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NUMERICAL OPTIMIZATION OF A HIGH PRESSURE STEAM TURBINE STAGE

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Abstract. Blading of a high-pressure (HP) steam turbine stage is optimized using an idea of direct constraint optimization. The objective function to be minimized is the enthalpy loss of the stage. A simplex method of deformed polyhedron proposed by Nelder-Mead is used for optimization. Current values of the objective function are found from 3D Reynolds-averaged Navier Stokes (RANS) computations. To secure global flow conditions, there are constraints imposed on the mass flow rate, exit swirl angle, and reaction. The optimized parameters here are the stator and rotor blade numbers and stagger angles, rotor blade twist angle and parameters of stator blade compound lean at root and tip. Blade profiles are not changed. Optimization gives a design with new 3D stacking lines of the blades and increased flow efficiency, compared to the original design.

Mathematical Subject Classification: 76F99
Keywords: HP axial-flow turbine stage, blade shape optimization, simplex method, blade lean and twist, 3D RANS solver

1. **Introduction**

Optimization of 3D blading in turbomachinery is a relatively new field of research. It has become possible only with the development of 3D Navier-Stokes codes for turbomachinery applications capable of locating and quantifying loss generation processes in complex turbomachinery geometries, and also thanks to increasing capabilities of computing machines. A robust 3D solver in collaboration with a reliable optimization technique can be a powerful design tool, also in the turbomachinery environment. At present two optimization/design approaches become conspicuous with respect to turbine blading systems. One approach concentrates on development of 3D inverse design using Euler or Navier-Stokes codes where the shape of the blading changes during an iterative procedure until the target distribution of, for example, blade surface static pressure, or downstream velocity is reached, guaranteeing the required performance, e.g. [1,2]. Another approach focuses on optimization of global characteristics of the stage, and the final shape is obtained from minimising/maximising an objective function, for example the total energy loss or efficiency, total pressure loss of the stage, etc. Values of the objective function for new geometries can be found directly from 3D viscous flow computations, e.g. [3,4]. However, as the direct optimization is time consuming and the main cost is time required for the Navier-Stokes solver, an interesting idea of approximate representation of the objective function using an artificial neural network trained over a data base of RANS solutions was put forward in [5]. In the process of optimization new geometries can be selected using a number of optimization methods, the most popular of which are the steepest gradient method, simplex methods and genetic algorithms.

Optimization of 3D blading in low-pressure (LP) steam turbine stages, especially for exit stages, is shown in [4,6] to bring considerable efficiency gains. There seems to be less room for efficiency improvements in low-load high-pressure (HP) steam turbine stages, [3,7]. Efficiency of cylindrical (radially stacked and straight) blades is relatively high here unless a mistake was made while selecting basic parameters of the blading, like stator and rotor blade numbers and stagger angles. This, however, makes the optimization task even more challenging.

A typical 3D design to raise the efficiency of HP stages is compound lean where some blade sections, usually at the root and tip, are linearly or non-linearly displaced in circumferential direction with respect to other sections. The effects of this design were investigated e.g. in [7-9]. It was found there that lean redistributes blade load, mass flow rate and loss span-wise, compared to the cylindrical blading, and can serve as a means of controlling secondary flow and tip leakage losses in HP turbines. Lean can change the state of boundary layers at the blade suction surface and at the end-walls, as well as can reduce span-wise variations of the exit swirl angle, which is likely to reduce downstream mixing losses and make stator/rotor matching easier. The quantitative effect of lean on the stage loss varies with turbine stage geometry and operation conditions. Due to a number of shape parameters involved in this design it is required that automatic optimization techniques are used to find the optimum design.

The procedure for optimization of an HP steam turbine stage is based on the concept of direct constrained optimization, using the Nelder-Mead method of deformed polyhedron and a 3D RANS solver for turbomachinery applications. Direct optimization is considered highly time consuming. The main cost is CPU time required for the Navier-Stokes solver that computes changing flow geometries. Therefore, 3D computational grids used during optimization will be relatively coarse. However, some experience of the author shows that major tendencies in changing flow patterns with changing geometry of the turbine stage can already be discovered on coarse grids. Anyway, at least the original and final geometries will be checked on refined grids, and possible changes in flow patterns and efficiency gains will be implied based on the comparison of post-optimization computations of the original and final geometries on refined grids.

2. **Method of optimization**

2.1. **Optimised functions and parameters.** Shape optimization of turbine blading is carried out with the help of a code Optimus, [10]. Optimization is understood here as an iterative procedure that seeks for an extremum of an objective function – a function of some shape parameters.

In this investigation, the optimized (minimized) objective function is the enthalpy loss of the stator/rotor stage (the leaving energy not considered a loss and assumed to be used in the subsequent stage), defined as $\xi = (h_2 - h_{2s})/(h_{0T} - h_{2s})$ where h_{0T}, h_2, h_{2s} are the inlet total enthalpy, exit static enthalpy and isentropic enthalpy referred to the exit static pressure, respectively. Alternatively, one can also choose the total enthalpy loss of the stage including the leaving energy (as for the exit turbine stages), or stage power as an objective function. The following parameters of blade shape can be considered during the optimization for each blade row: blade number, stagger angle, blade height, linear twist angle, linear lean angle, and linear sweep angle, 4 parameters of compound twist (2 at hub, 2 at tip), 4 parameters of compound lean (2 at hub, 2 at tip), and 4 parameters of compound sweep (2 at hub, 2 at tip). Each parameter is allowed to vary in a prescribed range of variation.

2.2. **Flow constraints.** To secure global flow conditions, there are also constraints imposed on the mass flow rate, exit angle, average reaction, reaction at tip and root. The exit angle and reactions are not allowed to assume values beyond the prescribed ranges, which is pronounced in the shape of the objective function f, that is $f = \zeta$, if the exit angle and reactions fall within the prescribed range, or $f = \infty$ (a very large number) otherwise, where ζ is a value of an optimized characteristic (enthalpy loss of the stage) obtained from the RANS solver. The penalty function is imposed on the mass flow rate if it falls beyond a required very narrow interval $[G_-, G_+]$: $f = \zeta$, if $G_- \leq G \leq G_+$, or $f = \zeta + min[(G - G_\pm)^2]/\varepsilon$ otherwise, where G - current mass flow rate, ε – penalty coefficient prescribed in a way that the objective function sharply rises to infinity with increasing distance from the limits of the assumed range of variation. As the pressure drop during the optimization is kept constant and the mass flow rate is constrained in a very narrow range of variation, any change of power of the optimized stage can be attributed to the reduced flow losses only.

2.3. **Method of deformed polyhedron.** It is generally accepted that none of the available optimization methods can be considered superior for all types of turbomachinery applications. The most popular seem to be the steepest gradient method, simplex-based methods and genetic algorithms. In this investigation, subsequent geometries in the course of optimization are selected using a simplex method of deformed polyhedron proposed by Nelder & Mead [11]. In this method, optimization of the objective function of n independent variables is performed using $n+1$ points being vertices of a polyhedron in the space of optimized parameters R^n. The initial polyhedron is usually generated in a random way from the prescribed range of variation of the optimized parameters, or the choice of initial vertices can also be supported by an educated guess, which is drawing on some engineering knowledge. The first aim of each iteration during the optimization procedure is to replace the least favorable vertex for which the objective function reaches the 'worst' value with a new vertex, and to form a new polyhedron. This is done with the help of the following operations: symmetrical reflection with respect to a gravity center, stretching, compression or reduction. These operations enable deformation of the polyhedron and its adapting to the topography of the objective function. An extremum of the objective function can be found even far away from the initial polyhedron. Of great importance for the effective operation of the algorithm is proper selection of the reflection ratio α, compression ratio β, stretching ratio γ and reduction ratio δ. An adequately chosen reflection ratio is decisive for the rate of convergence of the algorithm and its ability to go beyond the vicinity of local extrema and to find a path towards a global extremum. Certainly there is no guarantee that an extremum to which the algorithm would converge is a global extremum. Anyway, there are at least three ways of further treatment if the designer feels that the algorithm holds on to a local extremum. First, to recognise the fact that a local minimum is a solution whose objective function is 'better' than that of the original design. Second, to increase the reflection ratio, and third, to begin the optimization process from another initial polyhedron.

Usually, Nelder-Mead's method of deformed polyhedron enables efficient optimization of 5-10 geometrical parameters of 3D blading. Unlike for gradient-based methods, the efficiency of the deformed polyhedron method measured by the number of calculations of the objective function within one iteration, does not depend on the number of optimized parameters, and on average is limited to 2-3 RANS calculations per iteration. Yet it usually requires more iterations to converge, as compared to the steepest gradient method, however, fewer than in the case of genetic algorithms. Details of the algorithm used can be found in [3,6].

2.4. **3D RANS solver.** Values of the objective function are found from postprocessing CFD computations performed with the help of a code FlowER - solver of viscous compressible flows through multi-stage turbomachinery [12]. The solver draws on the set of Reynolds-averaged Navier-Stokes equations for perfect gas. During the optimization on coarse grids the effects of turbulence are taken into account with the help of a modified algebraic model of Baldwin-Lomax [13], whereas verifying computations of original and final geometries on refined grids are made using the Menter SST turbulence model [14]. This is a two-equation eddy-viscosity model where the standard $k-\omega$ model is activated in the sublayer and logarithmic region, and then switched to the $k-\varepsilon$ model in the wake region of the boundary layer, and where the eddy viscosity is redefined so as to guarantee the proportional relationship between the principal turbulent shear stress and the turbulent kinetic energy in the boundary layer. The governing equations are solved numerically based on the Godunov-type upwind differencing, high resolution ENO scheme and implicit operator δ of Beam & Warming, assuring second-order accuracy everywhere in space and time. An H-type multi-grid refined at the endwalls, blade walls and trailing and leading edges is used. The computations are carried out in one blade-to-blade passage of the stator and rotor, and converge to a steady state, with the condition of spatial periodicity, and mixing plane approach assumed. The assumed inlet/exit boundary conditions impose the pressure drop and let the mass flow rate be resultant.

The code has recently been validated on a number of turbomachinery test cases, including Durham Low Speed Turbine Cascade, NASA Rotor37, NASA Low Speed Centrifugal Compressor and a model air turbine of ITC Łódź. For details of the flow solver and its validation the reader is referred to [15-18].

3. **Optimization of a high-pressure steam turbine stage**

The optimized stage is an HP impulse stage of a 200 MW steam turbine with originally cylindrical blades and shrouded rotor blades. The stage power is about 5 MW, its average reaction 17%. The stage operates at a pressure drop $p_2/p_0 = 0.9$. Initial geometrical parameters of the stator and rotor are as follows: span/chord - 0.8 (stator) and 2 (rotor); stagger angles as defined between the profile chord and the normal to the cascade front – 46^o (stator) and 72^o (rotor), blade numbers – 50 (stator), 120 (rotor).

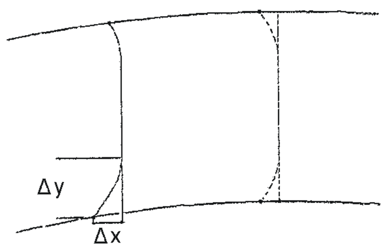

Figure 1. Compound lean geometry of stator blade – leading and trailing edge in circumferential view

In this investigation, the 3D-shaped compound leaned stator blade is optimized together with stator and rotor blade numbers and stagger angles as well as rotor blade twist. There are nine geometrical parameters for the Nelder-Mead method - stator blade number and stagger angle, rotor blade number and stagger angle, rotor blade linear twist angle, and four parameters of stator blade compound lean – two compound lean displacements at each endwall as defined in Figure 1. The blade sections (profiles) are assumed not to change during the optimization. The objective function is the enthalpy loss of the stage not including the leaving energy. A penalty is imposed on the mass flow rate if it changes by more than ±0.5%, compared to the original geometry. The average reaction is assumed not to exceed the original value. To control changes of the exit energy, the absolute exit swirl angle is not allowed to vary beyond the interval $(-10^o, 10^o)$. Flow computations (3D RANS) are carried out for perfect gas assuming the specific heat ratio $\gamma = 1.3$ and individual gas constant $R = 430\, J/kgK$. Tip leakage flow over shrouded rotor blades is not evaluated here. Due to time restrictions, RANS computations in the course of optimization are carried out on coarse grids of 100 000 cells (stator + rotor), also using the faster and less expensive turbulence model of Baldwin-Lomax. After optimization, the original and optimized geometries were recalculated on refined grids – 800 000 cells (stator + rotor), using the Menter SST turbulence model.

The optimization process was completed after 65 iterations (with 120 geometries calculated). The objective function was decreased by 0.4% on a coarse grid. The previously cylindrical stator blades acquired compound lean shapes with the lean direction opposite to that of rotation of the rotor blades at both endwalls. At the trailing edge of the new stator blade the compound lean displacements at the tip are larger, compared to those at the root. Some rotor blade twist was introduced, closing throats towards the tip. Stator and rotor blade numbers and stagger angles were also modified. Changes of the optimized parameters are given in Table 1.

Table 1. Change of the optimized parameters of the HP turbine stage (l - blade height)

Optimised parameter	Its change
Stator blade number	3
Rotor blade number	−3
Stator stagger angle at hub[1] [^{o}]	0.7
Rotor stagger angle at hub[1] [^{o}]	−0.5
Rotor twist angle[2] [^{o}]	−1.3
Stator compound lean displacement at tip[3] $\Delta x/l$	−0.08
Stator compound lean displacement at tip[1] $\Delta y/l$	0.28
Stator compound lean displacement at hub[3] $\Delta x/l$	−0.05
Stator compound lean displacement at hub[1] $\Delta y/l$	0.20

[1] positive value of stagger angle increment opens throats, negative value closes throats
[2] positive value of twist angle opens throats towards tip, negative value closes throats towards tip,
[3] positive value of stator blade at the hub/tip is protruded with rotation of the moving blades

A comparison of the flowfield results for the original and optimized stage (obtained on refined grids using the Menter SST turbulence model) is presented in Figures 2 to 6. The comparison exhibits significant differences in total pressure and entropy patterns in characteristic sections of the stator and rotor, as well as in span-wise distribution of pressures, velocities and losses. Changes of each optimized parameter have an effect on the stage performance. Two groups of parameters can be distinguished whose effects can clearly be separated – the first group consisting of stator and rotor blade numbers and stagger angles plus rotor blade twist (5 parameters), the other group consisting of four parameters of stator blade compound lean at the hub and tip.

The effect of the first group is best observed in Figure 2 showing the entropy function contours in the rotor 10% of the blade span from the root. The contours exhibit the presence of a separation zone at the front part of the rotor blade suction surface at the root in the original design. The size of the separation zone at the root in the optimized design is considerably reduced. This is owing to changes of stator blade number and stagger angle which decrease the incidence on the rotor blade by about 2 degrees (in the rotating frame of reference). At the same time changes in the rotor

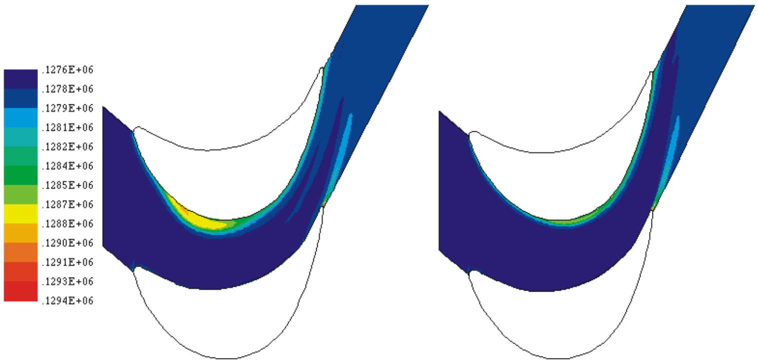

Figure 2. Entropy function contours in rotor 10% blade span from root; original (left), optimized (right)

blade number and stagger angle together with introduced rotor blade twist keep the well-streamlined flow patterns at the mid-span and tip, as well as the average reaction unchanged.

The effect of the second group of parameters is best observed in Figures 3 to 5. The chosen direction of compound lean (opposite to that of rotation of the moving blades) increases pressure at the endwalls in the stator, unloads stator blades and

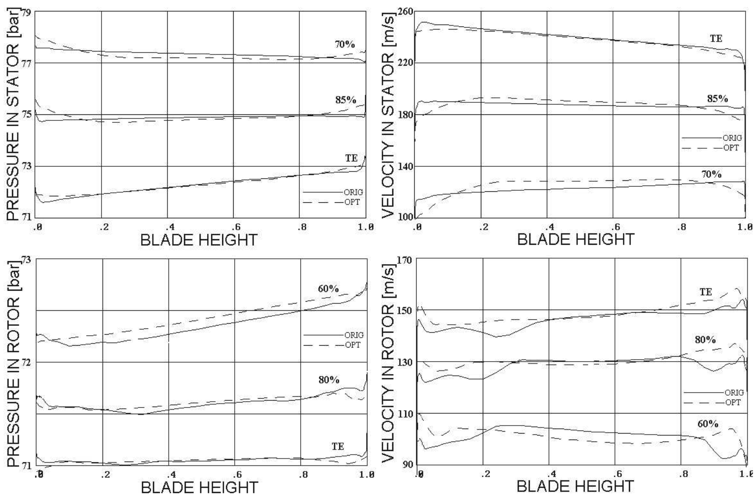

Figure 3. Span-wise distribution of pressure (right) and velocity (left) in the stator (top) and rotor (bottom)

reduces velocities there – Figure 3. This should reduce boundary layer losses near the endwalls. Let us note here that the boundary layer losses can be assumed proportional to the integral from the boundary layer edge velocity (relative to the exit isentropic velocity) in third power over the wall area, Denton [19]. However, as a result of additional span-wise pressure gradient there is increased convection of boundary layer fluid toward mid-span sections in the optimized design, which can be observed from total pressure contours downstream of the stator presented in Figure 4. The secondary flow maxima downstream of the stator at the mixing plane are less intensive in the optimized design, however, the wake seems to be slightly thicker at the mid-span.

The streamline curvature changes in the axial gap between the stator and rotor. In the optimized design, the pressure and velocity near the endwalls in the rotor exhibit opposite tendencies to those in the stator. The pressure decreases and velocity increases at the endwalls in the rotor, as compared to the original design. More mass is passed through the endwall regions, which is likely to increase endwall losses. However, the centres of loss due to secondary flows stay nearer the endwalls, see total pressure contours at the rotor trailing edge presented in Figure 4.

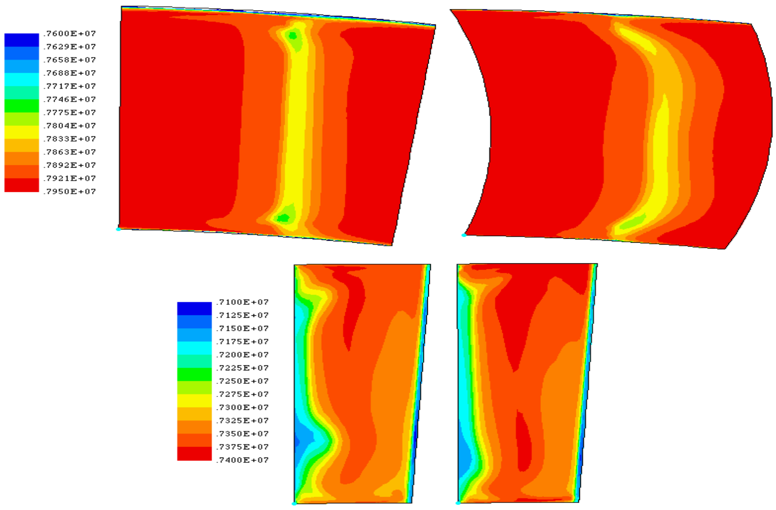

Figure 4. Total pressure contours downstream of the stator (top) and at the rotor trailing edge; original (left), optimized (right)

Figure 5 shows the comparison of span-wise distribution of enthalpy losses in the stator, rotor and stage for the original and optimized design. In the optimized design, stator losses are considerably decreased near the endwall sections, and negligibly increased at the mid-span. Quantitatively, the stator losses (mass-averaged) are decreased by 0.3%. Rotor losses are increased at the endwalls. However, peaks due to secondary flows and separation at the root are considerably lower. Quantitatively, the rotor losses are decreased by 0.8% here. The combination of stator and rotor losses shows that stage losses without the exit energy are decreased near the endwalls, and

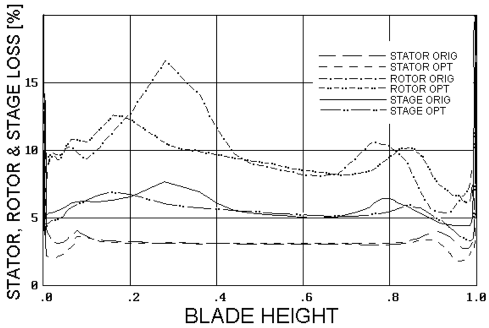

Figure 5. Span-wise distribution of enthalpy losses in stator, rotor and stage (without leaving energy)

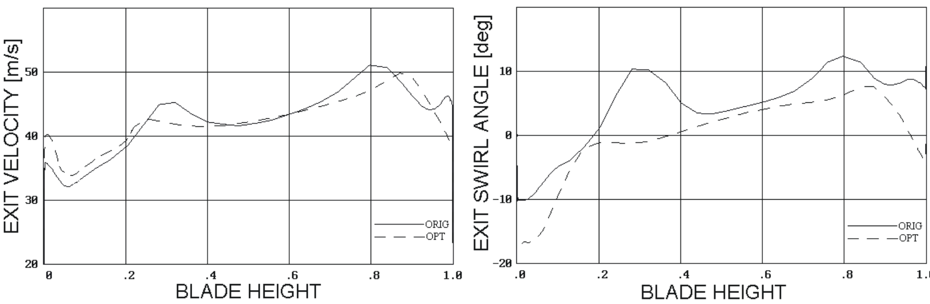

Figure 6. Span-wise distribution of exit velocity (left) and exit swirl angle (right)

3D peaks are lower than in the original design. Quantitatively, the stage losses are decreased from the calculated value of 5.9% down to 5.5%, that is by 0.4%. The level of loss decrease on a refined grid is here the same as on a coarse grid. Note that the stator losses are determined at the mixing plane, whereas the rotor and stage losses at a section located 40% of the rotor axial chord downstream of the rotor trailing edge.

The exit energy was not optimized in the process of optimization, but its value was controlled due to constraints imposed on the mass flow rate and mean exit swirl angle. Figure 6 shows the comparison of exit velocity and exit swirl angle in the non-rotating reference frame for the original and the optimized stage. For approximately the same mass flow rate (it turned out to be decreased by 0.1% compared to the original design), the mean exit velocity is slightly decreased in the optimized design. This is largely due to reduction of 3D peaks in exit swirl angle. Apart from the root section, the redistribution of the exit swirl angle should reduce downstream mixing loss and provide more favorable inlet conditions to the subsequent stage.

4. **Summary**

Stator blade compound lean was optimized together with stator and rotor blade numbers and stagger angles, as well as rotor blade twist in a low-load HP steam turbine stage. Nine geometrical parameters were optimized to reduce the stage loss (without the leaving energy). The idea of direct constrained optimization was presented. Geometries changing in the course of optimization were found using Nelder-Mead's method of deformed polyhedron, whereas the corresponding values of the objective function were found using a 3D RANS solver. There were also constraints imposed on the mass flow rate, exit swirl angle, and reaction. As a result of optimization, the stator of the tested stage acquired a new 3D stacking line, and the calculated stage loss without the leaving energy was decreased by 0.4%. The loss decrease can be attributed here to a reduced size of the separation zone at the rotor root, and decreased endwall and secondary flow losses. The obtained efficiency gains are not impressive here. However, further efficiency gains can still be expected due to a more favourable distribution of exit velocity and swirl angle, which is likely to reduce the downstream mixing loss and provide more favourable inlet conditions for the subsequent stage.

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