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THREE-COMPONENT DISCRETELY-FIBROUS COMPOSITES UNDER MATRIX MICRODAMAGING

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Abstract. In the present paper a model for the nonlinear deformations in stochastic composites under microdamaging is developed for three-component composites providing that the microdamage is accumulated in the matrix. The composite is treated as an isotropic matrix strengthened by two different types of spheroidal inclusions with transversally-isotropic symmetry of elastic properties. It is assumed that the loading process leads to accumulation of damage in the matrix. Fractured microvolumes are modeled by a system of randomly distributed quasispherical pores. The porosity balance equation and relations for determining the effective elastic modules for the case of transversally-isotropic components are taken as basic relations. The fracture criterion is assumed to be given as the limit value of the intensity of average shear stresses occurring in the undamaged part of the material. Based on the analytical and numerical approach the algorithm for determination of nonlinear deformative properties of such a material is constructed. The nonlinearity of composite deformations is caused by finiteness of component deformations. Using the numerical solution, the nonlinear stress-strain diagrams for three-component concrete for the case of uniaxial tension are obtained.

Mathematical Subject Classification: 74R99, 74M99
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1. Introduction

Deformation of composites may become nonlinear with increase in macrostrains or macrostresses. Physical nonlinearity of macrostress - macrostrain relations is due to the nonlinearity of physical deformations in the components or microdamages manifesting itself as micropores or microcracks. In this paper the nonlinear effective deformation properties of a composite attributed to the growth of microdamage in the deformed components are considered.

The idea on accumulation of scattered microscopic cavities occurring in a material under a load, resulting in a decrease in the effective (bearing) cross-sectional area, is based on the theory of material damage. The corresponding research is based on the introduction of a so-called ´damage' parameter with the associated evolutionary equation postulated in [1, 2]. Constants entering this equation are determined from long-term strength or creep tests. Such a formal approach is essentially equivalent to replacing time by another parameter (damage), which depends on time in a certain way so that it cannot provide knowledge or conceptions, supplementing the experiment, about the nature of the process, especially about structural changes in the material or the specific mechanism of the process. Moreover, any identification of damage with time will not encompass the extensive class of phenomena of short-period damage, manifested in the experimentally observed one-to-one dependence of the content of submicroscopic cracks on the given deformation, for example as it was shown in [3]. This also leaves aside the fundamental question on the structure of the relationship between damage and the physical and mechanical properties of a material.

The above indicates that a rational approach to the description of damage to a material and the phenomena accompanying it can be based only on the simulation of damage by specific structural microscopic elements in the form of the system of microcracks or micropores and on the construction of appropriate equations for the mechanics of a macroscopically inhomogeneous medium, allowing for the interaction of the processes of damage and deformation of the material. Such an informal approach applies to short-term damage of materials. The damage process is simulated by formation of a system of randomly distributed micropores, which are empty or are filled with destroyed material, in those microscopic volumes where microscopic destruction (microdestruction) occurs. The destruction criterion is taken in the form of the ultimate value of the intensity of shear stresses averaged over the undamaged part of the material or the Schleicher-Nadai criterion. The yield strength is assumed to be the random function of the coordinates, the single-point distribution of which is described by the power-law or Weibull distribution. The effective deformative properties and the stress-strain state of the material are determined from stochastic equations of the elasticity theory [4] - [6], which allow for the random nature of the arrangement of microdestruction. An equation of porosity balance is formulated for determining the porosity, which changes as a consequence of microdestruction. This fact makes it possible to describe the combined process of deformation and microdamage with allowance for their interaction, which leads to the nonlinear relationships between macrostresses and macrodeformations.

Microdestructions occur in the weakest microvolumes of a material under high-level loads, which reduces the bearing section of the material and leads to a redistribution of microstresses, and hence to nonlinear relationships between macrostresses and macrodeformations. The essence of this mechanism is described in [3] using the simplest working scheme for a material, in the form of the system of parallel, linearly elastic rods having a randomly distributed yield strength and acted upon by a uniform tensile load. The successive destruction of some of the rods under load results in a nonlinear deformation of a pattern.

The deformation and microdestruction of actual materials are considerably more complex processes, due primarily to the three-dimensional character of the stress-strain state and the random arrangement of the local microdestructions. The structural inhomogeneity of elastic properties, typical of composite materials, introduces additional complications. Experimental research [3] shows that during uniaxial stretching of polymer materials, submicrocracks develop with ratios of transverse to longitudinal dimensions ranging from 0.4 to 1.3. If we will assume that the part of the material in the vicinity of a disk-shaped crack does not carry a load along the major axis, as well as the disorientation of submicrocracks is not uniform and the stress state is complex due to the non-uniformity of the structure, then there are every reasons to simulate an individual act of microdestruction by a quasi-spherical pore. We take the limiting value of intensity of shear stresses in the microvolume as the condition for the appearance of individual microdestruction. Nor does invariance of the microdestruction criterion provide a basis for assuming a certain orientation for the individual microdestruction. We assume the arrangement of individual pieces of microdestruction in the material to be random, satisfying the criteria of statistical uniformity and isotropy.

The model of short-term microdamaging of composites of the stochastic structure, taking into account the distribution of material strength, is proposed by Khoroshun in [7]. This model is developed in papers [8, 9] for composite materials with isotropic components provided that microdestructions occur in the matrix or inclusions. The approach has received development in paper [10] for anisotropic materials, and then for composites with anisotropic components in [11]. The present work is devoted to the investigation of macrodeformations in three-component composites representing an isotropic matrix, randomly reinforced by two types of unidirectional spheroidal inclusions with various elastic properties and geometrical parameters. It is assumed that the accumulation of microdamage occurs in the matrix.

2. Mechanical model

Let us consider the representative volume V of a composite material subjected to uniform macrodeformations $\langle\varepsilon_{\alpha\beta}\rangle$. The composite is treated as an isotropic matrix strengthened by stochastically distributed unidirectional spheroids with various elastic constants and various geometrical parameters. Such a type of composite is shown in Figure 1. We suppose that the physical and mechanical properties of the material of inclusions have transversally-isotropic symmetry. It is assumed that the matrix shows initial microdamage, which is modelled by a system of randomly distributed micropores of quasispherical shape. The effective deformative properties and the stress-strain state of such a composite is determined on the basis of the stochastic equations of the elasticity theory by the method of conditional moment functions stated in [5].

Under homogeneous loading, the stresses and strains appearing in the representative volume will form statistically homogeneous random fields satisfying the ergodicity condition. In this case we can replace the operation of averaging over the representative volume by the operation of averaging over an ensemble of realizations. Then the macroscopic stresses $\langle\sigma_{ij}\rangle$ and strains $\langle\varepsilon_{\alpha\beta}\rangle$ of such a material will be related by Hooke's law:

$$\langle\sigma_{ij}\rangle = \lambda^{**}_{ij\alpha\beta}\,\langle\varepsilon_{\alpha\beta}\rangle\,, \quad (\,i,\, j,\, \alpha,\, \beta\,=\,1,\,2,\,3\,)\,. \tag{2.1}$$

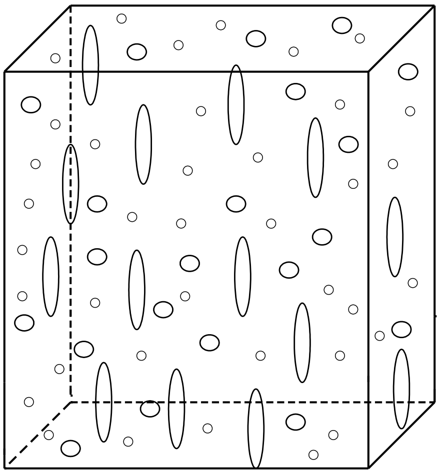

Figure 1. Type of the composite under consideration.

Here $\lambda^{**}_{ij\alpha\beta}$ is the tensor of effective elastic constants for the given composite which is determined in three steps. In the first step the elastic modules of the matrix weakened by micropores (microdamage) is determined on the basis of the results presented in paper [12]

$$\lambda^{p}_{ij\alpha\beta} = \lambda^{p}_{ij\alpha\beta}\left(\lambda^{[3]}_{ij\alpha\beta}, p_0\right). \tag{2.2}$$

Here $\lambda^{p}_{ij\alpha\beta}$ is the tensor of elastic modules of a porous matrix, which depends on the elastic constants of matrix material $\lambda^{[3]}_{ij\alpha\beta}$ and the initial porosity p_o of matrix. Then using the results, obtained in paper [6] for transversally-isotropic composites of stochastic structure, we can calculate the effective elastic modules of a two-component composite with porous matrix and spheroidal inclusions

$$\lambda^{*}_{ij\alpha\beta} = \lambda^{*}_{ij\alpha\beta}\left(\lambda^{[1]}_{ij\alpha\beta}, \lambda^{p}_{ij\alpha\beta}, c_1, t_1\right), \tag{2.3}$$

as the function of elastic module tensors of inclusions $\lambda^{[1]}_{ij\alpha\beta}$ and porous matrix $\lambda^{p}_{ij\alpha\beta}$, volume content of inclusions in matrix c_1 and their geometrical parameters t_1, which are characterized by the ratio of the spheroid semi-axes. In the last step we calculate the effective elastic properties of the composite material with a porous matrix stochastically reinforced by spheroidal fibres of two types which have different elastic properties and geometrical parameters:

$$\lambda^{**}_{ij\alpha\beta} = \lambda^{**}_{ij\alpha\beta}\left(\lambda^{*}_{ij\alpha\beta}, \lambda^{[2]}_{ij\alpha\beta}, c_2, t_2\right), \tag{2.4}$$

where $\lambda^{[2]}_{ij\alpha\beta}$ is the elastic module tensor of the material of inclusions of the second type, and c_2 and t_2 are their volume content and the ratio of the spheroid semi-axes. Considering equations (2.2) - (2.4), it is possible to make the conclusion that the effective elastic properties of such a composite $\lambda^{**}_{ij\alpha\beta}$ certainly depend on the elastic constants of components and geometrical parameters of fibers, and also on the initial porosity of matrix p_0

$$\lambda^{**}_{ij\alpha\beta} = \lambda^{**}_{ij\alpha\beta}\left(\lambda^{[1]}_{ij\alpha\beta}, \lambda^{[2]}_{ij\alpha\beta}, \lambda^{[3]}_{ij\alpha\beta}, c_1, c_2, p_0, t_1, t_2\right). \tag{2.5}$$

For the known effective elastic modules of such a composite, it is possible to calculate the matrix macrostrains $\langle \varepsilon_{\alpha\beta}|_3\rangle$ using consecutively the relations obtained in paper [11] for two-component composites:

$$\langle \varepsilon_{\alpha\beta}\,|_3\,\rangle = \left(I_{\alpha\beta kl} + (1-c_2)\left(c_2\lambda^{[2]}_{\alpha\beta kl} + (1-c_2)\,\lambda^{*}_{\alpha\beta kl} - \lambda^{**}_{\alpha\beta kl}\right)\left(\lambda^{[2]}_{kl\gamma\rho} - \lambda^{*}_{kl\gamma\rho}\right)^{-1}\right) *$$

$$* \left(I_{\gamma\rho mn} + c_3^{'}\,\left(\langle\lambda_{\gamma\rho mn}\rangle - \lambda^{*}_{\gamma\rho mn}\right)\,\lambda^{[4]-1}_{mnij}\right)\langle\varepsilon_{ij}\rangle\,, \tag{2.6}$$

in which

$$\langle\lambda_{\gamma\rho mn}\rangle = c_1^{'}\lambda^{[1]}_{\gamma\rho mn} + c_3^{'}\lambda^{[3]}_{\gamma\rho mn}; \quad \lambda^{[4]}_{mnij} = \lambda^{[1]}_{mnij} - \lambda^{[3]}_{mnij}, \tag{2.7}$$

where

$$(\, i,\, j,\, k,\, l,\, m,\, n,\, \alpha,\, \beta,\, \gamma,\, \rho\, =\, 1,\, 2,\, 3\,)\,.$$

Here c_3 is the volume content of the binding in a composite, and $c_1^{'}$ and $c_2^{'}$ are defined by the following ratios:

$$c_1^{'} = \frac{c_1}{c_1+c_3}, \quad c_3^{'} = \frac{c_3}{c_1+c_3}, \quad c_1+c_2+c_3 = 1\,. \tag{2.8}$$

Macrostrains and macrostresses in a matrix are related to each other by Hooke's law:

$$\langle\sigma_{ij}\,|_3\,\rangle = \lambda^{[3]}_{ij\alpha\beta}\,\langle\varepsilon_{\alpha\beta}\,|_3\,\rangle\,. \tag{2.9}$$

At the same time, stresses averaged over the matrix skeleton $\left\langle\sigma^{3p}_{ij}\right\rangle$ are connected with average stresses over all the matrix $\langle\sigma_{ij}\,|_3\,\rangle$ by the following dependence:

$$\left\langle\sigma^{3p}_{ij}\right\rangle = \frac{1}{1-p_0}\,\langle\sigma_{ij}\,|_3\,\rangle\,. \tag{2.10}$$

Thus, on the basis of expressions (2.6)-(2.10) stresses averaged over the matrix skeleton $\left\langle\sigma^{3p}_{ij}\right\rangle$ are connected with macrostrains by means of the relationships:

$$\left\langle\sigma^{3p}_{\alpha\beta}\right\rangle = \frac{1}{1-p_0}\lambda^{[3]}_{\alpha\beta mn} * \tag{2.11}$$

$$* \left(I_{\alpha\beta kl} + (1-c_2)\left(c_2\lambda^{[2]}_{\alpha\beta kl} + (1-c_2)\,\lambda^{*}_{\alpha\beta kl} - \lambda^{**}_{\alpha\beta kl}\right)\left(\lambda^{[2]}_{kl\gamma\rho} - \lambda^{*}_{kl\gamma\rho}\right)^{-1}\right) *$$

$$* \left(I_{\gamma\rho mn} + c_3^{'}\,\left(\langle\lambda_{\gamma\rho mn}\rangle - \lambda^{*}_{\gamma\rho mn}\right)\,\lambda^{[4]-1}_{mnij}\right)\,\langle\varepsilon_{ij}\rangle\,.$$

Let us take the strength condition for the microvolume of the undestroyed part of a matrix as the Huber-Mises criterion [3]:

$$I^3_\sigma = \left(\left\langle\sigma^{3p}_{ij}\right\rangle^{'}, \left\langle\sigma^{3p}_{ij}\right\rangle^{'}\right) = k_3, \tag{2.12}$$

where $\left\langle \sigma_{ij}^{3p} \right\rangle'$ is the deviator of stresses averaged over a matrix skeleton, and k_3 is the corresponding limit value of microstrength of the matrix, being a stochastic function of coordinates. One-point distribution function $F(k_3)$ of random variable k_3 can be described by the exponential distribution function in a semi-infinite domain, i.e. the Weibull distribution:

$$F(k_3) = \begin{cases} 0 & \text{if } k_3 < k_0 \ , \\ 1 - exp\left(-n\left(k_3 - k_0\right)^{\ \alpha}\right) & \text{if } k_3 \geq k_0 \ . \end{cases} \tag{2.13}$$

Here k_0 is the lower limit value of the intensity of the averaged tangential stresses over the matrix skeleton where destruction in some microvolumes begins; n and α are the factors chosen from a condition of the best approximation of strength distribution which is determined experimentally for each material.

If the stresses of the matrix skeleton $\left\langle \sigma_{ij}^{3p} \right\rangle$ are known, the function $F\left(I_\sigma^3\right)$ determines, according to formulas (2.12), (2.13), the relative content of the destroyed microvolumes in a matrix. If the destroyed microvolumes are modeled by the pores, it is possible to write down the balance porosity equation:

$$p = p_0 + F\left(I_\sigma^3\right)\ (1 - p_0)\ , \tag{2.14}$$

where p_0 is the initial porosity of the matrix. According to the formula (2.11), stresses of matrix skeleton $\left\langle \sigma_{ij}^{3p} \right\rangle$ can be expressed as a function of macrostrains of all composite $\langle \varepsilon_{\alpha\beta} \rangle$. These equations enable us to determine the current porosity of matrix p, generated by microdestructions, from nonlinear algebraic equations (2.12) - (2.14), as a dependence on macrostrains. Thus, we obtain the nonlinear dependence of macrostresses on macrostrains (2.2) - (2.14), caused by the matrixes microdestructions, with regard to strength distribution of a material.

3. **Iterative scheme of equation solution**

On the basis of the proposed model and the constructed solutions for the effective properties and stress-strain state of the elastic material, reinforced by two types of spheroidal inclusions with various elastic properties and geometrical parameters it is possible to investigate the accumulation of damage and the nonlinear deformation caused by microdamaging of transversally–isotropic composite under uniform macrodeformations. For given uniform macrostrains $\langle \varepsilon_{\alpha\beta} \rangle$ the matrix microdamaging, characterized by porosity p, is determined from the nonlinear system of equations (2.12) - (2.14). The solution of such a system of nonlinear equations can be obtained on the basis of the following iterative scheme.

Matrix porosity $p^{(n)}$ for the n-th approximation is determined as the function of the limiting value of averaged tangential stresses of a binding skeleton for the n-th approximation $I_\sigma^{3(n)}$. The averaged tangential stresses of the skeleton are related to the current porosity of a matrix in the $(n-1)$-th approximation $p^{(n-1)}$ via equations (2.5), (2.11).

Thus, on the basis of equations (2.13), (2.14) we can write

$$p^{(n)} = p_0 + F\left(I_\sigma^{3(n)}\right)\,(1 - p_0)\ , \tag{3.1}$$

where

$$F\left(I_\sigma^{3(n)}\right) = \begin{cases} 0 & \text{if } I_\sigma^{3(n)} < k_0\ , \\ 1 - exp\left(-n\left(I_\sigma^{3(n)} - k_0\right)\,\alpha\right) & \text{if } I_\sigma^{3(n)} \geq k_0\ . \end{cases} \tag{3.2}$$

According to expressions (2.6) – (2.12)

$$I_\sigma^{3(n)} = \varphi\left(\lambda_{ij}^{**(n-1)}, p^{(n-1)}, \langle\varepsilon_{\alpha\beta}\rangle\right), \tag{3.3}$$

while the tensor components of the effective elastic modules for the composite can be determined from formulas (2.5)

$$\lambda_{ij\alpha\beta}^{**(n-1)} = \lambda_{ij\alpha\beta}^{**}\left(\lambda_{ij\alpha\beta}^{[1]}, \lambda_{ij\alpha\beta}^{[2]}, \lambda_{ij\alpha\beta}^{[3]}, c_1, c_2, p^{(n-1)}, t_1, t_2\right)\ . \tag{3.4}$$

Hence, equations (3.1)-(3.4) allow us to investigate the effective elastic characteristics of porous transversally isotropic composites as a function of macrodeformations

$$\lambda_{ij\alpha\beta}^{**} = \lim_{n\to\infty}\ \lambda_{ij\alpha\beta}^{**(n)}. \tag{3.5}$$

Thus, giving the macrodeformation of the composite, and determining its effective elastic modules, from equations (3.1)-(3.5) it is possible to investigate the macrostresses arising in such a composite.

4. **Numerical results and discussion**

Using the above method and the relations obtained for determining the current porosity of a matrix material, as an example, we can construct the nonlinear diagram of macrodeformation and investigate the behavior of a concrete representing the cement matrix, strengthened by crushed stones and metal fibres under uniaxial extension

$$(\langle\varepsilon_{11}\rangle \neq 0, \quad \langle\varepsilon_{22}\rangle = 0, \quad \langle\varepsilon_{33}\rangle = 0)\,. \tag{4.1}$$

The elastic modulus and Poisson's ratios of crushed stone, metal and concrete are, respectively, equal to:

$$\begin{aligned} E^{[1]} &= 3\cdot 10^8\ \Pi a\,, \quad \nu^{[1]} = 0.4\,, \\ E^{[2]} &= 2000\cdot 10^8\ \Pi a\,, \quad \nu^{[2]} = 0.25\,, \\ E^{[3]} &= 300\cdot 10^8\ \Pi a\,, \quad \nu^{[3]} = 0.2\,, \end{aligned} \tag{4.2}$$

the volume contents of crushed stone and metal fibres are:

$$c_1 = 0.1\,, \quad c_2 = 0.3\,, \tag{4.3}$$

and the ratios of the spheroid semi-axes of crushed stone and metal are, respectively:

$$t_1 = 1.0\,, \quad t_2 = \infty\,, \tag{4.4}$$

the lower limit value of the intensity of the averaged tangential stresses over matrix skeleton is equal to:

$$k_0 = 0.8\cdot 10^8\ \Pi a\,. \tag{4.5}$$

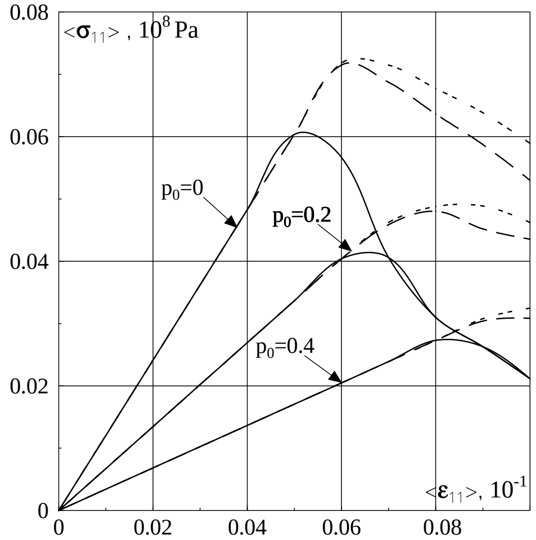

Figure 2. Stress-strain diagrams for various values of initial porosity and parameters characterizing the distribution function of the strength distribution.

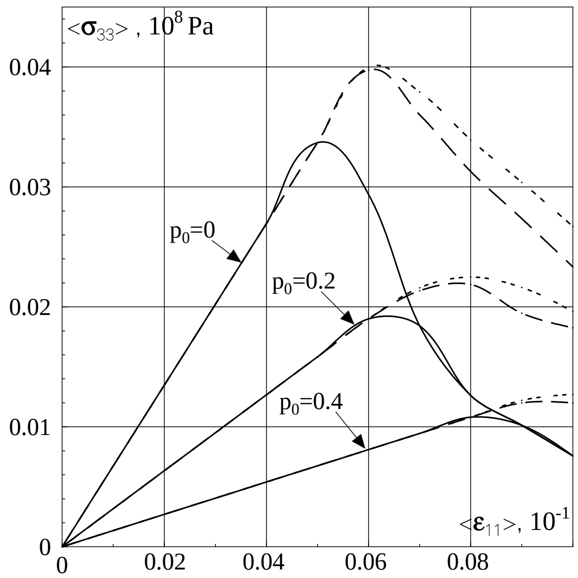

Figure 3. Stress-strain diagrams for various values of initial porosity and parameters characterizing the distribution function of the strength distribution.

In Figures 2–3 the nonlinear diagrams of stress-strain state for macrostresses $\langle\sigma_{11}\rangle$, and $\langle\sigma_{33}\rangle$ depending on macrostrain $\langle\varepsilon_{11}\rangle$ for various values of initial porosity of concrete p_0 ($p_0 = 0$; 0.2; 0.4) and parameters characterizing the distribution function of the strength distribution are represented. On the diagrams the continuous line designates the curves which take into account the strength distribution of a material with parameters $\alpha = 2; \quad n = 60$, the dotted line – the curves which take into account the strength distribution with parameters $\alpha = 2; \quad n = 30$, the dash-dot line – the curves which take into account the strength distribution with parameters $\alpha = 2; \quad n = 10$. For different values of matrix porosity, all three curves coincide up to the moment of the appearance of microdamage. Moreover, varying the parameters n and α, the experimental curve of macrodeformation can be fitted to the theoretical one for every specific material in the best way.

5. **Conclusions**

Thus, we may conclude that the above method and proposed numerical and analytical procedure based on the methods of conditional moment function and the iterative secant method allows us to investigate the nonlinear behavior of stochastic three-component composites representing an isotropic matrix, strengthened by spheroidal inclusion, under loading. It is assumed that the nonlinearity of composite deformations is caused by matrix microdamage. The numerical analysis demonstrates that the deformative properties of such a composite depend on elastic modules of components, shape and volume concentration of inclusions and initial matrix porosity. It is shown that the strength distribution of a matrix material has significant influence on the stress-strain diagrams of the composite.

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