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STABILITY ANALYSIS OF AN ORTHOTROPIC PLATE VIA MATHEMATICA

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Abstract. Stability analysis of an orthotropic plate is studied using the algebra system *Mathematica*. The critical force is computed for given material parameters, geometry and load type (*direct problem*). Then the critical force is assumed to be known and the material and geometric parameters are computed (*inverse problem*). The inverse problem can be treated similarly to the direct problem because the numerical solution of both problems can be reduced to the symbolic-numerical solution of a matrix eigenvalue problem.

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1. Introduction

Bridge construction is one of the most important application fields of orthotropic plates used as roadway plates. Orthotrophy is achieved by transversal ribs resulting in different stiffnesses in the directions x and y, respectively. Employing the Huber equation [1] , Szabó [2] developed a numerical technique based on matrix algebra to study such plates a load in their plane. Later, it was realized that stability analysis is also necessary because of the heat dilatation phenomena (Popper [3]). In this paper we shall consider a slightly modified, more general version of the Huber equation, namely:

$$\partial_{x,x}\left(A\partial_{x,x}w\left[x,y\right]+2H\partial_{y,y}w\left[x,y\right]\right)+\partial_{y,y}\left(B\partial_{y,y}w\left[x,y\right]+g\left(x,P\right)w\left[x,y\right]\right)=0 \quad (1)$$

Herer w is the displacement function. A and B denote bending stiffnesses in the directions x, y, respectively, and H is the torsional stiffness. The function $g(x, P)$ represents the distribution of load on the edges of plate (see Figure 1). The (1) is associated with linear boundary conditions on the boundary S:

$$lw(x, y)|_S = 0. \tag{2}$$

Concerning the stability problem in this model one may consider P, the parameter of the edge force distribution, to be a critical parameter. This is the classical direct problem, namely to compute the critical load. The other possibility is to consider one of the parameters A, B i.e. the stiffnesses of the plate, which is subjected to a constant load, to be a critical parameter. This is the inverse stability problem.

In this paper we suggest a general method to handle the direct and inverse stability problems which are of the same form from a mathematical point of view. The method fully utilizes the symbolic and numerical computation capabilities of the integrated computer algebra system *Mathematica*.

2. Mathematical background

In the following some introductory concepts from Functional Analysis and the Galerkin method, which we shall apply, are presented shortly.

Let Ω be a constrained domain in the plane xy, and let $L_2(\Omega)$ denote the set of all square-integrable (real) functions defined over the domain Ω. If $u \in L_2(\Omega)$, then $\int_\Omega u^2 d\Omega < \infty$.

The set $L_2(\Omega)$ is a vector space in which the *scalar product* is defined by the formula

$$(u|v) = \int_\Omega uv \, d\Omega ,$$

which induces a *norm* given by the formula

$$\|u\| = \sqrt{(u|u)} = \sqrt{\int_\Omega u^2 \, d\Omega} .$$

The vectors $u,\ v \in L_2$ are said to be *orthogonal*, if $(u \mid v) = 0$. Their *distance* is defined by the norm $|| \, u - v \, ||$. Any sequence of *linearly independent vectors* $\{\varphi_1, \varphi_2, \ldots\}$ in the function space $L_2(\Omega)$ form a *basis* for L_2 if every vector $u \in L_2$ can be approximated with arbitrary precision by a linear combination of functions $\{\varphi_1, \varphi_2, \ldots\}$. In other terms, if for every function $u \in L_2(\Omega)$ and arbitrary number $\varepsilon > 0$ there exists a positive integer N and scalar coefficients $c_1, c_2, \ldots, c_n$ such, that for every index $n > N$ it holds that

$$\left\| u - \sum_{k=1}^{n} c_k^{(n)} \varphi_k \right\| < \varepsilon.$$

The following *theorem* is a well-known result from the functional analysis:

If $v \in L_2$ is orthogonal to each element of a basis $\{\varphi_1, \varphi_2, \dots\}$ in L_2, then v is the null-element of the space L_2. In other terms if

$$(v|\varphi_k) = 0,\ k = 1, 2, ... \Rightarrow v = \Theta\,.$$

Suppose that in the function space $L_2(\Omega)$ is given the equation

$$Lu = f\,,$$

where L is a differential operator, which transforms the unknown function u into the given function f. The solution u is supposed to satisfy the prescribed boundary conditions on the boundary S of the domain Ω. If we find a function $u^* \in L_2$ which satisfies the (infinite) set of equations

$$(Lu^* - f|\varphi_k) = 0, \quad k = 1, 2, ...,$$

then it is a consequence of the previous theorem that $Lu^* - f = \Theta$. In other terms u^* is the solution of the equation $Lu = f$ in the space L_2. This simple idea forms the basis for the *Galerkin method*: The *approximate* solution of the equation $Lu = f$ is searched for in the form:

$$u_n = \sum_{k=1}^{n} c_k \varphi_k,$$

where n is an arbitrarily chosen, but fixed positive integer. In other terms, the approximate solution u_n of the exact solution $u^* \in L_2$ is expressed in the n-dimensional subspace of L_2 spanned by the base vectors $\varphi_1, \varphi_2, \dots, \varphi_n$. The unknown coefficients $c_1, c_2, \dots, c_n$ are computed from the orthogonality condition

$$(Lu_n - f|\varphi_k) = 0,\ k = 1, 2, ..., n,$$

which is a system of n equations. If the operator L is linear, then

$$Lu_n = \sum_{i=1}^{n} c_i L\varphi_i,$$

and hence this equation can be written in the usual form of a set of linear equations

$$\sum_{i=1}^{n} c_i \left(L\varphi_i|\varphi_k\right) = \left(f|\varphi_k\right), \quad k = 1, 2, ..., n,$$

or in matrix form

$$\mathbf{Mc} = \mathbf{z},$$

where

$$\mathbf{M} = \begin{bmatrix} (L\varphi_1|\varphi_1) & \cdots & (L\varphi_n|\varphi_1) \\ \vdots & \ddots & \vdots \\ (L\varphi_1|\varphi_n) & \cdots & (L\varphi_n|\varphi_n) \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} (f|\varphi_1) \\ \vdots \\ (f|\varphi_n) \end{bmatrix}.$$

In our problem the operator L is given by (1). With respect to the relation

$$H = \kappa\sqrt{AB},$$

and taking into account that there is no load perpendicular to the surface of the plate, we have

$$f = 0\,.$$

Hence our problem is reduced to solving the set of homogeneous linear equations

$$\mathbf{Mc} = \mathbf{0}. \tag{3}$$

It is well known that a non-trivial solution exists if and only if the matrix $\mathbf{M}$ is singular. Consequently, the condition for the lose of stability is given by the equation

$$det[\mathbf{M}(A, B, P)] = 0.$$

If two variables from among A, B, P are known, then the unknown one can be determined from the previous equation, which is a polynomial equation of degree n in the unknown variable. Its smallest root is the critical value sought. The solution $\mathbf{c}$ of the corresponding set of equations $\mathbf{Mc} = \mathbf{0}$ can be normalized to be unique.

3. Application to an orthotropic plate

3.1. **A simple problem.** To demonstrate the suggested method, a simple problem was considered using notations of *Mathematica*. Figure 1 shows the geometry and boundary conditions as well as the type of load.

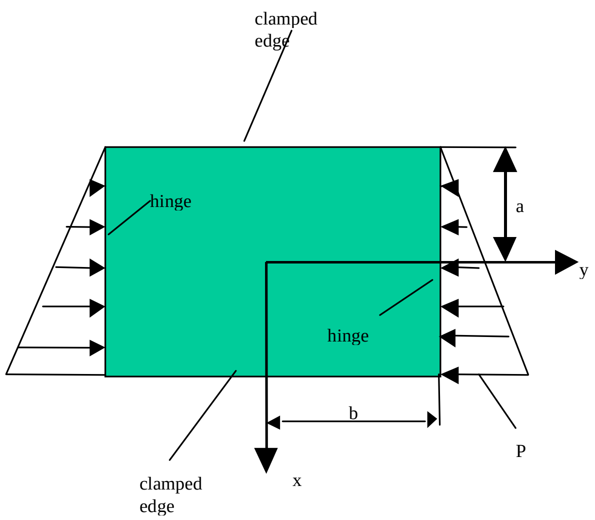

Figure 1. The geometry, boundary conditions and the type of load of the plate

The modified Huber equation is an elliptical PDF of order four associated with a linear force distribution on the edge

$$\partial_{x,x}\left(A\partial_{x,x}w\left[x,y\right]+2H\partial_{y,y}w\left[x,y\right]\right)+\partial_{y,y}\left(B\partial_{y,y}w\left[x,y\right]+\frac{P}{2}\left(x+a\right)w\left[x,y\right]\right)=0.$$

The homogeneous boundary conditions are the following:

– there is no displacement at the edges

$$w\left[a,y\right]=w\left[-a,y\right]=w\left[x,b\right]=w\left[x,-b\right]=0,$$

– edges parallel with the y axes are clamped

$$\partial_{x}w\left[a,y\right]=\partial_{x}w\left[-a,y\right]=0,$$

– there are hinges at the edges parallel with the x axes

$$\partial_{y,y}w\left[x,b\right]=\partial_{y,y}w\left[x,-b\right]=0.$$

The Galerkin method has been used to solve this homogeneous boundary value problem.

3.2. **Trial functions (basis).** Let us consider a polynomial of degree n in direction x and a polynomial of degree m in direction y, respectively, with dimensionless variables $\eta=x/a$ and $\xi=y/b$ (mind that the following formulas represent only one term).

$$\texttt{X[}\eta\texttt{\_,n\_]:=}\sum_{i=0}^{3}\alpha_i\eta^i+\eta^n$$
$$\texttt{Y[}\xi\texttt{\_,m\_]:=}\sum_{i=0}^{3}\beta_i\xi^i+\xi^m$$

The trial function of two variables is:

```
W[ξ_, η_,n_,m_]:=X[ξ,n]*Y[η,m]
W[ξ,η,4,5]
```
$$\left(\eta^4+\alpha_0+\eta\alpha_1+\eta^2\alpha_2+\eta^3\alpha_3\right)\left(\xi^5+\beta_0+\xi\beta_1+\xi^2\beta_2+\xi^3\beta_3\right)$$

The coefficients $\alpha[i]$ and $\beta[i]$ are defined by the eight boundary conditions.

```
eq1[n_]:=X[-1,n]=0
eq2[n_]:=X[1,n] =0
eq3[m_]:=Y[-1,m]=0
eq4[m_]:=Y[1,m] =0
eq5[n_]:=(D[X[η,n],η]/.η →-1)=0
eq6[n_]:=(D[X[η,n],η]/.η → 1)=0
eq7[m_]:=(D[Y[ξ,m],ξ,ξ]/.ξ →-1)=0
eq8[m_]:=(D[Y[ξ,m],ξ,ξ]/.ξ → 1)=0
```

Note, that the chain rule should be used because the dimensionless variables were introduced, but now, in case of homogeneous boundary conditions, it has no effect. Solving this set of equations, one obtains the unknown coefficients α[i] and β[i] depending on degrees n and m:

Let us consider max(n) = 5 and max(m) = 5:

```
maxn:=5;maxm=5;k=1;
Do[{α0,α1,α2,α3,β0,β1,β2,β3} =
 {α0,α1,α2,α3,β0,β1,β2,β3}/.solαβ[i,j]//Flatten;
 φ[k]=W[η,ξ,i,j];
 {α0,α1,α2,α3,β0,β1,β2,β3} = .;
 k=k+1,{i,4,maxn},{j,4,maxm}];
```

The number of the trial functions is:

```
K=(maxn-3)(maxm-3)
4
```

For example, the trial function φ[4]=W[η,ξ,5,5]:

```
φ[4]
```

$$\left(\eta-2\eta^3+\eta^5\right)\left(\frac{7\xi}{3}-\frac{10\xi^3}{3}+\xi^5\right)$$

```
Plot3D[φ[4],{η,-1,1},{ξ,-1,1}],PlotPoints→{30,30},
AxesLabel→{"η","ξ",None},
ColorFunction→(RGBColor[#,1-#,1-0.5#] & )];
```

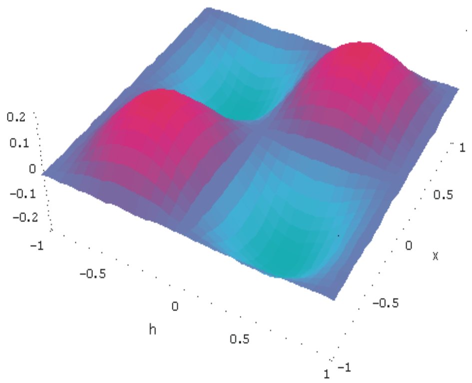

Figure 2. Trial function, n = 5, m = 5

4. Computing system matrix

A general element of the system matrix **M** is:

```
M[i_,j_]:  =
Simplify[
```

$$\int_{-1}^{1}\int_{-1}^{1}(\mathrm{b}^4\mathrm{AD}[\varphi[\mathrm{i}],\{\eta,4\}]+2\mathrm{a}^2\mathrm{b}^2\mathrm{HD}\ [\varphi[\mathrm{i}],\{\eta,2\},\{\xi,2\}]+$$
$$\mathrm{Ba}^4\mathrm{D}[\varphi[\mathrm{i}],\{\xi,4\}]\ +\ \frac{\mathrm{Pa}}{2}(\eta+1)\mathrm{a}^4\mathrm{b}^2\ \mathrm{D}[\varphi[\mathrm{i}],\{\xi,2\}])$$
$$\varphi[\mathrm{j}]\mathrm{d}\eta\mathrm{d}\xi$$

Keep in mind that the chain rule should be used for dη and dξ, because the dimensionless variables were introduced, but now, in case of homogeneous equation (see (3)), it has no effect.

The matrix can be developed easily

```
sys=Table[M[i,j],{i,K},{j,K}];
```

The plate loses stability if the equation system has a nontrivial solution, namely its determinant is zero:

```
sysdet=Det[sys]
```

$$\frac{1}{5294745225}\left(268435456\left(70Ab^4+220a^4B+132a^22b^22H-11a^5b^2P\right)\right.$$
$$\left(50Ab^4+20a^4B+44a^2b^2H-a^5b^2P\right)$$
$$\left(\frac{1031865892864A^2b^8}{3472875}+\frac{4127463571456a^4Ab^4B}{63669375}+\right.$$
$$\frac{4294967296a^8B^2}{3031875}+\frac{24897925414912a^2Ab^6H}{121550625}$$
$$\frac{146028888064a^6b^2BH}{9095625}+\frac{1241245548544a^4b^4H^2}{40516875}$$
$$\frac{17541720178688a^5Ab^6P}{1337056875}-\frac{36507222016a^9b^2BP}{63669375}$$
$$\left.\frac{620622774272a^7b^4HP}{191008125}+\frac{155155693568a^{10}b^4P^2}{2941525125}\right)-$$
$$\frac{1}{72765}\left(16384a^5b^2P\left(\frac{16906090788683776a^5A^2b^{10}P}{252703749375}+\right.\right.$$
$$\frac{67624363154735104a^9Ab^6BP}{4632902071875}$$
$$\frac{70368744177664a^{13}b^2B^2P}{220614384375}+\frac{37084328181628928a^7Ab^8HP}{804057384375}+$$
$$\frac{2392537302040576a^{11}b^4BHP}{661843153125}+\frac{20336567067344896a^9b^6H^2P}{2948210409375}$$
$$\frac{287403543407624192a^{10}Ab^8P^2}{97290943509375}-\frac{598134325510144a^{14}b^4BP^2}{4632902071875}-$$
$$\left.\left.\frac{1016828353367244840a^{12}b^6HP^2}{138987706215625}+\frac{2542070883418112a^{15}b^6P^3}{214040075720625}\right)\right)$$

5. **Direct stability problem**

For a direct stability problem, the geometrical parameters *a, b, A, B, H* are all known, and the load parameter *P* should be computed. Let us consider the following data taken from [3]:

```
κ=1.02848; a=5.7; b=11.4;
```

Then the system matrix **M** is

```
sysP=sys/.{A->190,B->230,H->κ√190230};
```

which can be decomposed as **sysP** = **Z0** $+P$**Z1**, where

```
Z0 = Map[Coefficient[#,P,0] & ,sysP];Z0//MatrixForm
```

$$\begin{pmatrix} 2.3752\times 10^{9} & 0 & 0 & 0 \\ 0 & 8.95871\times 10^{7} & 0 & 0 \\ 0 & 0 & 1.57284\times 10^{9} & 0 \\ 0 & 0 & 0 & 4.62149\times 10^{7} \end{pmatrix}$$

```
Z1=Map[Coefficient[#,P] & ,sysP];Z1//MatrixForm
```

$$\begin{pmatrix} -1.97549\times 10^{7} & 0 & -1.7959\times 10^{6} & 0 \\ 0 & -1.93675\times 10^{6} & 0 & -176068 \\ -1.7959\times 10^{6} & 0 & -1.7959\times 10^{6} & 0 \\ 0 & -176068 & 0 & -176068 \end{pmatrix}$$

Indeed

```
Simplify[sysP==Z0+PZ1]
True
```

Considering that **Z1** is not a diagonal matrix, the problem **Mc** = **0** is a *generalized (linear)* eigenvalue problem for P as eigenvalue.

Then the critical load parameter is:

```
solP=Solve[Det[sysP]==0,P]
{{P → 45.3935},{P → 118.547},{P → 294.219},{P → 977.085}}
Pcrit=Min[P/.solP]
45.3935
```

6. Inverse stability problem

In this case the critical load parameter is prespecified, and one of the geometrical parameters can be computed. For example, let us consider A_{crit} to be determined when

```
Pcrit = 55.;
```

Now, the system matrix, **M** is

```
sysA=sys/.{P->Pcrit,B->230,H->κ√(A230)};
```

which can be decomposed as **sysA** = **S0** $+\sqrt{A}$**S1** $+A$**S2**, where

```
S0=Map[Coefficient[#,A,0] & ,sysA];S0//MatrixForm
```

$$\begin{pmatrix} -1.05621\times 10^{9} & 0 & -9.87744\times 10^{7} & 0 \\ 0 & -9.44947\times 10^{7} & 0 & -9.68376\times 10^{6} \\ -9.87744\times 10^{7} & 0 & -9.60192\times 10^{7} & 0 \\ 0 & -9.68376\times 10^{6} & 0 & -8.59042\times 10^{6} \end{pmatrix}$$

```
S1=Map[Coefficient[#,√A ] & ,sysA];S1//MatrixForm
```

$$\begin{pmatrix} 1.9966\times 10^{7} & 0 & 0 & 0 \\ 0 & 1.95745\times 10^{6} & 0 & 0 \\ 0 & 0 & 6.65533\times 10^{6} & 0 \\ 0 & 0 & 0 & 652483. \end{pmatrix}$$

```
S2=Map[Coefficient[#,A] & ,sysA];S2//MatrixForm
```

$$\begin{pmatrix} 1.08931\times 10^{7} & 0 & 0 & 0 \\ 0 & 266204. & 0 & 0 \\ 0 & 0 & 7.78077\times 10^{6} & 0 \\ 0 & 0 & 0 & 190146. \end{pmatrix}$$

Indeed

```
Simplify[sysA==S0+√AS1+AS2]
True
```

Now, however **S1** and **S2** are diagonal matrices, the problem $\mathbf{Mc} = \mathbf{0}$ is a *nonlinear (quadratic) eigenvalue problem* for $\sqrt{A}$ as eigenvalue.

We can solve the equation, but now for A.

```
solA=Solve[Det[sysA]==0,A]
{{A → 8.4674},{A → 22.6257},{A → 81.8672},{A → 246.723}}
```

There arise a problem to determine which solution is acceptable from engineering point of view. To solve this problem, let us consider the following figure:

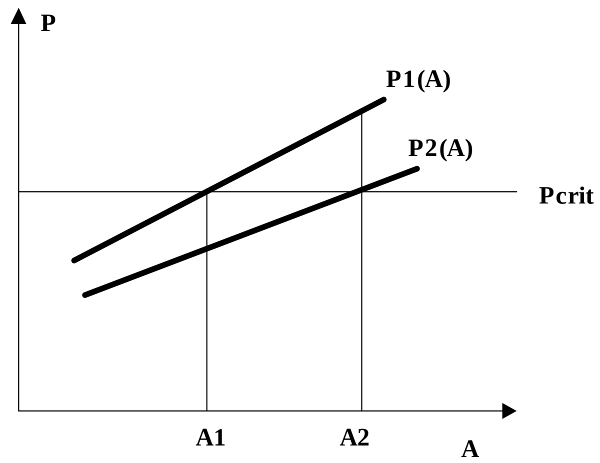

Figure 3. Selecting the proper critical geometrical parameter

It is easy to see that the index of the proper A_i can be computed as:

$$\min_j (P_j (A_i)) = P_{crit}.$$

Therefore, we can solve our stability equation symbolically for P with parameter A, namely $P_j(A)$:

```
solPA=Solve[sysdet==0,P]/.{B→230,H->κ√A230};
solPA=P/.solPA;
```

Then the matrix $m_{i,j} = P_j(A_i)$ can be computed as:

```
MPAcrit=Transpose[Map[Chop[N[#/.solA]] & ,solPA]]
```

{{8.96183, 9.78261, 30.3149, 55.0},
{13.6388, 18.5023, 55.0, 131.104},
{26.0052, 55.0, 144.231, 435.136},
{55.0, 151.326, 370.553, 1259.20}}
. . . .

Select the proper row:

```
rPAcrit=Select[MPAcrit,Chop[Min[#]-Pcrit=0 & ]
```

{{55., 151.326, 370.553, 1259.02}}

Select the index of the row:

```
pAcrit=First[Position[MPAcrit,First[rPAcrit]]]
```

{4}

Select the proper critical geometrical parameter:

```
Acrit=A/.solA[[First[pAcrit]]]
```

246.723

We remark that **Z0** is a positive definite diagonal and **Z1** is a real symmetric matrix. Hence the eigenvalue problem can be reduced to a special eigenvalue problem for a symmetric, i.e., selfadjoint matrix.

7. Computing eigenshapes

Let us return to the direct stability problem. After the critical load is determined, one may compute the solution of (3), i.e., the values of c_i's. Unfortunately, in both cases (direct and inverse stability problem) we have a non-standard eigenvalue problem, therefore the usual techniques cannot be directly applied [5]. One may solve the problem by definition. One of the rows of **M** can be left out, and instead of it the normalization can be considered to be a new equation:

$$\sum_{i=1}^{K} c_i = 1\,.$$

However, one should carefully select the row to be left out, because of the special chess table structure of **M** (see matrix **Z**). One should leave out a proper row, which really cancels the singularity of **M**. Fortunately, *Mathematica* has a built-in function to compute the nullspace [6] of **M**, so one can easily solve this unpleasant problem:

```
c=NullSpace[sysP/.P→Pcrit][[1]]//N
```

$\{0., 4.78239, 0., 1.\}$

Norming:

```
< < LinearAlgebra'MatrixManipulation'
c / VectorNorm[c,2]
```
$\{0., 0.97883, 0., 0.204674\}$

Here we have to mention that in case of computation with non rational numbers, *NullSpace* may result in an empty set because of the ill-conditioned feature of **M**. In that case the *Jordan decomposition* has been proved to be more robust and can provide acceptable approximation:

```
c = Transpose[
JordanDecomposition[(sysP//N)/.P→Pcrit][[1]]][[K]]
```
$\{0., -0.97883, 0., -0.204674\}$

Now we have the same result because we used rational numbers in the computation. In the following sections we employ Jordan decomposition in order to reduce computation time.

Then the first eigenshape:

```
wcrit = ∑_{i=1}^{K} c[[i]]φ[i]
```
$$0.\,(1-2\eta^2+\eta^4)(5-6\xi^2+\xi^4)+0.\,(\eta-2\eta^3+\eta^5)(5-6\xi^2+\xi^4)-$$
$$0.978783(1-2\eta^2+\eta^4)\left(\frac{7\xi}{3}-\frac{10\xi^3}{3}+\xi^5\right)-$$
$$0.204674(\eta-2\eta^3+\eta^5)\left(\frac{7\xi}{3}-\frac{10\xi^3}{3}+\xi^5\right)$$

```
Plot3D[wcrit,{η,-1,1},{ξ-1,1},PlotPoints->{30,30},
AxesLabel-> {"x/a","y/b",None},
ColorFunction->(RGBColor[#,1-#,0] & )];
```

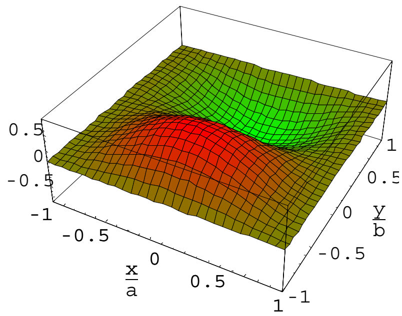

Figure 4. The first eigenshape

The contour plot shows clearly the effect antisymmetrical load distribution

```
ContourPlot[wcrit,{η,-1,1},{ξ-1,1},PlotPoints->50,
AxesLabel->{"a","b",None},
ColorFunction->(RGBColor[#,1-#,0] & ),Contours->15];
```

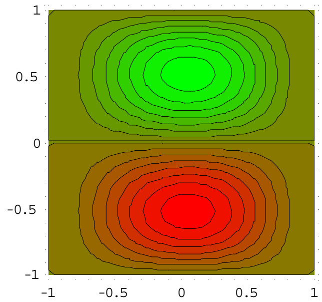

Figure 5. Contour plot of the first eigenshape

The further eigenshapes belonging to the next three roots are:

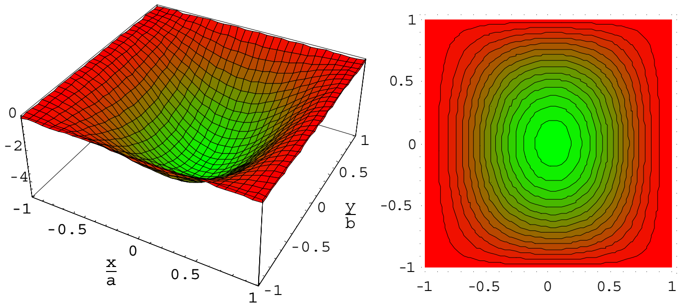

Figure 6a. Eigenshape for $P = 118.547$

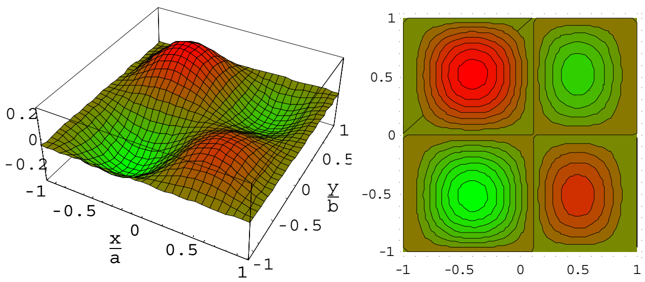

Figure 6b. Eigenshape for $P = 294.219$

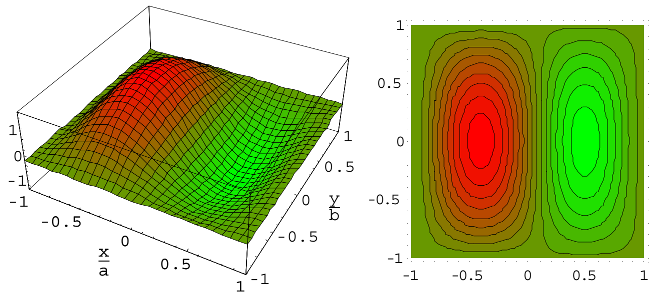

Figure 6c. Eigenshape for $P = 977.085$.

8. **Effect of higher order polynomials**

The value of the critical load parameter decreases and converges with an increase in the polynomial order of the trial functions:

$n = m$	Critical load parameter
4	120.234
5	45.3935
6	38.5191
7	38.5062

Table 1. The critical loads

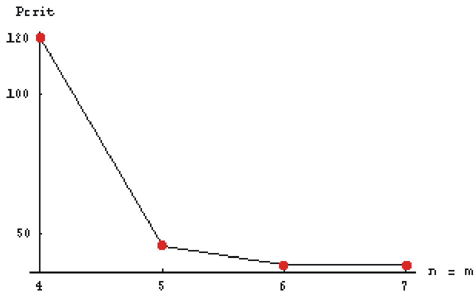

Figure 7. The critical load parameter as a function of the polynomial order of the trial functions

The first, critical eigenshapes are different but also converging to the true shape:

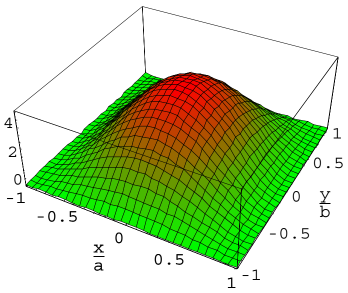

Figure 8a. Critical shape for $n = m = 4$

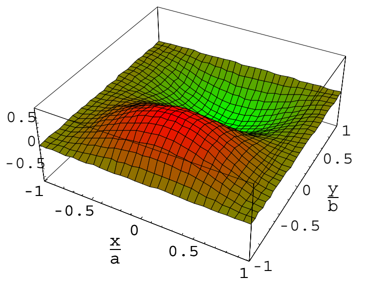

Figure 8b. Critical shape for $n = m = 5$

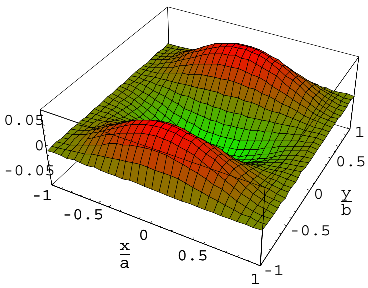

Figure 8c. Critical shape for $n = m = 6$

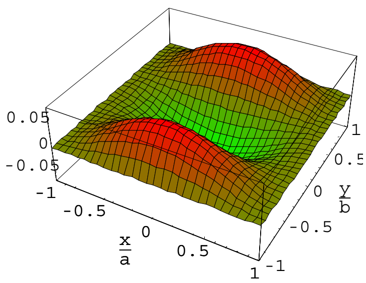

Figure 8d. Critical shape for $n = m = 7$

Employing higher order polynomials, new roots will emerge. For example, in case of $n = m = 6$, we obtain nine roots.

38.5191	45.3934	118.208	164.784	276.89
625.832	922.617	1203.99	4375.02	

Table 2. Roots for $n = m = 6$

The new roots result new eigenshapes. For example, for root 625.832, we obtain the following eigenshape

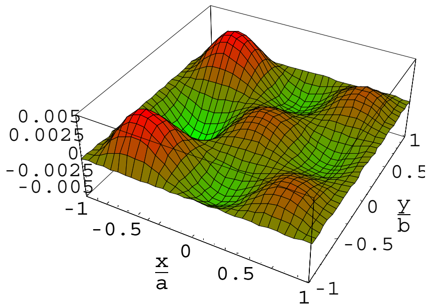

Figure 9. Eigenshape for root 625.832 in case of $n = m = 6$

A further increase of the order of the trial functions ($n, m \geqslant 7$) results in the appearance of complex and parasitic [7] (extremely big or small) roots.

9. Concluding remarks

A symbolic-numeric method has been developed to handle direct and inverse stability problems as a matrix eigenvalue problem. It was demonstrated that the direct stability problem can be considered to be a general, linear eigenvalue problem for the critical load parameter, and the inverse stability problem to be a nonlinear (quadratic) eigenvalue problem for a mechanical (stiffness) parameter. It has been shown that this method provides a simple, effective and elegant procedure for different boundary-value problems.

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