

LOCALLY ORTHOTROPIC FEMUR MODEL

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Abstract. The ability of spongy bone, which specifies the direction of trabeculae to be parallel to the local principal stress directions is called, Wolff's law [1]. Based on this statement we can create a simple model using an easy iterative method which is called locally orthotropic femur model [2]. The gist of the iteration is the following: in an FE model according to the actual loading, first the material directions are engaged to be parallel to the calculated principal stress for every element, then the principal stress directions are calculated again and the material directions are modified. (The initial material orientations are parallel to each other, or random.) The obtained iterative method is strongly convergent, after six iteration loops the material angles do not change more than a few tenths and after the fifteenth iteration are practically constant. Therefore the advantages of this iterative method are its speed, effectiveness and similarity to real bone's trabeculae structure. We are planning to extend the 2D model to 3D, or rather to use the model obtained to the validation process of femur prosthesis.

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1. Introduction

Remodelling theories represent one of the most important branches of bone biomechanical research. They are primarily aimed at the modelling of the processes happening in cancellous bone tissue, because this part of the bone remodels approximately six to eight times faster than compact bone tissue. Among the existing models there are micro and macro models. Micro models attempt to model the most complex microbiological properties of spongy bone while macro models reflect only the most fundamental properties of bones. This obviously means that the micro models are much more complex and therefore require a much more extensive computer capacity. (Some models takes several months to run on a 16-processor supercomputer, see Huiskes [3].)

The aim of our research team is to apply the created model to test the stability of femur prosthesis. Due to the large number of implantations to be tested there is no time for several months running. So we wanted to create a femur model that reflects as many properties of the bone as possible but at the same time allows a relatively fast

calculation. On the other hand, the exclusion of remodelling processes would be an unjustifiable negligence on our part, considering the fact that this is one of the most fundamental characteristics of bones. Accordingly, we concluded that the application of a macro model would be most reasonable.

The structure of spongy bone and at the same time the remodelling processes were first examined by Wolff. In his research he observed that the bone trabeculae form their orientation in such a way that they are parallel to the local principal stresses at every point. He stated this observation as a law that received the name Wolff's law. One of the most important conclusions of Wolff's law is that bones build up their internal structure according to the actual loading present for a longer period of time. This simple but very important property can easily be incorporated into a FEM model. The acquired model is the so-called locally orthotropic bone model, which was developed by Thomas J. Impelluso [2]. The model is fast and can easily be applied, therefore it was reasonable to choose it for our research. We improved this conception for 2D and 3D generic problems and three BASIC scripts were written, which are able to automate the iteration in Nastran for Windows systems.

2. Locally Orthotropic Bone Model

As mentioned above, the locally orthotropic bone model is aimed to apply Wolff's law in practice. This is achieved through a simple iteration procedure. As the name of the model suggests, the procedure requires the application of orthotropic materials; based on our knowledge of literature data this consideration fits well the real behavior of the bone, thus this is a reasonable simplification. Consequently, according to the present state of science, regarding bones as orthotropic materials we obtain a very good approximation [1].

The steps of the iteration are as follows:

1. In an orthotropic FEM model the original material orientations are chosen parallel to each other or even at random.
2. The principal stress directions are determined for each element according to actual loading.
3. Material orientations are modified to be parallel to the principal stress directions in each element.
4. If the magnitude of changes in material orientation exceeds the value preset by us, then the calculation is repeated from step 2.

3. Mathematical background

The iteration in 2D can easily be incorporated into the Nastran for Windows FEM code on Windows platforms, because the program makes the changing of material orientations and polling of principal stress directions possible. Consequently, creating a relatively simple BASIC script is enough. However, in 3D it is not possible to change material orientations and poll principal stress directions using a single BASIC function. Therefore first principal stress directions must be determined based on the

stress matrix, then when these are known the stiffness matrix (D) of the material must be transformed. Jacobi-transformation was applied to determine eigenvalues and eigenvectors, see Popper [4]. The principal stresses are obviously sorted according to magnitude. Then the components of the unit eigenvectors of the stress matrix give the direction cosines required for the transformation. The equation applied for transformation is the following [5], [6]:

$$D_{new} = TD_{original}T^T$$

in which the form of T rotation matrix is:

$$T = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix}$$

with Λ being the matrix containing direction cosines:

$$\Lambda = \begin{bmatrix} \cos(\sigma_1, x) & \cos(\sigma_1, y) & \cos(\sigma_1, z) \\ \cos(\sigma_2, x) & \cos(\sigma_2, y) & \cos(\sigma_2, z) \\ \cos(\sigma_3, x) & \cos(\sigma_3, y) & \cos(\sigma_3, z) \end{bmatrix},$$

where $\cos(\sigma_1, x)$ is the cosine of the angle between σ_1 principal stress direction and the x axis that is the first component of the eigenvector belonging to the highest eigenvalue of the stress matrix and so on. The transformation must always be carried out with the original (not rotated) material stiffness matrix, because the eigenvectors of the stress matrix always give the directions related to the global (x, y, z) axes.¹ The material stiffness matrix is not part of the input data of the program, so it can only be determined through the values appearing in it. Its form is as follows:

$$D = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & & & \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & & 0 & \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & & & \\ & 0 & & \frac{1}{G_{12}} & & \\ & & & & \frac{1}{G_{23}} & \\ & & & & & \frac{1}{G_{13}} \end{bmatrix}.$$

For an orthotropic material, only 9 of the 12 nonzero terms are independent, due to the symmetry of the D tensor:

$$\frac{\nu_{12}}{E_1} = \frac{\nu_{21}}{E_2} \quad \frac{\nu_{13}}{E_1} = \frac{\nu_{31}}{E_3} \quad \frac{\nu_{23}}{E_2} = \frac{\nu_{32}}{E_3}.$$

We can easily modify these parameters automated after FE analysis in the Nastran for Windows system, thus we can set the calculated material orientations. Explained details of procession are in the next section.

4. Programming of the Nastran for Windows GUI

The Nastran for Windows system is a very effective and fast final element program that at the same time can be easily handled. It contains its own internal BASIC script language, which makes the running of programs written by us possible as well as incorporates them into the menu, which makes their activation easier.

¹This way there is no need to store the angles of the previous transformation.

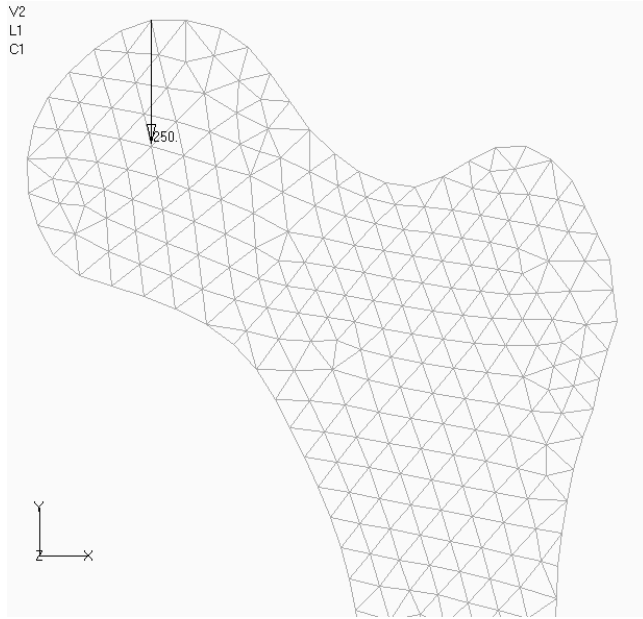


Figure 1. (a). The force was applied in bilateral body posture

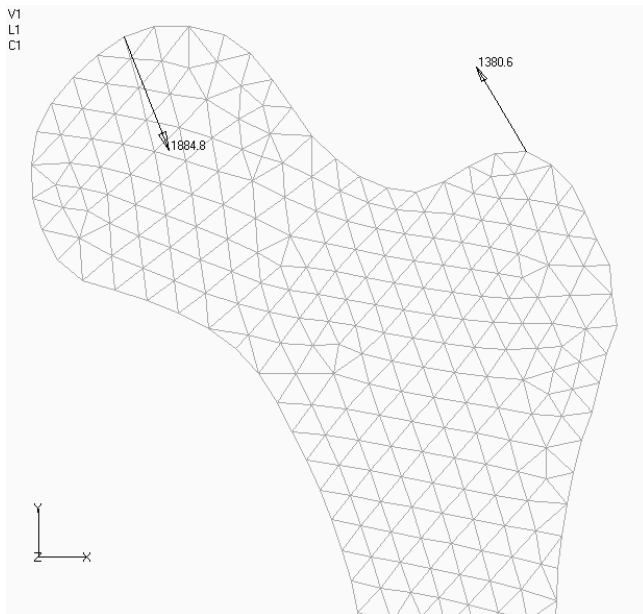


Figure 1. (b). The force was applied in unilateral body posture

As already mentioned, in 2D it is possible to poll principal stress directions and to set material orientations using simple functions. Therefore to realize local orthotropy, three Basic scripts were written. The first nullifies material orientations for every element (or sets them to any other angle), creating original material orientations parallel to each other. The second sets original material directions at random for every element. The third polls the direction of the highest principal stress for each element and modifies material orientations accordingly.

Unfortunately, in 3D the setting of material directions according to principal stress directions is much more complicated, because in the program the direction and coordinates of principal stress vectors cannot be polled, the material orientations can not be set, so in this case the transformation mentioned earlier must be applied.

5. Results

5.1. Load states. The 2D model was tested for two fundamental load states: based on literature data examining bilateral and unilateral body postures. The applied forces are shown in Figure 1a.

For bilateral body posture (when we are standing on two legs) we used a very simple model. The weight of the upper body is split equally into the two legs. The bodyweight of an average person is 70 *kg* and the weight of the upper body is two thirds of the total bodyweight, thus 50 *kg*, consequently 250 *N* acts on each leg. We applied this load on one node in the midsection of the femur head and the hip joint.

The unilateral body posture is very similar to the bilateral body posture. When standing on one leg we need a muscle force to keep the balance, thus based on literature data we applied two loads: a contact force at the midsection of the femur head and the hip joint (1884.8*N*) and a muscle force (1380.6*N*).

In both cases, the original material orientation was taken in five different ways: parallel (0°, 45°, 75°, 90°) and randomly. The acquired results are alike, the only difference is in the number of steps required for the final result. (The originally parallel material orientations reached the final results in 10 or 11 iteration steps while the random running required 12 or 13 steps.) However, in the presentation of results only the data acquired from random original orientation are described because ‘changes’ are most striking here.

The material parameters were taken from the book [2]:

$$E_1 = 17 \text{ GPa}, \quad E_2 = 11 \text{ GPa}, \quad G_{12} = 3.3 \text{ GPa}, \quad \nu_{12} = 0,41 .$$

5.2. Bilateral body posture. In a bilateral body posture the bone carries smaller loads than in a unilateral body posture, therefore the stresses in the bone are also smaller. The result of the procedure is the decrease of stresses and ‘smoothing’ of local stress peaks in the bone model as described later. First take a look at the distribution of material orientations as these show the method’s effectiveness to a greater extent. Figure 2 shows the material orientation in every element, with random original orientation (a), after the first iteration step (b), and after 20 iteration steps (c).

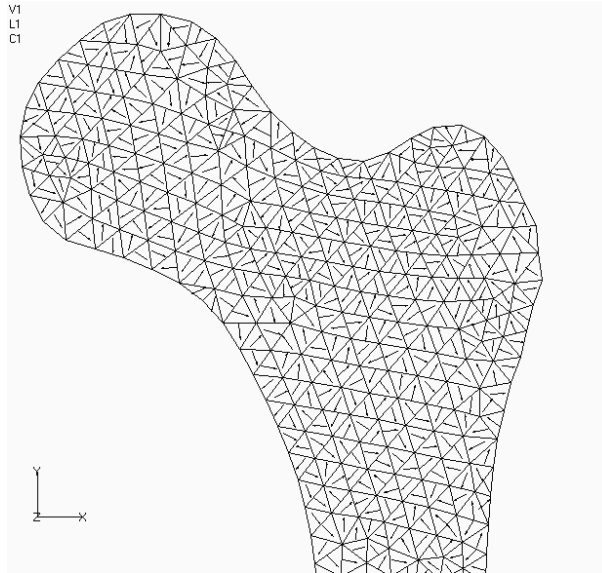


Figure 2. (a). Random material directions in the elements

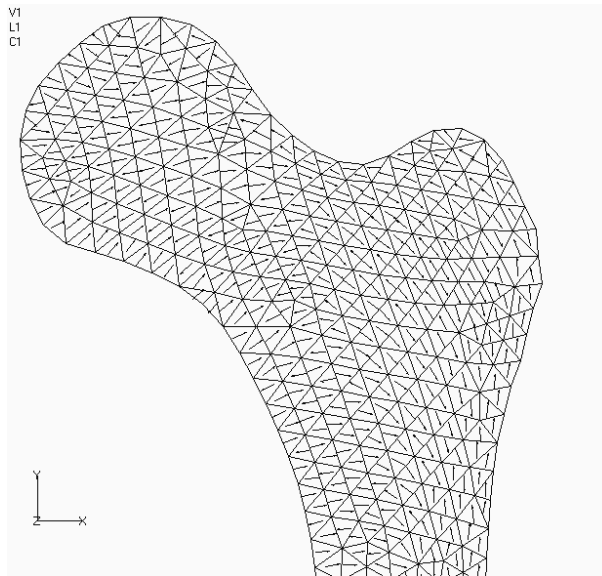


Figure 2. (b). Material directions in the elements after the first iteration step

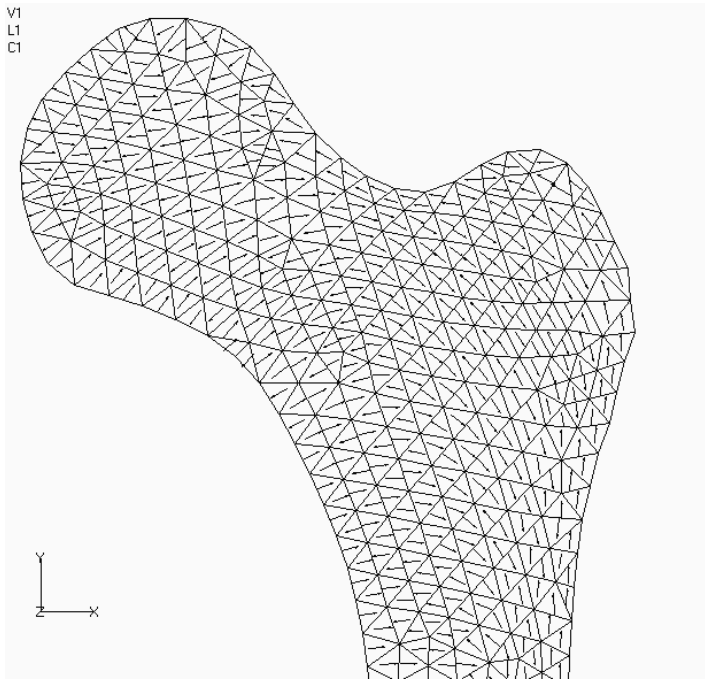


Figure 2. (c). Material directions in the elements after the 20th iteration steps

As can be seen, the material orientations do not change considerably (c) compared to the first iteration step (b). If calculations are carried out twice (only the first iteration step is finished), it already gives a good approximation, therefore this procedure provides very fast results. 15 iteration steps are enough for a very accurate calculation. Besides it also can be seen that the resulting material orientation diagram is very similar to the Culmann-trajectory and the actual form of trabeculae in real bones.

Figure 3 shows the above mentioned stress decreasing effect. Figure 3a shows top major principal stresses calculated with original material orientations set at random, while Figure 3b shows top major principal stress distribution after the 20th iteration.

The changes in stress values are more visible if a stress diagram is produced. This is shown in Figure 4. The red diagram ‘in the background’ shows the top major principal stress distribution with original material orientations set at random, while the blue diagram shows the situation after the 20th iteration. In both cases the x coordinates of the elements are on the horizontal axis and all elements are represented. It is possible to realise that this method ‘smoothens’ and decreases by approximately

10% the stress peaks critical for fracture, so really an effect similar to real behavior is shown.

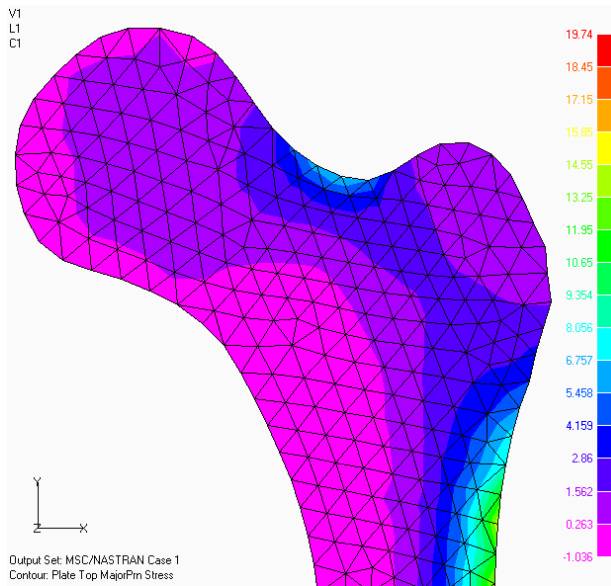


Figure 3. (a). Distribution of the top major principal stress with original material directions [MPa]

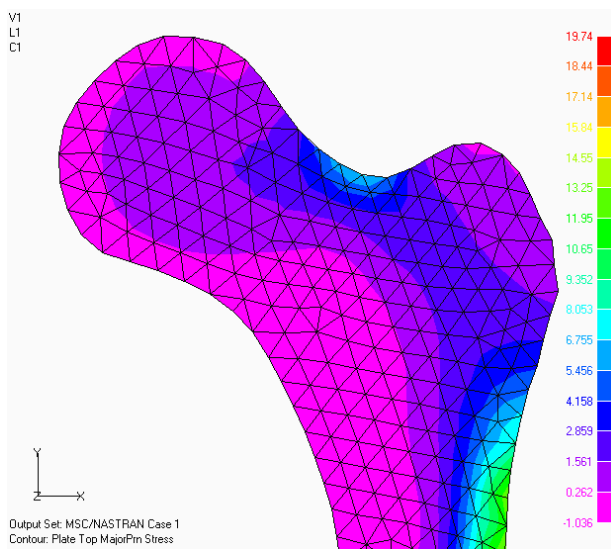


Figure 3. (b). Distribution of the top major principal stress after the 20th iteration steps [MPa]

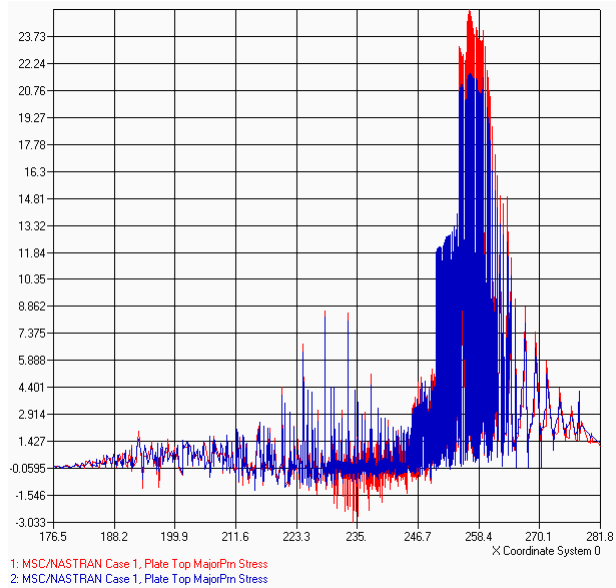


Figure 4. Top major principal stress distribution diagram versus the x coordinate. [MPa]

5.3. **Unilateral body posture.** Unilateral body posture is very similar to bilateral, but the forces are greater. Because a force exerted by the muscles is taken into

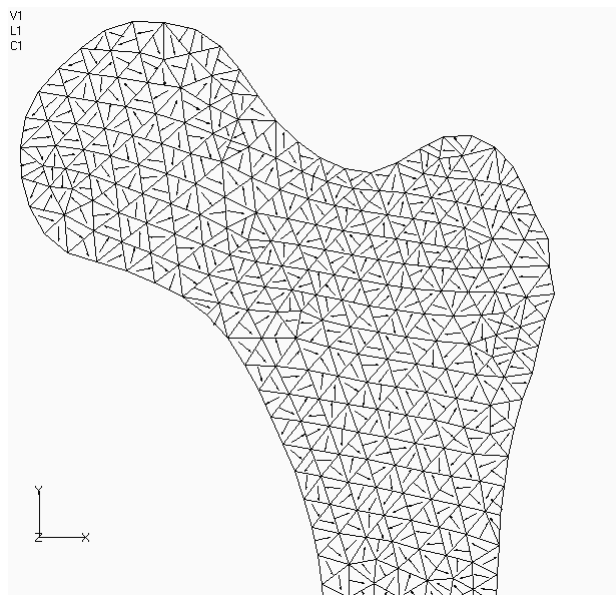


Figure 5. (a). Random Material directions in the elements

consideration and with this, balance can only be maintained if the forces acting at the hip-joint are increased. The top major principal stress distributions are a little bit different here, but their form is very similar and the evaluations of the results are also the same. The procedure was repeated for 20 iterations here as well because in

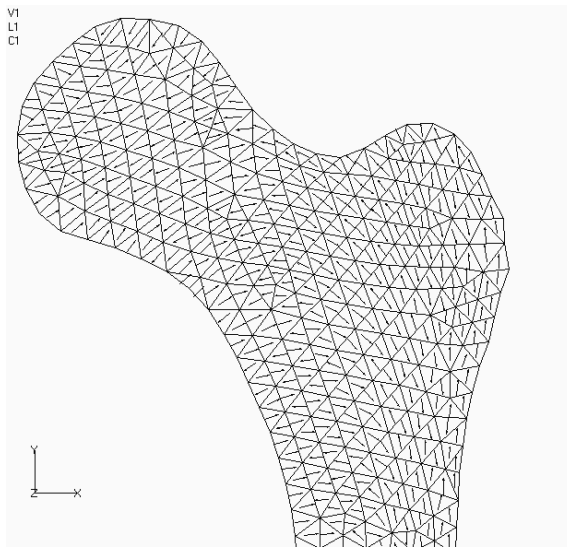


Figure 5. (b). Material directions in the elements after the first iteration step

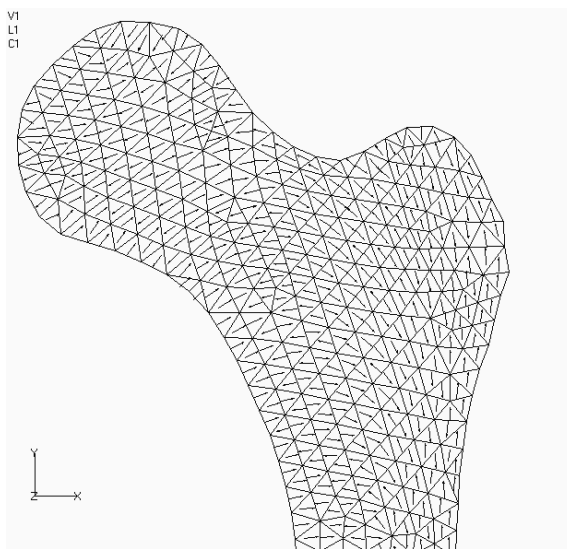


Figure 5. (c). Material directions in the elements after the 20th iteration step

this case iteration converged perfectly for any original condition and any load situation. The results here also show that the method strives for the decreasing of high stress peaks; we also emphasize that stress levels are much smoother, rounder.

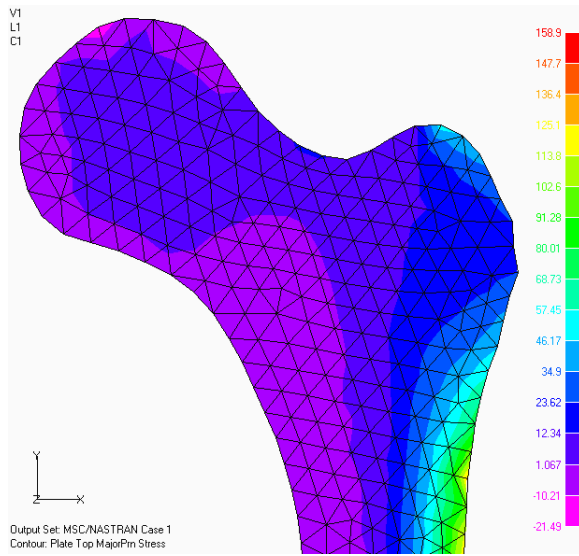


Figure 6. (a). Distribution of the top major principal stress with original material directions [MPa]

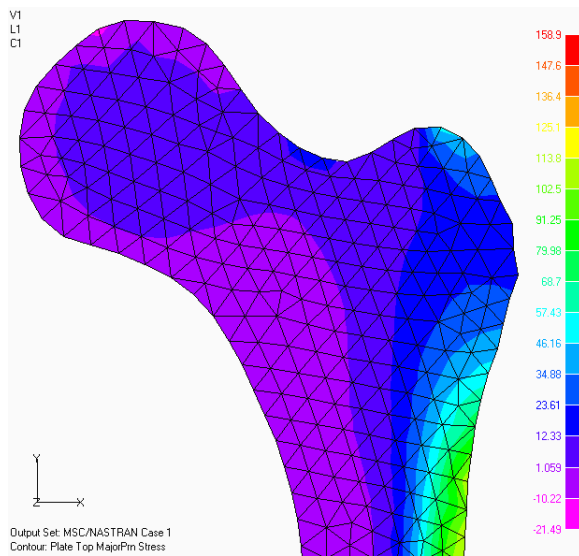


Figure 6. (b). Distribution of the top major principal stress after the 20th iteration step [MPa]

The material orientations follow the Culmann-trajectory again, so they are very similar to the real case. Therefore they are only shown here, because the evaluations of the results are the same as for bilateral body posture. (Please refer to them.)

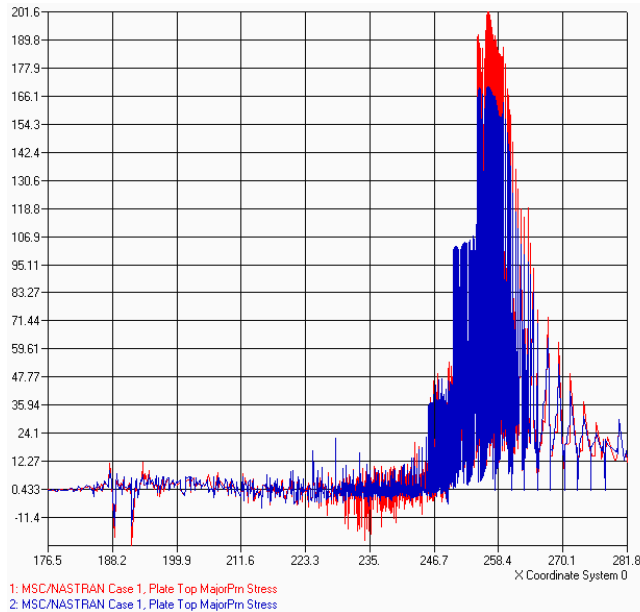


Figure 7. Top major principal Stress distribution diagram as the function of the x coordinate [MPa].

It can be recognized that the method is not sensitive to the applied loads and constraints comparing the two cases of loading. As we can see in Figures 2*a*, *b* and *c* and Figures 5*a*, *b* and *c* the evolving material orientations are similar to each other and also to the Culmann-trajectory. So we can apply any kind of loads and constraints: nodal forces, forces on surface, pressure, nonlinear forces, nodal constraints, constraints on surface and any combination of these. (Note: this property of the method follows from the way of realization.)

6. Concluding remarks

The creation steps of a local orthotropic femur model were described in detail and the results acquired from application were analyzed for two types of load. Based on these it can be seen that the model shows very well what we expect from it on the basis of physiological considerations, and that the acquired material direction ‘trajectories’ obtained are very similar to the real ones and correlate well with the most fundamental assumptions of bone biomechanics. Based on this the model is considered suitable to be included in implantation stability investigation. In the near future by building up the 3D model and adding properties like the separation of the mechanical properties of solid and spongy bone tissues and their marking based

on CT diagrams, and taking the dependence of material properties on density into consideration (thus creating an opportunity to characterize mechanical properties as function of position), probably one of the most detailed femur models will be acquired, which will hopefully be a suitable basis for a numerical implantation investigation. Obviously in the case of such a serious project we cannot rely solely on numerical calculations, so simultaneously with these a research series is also started, which will partly be a reference to simulations, partly provide those with material properties.

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