

OPTIMUM DESIGN OF A UNIPLANAR CHS TRUSS FOR FATIGUE

KÁROLY JÁRMAI

Department of Materials Handling and Logistics, University of Miskolc
3515 Miskolc–Egyetemváros, Hungary
altjar@gold.uni-miskolc.hu

[Received: December 20, 2003]

Dedicated to Professor József FARKAS on the occasion of his seventy-fifth birthday

Abstract. The new IIW (International Institute of Welding) fatigue design recommendations are used for the determination of the optimum strut dimensions and truss height minimizing the structural mass or cost. In an illustrative numerical example a simply supported uniplanar CHS truss with parallel chords is designed, which is loaded by a pulsating force. An advanced cost function is minimized which contains the costs of material, cutting and grinding of strut ends, assembly, welding and painting. Fatigue design constraints are formulated for governing X- and K-gap joints. Six strut dimensions are optimized for a series of discrete truss height ratios and the optimum height ratio is selected considering the minimum cost. A parametric investigation is made to find the relation between the optimum truss height ratios and the span length.

Mathematical Subject Classification: 73A05

Keywords: truss structures, fatigue design, optimization, tubular structures, welded structures, cost function

1. Introduction

Tubular trusses are recently more popular, due to their high strength and low weight. They are in many cases subject to fluctuating loads, e.g. cranes, vehicles, bridges, offshore structures, bodies of agricultural machines, etc. Since high stress concentrations arise in their welded joints, it is important to have a reliable fatigue design method. The IIW Subcommission XV-E for welded tubular joints has made great efforts to give designers such methods.

In 1985 design rules were given for fatigue design [1], which made it possible to work out some optimum design applications in this field [2], [3]. Based on a wide international experimental work, the subcommission has developed a modern version of design rules [4], [5]. Our aim is to show how to apply these rules for the optimum fatigue design of a simply supported uniplanar truss constructed from circular hollow section (CHS) rods subject to a fluctuating force (Figure 1).

For the optimization continuous functions are necessary, therefore we use approximate polynomials for stress concentration factors instead of diagrams given in [1]. For correction factors we use the formulae given by Zhao et al. [4] instead of diagrams.

The optimum height (distance between the parallel chords) is determined, which minimizes the mass or cost of the structure. From the point of view of economy it is important to formulate a realistic cost function. For welded plated structures we have developed and applied a relatively simple cost function containing material and welding costs, based on welding times given by the Netherlands Welding Institute [6], [7], [8], [9]. On the basis of cost data given by Tizani et al. [10], we have developed a modified cost function, which considers the specialties of tubular trusses.

2. Problem formulation

A simply supported uniplanar truss with parallel chords is designed (Figure 1). The truss is welded from CHS rods with K-type gap joints and loaded by a pulsating force at midspan.

Data: $a = 2$ m, $L = 12 \times 2 = 24$ m, the range of the pulsating force is $\Delta F = 160$ kN, the number of cycles is $N_F = 10^5$. Three groups of rods are considered having the same cross-sectional area, one for lower chords (d_0, t_0), one for upper chords (d_2, t_2) and one for braces (d_3, t_3). Thus, the number of unknown strut dimensions is 6. The truss height ratio of $\omega = h/a$ is discretely varied with steps of 0.1.

The truss mass as well as cost is minimized for each h/a ratio to obtain the optimum h/a ratio. Design constraints relate to the fatigue strength of governing joints E, F and A. Ranges of validity defined by [5] are related to zero joint eccentricity and limit the main ratios of strut dimensions.

3. Design constraints

3.1. Fatigue strength. The fatigue strength constraints have the following form

$$(MF) \frac{\Delta F_i}{A_i} SCF_0(\beta, \theta) CF(\gamma, \tau) \leq \frac{S_{rhs}}{\gamma_{Mf}}, \quad (3.1)$$

where (MF) is the magnification factor expressing the effect of additional bending moments. Note that other bending effects are not considered, since a geometrical constraint on zero eccentricity is taken into account. SCF_0 is the stress concentration factor depending on $\beta = d_{brace}/d_{chord}$ and on $\theta = \arctan \omega$; CF is the correction factor depending on $\gamma = d_i/2t_i$ and on $\tau = t_{brace}/t_{chord}$, $A_i = \pi(d_i - t_i)t_i$ is the cross-sectional area of rods. Note that, in some cases, instead of $SCF_0 \times CF$ other formulae are used. S_{rhs} is the hot spot stress range depending on the number of cycles and the member thickness. For $N_F = 10^5$ equation (3.2) is used:

$$\log S_{rhs} = \frac{1}{3} (12.476 - \log N_F) + 0.06 \log N_F \log \frac{16}{t_i}, \quad (3.2)$$

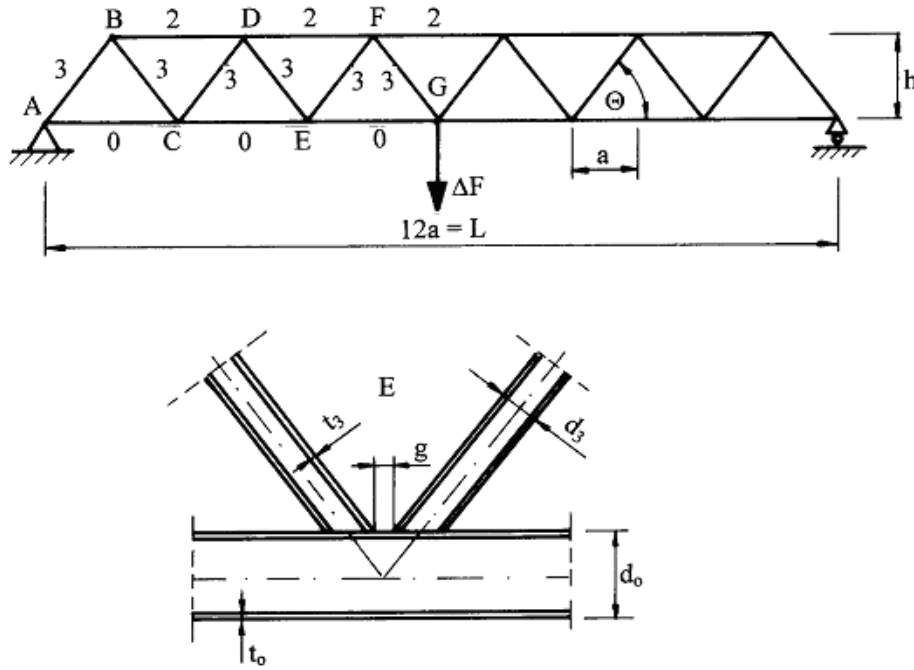


Figure 1. Simply supported uniplanar CHS truss with parallel chords subject to a fluctuating force

where $\gamma_{Mf} = 1.25$ is the fatigue safety factor. It should be mentioned that, for K-gap joints, in the case of axial balanced brace, the values of SCF_0 are given in diagrams. Since for the optimization continuous functions are needed, we have replaced these diagrams by approximate second order polynomials. For CF we have used the formulae given in Zhao et al. [4] instead of the diagrams in [5].

3.2. Fatigue strength of the chord of joint E. The joint E is selected instead of G, since in the chord wall at joint E stress concentration arises also from the balanced axial loading.

$$1.5 \frac{N_{E0}}{A_0} SCF_{CH,CH} + 1.3 \frac{N_{E3}}{A_3} SCF_{0,CH,AX} CF_{CH,AX} \leq \frac{S_{rhs}}{\gamma_{Mf}}. \quad (3.3)$$

The axial member forces are as follows:

$$N_{E0} = \frac{2\Delta F}{\omega}; N_{E3} = \frac{\Delta F(1 + \omega^2)^{0.5}}{\omega}. \quad (3.4)$$

In the calculation of SCF for chord loading the formula given by Zhao et al. [4] in Table D.3 is used instead of Figure D.8 in [5]:

$$SCF_{CH,CH} = 1.2 \left(\frac{t_3}{t_0} \right)^{0.3} (\sin \theta)^{-0.9}. \quad (3.5)$$

In the calculation of SCF_0 for balanced axial loading in the two braces the following approximate continuous formula is used instead of the diagram of Figure D.6 given by [5]

$$SCF_{0,CH,AX} = 0.217 + 0.1171\theta - 0.0009311\theta^2 + (2.99 - 0.173\theta + 0.0017111\theta^2) \frac{d_3}{d_0}. \quad (3.6)$$

In the calculation of CF for balanced axial loading the formula given by Zhao et al. [4] in Table D.3 is used instead of the diagram in Figure D.6 in [5]

$$CF_{CH,AX} = \left(\frac{d_0}{24t_0} \right)^{0.4} \left(\frac{t_3}{0.5t_0} \right)^{1.1}. \quad (3.7)$$

In S_{rhs} (equation 3.2) $t_i = t_0$.

3.3. Fatigue strength of the brace of joint E.

$$1.3 \frac{N_{E3}}{A_3} SCF_{0,B,AX} CF_{B,AX} \leq \frac{S_{rhs}}{1.25}, \quad (3.8)$$

where

$$SCF_{0,B,AX} = 2.49 - 0.078\theta + 0.001664\theta^2 - (3.6 - 0.186\theta + 0.0029333\theta^2) \frac{d_3}{d_0}, \quad (3.9)$$

$$CF_{B,AX} = \left(\frac{d_0}{24t_0} \right)^{0.5} \left(\frac{t_3}{0.5t_0} \right)^{0.5}. \quad (3.10)$$

In S_{rhs} (equation 3.2) $t_i = t_3$.

3.4. Fatigue strength of the chord of joint F.

$$1.5 \frac{N_{F2}}{A_2} SCF_{CH,CH} + 1.3 \frac{N_{F3}}{A_3} SCF_{0,CH,AX} CF_{CH,AX} \leq \frac{S_{rhs}}{1.25}, \quad (3.11)$$

where

$$N_{F2} = \frac{3\Delta F}{\omega}, N_{F3} = N_{E3}. \quad (3.12)$$

For $SCF_{CH,CH}$ equation (3.4) is used, but with t_2 instead of t_0 .

For $SCF_{0,CH,AX}$ equation (3.5) is used, but with d_2 instead of d_0 .

For $CF_{CH,AX}$ equation (3.6) is used, but with d_2 and t_2 instead of d_0 and t_0 .

In S_{rhs} (equation 3.2) $t_i = t_2$.

3.5. Fatigue strength of the brace of joint F.

$$1.3 \frac{N_{F3}}{A_3} SCF_{0,B,AX} CF_{B,AX} \leq \frac{S_{rhs}}{1.25}. \quad (3.13)$$

For $SCF_{0,B,AX}$ equation (3.8) is used, but with d_2 instead of d_0 .

For $CF_{B,AX}$ equation (3.9) is used, but with d_2 and t_2 instead of d_0 and t_0 .

In S_{rhs} (equation 3.2) $t_i = t_3$.

3.6. Fatigue strength of the chord of joint A. Joint A is calculated as an X-joint.

$$\gamma = \frac{d_0}{2t_0}; \tau = \frac{t_3}{t_0}; \beta = \frac{d_3}{d_0}, \quad (3.14)$$

$$1.5 \frac{N_{A0}}{A_0} X_{1,2 \max} \leq \frac{S_{rhs}}{1.25}, \quad (3.15)$$

$$N_{A0} = \frac{\Delta F}{2\omega}, \quad (3.16)$$

$$X_1 = 3.87\gamma\tau\beta (1.1 - \beta^{1.8}) \sin^{1.7} \theta, \quad (3.17)$$

$$X_2 = \gamma^{0.2}\tau [2.65 + 5(\beta - 0.65)^2] - 3\tau\beta \sin \theta. \quad (3.18)$$

Note that the approximate value of α is calculated as

$$\alpha = \frac{2a}{d_0} = \frac{2 \times 4000}{400} = 80 > 12, \quad (3.19)$$

thus, $F_2 = 1$. In S_{rhs} (equation 3.2) $t_i = t_0$.

3.7. Fatigue strength of the brace of joint A.

$$1.3 \frac{N_{A3}}{A_3} X_{3,4 \max} \leq \frac{S_{rhs}}{1.25}, \quad (3.20)$$

where $N_{A3} = N_{E3}$;

$$X_3 = 1 + 1.9\gamma\tau^{0.5}\beta^{0.9} (1.09 - \beta^{1.7}) \sin^{2.5} \theta, \quad (3.21)$$

$$X_4 = 3 + \gamma^{1.2} 0.12 \exp(-4\beta) + 0.011\beta^2 - 0.045, \quad (3.22)$$

In S_{rhs} (equation 3.2) $t_i = t_3$. γ, τ, β are defined in Section 3.5.

3.8. Size constraints. The ranges of validity are as follows:

$$0.3 \leq \frac{d_3}{d_0}, \frac{d_3}{d_2} \leq 0.6, \quad (3.23)$$

$$24 \leq \frac{d_0}{t_0}, \frac{d_2}{t_2} \leq 60, \quad (3.24)$$

$$0.25 \leq \frac{t_3}{t_0}, \frac{t_3}{t_2} \leq 1.0, \quad (3.25)$$

$$30^\circ \leq \theta \leq 60^\circ, \quad (3.26)$$

$$4 \leq t_{0,2,3} \leq 50 \text{ mm}. \quad (3.27)$$

3.9. Constraint on zero joint eccentricity. From the limitation for the gap g that

$$g = \frac{d_{0,2}}{\tan \theta} - \frac{d_3}{\sin \theta} \geq 2t_3, \quad (3.28)$$

one obtains

$$d_{0,2} \geq 2t_3\omega + d_3(1 + \omega^2)^{0.5}. \quad (3.29)$$

4. The cost function

4.1. Cost function components. The cost function contains the costs of material, cutting and grinding of strut ends, assembly, welding and painting

$$K = K_M + K_C + K_A + K_W + K_P \quad (4.1)$$

In the material cost

$$K_M = \rho \sum_i k_{Mi} A_i L_i \quad (4.2)$$

the material cost factors of Price List [11] are used as given in Table 1. The material density is $\rho = 7.85 \times 10^{-6} \text{ kg/mm}^3$. The hot formed CHS profiles are selected according to prEN 10210-2 [12]. Some new profiles are given in [13].

The strut lengths are as follows $L_0 = 24000$, $L_2 = 20000$, $L_3 = 24000(1 + \omega^2)^{0.5}$ mm.

For the calculation of cutting and grinding times of strut ends an empirical formula is developed on the basis of measurements in a Hungarian steel construction factory as follows:

$$T_i = 3.0442x1.007^{di} \text{ (min)}, \quad (4.3)$$

di in mm.

This formula is valid for diagonals. In our example

$$K_C = k_F \Theta_C x 3.0442(2x1.007^{d0} + 2x1.007^{d2} + 24x1.007^{d3}), \quad (4.4)$$

where the difficulty factor is taken as $\Theta_C = 2$ and the fabrication cost factor is selected using the data of Tizani et al [10] as $k_F = 0.6667$ \$/min. Note that the cutting time data of Tizani et al. [10] cannot be used here, since they are related to too small a diameter of 60 mm [14]. It should be mentioned that in our other paper [15] another formula is used which contains also the effect of strut thickness.

$$K_A = C_A k_F \Theta_A (\kappa \rho V)^{0.5}, \quad (4.5)$$

where $C_A = 1.0 \text{ min/kg}^{0.5}$; $\Theta_A = 3.5$; the number of structural elements to be assembled is $\kappa = 14$.

The cost calculation of welding is based on welding times developed from the COST-COMP [6] software for different welding technologies and weld types.

$$K_W = k_F \Theta_W \sum_i C_{Wi} a_{Wi}^n L_{Wi}, \quad (4.6)$$

where the difficulty factor is taken as $\Theta_W = 2$. The fillet weld size is $a_W = t_3$. For fillet welds performed by SMAW (shielded metal arc welding)

$$C_W a_W^n = 0.7889x10^{-3} a_W^2. \tag{4.7}$$

d (mm)	k_M (\$/kg)
88.9, 101.6, 114.3	1.0553
139.7, 168.3, 177.8, 193.7	1.1294
219.1, 244.5, 273.0, 323.9	1.2922
355.6, 406.4	1.3642
457.0, 508.0	1.4081

Table 1. Material cost factors for available hot formed CHS profiles

The weld length in our example is

$$L_W = \pi d_3 (1 + \omega^2)^{0.5} / \omega. \tag{4.8}$$

The painting cost is calculated as

$$K_P = k_P S_P, \tag{4.9}$$

where, according to Tizani et al. [10] the cost factor is $k_P = 14.4$ \$/m². The painted surface in our example is

$$S_P = 10^{-6} \pi (24.000d_0 + 20.000d_2 + 24.000d_3 (1 + \omega^2)^{0.5}). \tag{4.10}$$

5. Mathematical optimization and results

5.1. Constrained function minimization. The constrained function minimization is performed using the Rosenbrock’s hillclimb method with additional discretization to find the corresponding available cross-sectional dimensions [8]. The results are summarized in Tables 2, 3 and Figure 2.

The optimum solution of $h/a = 1.5$ is marked by bold letters.

The optimum strut dimensions in the case of $h/a = 1.5$ are given in Table 3.

$\omega = h/a$	K (\$)
1.0	27599.3
1.1	27344.5
1.2	27061.8
1.3	26592.2
1.4	26061.3
1.5	25942.8
1.6	26491.1
1.7	26912.8
1.8	27112.5

Table 2. The cost of continuous (nondiscrete) solutions against the truss height ratio h/a

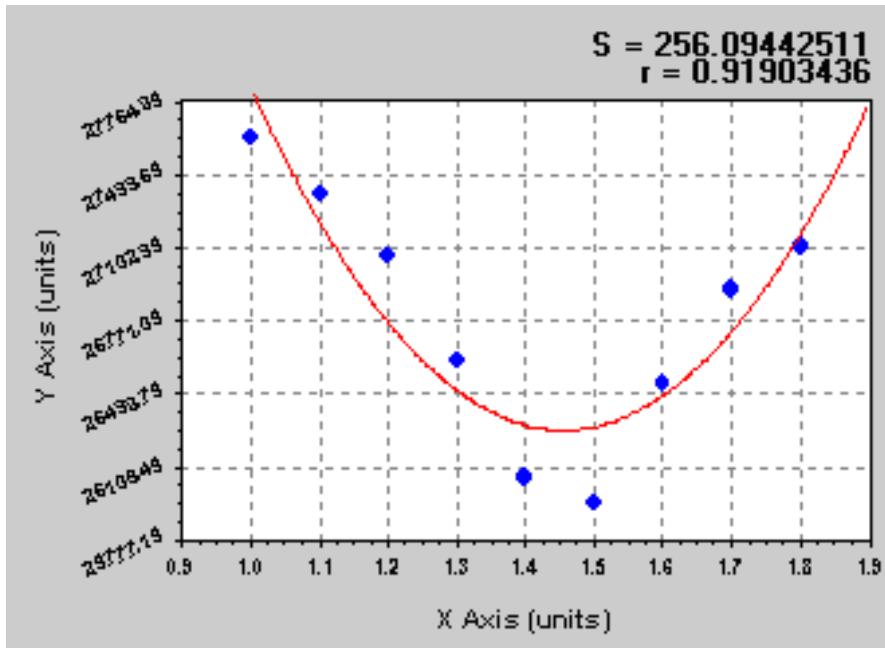


Figure 2. Cost against h/a ratio

d_0, t_0	323.9x12.5
d_2, t_2	323.9x12.5
d_3, t_3	168.3x5

Table 3. Optimum strut dimensions in mm for $h/a = 1.5$

L (m)	optimum value of $\omega = h/a$
15	1.3
20	1.4
24	1.5
30	1.5
35	1.6
40	1.6
45	1.7
50	1.7

Table 4. Optimum value of $\omega = h/a$ for different span lengths

We have made a parametric survey changing the span length keeping all other parameters at the same values and calculating the optimum values of $\omega = h/a$. Due to the limits on θ , i.e. $30^\circ \leq \theta \leq 60^\circ$, ω can be varied between 0.6 and 1.8 .

It is visible from Table 4 that the optimum value of ω is proportional to the span length. All optima in the calculation were nondiscrete, so the discrete solution distances can be different from the nondiscrete ones. The optimum value of ω is changing, depending on the shape of structure, loadings, supports, etc. so it is difficult to arrive at a general conclusion. At least we can say that ω has an optimum value and it is worth finding.

6. Conclusions

In the welded joints of tubular trusses high stress concentrations occur. The new IIW fatigue design rules enable designers to calculate the stress concentration factors more precisely than previously. This calculation method is used for the optimum design of a uniplanar CHS truss subject to a fluctuating force.

In the optimization process the cross-sectional dimensions and the distance between the parallel chords (truss heights) are optimized, which minimizes the structural cost. The height is discretely varied. Three rod groups are defined having the same cross-sectional area, thus six unknown variables are optimized for each truss height ratio.

The existence of an optimum height can be explained by the fact that, increasing the height, the chord forces decrease, but the branch length increases and this tendency turns back when the height decreases.

The difference between the cost corresponding to the best and worst solution, indicated in Table 2, is 6.4%.

After a parametric survey, changing the span length, we have found that ω has an optimum value and it is proportional to the span length.

The advanced cost function, which contains the costs of material, cutting and grinding of strut ends, assembly, welding and painting, enables designers to calculate the costs more realistically than previously.

Acknowledgement. The research work was supported by the Hungarian Scientific Research Foundation grants OTKA T38058, and T37941. The author gratefully acknowledges the help of Prof. József Farkas (University of Miskolc, Hungary) and Prof. Erkki Niemi (Lappeenranta University of Technology, Finland) in formulating the problem.

REFERENCES

1. Recommended fatigue design procedure for hollow section joints: Part 1. Hot spot stress method for nodal joints, IIW Subcomm. XV-E. IIW-Doc. Strasbourg, 1985, pp. 582–585.
2. FARKAS, J.: Optimum design for fatigue of trusses welded from circular hollow sections. *Acta Technica Acad. Sci. Hung.* **100**(1-2), (1987)
3. FARKAS, J.: Minimum cost design of tubular trusses considering buckling and fatigue constraints. Niemi, E. and Mäkeläinen, (eds.) *Tubular Structures*. P. Elsevier, London-New York, 1990, pp. 451–459.
4. ZHAO, X.L. AND HERION, S. ET AL.: Design guide for circular and rectangular hollow section joints under fatigue loading, IIW-Doc. XV-E, 1998, pp. 98–236.

5. Recommended fatigue design procedure for welded hollow section joints: Part 1-2, IIW-Doc.XIII-1804-99, IIW-Doc.XV, 1999, pp. 1035–1099.
6. COSTCOMP: Programm zur Berechnung der Schweisskosten, Düsseldorf: Deutscher Verlag für Schweißtechnik, 1990.
7. BODT, H.I.M.: *The global approach to welding costs*. The Hague: The Netherlands Institute of Welding, 1990.
8. FARKAS, J. AND JÁRMAI, K.: *Analysis and Optimum Design of Metal Structures*. Rotterdam-Brookfield, Balkema, 1997.
9. JÁRMAI, K. AND FARKAS, J.: Cost calculation and optimisation of welded steel structures. *J. Constructional Steel Research*, **50**, (1999), 115–135.
10. TIZIANI, W.M.K. AND YUSUF, K.O. ET AL.: A knowledge based system to support joint fabrication decision making at the design stage – Case studies for CHS trusses, Farkas, J., and Jármai, K., (eds.) *Tubular Structures VII.*, Balkema, Rotterdam-Brookfield, 1996, 483–489.
11. *PRICE LIST 20*. Steel tubes, pipes and hollow sections. Part 1b. Structural hollow sections, British Steel Tubes and Pipes, 1995.
12. *prEN 10210-2*. Hot finished structural hollow sections, 1996.
13. DUTTA, D.: *Structures with hollow sections*. Ernst & Sohn, Berlin, 2002.
14. FARKAS, J. AND JÁRMAI, K.: *Economic design of metal structures*. Millpress Science Publisher, Rotterdam, 2003.
15. FARKAS, J. AND JÁRMAI, K.: Weight optimization of a triangular CHS truss using an improved cost function, R. Puthli, and H. Herion, (eds) 9th International Symposium on Tubular Structures, April 4-8. 2001. Düsseldorf, Balkema Publishers, Rotterdam, 2001, 429-435.