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CHANGING THE CONNECTIONS OF STRUCTURAL ELEMENTS DURING AN OPTIMIZATION PROCESS

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Abstract. Optimization of internal joint connections is the inverse problem of structural optimization. There are three types of internal joints: rigid connection, flexible connection and no connection. A continuous function is chosen to design the type of every joint. These functions are determined by the optimization. The methods presented in this paper can be used for topological design as well. The paper presents the mathematical formulation. The examples shown are compared with the usual topological optimization forms.

Mathematical Subject Classification: 74P10
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1. Introduction

The boundary conditions, defined as external foundations and/or internal joint connections, basically determine the shape and design of structures. Earlier analysis is presented in [1] and [2], and a summary is given in [3]. In practice the boundaries are given, and the calculations are carried out in order to find a minimal weight design without changing the earlier defined form and stress distribution of the structure. The topology optimization methods modify the stress distribution by changing the value of the cross-sectional area. By using the method the area of all unnecessary elements will converge to zero while the other elements will approach full stress. The disadvantage of those optimization forms – ***cross-section optimization*** in the following - is that they keep and use the unnecessary elements. The aim of this paper is to define a topological optimization form based on the internal joint connections – referred to as ***connection optimization*** in the following - and compare it to cross-section optimization. Both optimizations are analyzed in the case of ***bar structures***. Section 2 presents the mathematical background. Section 3 is devoted to topological optimization and the corresponding examples.

The analysis we have presented is based on the equilibrium and compatibility equations of bar structures. The problem formulations are based on the following preconditions:

- the geometrical data are known,
- the external loads are given,
- the external supports are given,
- the material is homogeneous and linearly elastic,
- the displacements are small,
- buckling is not treated.

The state equation of FEM analysis concerns the displacements only. To take the internal forces as optimization limits into consideration, it is useful to separate the equilibrium and compatibility equations (1.1). Thus the equation system is:

$$\begin{bmatrix} \mathbf{C} & \mathbf{G} \\ \mathbf{G}^{\mathrm{T}} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \mathbf{s} \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{0} \end{bmatrix}, \tag{1.1}$$

where $\mathbf{C}$ is the diagonal matrix of the displacement supports, $\mathbf{G}$ and its transpose are the geometrical matrices, $\mathbf{F}$ is the flexibility matrix, $\mathbf{v}$ is the vector of node displacements, $\mathbf{s}$ is the vector of the internal forces acting in the bars and $\mathbf{q}$ is the external load vector [4]. We assume that there are no permanent deformations and initial displacements at the supports: $\mathbf{0}$.

2. The connection/disconnection problem

2.1. General formulations for the connection modification. In FEM design the connections between the nodes and elements are defined fix as default. Other types of connections (e.g. hinge, elastic, etc. can be taken into consideration by subtracting a suitable dyad from the stiffness matrix $\mathbf{K}$:

$$\tilde{\mathbf{K}} = \mathbf{K} - \frac{1}{k_{ii}}\mathbf{k}_i\mathbf{k}_i^T\,, \tag{2.1}$$

where k_{ii} is an element in the main diagonal of $\mathbf{K}$, $\mathbf{k}_i$ and $\mathbf{k}_i^T$ are the column and the row in $\mathbf{K}$ that involve k_{ii}. The flexible connections are defined by the spring constants ρ_i and are taken into consideration via a dyad which is also to be subtracted from $\mathbf{K}$:

$$\tilde{\mathbf{K}} = \mathbf{K} - \frac{\rho_i}{1+\rho_i k_{ii}}\mathbf{k}_i\mathbf{k}_i^T\,. \tag{2.2}$$

For $\rho_i \to \infty$ the limit of equation (2.2) coincides with equation (2.1) [4] – see Figure 1 for details which graphically represent the connection.

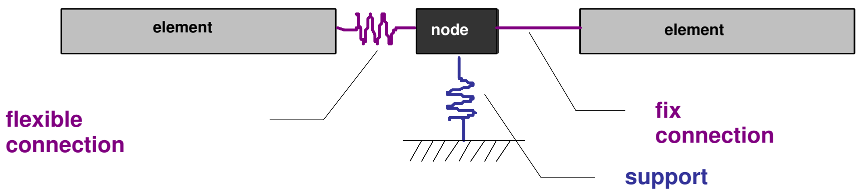


Figure 1. Model of supports and connections in FEM design

To present the same effect in the separated equation system (1.1), the symmetric flexibility matrix $\mathbf{F}$ of a structural element has to be diagonalized:

$$\mathbf{F} = \mathbf{U}\langle\hat{\mathbf{F}}\rangle\mathbf{U}^T\,, \tag{2.3}$$

where $\langle\hat{\mathbf{F}}\rangle$ is the diagonal flexibility matrix and $\mathbf{U}$ and its transpose $\mathbf{U}^T$ are the matrices of the transformation. By adding the spring variable ρ_i to $\left\langle \hat{F}_i \right\rangle$ we have

$$\langle\tilde{\mathbf{F}}\rangle = \langle\hat{\mathbf{F}}\rangle + \langle\boldsymbol{\rho}\rangle\ . \tag{2.4}$$

Generating the stiffness matrix we obtain (a) equation (2.1) if $\rho_i = -\frac{1}{\langle F_i\rangle}$ (b) $\mathbf{K}$ if $\rho_i = 0$ and (c) equation (2.2) otherwise. The system we have generated can be used for ***connection optimization*** design.

2.2. **The example structure.** The structure we shall analyze as an example is a well known nine-bar truss [5]. The optimal form of the structure is taken from literature [6] – see Figure 2. These two forms are used for making comparisons. The advantage of the example structure is that the flexibility matrix is a diagonal one. Consequently, there is no need for a diagonalization. The values of the vertical loads acting on the nodes 2 and 3 are the same, i.e., 400kN. The Young modulus of all elements is $2.1\cdot10^5$ MPa. The initial cross-sectional area is 85 cm^2, σ_e = 160 MPa is the elastic stress limit and the largest bar force equals 1333 kN.

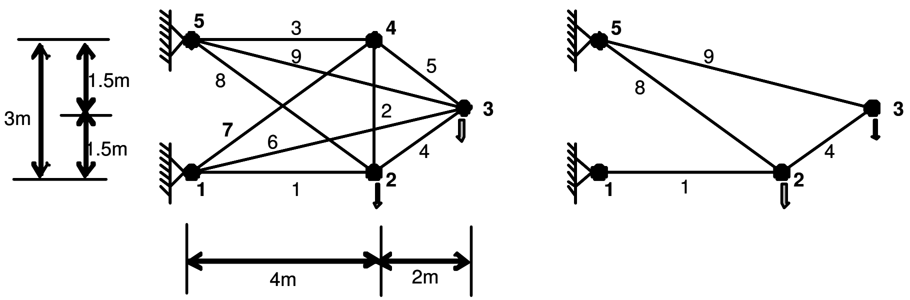


Figure 2. The nine bar truss to be investigated and its optimal form

2.3. **Cross-section modification in case of the example structure.** In case of the example structure the flexibility matrix of an element contains only one element, $\langle F_i\rangle = \frac{l_i}{EA_i}$, where the Young modulus E and the length of the bar l_i are constant values. A_i is the cross-sectional area. Multiplying the element $\langle F_i\rangle$ by $\rho_i \in 1..1e4$ or regarding the problem as a simple inverse one and dividing by spring variable $\hat{\rho}_i \in 1e-4..1$, we obtain

$$\langle\tilde{F_i}\rangle = \langle F_i\rangle\langle\rho_i\rangle = \frac{l_i\rho_i}{EA_i} \tag{2.5}$$

or

$$\langle \tilde{F}_i \rangle = \frac{\langle F_i \rangle}{\langle \hat{\rho}_i \rangle} = \frac{l_i}{EA_i \hat{\rho}_i}, \quad i = 1..9. \tag{2.6}$$

The system we have generated is adaptable for cross-section modification, thus usable for a ***cross-section optimization*** design.

2.4. **Comparison.** To check formulas (2.4) and (2.5), the admissible form of the example structure is solved in three different ways, see Figure 2 and Table 1. The results obtained by comparing the different techniques (Section 2.1, 2.3) for the example structure (presented in Section 2.2) show that disconnecting the elements unused the bar forces and the displacements at the fixed points are much closer to the real values in Table 1.

		9 bar structure	**4 bar structure**		
			using eq. (2.5)	**using eq. (2.4)**	**analytical solution**
Displacements (cm)	**1x**	.1333e-2	**.1333e-2**	**.1333e-3**	**.1333e-2**
	1y	-.3879e-3	**-.1037e-6**	**-.2824e-11**	**0**
	2x	-.1188	**-.2974**	**-.2987**	**-.2975**
	2y	.4027	**.9385**	**.9426**	**.9387**
	3x	-.7124e-2	**-.1629**	**-.1629**	**-.1629**
	3y	.5847	**1.235**	**1.240**	**1.235**
	4x	.1173	**.1862**	**.2154**	
	4y	.3743	**.8034**	**.7621**	
	5x	-.1333e-2	**-.1333e-2**	**-.1333e-3**	**-.1333e-2**
	5y	-.4121e-3	**-.7999e-3**	**-.8000e-4**	**-.8000e-3**
Forces in Members (kN)	**1**	-536.0	**-1333.**	**-1333.**	**-1333.**
	2	168.5	**.8037e-1**	**.1805e-4**	
	3	529.5	**.8370e-1**	**.2155e-4**	
	4	-142.1	**-499.9**	**-500.0**	**-500.0**
	5	190.5	**-.1466e-1**	**-.1571e-5**	
	6	-433.2	**-.1324**	**-.4590e-4**	
	7	-471.4	**-.1193**	**-.2851e-4**	
	8	527.9	**1166.**	**1167.**	**1167.**
	9	393.3	**412.4**	**412.3**	**412.3**

Table 1. Results obtained by applying different techniques

3. The optimization problem with examples

The optimization was carried out for the example structure in Figure 2 with both methods mentioned in Section 2. The aim of the design was to find the best statically determinate form of the structure.

The optimization is implemented by a sequential quadratic programming method for solving nonlinear problems. The following variables are the unknowns in the program developed:

- $\rho_{1..9}$ are the variables to be modified,
- $\mathrm{v}_{1..10}$ are the node displacements,
- $\mathrm{s}_{1..9}$ are the values of the bar forces.

$$obj1 = \sum_{i=1}^{9} \mathrm{s_i} \qquad obj4 = \sum_{i=1}^{9} (\mathrm{s}_i - \rho_i)$$

$$obj2 = -\sum_{i=1}^{9} \rho_\mathrm{i} \qquad obj5 = \sum_{i=1}^{9} \left(\mathrm{s}_i - \mathrm{s}_i^2 \left\langle \tilde{\mathrm{F}}_i \right\rangle\right)$$

$$obj3 = -\mathbf{s}^\mathbf{T} \left\langle \tilde{\mathbf{F}} \right\rangle \mathbf{s} = -\sum_{i=1}^{9} \mathrm{s}_i^2 \left\langle \tilde{\mathrm{F}}_i \right\rangle \qquad obj6 = -\sum_{i=1}^{9} \left(\mathrm{s}_i^2 \left\langle \tilde{\mathrm{F}}_i \right\rangle + \rho_i\right)$$

Table 2. The objective functions

There are six objective functions given in Table 2, defined as a summation of the internal bar forces, of the spring variables, of the compliance and of three of their variations, respectively.

In both cases of topological optimization, the mathematical programming problems are formulated as follows:

$$\begin{array}{rll} \text{obj n} = \text{min!} & & \text{Objective function from Table 2} \\ \mathbf{Cv} + \mathbf{Gs} - \mathbf{q} = \mathbf{0} & & \text{Equalities} \\ \mathbf{G^T v} + \left\langle \tilde{\mathbf{F}} \right\rangle \mathbf{s} = \mathbf{0} & & \\ LL \le \rho_{i=1\ldots9} \le 1e4 & & \text{Inequalities} \\ -1.8 \le \mathrm{v_{i=1\ldots10}} \le 1.8 & [\text{cm}] & \\ -1335 \le \mathrm{s_{i=1\ldots9}} \le 1335 & [\text{kN}] & \end{array} \tag{3.1}$$

The lower limit LL is zero for connection optimization and is equal to one for cross-sectional optimization.

Note: The value of the variables to be modified should fall between two positive limits. The admissible form needs a higher value. Therefore a negative sign is used to ensure the minimal optimum.

	9 bar structure	**4 bar structure**		
		using eq. (2.5)	**using eq. (2.4)**	**analytical solution**
Compliance	398.8	**873.0**	**873.6**	**873.6**
$\sum \mathrm{s_i}$	227.0	**-254.6**	**-253.7**	**-253.7**
$\sum \mathrm{s_i}, \mathrm{s_i} > 0$	1809.7	**1578.4**	**1579.3**	**1579.3**
$\sum \mathrm{s_i}, \mathrm{s_i} < 0$	-1582.7	**-1833.1**	**-1833**	**-1833**

Table 3. Results obtained by the three different techniques

The comment on the positive and negative signs of other objective functions is in Table 3. The compliance of a statically determinate structure is much higher than that of an indeterminate one. A negative sign is used to present the maximum value in a minimization process, *obj3*. The sum of the internal bar forces is less, negative, in case of a statically determinate structure. The number of bars in compression is more than that of the bars in tension. In addition the compressive stress is quite large. Since the bars are in compression the sign of the normal stress is neglected when we seek for a minimum, *obj1*.

Results: The ***cross-section optimization*** gives four different results, three statically determinate structures and an indeterminate one – see Table 4, 6 – 14. The statically indeterminate form is due to compliance *obj3*. The simple summation *obj1* and *obj2*, and the combination *obj4* are the dominant objective functions. Compliance *obj3* only slightly modifies the result.

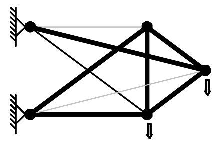
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	1	**4**	1	**7**	1	**1x**	.133e-2	**1**	-725.5
2	1	**5**	1	**8**	57	**1y**	-.451e-3	**2**	894.0
3	557	**6**	120	**9**	1	**2x**	-.161	**3**	4.953
						**2y**	1.32	**4**	-865.1
						3x	.031	**5**	-742.0
						3y	1.78	**6**	-9.635
						4x	.617	**7**	-748.2
						4y	1.17	**8**	41.7
						5x	-.133e-2	**9**	1335
						5y	-.348e-3		

Table 4. Results of the mathematical programming problem (3.1) with (2.5) and *obj1* from Table 2

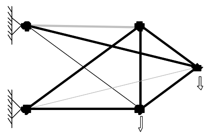
Connection optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	2e-5	**4**	0	**7**	0	**1x**	.133e-2	**1**	-729.0
2	0	**5**	0	**8**	.016	**1y**	-.453e-3	**2**	900.6
3	.083	**6**	2.1	**9**	0	**2x**	-.174	**3**	7.52
						**2y**	1.33	**4**	-872.8
						3x	.025	**5**	-745.8
						3y	1.8	**6**	-.197
						4x	.622	**7**	-755.2
						4y	1.18	**8**	38.49
						5x	-.133e-2	**9**	1335
						5y	-.347e-3		

Table 5. Results of the mathematical programming problem (3.1) with (2.4) and *obj1* from Table 2

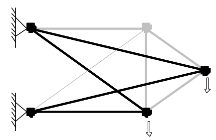
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	9.4	**4**	1e4	**7**	1e4	**1x**	.133e-2	**1**	-532.0
2	1e4	**5**	1e4	**8**	1	**1y**	-.200e-3	**2**	.144
3	1e4	**6**	1	**9**	2	**2x**	-1.12	**3**	.163
						**2y**	1.8	**4**	.728
						3x	.156	**5**	-.016
						3y	1.8	**6**	-826.0
						4x	.363	**7**	-.219
						4y	1.56	**8**	655.7
						5x	-.133e-2	**9**	825.4
						5y	-.599e-3		

Table 6. Results of the mathematical programming problem (3.1) with (2.5) and *obj2* from Table 2

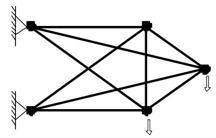
Connection optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	0	**4**	3e-5	**7**	0	**1x**	.133e-2	**1**	-536.0
2	2e-5	**5**	2e-5	**8**	~ 0	**1y**	-.388e-3	**2**	168.5
3	0	**6**	~ 0	**9**	~ 0	**2x**	-.119	**3**	529.5
						**2y**	.406	**4**	-142.1
						3x	-.008	**5**	190.5
						3y	.593	**6**	-433.2
						4x	.117	**7**	-471.4
						4y	.374	**8**	527.9
						5x	-.133e-2	**9**	393.3
						5y	-.412e-3		

Table 7. Results of the mathematical programming problem (3.1) with (2.4) and *obj2* from Table 2

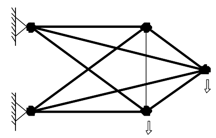
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	~ 1	**4**	~ 1	**7**	5.69	**1x**	.133e-2	**1**	-536.0
2	13.8	**5**	~ 1	**8**	6.67	**1y**	-.388e-3	**2**	168.6
3	~ 1	**6**	3.85	**9**	2.19	**2x**	-.119	**3**	529.4
						**2y**	1.8	**4**	-142.1
						3x	-.144	**5**	190.4
						3y	1.8	**6**	-433.2
						4x	.117	**7**	-471.4
						4y	1.41	**8**	527.8
						5x	-.133e-2	**9**	393.4
						5y	-.412e-3		

Table 8. Results of the mathematical programming problem (3.1) with (2.5) and *obj3* from Table 2

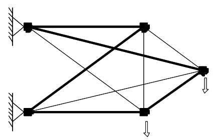
Connection optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	0	**4**	~ 0	**7**	5e-5	**1x**	.133e-2	**1**	-536.0
2	.008	**5**	.003	**8**	.002	**1y**	-.388e-3	**2**	168.5
3	0	**6**	.001	**9**	4e-4	**2x**	-.119	**3**	529.5
						**2y**	1.8	**4**	-142.1
						3x	-.144	**5**	190.5
						3y	1.8	**6**	-433.2
						4x	.117	**7**	-471.4
						4y	.414	**8**	527.9
						5x	-.133e-2	**9**	393.3
						5y	-412e-3		

Table 9. Results of the mathematical programming problem (3.1) with (2.4) and *obj3* from Table 2

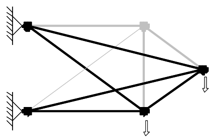
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	144	**4**	23	**7**	1e4	**1x**	.133e-2	**1**	-38.18
2	1e4	**5**	1e4	**8**	1	**1y**	-.324e-3	**2**	.191
3	1e4	**6**	1	**9**	1	**2x**	-1.23	**3**	.125
						**2y**	1.8	**4**	309.3
						3x	-.046	**5**	-.082
						3y	1.72	**6**	-1335
						4x	.278	**7**	-.238
						4y	1.48	**8**	357.0
						5x	-.133e-2	**9**	1080
						5y	-.476e-3		

Table 10. Results of the mathematical programming problem (3.1) with (2.5) and *obj4* from Table 2

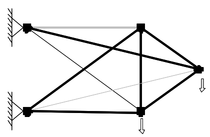
Connection optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	2e-5	**4**	0	**7**	0	**1x**	.133e-2	**1**	-735.8
2	0	**5**	0	**8**	.012	**1y**	-.448e-3	**2**	895.0
3	1.6	**6**	.56	**9**	0	**2x**	-.176	**3**	.396
						**2y**	1.33	**4**	-872.4
						3x	.025	**5**	-745.6
						3y	1.8	**6**	-.735
						4x	.623	**7**	-746.1
						4y	.118	**8**	47.40
						5x	-.133e-2	**9**	1335
						5y	-.352e-3		

Table 11. Results of the mathematical programming (3.1) with (2.4) and *obj4* from Table 2

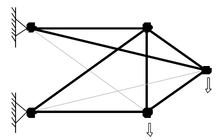
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	1	**4**	1	**7**	1.07	**1x**	.133e-2	**1**	-680.0
2	1	**5**	1	**8**	1e4	**1y**	-.476e-3	**2**	923.2
3	71.1	**6**	1317	**9**	1	**2x**	-.155	**3**	38.12
						**2y**	1.36	**4**	-872.3
						3x	.025	**5**	-745.5
						3y	1.8	**6**	-.895
						4x	.606	**7**	-793.2
						4y	1.2	**8**	.243
						5x	-.133e-2	**9**	1335
						5y	-.324e-3		

Table 12. Results for the mathematical programming problem (3.1) with (2.5) and *obj5* from Table 2

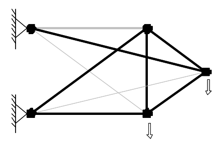
Connection optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	0	**4**	0	**7**	3e-5	**1x**	.133e-2	**1**	-705.5
2	0	**5**	0	**8**	.04	**1y**	-.466e-3	**2**	908.8
3	.024	**6**	.04	**9**	0	**2x**	-.158	**3**	24.76
						**2y**	1.36	**4**	-864.9
						3x	.025	**5**	-741.8
						3y	1.8	**6**	-9.99
						4x	.604	**7**	-772.8
						4y	1.2	**8**	16.95
						5x	-.133e-2	**9**	1335
						5y	-.334e-3		

Table 13. Results of the mathematical programming problem (3.1) with (2.4) and *obj5* from Table 2

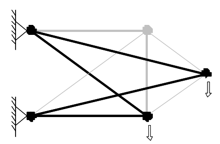
Cross-section optimization									
ρ_i [dimensionless]						$\mathrm{v_i}$ [cm]		$\mathrm{s_i}$ [kN]	
1	9.4	**4**	1e4	**7**	1e4	**1x**	.133e-2	**1**	-532.0
2	1e4	**5**	1e4	**8**	1	**1y**	-.200e-3	**2**	.147
3	1e4	**6**	1	**9**	2.1	**2x**	-1.12	**3**	.167
						**2y**	1.8	**4**	.728
						3x	.156	**5**	-.018
						3y	1.8	**6**	-826.0
						4x	.373	**7**	-.227
						4y	1.55	**8**	665.7
						5x	-.133e-2	**9**	825.4
						5y	-.600e-3		

Table 14. Results of the mathematical programming problem (3.1) with (2.5) and *obj6* from Table 2

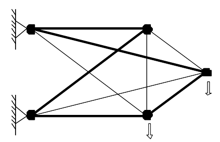
Connection optimization									
ρ_i [dimensionless]						v_i [cm]		s_i [kN]	
1	0	**4**	~ 0	**7**	3e-5	**1x**	.133e-2	**1**	-536.0
2	.009	**5**	.003	**8**	.002	**1y**	-.388e-3	**2**	168.5
3	0	**6**	.001	**9**	4e-4	**2x**	-.119	**3**	529.5
						**2y**	1.8	**4**	-142.1
						3x	-.145	**5**	190.5
						3y	1.8	**6**	-433.2
						4x	.117	**7**	-471.4
						4y	.399	**8**	527.9
						5x	-.133e-2	**9**	393.3
						5y	-.412e-3		

Table 15. Results of the mathematical programming problem (3.1) with (2.4) and *obj6* from Table 2

The ***connection optimization*** gives three different solutions: a statically determinate structure and two indeterminate ones – see Tables 5, 7 and 15. In this case the sum of internal bar forces *obj1*, and the compliance *obj3* are the useful functions.

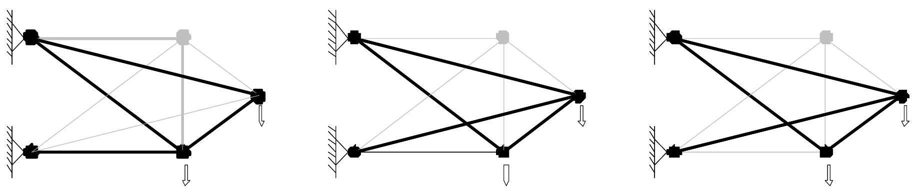


Figure 3. The results of limit modifications

The limits, used in inequalities, are important for the optimal form as well. Increasing the positive displacement limit and/or reduces the negative internal force limit the optimal statically determinate form can change. In the case of the example structure Figure 3 using *obj4* the following happens: the result is the left figure if the vertical displacement limit is 1.3cm and the internal bar force limit is 1335kN, the middle structure if the displacement limit is increased to 1.8 cm, and finally the right form if the internal force limit is changed to -1375kN.

The results of the optimization process with limits of (2.5) are presented in Tables 4 – Table 15.

4. **Conclusion**

The method we have presented in this paper is capable of solving the connection – disconnection problem in structural design. From a mechanical point of view the problem is an inverse one. Mathematically it is a not a convex problem. The optimization algorithm can find as many solutions as the number of the possible statically determinate structures. The results of the optimization process are determined by the objective functions. The limits set up for the displacement and the internal forces influence the local optima.

The main objective of the analysis was to formulate and try a solution technique. Thus the stability, safety and economy requirements are not fulfilled.

The method we have presented is developed for numerical optimization problems. The example structure is a well–known one from the literature, and is practical for demonstrating that the connection optimization technique developed gives good solutions. The advantages of that simple structure are that the calculations can easily be controlled, and the results obtained can easily be compared with those found in the literature [5], [6] and [7].

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