RELATIONS FOR WATER-VAPOR TRANSPORT THROUGH FIBERS

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Abstract. The present work is devoted to the combined microwave and convective drying behavior of carpets. The mathematical relations developed may be used to describe the moisture sorption mechanisms of carpets. The effects of important parameters during the drying process are discussed and the mathematical relations are formulated to illustrate the range of their applications. According to the results presented in this work, it is shown that combined microwave and convective drying can provide higher drying rates and, consequently, faster drying times.

Mathematical Subject Classification: 80A20

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1. Introduction

The cost of drying carpets, which is done toward the end of the manufacturing process, is quite high. A very common method of removing water from carpets is convective drying. Hot air is used as the heat transfer medium and is exhausted to remove vaporized water. Considerable thermal energy, about 30% of the total energy used, is required to heat make-up air as the hot air is exhausted. Thus, reducing the amount of exhaust is clearly a way to conserve energy, but there is little in the literature that can be used as guidelines for optimizing dryer exhaust flow. In this case the effect of humidity on the drying rates of carpet tiles should be studied.

When the water is deep within the carpet, combined microwave and convective drying, which have faster heat and mass transfer, may be a better choice. To fully understand the heat and mass transfer phenomena occurring within the carpet during combined microwave and convective drying, it is required to analyze the moisture, temperature, and pressure distributions generated throughout the process. The water remaining in the carpet is 50-60% times the weight of the carpet and after mechanical removal is in its thermodynamically favorable position, largely because the low viscosity of the water and connectivity of the pores ensure rapid equilibration. The pores are approximated as essentially cylindrical, with their axis vertical to the carpet

backing. The amount of porosity, i.e., the volume fraction of voids within the carpet, determines the capacity of a carpet to hold water; the greater the porosity, the more water the carpet can hold. The carpet is able to absorb 6 times it weight in water. When the water content is reduced to roughly 50-60% by mechanical extraction, free water is probably situated in pores with an effective capillary diameter of up to 200 μ m. The occupied pores are distributed throughout the carpet pile thickness and largely within the yarns.

The flow of air through carpet dryers is very complex, and neither simple concurrent or countercurrent flow of air and material takes place. Usually there are a set of well mixed zones, throughout which the drying conditions are effectively uniform. The commonest strategy for regulating a dryer's behavior is feedback control, in which the controlled variables (such as the moisture content of the material) are compared with the desired set point. The difference signal, suitably processed by the controller, is then used to modulate the input parameter being used as the control variable. An ideal three-term controller incorporates proportional, integral and derivative action. The proportional term gives a rapid response to an error signal, but the controlled variable is permanently offset from its desired value. The integral function compensates for the steady-state error. Derivative action is rarely used, except with well defined signals, because noise in the signal can confuse the controller, resulting in a hunting action about some mean value. The difficulties in obtaining online measures of a material's moisture content have led to the use of environmental control of the humidity or temperature within the drying chamber. This strategy, however, can lead to inadequate control, as the wet-bulb depression is the driving force for drying. Use of the dry-bulb temperature or relative humidity of the bulk air as the control variable will only be satisfactory as long as the surface conditions of the material remain constant. The difference in temperature between the surface and the bulk air will give a measure of the surface moisture content, provided the material is hygroscopic and all unbound moisture has been driven out.

2. Notational conventions

Notations
Specific heat
Diffusivity $(m^2 s^{-1})$
Gravitational constant (ms^{-2})
Intrinsic averaged enthalpy (Jkg^{-1})
Latent heat of evaporation (Jkg^{-1})
Differential heat of sorption (Jkg^{-1})
Intrinsic permeability (m^2)
Thermal conductivity (kW $m^{-1} K^{-1}$)
Relative permeability of gas
Mass transfer coefficient (ms^{-1})
Relative permeability of water
Thickness of the fabric (m)

M Molar mass $(kg \ mol^{-1})$

Power absorbed in the material per unit area (Wm^{-2}) $P_{\rm abs}$ Heat transfer coefficient $(Wm^{-2}K^{-1})$ QRUniversal gas constant $(J \ mol^{-1} \ K^{-1})$ SVolume saturation tTime (s) TTemperature (K)Averaged velocity (ms^{-1}) vDistance (m) z θ Non-dimensional temperature Density of the fibers (kg m^{-3}) ρ Surface tension (Nm^{-1}) σ Non-dimensional time au ϕ Porosity (m^3m^{-3}) Surface porosity (m^2m^{-2}) χ Internal microwave power source (Wm^{-3}) Φ Ψ Relative humidity Dynamic viscosity of gas (kg $m^{-1}s^{-1}$) μ_g Dynamic viscosity of water (kg $m^{-1}s^{-1}$) μ_W Permeability of free space (Hm^{-1}) μ

Subscripts

- a Air
- c Capacity
- g Gas
- v Vapor
- W Liquid
- o Atmospheric
- 1 Initial

3. Background

In general, drying means disposing of the liquid completely or partially. We define it more narrowly in this paper as the vaporization and removal of water from a material. The typical drying curve begins with a warm-up period, where the material is heated and the drying rate is usually low. The drying rate can be negative in the warmup period if the gas stream is humid enough. As the material heats up, the rate of drying increases to peak rate that is maintained for a period of time known as the constant rate period. Eventually, the moisture content of the material drops to a level, known as the critical moisture content, where the high rate of evaporation cannot be maintained. This is the beginning of the falling rate period. During the falling rate period, the moisture flow to the surface is insufficient to maintain saturation at the surface. This period can be divided into the first and second falling rate periods. The first falling rate period. In the constant rate period, external variables such as gas stream humidity, temperature, and flow rate dominate. In the second falling rate period, internal factors such as moisture and energy transport in the carpet dominate. Although much of the water is removed in the constant rate period of drying, the time required to reduce the moisture in the product to the desired value can depend on the falling rate period. If the target moisture content is significantly lower than the critical moisture content, the drying rates in the falling rate period become important.

It has been recognized that microwave could perform a useful function in carpet drying in the leveling out of moisture profiles across a wet sample. This is not surprising because water is more reactive than any other material to dielectric heating so that water removal is accelerated.

Many investigators have attempted to explain the effect of humidity drying rates and the existence of inversion temperatures. The explanations are usually based on changes that occur in convective heat transfer, radiative heat transfer, and mass transfer as the humidity and temperature of the gas stream change. We will briefly discuss these explanations.

At a given gas stream temperature, convective heat transfer rate can change as the humidity in the gas stream is varied, because product temperature and fluid properties vary with humidity. These effects can be explained using the following relationship for the convective heat transfer rate:

$$\frac{q}{A} = h \left(T_{\infty} - T_S \right) = h \Delta T ,$$

where $\frac{q}{A}$ = convective heat transfer per unit surface area A, h = heat transfer coefficient, T_{∞} = free stream temperature of the drying medium, T_S = surface temperature of material being dried.

Since the product temperature is dependent on humidity, clearly ΔT is also dependent. Further, the heat transfer coefficient h is a function of both product temperature and fluid properties. Thus, the convective heat transfer rate changes with humidity, as does the drying rate of a material. However, drying in air will always have an advantage over drying in steam because ΔT is larger for drying in air; this is a consequence of T_S being very near the wet bulb temperature. The wet bulb temperature is lowest for dry air, increases with increasing humidity, and reaches the saturation temperature of water for a pure steam environment. Thus, ΔT_{AIR} will be larger than ΔT_{STEAM} , but $\Delta T_{AIR}/\Delta T_{STEAM}$ decreases with increasing T_{∞} . Further, the heat transfer coefficient increases with humidity. Apparently, the net effect of the changes in h and ΔT is that the convective heat transfer rate increases faster for steam than for air with increasing temperature.

King and Cassie [1] conducted an experimental study on the rate of absorption of water vapor by wool fibers. They observed that, if a textile is immersed in a humid atmosphere, the time required for the fibers to come to equilibrium with this atmosphere is negligible compared with the time required for the dissipation of heat generated or absorbed when the regain changes. McMahon and Watt [2] investigated the effects of heat of sorption in the wool-water sorption system. They observed that the equilibrium value of the water content was directly determined by the humidity but that the rate of absorption and desorption decreased as the heat-transfer efficiency decreased. Heat transfer was influenced by the mass of the sample, the packing density of the fiber assembly, and the geometry of the constituent fibers. Crank [3] pointed out that the water-vapor-uptake rate of wool is reduced by a rise in temperature that is due to the heat of sorption.

The dynamic-water-vapor-sorption behavior of fabrics in the transient state will therefore not be the same as that of single fibers owing to the heat of sorption and the process to dissipate the heat released or absorbed.

Henry [4,5] was the first to start theoretical investigation of this phenomenon. He proposed a system of differential equations to describe the coupled heat and moisture diffusion into bales of cotton. Two of the equations involve the conservation of mass and energy, and the third relates fiber moisture content to moisture in the adjacent air. Since these equations are non-linear, Henry made a number of simplifying assumptions to derive an analytical solution.

In order to model the two-stage sorption process of wool fibers, David and Nordon [6] proposed three empirical expressions for a description of the dynamic relationship between fiber moisture content and the surrounding relative humidity. By incorporating several features omitted by Henry [4] into the three equations, David and Nordon [6] were able to solve the model numerically. Since their sorption mechanisms (i.e. sorption kinetics) of fibers were neglected, the constants in their sorption-rate equations had to be determined by comparing theoretical predictions with experimental results.

Farnworth [7] reported a numerical model describing the combined heat and watervapor transport through fibers. The assumptions in the model did not allow for the complexity of the moisture-sorption isotherm and the sorption kinetics of fibers. Wehner *et al* [8] presented two mechanical models to simulate the interaction between moisture sorption by fibers and moisture flux through the void spaces of a fabric. In the first model, diffusion within the fiber was considered to be so rapid that the fiber moisture content was always in equilibrium with the adjacent air. In the second model, the sorption kinetics of the fiber were assumed to follow Fickian diffusion. In these models, the effect of heat of sorption and the complicated sorption behavior of the fibers were neglected.

Li and Holcombe [9] developed a two-stage model, which takes into account watervapor-sorption kinetics of wool fibers and can be used to describe the coupled heat and moisture transfer in wool fabrics. The predictions from the model showed good agreement with experimental observations obtained from a sorption-cell experiment. More recently, Li and Luo [10] further improved the method of mathematical simulation of the coupled diffusion of the moisture and heat in wool fabric by using a direct numerical solution of the moisture-diffusion equation in the fibers with two sets of variable diffusion coefficients. These research publications were focused on fabrics made from one type of fiber. The features and differences in the physical mechanisms of coupled moisture and heat diffusion into fabrics made from different fibers have not been systematically investigated.

Downes and Mackay [11] found experimentally that the sorption of water vapor by wool is a two-stage process, the first stage obeys Fick's law of diffusion with a concentration-dependent diffusion coefficient. The second stage, which involves structural changes within fibers, is much slower than the first. To simulate the twostage-sorption process, David and Nordon [6] proposed an exponential function to describe the rate of change of fiber water content, which needed to be adjusted according to the measured fabric moisture content. Li and Holcombe [9] developed a two stage sorption-rate equation to describe the moisture sorption of wool. The first stage is represented by a Fickian diffusion with a constant coefficient. The diffusion equation was solved by using Crank's truncated solution [3]. The second stage is described by an exponential relationship, which also needs to be adjusted according to the experimental measurements. The relative contributions of the two stages to the total moisture sorption are functions of the sorption time and the initial regain of the fibers.

Before a carpet is conveyed into the drying oven, most of the water is typically vacuum extracted. After vacuum extraction, the moisture regain is about 50-60%, indicating that there is still a significant amount of water inside the carpet. This water is usually removed with heat in industrial manufacturing processes. As a general rule, the water is distributed in larger pores.

It seems reasonable to anticipate that many of these pores are formed within the pile yarns throughout the thickness of the carpet. The location of this water is of interest to people in the carpet industry.

4. Mathematical Formulation

It was shown by Ilic and Turner [12] that a theory based on a continuum approach led to the following equations of motion governing the drying of a slab of material:

Total mass:

$$\frac{\partial}{\partial t} \left(\phi S_g \rho_g + \phi S_W \rho_W \right) + \nabla \left(X_g \rho_g V_g + X_W \rho_W V_W \right) = 0 .$$
(4.1)

Total liquid:

$$\frac{\partial}{\partial t} \left(\phi S_g \rho_{gv} + \phi S_W \rho_W \right) + \nabla \left(x_g \rho_{gv} V_{gv} + X_W \rho_W V_W \right) \ . = 0 \tag{4.2}$$

Total enthalpy:

$$\begin{aligned} \frac{\partial}{\partial t} \left[\phi S_g \rho_{gv} h_{gv} + \phi S_g \rho_{ga} h_{ga} + \phi S_g \rho_{ga} h_{ga} + \phi S_W \rho_W h_W + \\ &+ (1 - \phi) \rho_S h_S - \phi \rho_W \int_0^{S_W} \Delta h_W(S) dS \right] + \\ &+ \nabla . \left(X_g \rho_{gv} V_{gv} h_{gv} + X_g \rho_{ga} V_{ga} h_{ga} + X_W \rho_W V_W h_W \right) = \\ &= \nabla . \left((K_g X_g + K_W X_W + K_S (1 - X)) \nabla T \right) + \phi , \quad (4.3) \end{aligned}$$

where ϕ is the internal microwave power dissipated per unit volume. In equation (3) the effects of viscous dissipation and compressional work have been omitted.

Equations (1-3) are augmented with the usual thermodynamic relations and the following relations:

Flux expressions are given as follows:

• Gas flux:

$$X_g \rho_g V_g = -\frac{K K_g(S_W) \rho_g}{\mu_g(T)} \left[\nabla P_g - \rho_g g \right] , \qquad (4.4a)$$

• Liquid flux:

$$X_W \rho_W V_W = -\frac{K K_W(S_W) \rho_W}{\mu_W(T)} \left[\nabla \left(P_g - P_C(S_W, T) \right) - \rho_W g \right] , \qquad (4.4b)$$

 $\bullet~{\rm Vapor~flux}$:

$$Xg\rho_{gv}V_{gv} = X_g\rho_{gv}V_g - \frac{X_g\rho_g D\left(T, P_g\right)M_aM_v}{M^2}\nabla\left(\frac{P_{qv}}{P_g}\right) , \qquad (4.4c)$$

• Air flux :

$$X_g \rho_{ga} V_{ga} = X_g \rho_g V_g - X_g \rho_g V_{gv} . \qquad (4.4d)$$

Relative humidity (Kelvin effect):

$$\psi\left(S_W, T\right) = \frac{P_{gv}}{P_{gvs}(T)} = \exp\left(\frac{2\sigma(T)M_v}{r(S_W)\rho_W RT}\right)$$

where $P_{gvs}(T)$ is the saturated vapour pressure given by the 0 Clausius-Clapeyron equation.

Differential heat of sorption:

$$\Delta h_W = R_v T^2 \frac{\partial (\ln \psi)}{\partial T} \; .$$

Enthalpy-Temperature relations:

$$h_{ga} = C_{pa}(T - T_R) ,$$

$$h_{gv} = h_{vap}^0 + C_{pv} (T - T_R) ,$$

$$h_W = C_{pW} (T - T_R) ,$$

$$h_s = C_{ps} (T - T_R) .$$

The expressions for K_g , K_W are those given by Turner and Ilic [12], and μ_g , μ_W have had functional fits according to the data by Holman [13]. The diffusivity

$$D\left(T,P_{g}\right)$$

is given by Quintard and Puiggali [14] and the latent heat of evaporation given by

$$h_{vap}\left(T\right) = h_{gv} - h_W$$

After some mathematical manipulations, the one-dimensional system of three nonlinear coupled partial differential equations which model the drying process in a thermal equilibrium environment are given by :

$$a_{s1}\frac{\partial S_W}{\partial t} + a_{s2}\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z} \left[K_{S1}\frac{\partial S_W}{\partial Z} + K_{T1}\frac{\partial T}{\partial Z} + K_{T1}\frac{\partial T}{\partial Z} + K_{P1}\frac{\partial P_g}{\partial Z} + K_{gr1} \right],$$
(4.5)

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$$a_{T1}\frac{\partial S_W}{\partial t} + a_{T2}\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z}\left(K_e\frac{\partial T}{\partial Z}\right) - \phi\rho_W h_{vap}\frac{\partial}{\partial Z}\left[K_S\frac{\partial S_W}{\partial Z} + K_T\frac{\partial T}{\partial Z} + K_P\frac{\partial P_g}{\partial Z}K_{gr} + \left[\phi\rho_W C_{pW}\left(K_{S2}\frac{\partial S_W}{\partial Z} + K_{T2}\frac{\partial T}{\partial Z} + K_{P2}\frac{\partial P_g}{\partial Z} + K_{gr2}\right)\right]\frac{\partial T}{\partial Z} + \Phi\left(S_W, T\right), \quad (4.6)$$

$$a_{P1}\frac{\partial S_W}{\partial T} + a_{P2}\frac{\partial T}{\partial t} + a_{P3}\frac{\partial P_g}{\partial t} = \frac{\partial}{\partial Z} \left[K_S \frac{\partial S_W}{\partial Z} + K_T \frac{\partial T}{\partial Z} + K_{P3}\frac{\partial P_g}{\partial Z} + K_{gr3} \right] .$$
(4.7)

The capacity coefficients a_{S1} , a_{T1} , a_{p1} and the kinetic coefficients K_{S1} , K_{T1} , K_{P1} , K_{gr1} all depend on the independent variables : Saturation S_W , Temperature T and total pressure P_g . The boundary conditions are written in one dimension as:

At z = 0 (Drying surface) :

$$K_{S1}\frac{\partial S_W}{\partial Z} + K_{T1}\frac{\partial T}{\partial Z} + K_{P1}\frac{\partial P_g}{\partial Z} + K_{gr1} = \frac{K_m M_V}{R\phi\rho_W} \left(\frac{P_{gV}}{T} - \frac{P_{gV0}}{T_0}\right) , \qquad (4.8a)$$

$$K_e \frac{\partial T}{\partial Z} - \phi \rho_W h_{Vap} \left(K_S \frac{\partial S_W}{\partial Z} + K_T \frac{\partial T}{\partial Z} + K_P \frac{\partial P_g}{\partial Z} + K_{gr} \right) = Q \left(T - T_0 \right) , \quad (4.8b)$$

$$P_g = P_o . (4.8c)$$

At z = L (Impermeable surface):

$$K_{S1}\frac{\partial S_W}{\partial Z} + K_{T1}\frac{\partial T}{\partial Z} + K_{P1}\frac{\partial P_g}{\partial Z} + K_{gr1} = 0 , \qquad (4.9a)$$

$$K_e \frac{\partial T}{\partial Z} - \phi \rho_W h_{Vap} \left(K_S \frac{\partial S_W}{\partial Z} + K_T \frac{\partial T}{\partial Z} + K_P \frac{\partial P_g}{\partial Z} + K_{gr} \right) = 0 , \qquad (4.9b)$$

$$(K_{S1} - K_S)\frac{\partial S_W}{\partial Z} + (K_{T1} - K_T)\frac{\partial T}{\partial Z} + (K_{P1} - K_{P3})\frac{\partial P_g}{\partial Z} + (K_{gr1} - K_{gr3}) = 0.$$
(4.9c)

Initial conditions:

$$T(z,0) = T_1$$
, (4.10a)

$$P_a(z.0) = P_0$$
, (4.10b)

$$\frac{\partial P_c}{\partial Z} = -\rho_W g . \qquad (4.10c)$$

The numerical solution technique employed to solve the resulting system over the control volume is discussed by Patankar [15].

5. Discussion

Figures 1 and 2 show a comparison of convective drying with and without microwave. Whilst for convective drying there are definite constant rate periods, when microwaves are added, the form of the curves change and the use of the words constant and falling rate may no longer apply. In hot air drying the constant rate period is the period of drying before the drying front recedes below the outer boundary. In our calculations the SMER (Figure 3) is defined as the energy required to extract a unit of water from the product. Figure 3 shows that the SMER for the microwave case is much lower than the convective case.



Figure 1. Average saturation profiles in time for drying



Figure 2. Drying rate curve corresponding to the profile plotted in Figure 1



Figure 3. Specific moisture extraction rate time for drying



Figure 4. Liquid flux as function of distance



Figure 5. Rate of evaporation as function of distance

The analysis of liquid and gas flux as given by equations (4.4a)-(4.4d) has been exhibited in Figures 4 and 5, for both the combined microwave and convective and convective drying cases. During this period the convective heat transfer is used for evaporation only, resulting in a constant surface temperature and drying rate (see horizontal plateau in Figure 4). The liquid flows to the surface initially in a linear fashion. As the fibers begin to dry out, there is a liquid flux up to what can confidently be explained as the position of evaporation front and there is no movement of liquid from that point to the surface.

6. Conclusion

The primary parameter monitored during the drying tests was moisture content, which may be calculated as a ratio of wet weight minus the final dry weight over the final dry weight. The weights include the weight of the entire sample, tufts, and backing. Weight loss during drying is predominately due to water evaporation. Although other materials such as finish may be driven off during drying, the associated weight loss is insignificant compared to that of water. Thus, the determination of moisture content is simple yet accurate.

For most industrial carpet tile operations, the target moisture content is set so that the weight of moisture on the tiles is approximately 5% of the weight of the face yarns. Whilst for convective drying there are definite constant rate and falling rate periods, when microwaves are added, the use of the words constant and falling rate may no longer apply.

In hot air drying the constant rate period is the period of drying before the drying front recedes below the outer boundary. During this period the convective heat transfer can be used for evaporation only, resulting in a constant surface temperature and drying rate.

As the material begins to dry out, there is a liquid flux up to what can confidently be explained as the position of evaporation front and then there is no movement of liquid from that point to the surface.

According to the results presented in this work, it is concluded that combined microwave and convective drying can provide higher drying rates and consequently faster drying times. Moreover, there are higher fluxes of liquid to the drying surface from the interior of the carpet. A good dryer design should take advantage of this feature and optimize excess water removal from the drying surface.

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