leads to a considerable domain of feasible solutions. In process of design of the structure many requirements and conditions must be satisfied. It leads to a considerable number of constraints and criteria. In order to obtain the solution in a reasonable period of time, application of the quasi–evolutionary approach to the polyoptimization is highly recommended.

4. Solution of the problem

4.1. **General algorithm.** The problem is solved in six evolution cycles. Each cycle is a standard polyoptimization problem. In most cases the enumeration method is employed to analyze the problem. The objective result is a set of nondominated evaluations and nondominated solutions. From these sets a preferred evaluation and its inverse image – a preferred solution are selected. The preferred solution from the former cycle is the starting point for the next one. The starting point of the first cycle is chosen arbitrarily. In this cycle the polyoptimal catalogue is searched. In the second one, shape of the cover and the manner of support are analyzed. The third cycle concerns the rise of the truss, the fourth one – the depth of the truss and the distance between nodes. The next one chooses the grade of steel and the last one – selects catalogue again. In this way, the complex problem is divided into several relatively simple problems coordinated naturally by the idea of evolution.

4.2. **The OPTYTRUSS system.** Numerical analysis of the structure is performed by the OPTYTRUSS system. The system enables computation of internal forces in the truss bars on the basis of the matrix displacement method, the choice of the appropriate profile for each bar, and optimization analysis of different variants of the structure. The profiles are selected from the catalogue provided. Such an approach makes the problem a discrete one, because the space of solutions contains a finite number of variants. Since every analyzed variant of structure is built of profiles that are commercially available this way of optimum design is technologically correct.

In the system all loadings typical of the covers may be applied, i.e. deadweight, snow load, wind load and loads arising from mounted installations or devices. The loads are realized by several simple types of loads: uniformly distributed load, trapezoidal load, triangular load and concentrated load. The loads may be imposed in the upper or lower layer. There are two means of imposing the distributed loads: they can be vertical or normal to the surface of the cover. The concentrated load is expressed by its three components, so the force may be imposed in any direction. Every load is reduced to a nodal load.

There are four types of profiles available: circular tube, circular, square tube and square (Figure 1). For every type a catalogue with cross–sections of different sizes may be applied.

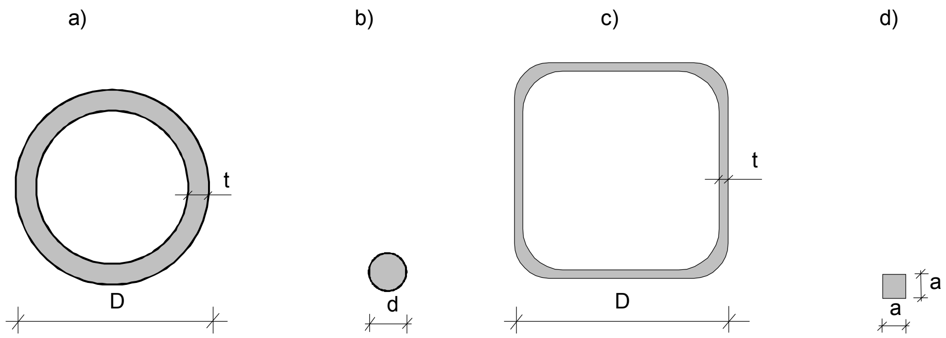

Figure 1. Types of profiles available in the OPTYTRUSS system

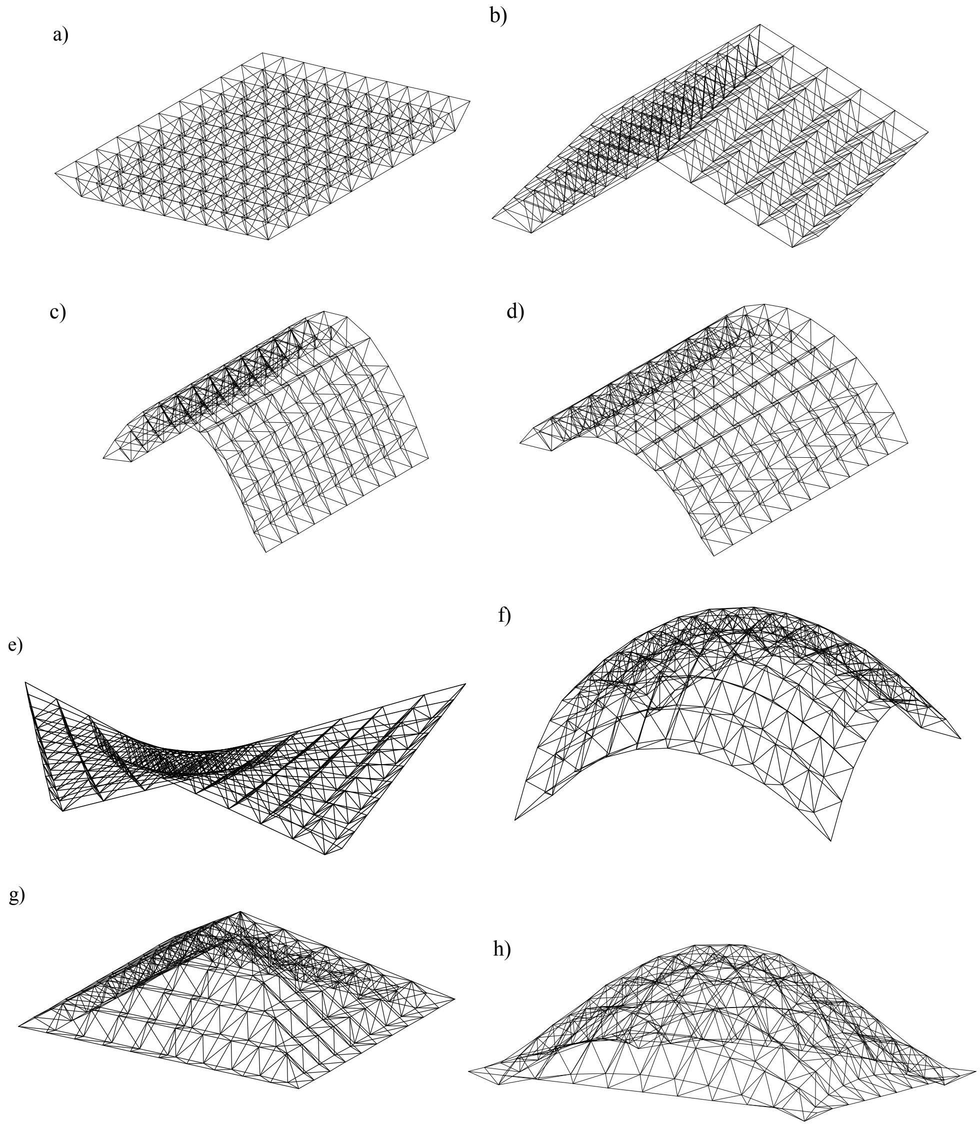

Figure 2. Analyzed shapes of the cover: a) plane, b) double slope, c) a parabolic arc, d) circular arc, e) hyperbolic paraboloid, f) dome–shaped with arc edges, g) four slope, h) dome–shaped with flat edges

The OPTYTRUSS system facilitates analysis of the two–layer spatial trusses based on the orthogonal grid of nodes in both layers. There are eightshapes of analyzed covers available: plane, double slope, parabolic arc, circular arc, hyperbolic paraboloid, dome–shaped with arc edges, four slope, and dome–shaped with flat edges (Figure

2). Six objective functions are computed for every solution: the volume of material of truss members per one square meter of projection of the cover, the strain energy per one square meter of projection of the cover, the greatest displacement, average use of cross–sections and two objective functions that may be created by the user. There are three methods of optimization built–in: exhaustive search, crude Monte Carlo and modified Monte Carlo. The system computes values of objective functions for every analyzed solution, therefore other methods may also be used, but this cannot be performed automatically. In order to choose the preferred evaluation the distance method is applied. As a reference point the ideal evaluation or the Nadir evaluation may be used.

4.3. **The first evolution cycle.** The starting point in the first evolution cycle is the plate spatial truss with an orthogonal grid of nodes and a depth of 3 m. The distance between nodes is 3.75 m. The truss is made of steel with the yield stress equal to 225 MPa. The cover is supported in the upper layer by posts placed on the external longer edges of the truss. The distance between the supports is 15 m.

The first evolution cycle considers the catalogue of steel profiles. At the start, a catalogue containing 72 elements is assumed. This catalogue is a representative sample of the metallurgical assortment available in Poland. On the basis of the starting catalogue, sixteen other catalogues are created. They are formed with regard to the frequency of choice of particular profiles by the designing system. The initial catalogue is used for designing the analyzed structure. Then the elements that are not chosen or are rarely chosen by the designing system are discarded from the initial catalogue. In that way the second catalogue (second solution analyzed in this evolution cycle) is obtained. It is applied in the system during designing the analyzed structure and the process is repeated. It is stopped when the one–element catalogue is obtained. In this way the discrete 17–element domain of feasible solutions is obtained.

$$\mathbf{x}_{(l)} = \left\{x_{i(l)}\right\} \qquad i = 1, \ldots, 17 \tag{4.1}$$

where the number in brackets in subscript denotes the number of the evolution cycle.

Definition of the vector of constraints is not required in this cycle, because all created catalogues facilitate designing a structure correctly. Each solution is evaluated with regard to three criteria of optimization that are expressed formally as the vector of objective functions

$$f_{(l)}\left(\mathbf{x}_{(l)}\right) = \left\{f_{1(l)}\left(\mathbf{x}_{(l)}\right), f_{2(l)}\left(\mathbf{x}_{(l)}\right), f_{3(l)}\left(\mathbf{x}_{(l)}\right)\right\} \tag{4.2}$$

where

$f_{1(l)}\left(\mathbf{x}_{(l)}\right)$ – mass of the structure per one square meter of projection of the cover (in kg/m^2),
$f_{2(l)}\left(\mathbf{x}_{(l)}\right)$ – the greatest displacement (in cm),
$f_{3(l)}\left(\mathbf{x}_{(l)}\right)$ – the number of profiles in the catalogue (as the technological criterion).

All the functions are minimized. The diagram of the objective functions is presented in Figure 3.

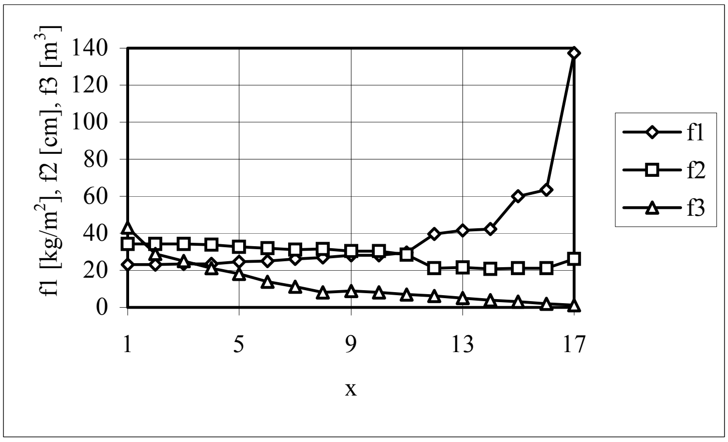

Figure 3. Diagram of the objective functions

From the set of evaluations the nondominated ones are selected and normalized. Then the preferred evaluation is chosen with the use of distance function method with the norm $||p|| = 2$ [6]. The inverse image of the preferred evaluation is the preferred solution that determines the catalogue consisting of four elements (Table 1).

Table 1. The preferred catalogue

Number of element	Diameter[mm]	Thickness of the wall[mm]
1	51.0	2.9
2	88.9	3.6
3	159.0	4.5
4	323.9	8.0

The values of the objective functions for the preferred solution form the vector

$$\mathbf{f}p_{(l)} = \{42.17, 20.96, 4\} \ . \tag{4.3}$$

During the analysis it has been noticed that from profiles with the same diameter those with the thinnest walls are most frequently chosen by the designing system. It is due to the smaller slenderness ratio of bars made of such profiles (when bars with the same cross–sectional area are considered).

4.4. **The second evolution cycle.** In the second evolution cycle, the polyoptimal manner of support and shape of the cover are selected. Four shapes are considered: plane, double slope, parabolic arc, and circular arc (Fig. 2), and three manners of support: one–point supports at the external edge of the upper or lower layer and four–point supports in the lower layer (Fig. 4). Every shape (excluding the planar one) is analyzed with three different rises of 3, 6 and 9 m in order to establish its approximate value.

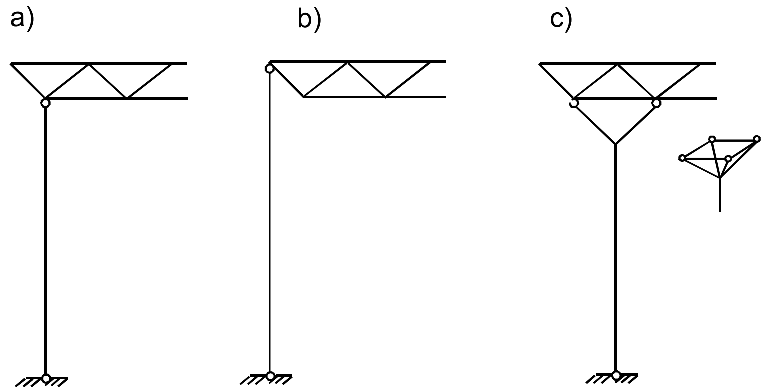

Figure 4. Analyzed manners of support: a) one–point support in the lower layer, b) one–point support in the upper layer, c) four–point support

The analyzed variants of the cover are represented by the vector of decision variables $\mathbf{x}_{(2)}$:

$$\mathbf{x}_{(2)} = \{x_{1(2)}, x_{2(2)}\}, \tag{4.4}$$

where:

$x_{1(2)}$ – shape of the cover,
$x_{2(2)}$ – manner of support.

Three criteria of evaluation are assumed: minimum of the mass of the structure per one square meter of projection, minimum of the greatest displacement and minimum of increase of the space volume arising from changing the shape of the cover (in reference to the volume of the starting point structure). Minimizing the volume is important because of maintenance costs (space that needs heating in the winter). The vector of objective functions is presented below:

$$\mathbf{f}_{(2)}\left(\mathbf{x}_{(2)}\right) = \left\{f_{1(2)}\left(\mathbf{x}_{(2)}\right), f_{2(2)}\left(\mathbf{x}_{(2)}\right), f_{3(2)}\left(\mathbf{x}_{(2)}\right)\right\} \tag{4.5}$$

where

$f_{1(2)} = f_{1(1)}$and $f_{2(2)} = f_{2(1)}$,
$f_{3(2)}(\boldsymbol{x}_{(2)})$ – increase in volume in reference to the volume of the starting point structure (in m^3).

In order to solve the problem, the exhaustive search method is employed. The feasible domain contains 30 elements. The computations are performed with the use of the catalogue obtained as a result of the former cycle. Diagrams of the 3D–space of evaluations are presented in Figure 5.

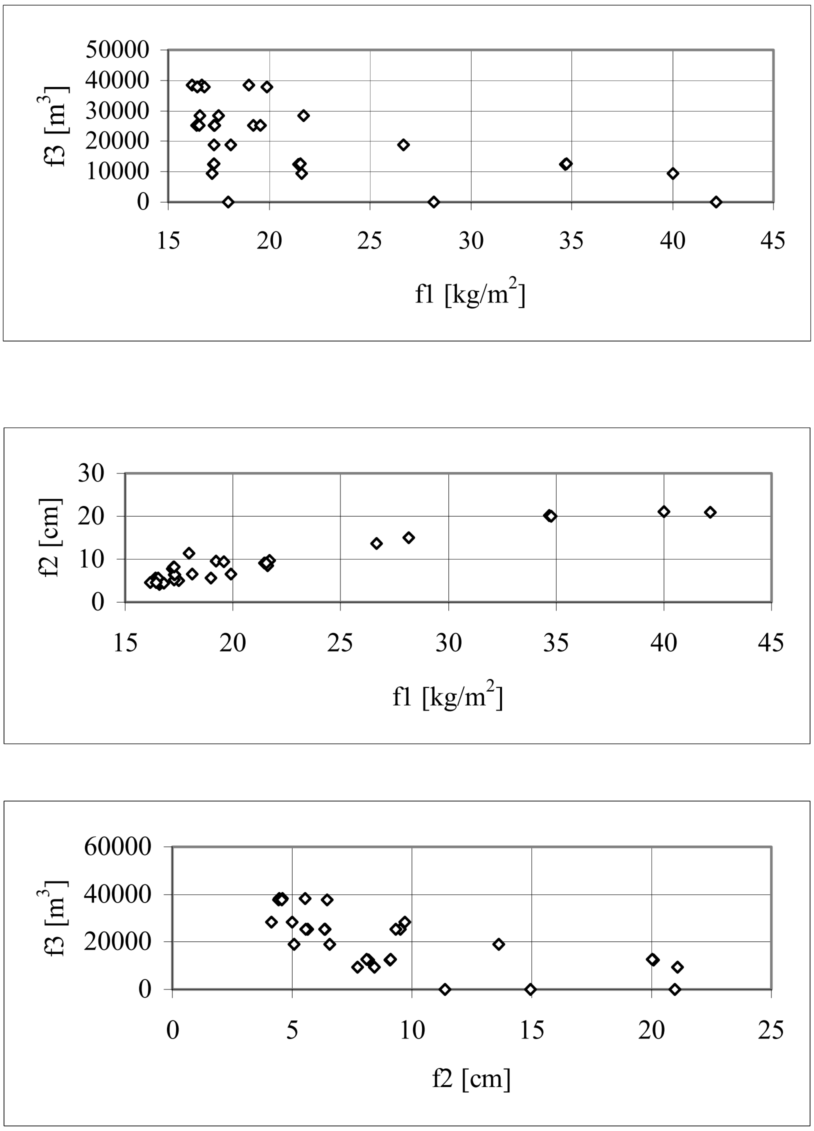

Figure 5. Diagrams of the 3D space of evaluations

After the polyoptimization analysis, the preferred solution is obtained: – a double slope cover with a rise of 3 m, supported in the lower layer by the four–point supports. The vector of objective functions for the preferred solution is presented below:

$$\mathbf{f}p_{(2)} = \{17.18, 7.72, 9450\} \ . \tag{4.6}$$

A fragment of the obtained structure is presented in Figure 6.

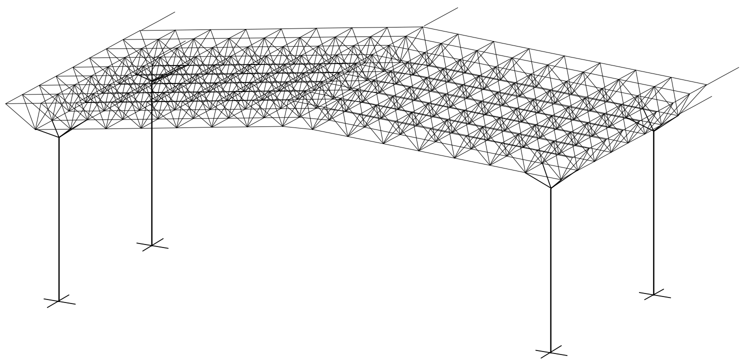

Figure 6. A fragment of the structure being the result of the second evolution cycle

4.5. **The third evolution cycle.** In this cycle, only one decision variable is analyzed: the rise of the cover in order to establish its precise value. The vector of decision variables is therefore a one–element one:

$$\mathbf{x}_{(3)} = \left\{ x_{l(3)} \right\} \ , \tag{4.7}$$

where $x_{l(3)}$ is the rise. The decision variable is discretized and some constraints are imposed on it:

$$x_{1(3)} = n \cdot 0.6m, \qquad n = 2, 3, ..., 8 \ . \tag{4.8}$$

The vector of objective functions is the same as in the former cycle. The diagram of the objective functions is presented in Figure 7.

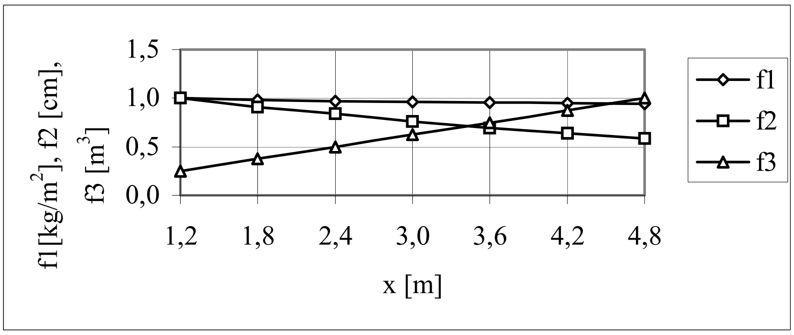

Figure 7. The diagram of objective functions

After the polyoptimization analysis the preferred solution is obtained:

$$\mathbf{x}_{p(3)} = \{1.8\} \ . \tag{4.9}$$

It means that a rise of 1.8 m satisfies the assumed criteria in the best way. The values of objective functions are as follows:

$$\mathbf{f}_{P(3)} = \{17.57, 9.20, 5670\} \ . \tag{4.10}$$

4.6. **The fourth evolution cycle.** The fourth cycle concerns the depth of the truss and the distance between nodes. A two–element vector of decision variables is assumed: The fourth cycle concerns the depth of the truss and the distance between nodes.

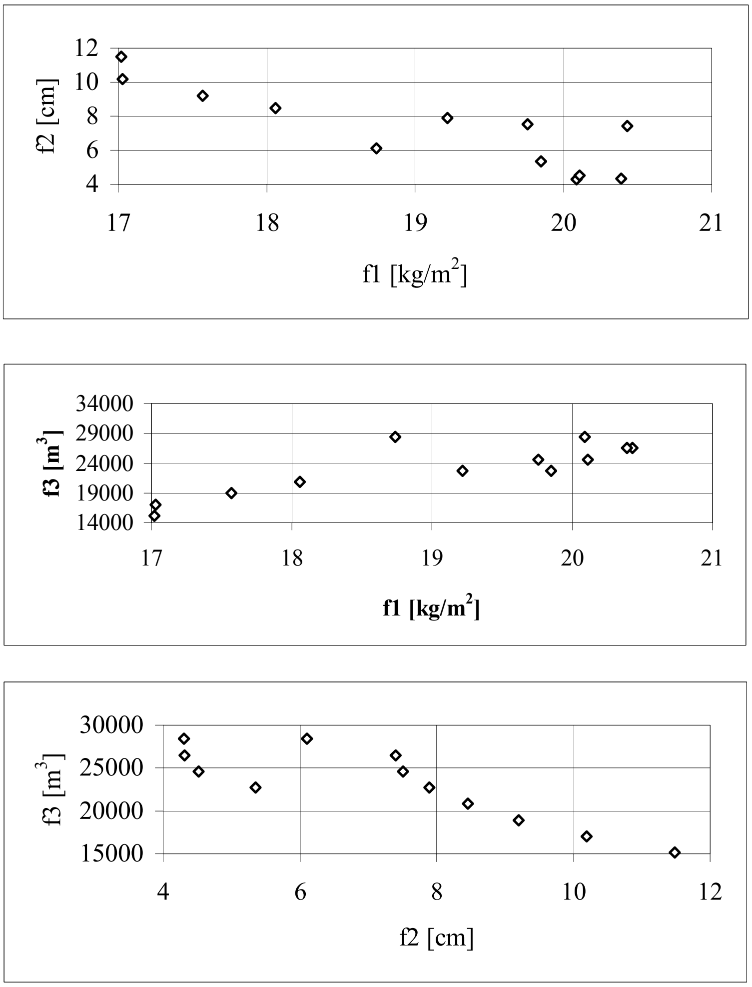

Figure 8. Diagrams of the 3D space of evaluations

A two–element vector of decision variables is assumed:

$$\mathbf{x}_{(4)} = \left\{x_{1(4)}, x_{2(4)}, \right\} \tag{4.11}$$

where

$x_{1(4)}$ – depth of the truss (in m)

$$x_{1(4)} = n \times 0.3m, \qquad n = 8, 9, ..., 15 \tag{4.12}$$

$x_{2(4)}$ – distance between nodes (in m)

$$x_{2(4)} = 3.00, 3.75, 5.00\ m\ . \tag{4.13}$$

In order to make a solution technologically correct, angles between a cross brace and the layer of the truss must be included between 30^o and 60^o. This fact is one of the constraints of the feasible domain.

The objective functions are analogous to those from the former cycle. The only difference is that the considered increase of the volume arises here from changing the value of the truss depth. In order to solve the problem, the Gauss–Seidel method is used. Diagrams of the 3D space of evaluations are presented in Figure 8.

As a result of the analysis, the 12–element set of nondominated evaluations and the set of nondominated solutions are obtained. The preferred solution is chosen by the use of the distance method:

$$\mathbf{x}_{p(4)} = \{3.6, 5.0\}\ . \tag{4.14}$$

The preferred evaluation is:

$$\mathbf{f}p_{(4)} = \{19.85; 5.35; 22, 720\}\ . \tag{4.15}$$

4.7. **The fifth evolution cycle.** The aim of this cycle is the choice of the grade of steel. Two grades of steel are considered, with the yield stress of the first equal to 225 MPa and that of the second one 210 MPa. Thus, only one decision variable is considered here:

$$\mathbf{x}_{(5)} = \left\{x_{1(5)}\right\} \tag{4.16}$$

where

$$x_{1(5)} = 210, 225\ MPa\ . \tag{4.17}$$

The problem is analyzed with regard to two criteria: minimum of the structure mass and minimum of the greatest displacement, defined in the vector of objective functions

$$\mathbf{f}_{(5)}\left(\mathbf{x}_{(5)}\right) = \left\{f_{1(5)}\left(\mathbf{x}_{(5)}\right), f_{2(5)}\left(\mathbf{x}_{(5)}\right)\right\}\ , \tag{4.18}$$

where $f_{1(5)} = f_{1(4)}$ and $f_{2(5)} = f_{2(4)}$.

The evaluations of the solutions analyzed are contained in Table 2.

Table 2. Evaluations of the solutions analyzed

Yield stress [MPa]	Structure mass [kg/m^2]	Greatest displacement [cm]
210	20.02	4.92
225	19.85	5.35

The preferred solution is chosen on the basis of the results discussion. Using steel with the lower yield stress leads to an increase of structure mass of 0.9 % and a decrease of the greatest displacement of 8.0 %. Therefore, the preferred solution is:

$$\mathbf{x}_{p(5)} = \{210\} \tag{4.19}$$

and the preferred evaluation:

$$\mathbf{f}p_{(5)} = \{18.70, 6.07\} \ . \tag{4.20}$$

4.8. **The sixth evolution cycle.** In this last cycle the catalogue is considered again. 24 three–, four– and five–element catalogues, similar to the one obtained in the first cycle, are analyzed with the exhaustive search method. The criteria of evaluation are analogous to the ones from the former cycle. After the polyoptimization analysis the preferred catalogue is established (Tab. 3). The catalogues obtained in the first (T1) and discussed sixth cycle (T2) are presented in Figure 9.

Table 3. The preferred catalogue

Number of element	Diameter [mm]	Thickness of the wall [mm]
1	54.0	2.9
2	101.6	3.6
3	168.3	5.0
4	273.0	7.1

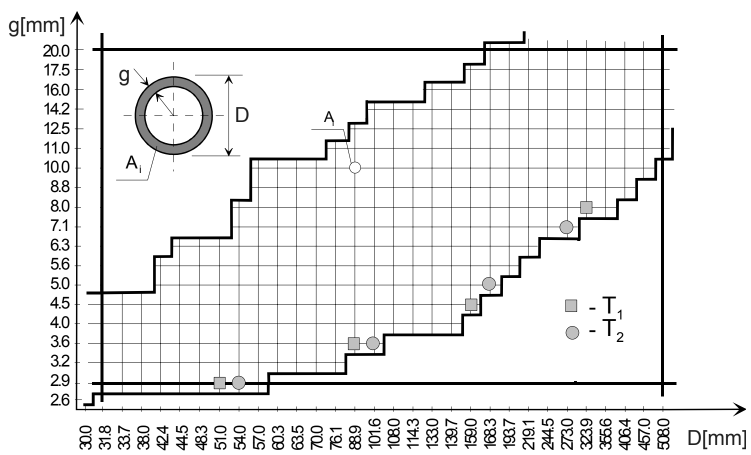

Figure 9. The catalogues obtained in the first and sixth cycles of evolution

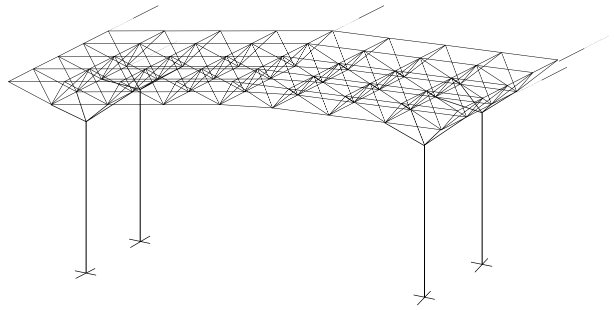

Figure 10. A fragment of the structure being the result of quasi–evolutionary polyoptimization

The preferred evaluation is:

$$\mathbf{f}p_{(6)} = \{17.00, 5.33\} \ . \qquad (4.21)$$

5. **Conclusions**

The quasi–evolutionary approach to optimization problems leads to better results than the ones obtained with a traditional analysis. In case of complex problems, it is possible to divide them into several simple ones. The numerical example shows that the proposed method is efficient enough to analyze large–scale truss systems (Figure 11). The structure that fulfils the assumed criteria best is a double–sloped truss supported by four–point supports with a rise of 1.8 m, a depth of 3.6 m and a distance between nodes 5.0 m, made of steel with a yield stress 210 MPa and four kinds of profiles. The fact that every next cycle is based on the information obtained in the former ones leads to permanent development of the problem formulation. It also facilitates analysis from general issues to details and may be easily applied in engineering practice.

REFERENCES

1. Makowski, Z. S.: *Space structures of today and tomorrow: A brief review of their present development and a glance into their future use*, In H. Nooshin (ed.), *Space structures;* Proc. of the Third Int. Conf., University of Surrey, Guildford, UK, 11–14 Sept. 1984, London – New York, Elsevier Applied Science Publisher, (1984), 1–8.
2. Rêbialak, J.: *Space Structures of Large Span*, Scientific Works of Technical University of Wroclaw, No 27, Wroclaw, 1992, (in Polish)
3. Ameljañczyk, A.: *Multicriteria Optimization in the Control and Management Problems*, Ossolineum, Wrocşaw, 1984, (in Polish)
4. Jendo, S.: *Multiobjective optimization*, In M. Save & W. Prager (eds), *Structural Optimization, Vol. 2: Mathematical Programming*, Plenum Press, New York, (1990), 311–342.
5. Jendo, S. and Paczkowski, W. M.: *Multicriterion discrete optimization of space trusses with stiffness variations,* In J. B. Obrêbski (ed) *Lightweight Structures in Civil Engineering*, Int. Conf. on LSCE, Warsaw, Poland, 25–29 September 1995, (1995), 109–114.

6. Paczkowski, W. M.: *Chosen Problems of Discrete Evolutionary Optimization*, Scientific Works of Technical University of Szczecin, No 544, Szczecin, 1999 (in Polish).