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# CP ASYMMETRIES IN $B \to K\pi, K^*\pi, \rho K$ DECAYS

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We show that ratios of tree and penguin amplitudes in $B \to K^*\pi$ and $B \to \rho K$ are two to three times larger than in $B \to K\pi$. This allows for considerably larger CP asymmetries in the former processes than the 10% asymmetry measured in $B^0 \to K^+\pi^-$. We study isospin sum rules for rate asymmetries in $B \to K\pi, K^*\pi, \rho K$, estimating small violation from interference of tree and electroweak penguin amplitudes. The breaking of the $K\pi$ asymmetry sum rule is estimated to be one to two percent and negative. Violation of $K^*\pi$ and $\rho K$ sum rules can be estimated from $B \to \rho\pi$ amplitudes using flavor SU(3), while breaking of a sum rule combining $K^*\pi$ and $\rho K$ asymmetries can be measured directly in a Dalitz analysis of $B^0 \to K^+\pi^-\pi^0$. The three sum rules can be tested at the LHCb and at a future Super Flavor Factory, providing precision searches for new $\Delta S = \Delta I = 1$ operators in the low energy effective Hamiltonian.

## I. INTRODUCTION

The Cabibbo-Kobayashi-Maskawa (CKM) framework for flavor physics and CP violation has been confirmed successfully in numerous experiments involving a variety of $B$ meson decays [1, 2, 3, 4]. The measured oscillating CP asymmetry in $B^0 \to J/\psi K_{S,L}$, which has a theoretically clean interpretation including short distance perturbative corrections [5] and long distance rescattering effects [6], provided a precise value for $\beta$ with an accuracy of $1^\circ$. Using isospin symmetry [7, 8], time-dependent CP symmetries measured in $B^0 \to \pi^+\pi^-, \rho^+\rho^-, \rho^\pm\pi^\mp$ led to a determination of the phase $\alpha$ with an error of $4^\circ$. The values obtained for the two CP-violating phases are in good agreement with CP-conserving CKM constraints. Other asymmetries probing the phase $\beta$ were measured in loop-dominated processes from $b \to q\bar{q}s$ $(q = u, d, s)$ including $B^0 \to \pi^0 K_S, \eta' K_S, \phi K_S, \rho^0 K_S$. These processes are sensitive to corrections from flavor changing $\Delta S = 1$ operators appearing in extensions of the low energy theory [9, 10]. Uncertainties in calculating hadronic amplitudes by applying QCD dynamics [11, 12] prohibit distinguishing between New Physics and hadronic effects on these asymmetries.

Direct CP asymmetries have been observed in several charmless $\Delta S = 1$ $B$ decays, with a high confidence level above $8\sigma$ in $B^0 \to K^+\pi^-$ and at a lower confidence level of 3-4$\sigma$ in $B^+ \to \eta K^+, \rho^0 K^+$ and $B^0 \to \eta K^{*0}$ [4]. These observations are interesting by themselves but do not provide clean tests for the CKM framework. An interpretation of direct asymmetries in terms of fundamental CP phases requires calculating nonleptonic decay amplitudes and strong phases. Remarkable progress has been achieved in the past ten years in the theory of hadronic $B$ decays, starting with work advocating QCD factorization [13] and perturbative QCD [14]. While great progress has been made recently in calculating higher order terms in $\alpha_s$ [15, 16], $1/m_b$ corrections remain a serious difficulty. Thus, nonfactorizable hadronic matrix elements of charming penguin operators [17, 18] and of color-suppressed tree amplitudes lead to difficulties in calculating the decay rate for $B^0 \to \pi^0\pi^0$ [13, 16, 19]. This example and an intrinsic failure to account for individual strong phases (as discussed briefly in Sec. II for one example), seem to originate in incalculable long distance final state rescattering effects.

Whereas individual direct CP asymmetries in hadronic $B$ decays cannot be calculated reliably, there are certain classes of decays in which asymmetries can be related to each other within the CKM framework in a model independent way using symmetry arguments. Isospin symmetry, which is expected to hold within a couple of percent, has been shown to imply an approximate sum rule for asymmetries in the four $B \to K\pi$ decay processes [20],

$$A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+) \approx A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) \ . \tag{1}$$

A violation of this sum rule would be evidence for a new $\Delta S = \Delta I = 1$ term in the low enery effective Hamiltonian. A similar sum rule may hold approximately in the CKM framework for $B \to K^*\pi$ and $B \to \rho K$ decays, where individual CP asymmetries may be larger than in $B \to K\pi$.

In this paper we will study asymmetries in penguin-dominated $B \to K\pi, K^*\pi$ and $B \to \rho K$ and will compare their three isospin sum rules. These sum rules involve corrections from interference of subleading tree and electroweak penguin (EWP) amplitudes which are related to each other in a flavor SU(3) limit. Our aim will be to estimate these second order corrections in a model-independent way, or to propose methods in which the corrections can be measured elsewhere.

Section II gives $B \to K\pi$ decay amplitudes in terms of their diagramatic contributions, discussing the role of these contributions in $B \to K\pi$ asymmetries. Sec. III generalizes this description to $B \to K^*\pi$ and $B \to \rho K$ decays. In Sec. IV we use broken flavor SU(3) to calculate ratios of tree-to-penguin amplitudes in $B \to K\pi, K^*\pi, \rho K$, providing estimates for maximal possible CP asymmetries in these three classes of processes. Sec. V studies experimental tests for the broken SU(3) symmetry assumption. SU(3) relations between subleading tree and EWP amplitudes are discussed in Sec. VI, and are used in Sec. VII for estimating deviations from exact sum rules among asymmetries for $B \to K\pi, K^*\pi$ and $B \to \rho K$ decays. Sec. VIII concludes.

## II. AMPLITUDES AND ASYMMETRIES IN $B \to K\pi$

Decay amplitudes fot $B \to K\pi$ may be described generally in a model-independent way using topological graphical contributions [21],

$$-A(K^+\pi^-) = \lambda_t^{(s)}(P_{tc} + \frac{2}{3}P^C_{EW}) + \lambda_u^{(s)}(P_{uc} + T) , \quad (2)$$

$$A(K^0\pi^+) = \lambda_t^{(s)}(P_{tc} - \frac{1}{3}P^C_{EW}) + \lambda_u^{(s)}(P_{uc} + A) , \quad (3)$$

$$-\sqrt{2}A(K^+\pi^0) = \lambda_t^{(s)}(P_{tc} + P_{EW} + \frac{2}{3}P^C_{EW}) + \lambda_u^{(s)}(P_{uc} + T + C + A) , \quad (4)$$

$$\sqrt{2}A(K^0\pi^0) = \lambda_t^{(s)}(P_{tc} - P_{EW} - \frac{1}{3}P^C_{EW}) + \lambda_u^{(s)}(P_{uc} - C) , \quad (5)$$

where $\lambda_q^{(q')} \equiv V^*_{qb}V_{qq'}$ ($q = u, t; q' = d, s$) are Cabibbo-Kobayashi-Maskawa (CKM) factors with a small ratio $|\lambda_u^{(s)}|/|\lambda_t^{(s)}| \sim 0.02$. Each of the seven amplitudes, $P_{tc}, P_{EW}, P^C_{EW}, T, C, P_{uc}, A$ involves an unknown strong phase.

The common penguin amplitude $P' \equiv \lambda_t^{(s)} P_{tc}$ [or the linear combination $\lambda_t^{(s)}(P_{tc} - P^C_{EW}/3)$] with weak phase $\arg(\lambda_t^{(s)}) = \pi$, dominates all four $B \to K\pi$ amplitudes. This dominance is tested by simple approximate ratios,

$$\mathcal{B}(K^+\pi^-) : \mathcal{B}(K^0\pi^+)/r_\tau : \mathcal{B}(K^+\pi^0)/r_\tau : \mathcal{B}(K^0\pi^0) \simeq 1 : 1 : \frac{1}{2} : \frac{1}{2} , \quad (6)$$

The four branching ratios given in Table I, and the $B^+$ to $B^0$ lifetime ratio, $r_\tau \equiv \tau_+/\tau_0 = 1.071 \pm 0.009$ [4], imply experimental ratios:

$$(0.899 \pm 0.048) : 1 : (0.558 \pm 0.035) : (0.454 \pm 0.034) . \quad (7)$$

Thus Eq. (6) holds reasonably well with several percent errors leaving little space for non-penguin amplitudes.

Let us now discuss the subdominant terms occurring in Eqs. (2)-(5), their relative magnitudes with respect to $P_{tc}$ or $P'$, and their effects on CP asymmetries in $B \to K\pi$ decays.

- The largest subdominant term with weak phase $\arg(\lambda_u^{(s)}) \equiv \gamma$ is expected to be the color-favored tree amplitude $T' \equiv \lambda_u^{(s)} T$. An interference between $P'$ and $T'$ dominates the asymmetry in $B^0 \to K^+\pi^-$ for which a negative value has been measured at a level of 10%. We will show in the next section that $T'$ is about an order of magnitude times smaller that $P'$.

- The difference between the asymmetry in $B^0 \to K^+\pi^-$ and the preferably positive asymmetry measured in $B^+ \to K^+\pi^0$, obtaining contributions from interference of $P'$ with both $T'$ and and with the formally color-suppressed $C' \equiv \lambda_u^{(s)} C$, has shown [22, 23] that the ratio $|C|/|T|$ is not much smaller than one as naively anticipated, and that the relative strong phase $\mathrm{Arg}(CT^*)$ is sizable and negative. (We use a convention in which strong phase differences lie between $-\pi$ and $\pi$.) We note that $\mathrm{Arg}(CT^*) \simeq 0$ is predicted at leading order in $1/m_b$ by QCD factorization [24] and in a Soft Collinear Effective Theory approach [18]. The large strong phase may be interpreted as either large $1/m_b$ corrections or sizable nonfractorizable contributions to $C$ from rescattering through color-favored intermediate states.

Table I: Branching fractions and CP asymmetries for $B \to K\pi, K^*\pi, \rho K$ [4].

| Mode | $\mathcal{B}$ $(10^{-6})$ | $A_{CP}$ |
|---|---|---|
| $B^0 \to K^+\pi^-$ | $19.4 \pm 0.6$ | $-0.098^{+0.012}_{-0.011}$ |
| $B^+ \to K^0\pi^+$ | $23.1 \pm 1.0$ | $0.009 \pm 0.025$ |
| $B^+ \to K^+\pi^0$ | $12.9 \pm 0.6$ | $0.050 \pm 0.025$ |
| $B^0 \to K^0\pi^0$ | $9.8 \pm 0.6$ | $-0.01 \pm 0.10$ |
| $B^0 \to K^{*+}\pi^-$ | $8.6^{+0.9}_{-1.0}$ | $-0.18 \pm 0.08$ |
| $B^+ \to K^{*0}\pi^+$ | $9.9^{+0.8}_{-0.9}$ | $-0.038 \pm 0.042$ |
| $B^+ \to K^{*+}\pi^0$ | $6.9 \pm 2.3$ | $0.04 \pm 0.29$ |
| $B^0 \to K^{*0}\pi^0$ | $2.4 \pm 0.7$ | $-0.15 \pm 0.12$ |
| $B^0 \to \rho^- K^+$ | $8.6^{+0.9}_{-1.1}$ | $0.15 \pm 0.06$ |
| $B^+ \to \rho^+ K^0$ | $8.0^{+1.5}_{-1.4}$ | $-0.12 \pm 0.17$ |
| $B^+ \to \rho^0 K^+$ | $3.81^{+0.48}_{-0.46}$ | $0.37 \pm 0.11$ |
| $B^0 \to \rho^0 K^0$ | $4.7 \pm 0.7$ | $0.06 \pm 0.20$ |

- EWP terms $P_{EW}, P^C_{EW}$, which are higher order in the electroweak coupling, are an order of magnitude times smaller than $P_{tc}$. A quantitative discussion of these contributions relating them to penguin and tree amplitudes will be presented in Section VI. Effects of EWP terms on CP asymmetries through their interference with tree amplitudes are of second order because these two kinds of amplitudes are an order of magnitude smaller than $P_{tc}$. Interference of EWP amplitudes with $P_{tc}$ leads to no CP asymmetry as these two contributions carry the same weak phase.

- The very small asymmetry measured in $B^+ \to K^0\pi^+$, consistent with zero, indicates that both $P_{uc}$ and the annihilation amplitude $A$ are much smaller than $T$. A large enhancement by rescatterring of these amplitudes, which are expected to be intrinsically very small [25], would have created a sizable strong phase in these amplitudes relative to $P_{tc}$, thereby leading to a non-nlegligible CP asymmetry in $B^+ \to K^0\pi^+$.

## III. AMPLITUDES IN $B \to K^*\pi$ and $B \to \rho K$

Amplitudes for the eight processes $B \to K^*\pi$ and $B \to \rho K$ with final charges as in $B \to K\pi$ may be decomposed into expressions similar to Eqs. (2)–(5), where $P, T, C, A$ are now replaced by $P_P, T_P, C_V, A_P$ in $B \to K^*\pi$ and by $P_V, T_V, C_P, A_V$ in $B \to \rho K$ [26]. The suffix $M = P, V$ denotes whether the spectator quark is included in a pseudoscalar ($P$) or vector meson ($V$). Electroweak contributions, $P'_{EW,M} \equiv \lambda^{(s)}_t P_{EW,M}$ and $P'^C_{EW,M} \equiv \lambda^{(s)}_t P^C_{EW,M}$, are introduced as in $B \to K\pi$ through the substitution [21]

$$T'_M \to T'_M + P'^C_{EW,M}~, \quad C'_M \to C'_M + P'_{EW,M}~, \quad P'_M \to P'_M - \frac{1}{3}P'^C_{EW,M}~. \tag{8}$$

Thus, $B \to K^*\pi$ amplitudes analogous to Eqs. (2)-(5) are given by:

$$-A(K^{*+}\pi^-) \;=\; \lambda^{(s)}_t(P_{tc,P} + \frac{2}{3}P^C_{EW,P}) + \lambda^{(s)}_u(P_{uc,P} + T_P)~, \tag{9}$$

$$A(K^{*0}\pi^+) = \lambda_t^{(s)}(P_{tc,P} - \frac{1}{3}P^C_{EW,P}) + \lambda_u^{(s)}(P_{uc,P} + A_P) , \quad (10)$$

$$-\sqrt{2}A(K^{*+}\pi^0) = \lambda_t^{(s)}(P_{tc,P} + P_{EW,V} + \frac{2}{3}P^C_{EW,P}) + \lambda_u^{(s)}(P_{uc,P} + T_P + C_V + A_P) , \quad (11)$$

$$\sqrt{2}A(K^{*0}\pi^0) = \lambda_t^{(s)}(P_{tc,P} - P_{EW,V} - \frac{1}{3}P^C_{EW,P}) + \lambda_u^{(s)}(P_{uc,P} - C_V) . \quad (12)$$

Corresponding amplitudes for $B \to K\rho$ are obtained by interchanging subscripts $P \leftrightarrow V$.

Penguin dominance in $B \to K^*\pi$ and $B \to \rho K$ decays leads to approximate ratios of branching ratios in these processes which are similar to the ratios $1 : 1 : 1/2 : 1/2$ in $B \to K\pi$. Using branching ratios in Table I, involving experimental errors considerably larger than in $B \to K\pi$, one finds,

$$\mathcal{B}(K^{*+}\pi^-) : \mathcal{B}(K^{*0}\pi^+)/r_\tau : \mathcal{B}(K^{*+}\pi^0)/r_\tau : \mathcal{B}(K^{*0}\pi^0)$$
$$= (0.93 \pm 0.13) : 1 : (0.70 \pm 0.24) : (0.26 \pm 0.08) , \quad (13)$$

$$\mathcal{B}(\rho^- K^+) : \mathcal{B}(\rho^+ K^0)/r_\tau : \mathcal{B}(\rho^0 K^+)/r_\tau : \mathcal{B}(\rho^0 K^0)$$
$$= (1.15 \pm 0.25) : 1 : (0.48 \pm 0.10) : (0.63 \pm 0.15) . \quad (14)$$

Noting that $\mathcal{B}(K^{*0}\pi^0)$ in (13) seems somewhat small [27], large experimental errors in branching ratio measurements leave ample space for subdominant non-penguin amplitudes in these processes.

Magnitudes of dominant penguin amplitudes and of subdominant amplitudes, and certain relative strong phases between them, have been studied numerically in Ref. [28], applying a global flavor SU(3) fit to data of these processes and corresponding $\Delta S = 0$ $B \to VP$ decays. Recently $\chi^2$ fits were performed for $B \to K^*\pi$ and $B \to \rho K$ data, concluding that current experimental errors are too large for showing an inconsistency with the CKM framework [29]. The purpose of the next section is limited to estimating ratios of tree and penguin amplitudes determining the potentially largest CP asymmetries in $B \to K^*\pi$ and $B \to \rho K$.

## IV. RATIOS OF TREE-TO-PENGUIN AMPLITUDE IN $B \to K\pi, K^*\pi, \rho K$

The ratios $|T'/P'|, |T'_P/P'_P|$ and $|T'_V/P'_V|$ determine the maximal potential asymmetries in $B^0 \to K^+\pi^-$, $B^0 \to K^{*+}\pi^-$ and $B^0 \to \rho^- K^+$, given roughly by $2|T'/P'|\sin\gamma$, $2|T'_P/P'_P|\sin\gamma$ and $2|T'_V/P'_V|\sin\gamma$. Values of these three ratios may be estimated by relating within broken flavor SU(3) the amplitudes for these processes to those for $B^0 \to \pi^+\pi^-, B^0 \to \rho^+\pi^-$ and $B^0 \to \rho^-\pi^+$. One way of using flavor SU(3) is by applying group theory for decomposing $\Delta S = 1$ and $\Delta S = 0$ decay amplitudes into five SU(3) reduced matrix elements in $B \to PP$ and ten reduced matrix elements in $B \to VP$ [30]. In our more powerful SU(3) method based on a diagramatic approach [21, 26] a certain hierarchy between amplitudes can be shown to exist [31] and SU(3) breaking factors in tree amplitudes may be motivated by a factorization assumption. We keep SU(3) invariant penguin amplitudes as a default because factorization is not expected to hold for these contributions [17, 18]. Tests of these assumptions will be presented in the next section. The effect of possible SU(3) breaking in penguin amplitudes, at an expected level of 20%, can be easily included in our estimates for the ratios of tree and penguin amplitudes.

Table II: Amplitudes, branching fractions and asymmetries for certain $B \to PP, VP$ [4].

| Mode | Amplitude | $\mathcal{B}$ ($10^{-6}$) | $A_{CP}$ |
|---|---|---|---|
| $B^+ \to K^0\pi^+$ | $P'$ | Table I | Table I |
| $B^0 \to K^+\pi^-$ | $-(P'+T')$ | Table I | Table I |
| $B^0 \to \pi^+\pi^-$ | $\tilde{\lambda}P' - \tilde{\lambda}^{-1}T'\left(\frac{f_\pi}{f_K}\right)$ | $5.16 \pm 0.22$ | $0.38 \pm 0.06$ |
| $B^+ \to K^{*0}\pi^+$ | $P'_P$ | Table I | Table I |
| $B^0 \to K^{*+}\pi^-$ | $-(P'_P+T'_P)$ | Table I | Table I |
| $B^0 \to \rho^+\pi^-$ | $\tilde{\lambda}P'_P - \tilde{\lambda}^{-1}T'_P\left(\frac{f_\rho}{f_{K^*}}\right)$ | $15.7 \pm 1.8^a$ | $0.12 \pm 0.06^b$ |
| $B^+ \to \rho^+K^0$ | $P'_V$ | Table I | Table I |
| $B^0 \to \rho^-K^+$ | $-(P'_V+T'_V)$ | Table I | Table I |
| $B^0 \to \rho^-\pi^+$ | $\tilde{\lambda}P'_V - \tilde{\lambda}^{-1}T'_V\left(\frac{f_\pi}{f_K}\right)$ | $7.3 \pm 1.2^c$ | $-0.15 \pm 0.12^d$ |

$^{a,c}$ We use $\mathcal{B}(\rho^\pm\pi^\mp) = \frac{1}{2}\mathcal{B}(1 \pm A^{\rho\pi}_{CP}C \pm \Delta C)$, $\mathcal{B} \equiv \mathcal{B}(\rho^+\pi^-) + \mathcal{B}(\rho^-\pi^+)$.
$^{b,d}$ We use $A_{CP}(\rho^\pm\pi^\mp) = \mp[A^{\rho\pi}_{CP}(1 \pm \Delta C) \pm C]/[1 \pm A^{\rho\pi}_{CP}C \pm \Delta C]$.

Table II lists dominant terms in amplitudes (in the $c$-convention [32]), branching ratios and asymmetries for $\Delta S = 1$ penguin-dominated amplitudes, and for $\Delta S = 0$ processes involving the three pairs $(P', T'), (P'_P, T'_P), (P'_V, T'_V)$. We define $\tilde{\lambda} \equiv \lambda/(1-\lambda^2/2) = 0.232$, where $\lambda$ is the Wolfenstein parameter [33], and use the following ratios of meson decay constants to represent SU(3) breaking factors in color-favored tree amplitudes, $f_\pi/f_K = 0.84$, $f_\rho/f_{K^*} = 0.96$ [1]. Other SU(3) breaking factors in tree amplitudes involving ratios of kernels and ratios of their overlaps with meson wave functions (given roughly by ratios of a given form factor at slightly different values of $q^2$) will be neglected.

Assuming

$$\tilde{\lambda}^2|P'_{(V,P)}| \ll |T'_{(V,P)}| \tag{15}$$

we estimate the three ratios $|T'_{(V,P)}|/|P'_{(V,P)}|$ using central values for branching ratios given in Tables I and II,

$$\frac{|T'|}{|P'|} \simeq \tilde{\lambda}\left(\frac{f_K}{f_\pi}\right)\sqrt{\frac{r_\tau\mathcal{B}(\pi^+\pi^-)}{\mathcal{B}(K^0\pi^+)}} = 0.13\ , \tag{16}$$

$$\frac{|T'_P|}{|P'_P|} \simeq \tilde{\lambda}\left(\frac{f_{K^*}}{f_\rho}\right)\sqrt{\frac{r_\tau\mathcal{B}(\rho^+\pi^-)}{\mathcal{B}(K^{*0}\pi^+)}} = 0.31\ , \tag{17}$$

$$\frac{|T'_V|}{|P'_V|} \simeq \tilde{\lambda}\left(\frac{f_K}{f_\pi}\right)\sqrt{\frac{r_\tau\mathcal{B}(\rho^-\pi^+)}{\mathcal{B}(K^0\rho^+)}} = 0.27\ . \tag{18}$$

We note that our assumption (15) is a good approximation for the last two cases where indeed $|T'_{P,V}|/|P'_{P,V}| \gg \tilde{\lambda}^2 = 0.05$, but may provide only a crude estimate for $|T'|/|P'|$.

One may avoid the assumption (15) for $|T'|/|P'|$ by replacing (16) with a quadratic relation following from the amplitudes in Table II,

$$R \equiv \frac{r_\tau \mathcal{B}(\pi^+\pi^-)}{\mathcal{B}(K^0\pi^+)} = \tilde{\lambda}^2 + \left(\frac{|T'|}{|P'|}\right)^2 \left(\tilde{\lambda}\frac{f_K}{f_\pi}\right)^{-2} + 2\frac{|T'|}{|P'|}\frac{f_\pi}{f_K}\cos\delta\cos\gamma \ , \tag{19}$$

where $\delta$ is the unknown strong phase difference between $P'$ and $T'$. Denoting $z = \cos\delta\cos\gamma$ one solves for $|T'|/|P'|$,

$$\frac{|T'|}{|P'|} = \tilde{\lambda}^2 \frac{f_K}{f_\pi}\left(\sqrt{z^2 + (R - \tilde{\lambda}^2)/\tilde{\lambda}^2} - z\right) \ , \tag{20}$$

which is a monotonically decreasing function of $z$. Taking conservative bounds $-0.6 \le z \le 0.6$, based on a lower bound on $\gamma$ [2] and on no restrictions on $\delta$, one obtains

$$0.09 \le \frac{|T'|}{|P'|} \le 0.16 \ , \tag{21}$$

which describes a rather narrow range around the value in (16). Similarly, including quadratic corrections in estimates of $|T'_P|/|P'_P|$ and $|T'_V|/|P'_V|$, one obtains the following bounds,

$$0.28 \le \frac{|T'_P|}{|P'_P|} \le 0.35 \ , \qquad 0.23 \le \frac{|T'_V|}{|P'_V|} \le 0.31 \ . \tag{22}$$

We do not include errors in input branching ratios. The resulting uncertainties in the above two ratios from errors in branching ratios are somewhat smaller than the ones shown, which are caused by our conservative assumption of completely arbitrary strong phases $\delta$.

Our conclusion is that the ratios $|T'_P|/|P'_P|$ and $|T'_V|/|P'_V|$ of tree-to-penguin amplitudes in $B \to K^*\pi$ and $B \to \rho K$, respectively, are between two to three times larger than the corresponding ratio $|T'|/|P'|$ in $B \to K\pi$. The processes $B^0 \to K^{*+}\pi^-$ and $B^+ \to K^{*+}\pi^0$ or $B^0 \to K^+\rho^-$ and $B^+ \to K^+\rho^0$ involve interference of $P'_P$ and $T'_P$ or interference of $P'_V$ and $T'_V$. Thus, these processes may potentially involve asymmetries between two to three times larger than the 10% asymmetry measured in $B^0 \to K^+\pi^-$.

Maximal CP asymmetries $A^{\max}_{CP} \simeq 2|T'_{(P,V)}|/|P'_{(P,V)}|\sin\gamma$ in $B^0 \to K^+\pi^-, B^0 \to K^{*+}\pi^-$ and $B^0 \to \rho^- K^+$, are obtained for $\delta = 90^\circ$ or $z = 0$ corresponding to the central values of $|T'_{(P,V)}|/|P'_{(P,V)}|$ given in Eqs. (16), (17) and (18). The actual values of these asymmetries depend, of course, on $\delta$. The measured asymmetry in $B^0 \to K^+\pi^-$ indicates $|T'/P'| \sim 0.10$ within the range (21) and $\delta(K\pi) \sim 30^\circ$. The strong phases in $B^0 \to K^{*+}\pi^-$ and $B^0 \to \rho^- K^+$ may be different which would affect the asymmetries in these processes.

The asymmetry in $B^+ \to K^{*+}\pi^0$ ($B^+ \to K^+\rho^0$) depends also on the interference of $P'_P$ and $C'_V$ ($P'_V$ and $C'_P$). Unlike $B^+ \to K^+\pi^0$, where the effect of $C'$ on the CP asymmetry is destructive relative to the effect of $T'$, these interference effects do not have to act destructively with respect to the interference between $P'_P$ and $T'_P$ ($P'_V$ and $T'_V$) . Thus they may increase the asymmetry in $B^+ \to K^{*+}\pi^0$ ($B^+ \to K^+\rho^0$) relative to that in $B^0 \to K^{*+}\pi^-$ ($B^0 \to K^+\rho^-$).

We note in passing that one may use the ranges of value in Eqs. (21) and (22) to estimate ratios of penguin and tree amplitudes in $B^0 \to \pi^+\pi^-$, $B^0 \to \rho^+\pi^-$ and $B^0 \to \rho^-\pi^+$ [34, 35].

Taking into account CKM and SU(3) breaking factors, one finds for these three ratios

$$0.40 \leq \tilde{\lambda}^2 \frac{f_K}{f_\pi} \frac{|P'|}{|T'|} \leq 0.71 \ , \quad 0.16 \leq \tilde{\lambda}^2 \frac{f_{K^*}}{f_\rho} \frac{|P'_P|}{|T'_P|} \leq 0.20 \ , \quad 0.21 \leq \tilde{\lambda}^2 \frac{f_K}{f_\pi} \frac{|P'_V|}{|T'_V|} \leq 0.28 \ . \quad (23)$$

Thus, whereas the penguin contribution in $B^0 \to \pi^+\pi^-$ is only somewhat smaller than that of the tree amplitude, penguin terms contribute only about 20% to the amplitudes for $B^0 \to \rho^+\pi^-$ and $B^0 \to \rho^-\pi^+$ which are largely dominated by tree amplitudes. We note that while calculations in Ref. [36] using QCD factorization obtain values for the ratios (23) which are about a factor two smaller than the above, a more recent update agrees with the above ranges [37].

## V. TESTS OF FLAVOR SU(3)

In the previous section we assumed that penguin amplitudes are SU(3)-invariant, while SU(3) breaking factors involving ratios of meson decay constants were assumed for tree amplitudes. In this section we will present experimental tests for these assumptions based on measurements of decay rates and CP asymmetries in pairs of $\Delta S = 1$ and $\Delta S = 0$ decays which are related to each other by flavor SU(3).

Consider the three pairs of $B^0$ decay processes listed in Table II. The structure of these amplitudes, involving for a given pair the same penguin and tree amplitudes with different CKM factors, leads to simple relations between CP rate differences defined by

$$\Delta(B \to f) \equiv \Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f) \equiv 2\bar{\Gamma}(B \to f) A_{CP}(B \to f) \ , \quad (24)$$

where $\bar{\Gamma}$ is a CP-averaged decay rate. Denoting by $\mathcal{B}A_{CP}$ the product of a charge-averaged branching ratio and a CP asymmetry for a given process, one finds [38, 39]

$$\Delta(K^+\pi^-) = -\frac{f_K}{f_\pi}\Delta(\pi^+\pi^-) \qquad \text{or} \qquad [\mathcal{B}A_{CP}](K^+\pi^-) = -\frac{f_K}{f_\pi}[\mathcal{B}A_{CP}](\pi^+\pi^-) \ , \quad (25)$$

and

$$\Delta(K^{*+}\pi^-) = -\frac{f_{K^*}}{f_\rho}\Delta(\rho^+\pi^-) \quad \text{or} \quad [\mathcal{B}A_{CP}](K^{*+}\pi^-) = -\frac{f_{K^*}}{f_\rho}[\mathcal{B}A_{CP}](\rho^+\pi^-) \ , \quad (26)$$

$$\Delta(\rho^-K^+) = -\frac{f_K}{f_\pi}\Delta(\rho^-\pi^+) \qquad \text{or} \qquad [\mathcal{B}A_{CP}](\rho^-K^+) = -\frac{f_K}{f_\pi}[\mathcal{B}A_{CP}](\rho^-\pi^+) \ . \quad (27)$$

Using branching ratios and asymmetries in Tables I and II, these three equalities read respectively in units of $10^{-6}$,

$$-1.90 \pm 0.23 = -2.33 \pm 0.38 \ , \quad (28)$$

$$-1.54 \pm 0.71 = -2.01 \pm 1.07 \ , \quad (29)$$

$$1.29 \pm 0.54 = 1.27 \pm 1.03 \ . \quad (30)$$

The first test involves reasonably small errors and works well within $1\sigma$. It would have worked somewhat less well if penguin amplitudes were assumed to factorize like tree amplitudes and to involve an SU(3) breaking factor $f_K/f_\pi$. The current experimental errors in $B \to VP$ decays are still very large and do not provide useful SU(3) tests.

An independent test for SU(3) invariance of penguin amplitudes is provided by the ratio of rates of penguin dominated $\Delta S = 0$ and $\Delta S = 1$ $B \to PP$ and $B \to VP$ decays,

$$\sqrt{\frac{\mathcal{B}(\bar{K}^0 K^+)}{\mathcal{B}(K^0\pi^+)}} \simeq \sqrt{\frac{\mathcal{B}(\bar{K}^{*0} K^+)}{\mathcal{B}(K^{*0}\pi^+)}} \simeq \tilde{\lambda} \ . \tag{31}$$

Using $\mathcal{B}(\bar{K}^0 K^+) = (1.36^{+0.29}_{-0.27}) \times 10^{-6}$, $\mathcal{B}(\bar{K}^{*0} K^+) = (0.68 \pm 0.19) \times 10^{-6}$ [4], and taking $\Delta S = 1$ branching ratios in Table I, the above ratios of amplitudes are found to be $0.243 \pm 0.026$ and $0.262 \pm 0.038$, consistent with $\tilde{\lambda}$ within reasonable experimental errors. These errors must be reduced somewhat in order to distinguish between the assumed SU(3) invariant penguin amplitudes and SU(3) breaking in these amplitudes given by a factor $f_K/f_\pi$.

## VI. SU(3) RELATIONS BETWEEN TREE AND EWP AMPLITUDES

It has been noted that in the limit of flavor SU(3) symmetry approximate relations hold between the subdominant tree and electroweak penguin (EWP) amplitudes in $B \to K\pi$ transforming as $\Delta I = 1$. Neglecting EWP operators $\mathcal{O}_7$ and $\mathcal{O}_8$ with tiny Wilson coefficients in the effective weak Hamiltonian, and using a quite precise relation for Wilson coefficients (true within a few percent) [40],

$$\mathcal{K} \equiv \frac{c_9 + c_{10}}{c_1 + c_2} \approx \frac{c_9 - c_{10}}{c_1 - c_2} \approx -0.0087 \ , \tag{32}$$

one obtains [41, 42, 43],

$$\mathrm{EWP}(B^+ \to K^0\pi^+) + \sqrt{2}\mathrm{EWP}(B^+ \to K^+\pi^0) = -(P_{EW} + P^C_{EW}) = \frac{3\mathcal{K}}{2}(T + C) \ , \tag{33}$$

and [42]

$$\mathrm{EWP}(B^0 \to K^+\pi^-) + \mathrm{EWP}(B^+ \to K^0\pi^+) = -P^C_{EW} = \frac{3\mathcal{K}}{2}(C - E) \ . \tag{34}$$

These relations follow from corresponding properties of operators in the $\Delta S = 1, \Delta I = 1$ effective Hamiltonian behaving as $\overline{\mathbf{15}}$ and $\mathbf{6}$ under SU(3) transformation [42, 44],

$$\frac{1}{\lambda^{(s)}_t}\mathcal{H}^{(s)}_{\mathrm{EWP}}(\overline{\mathbf{15}}) = -\frac{3\mathcal{K}}{2}\frac{1}{\lambda^{(s)}_u}\mathcal{H}^{(s)}_{\mathrm{Tree}}(\overline{\mathbf{15}}) \ , \tag{35}$$

$$\frac{1}{\lambda^{(s)}_t}\mathcal{H}^{(s)}_{\mathrm{EWP}}(\mathbf{6}) = \frac{3\mathcal{K}}{2}\frac{1}{\lambda^{(s)}_u}\mathcal{H}^{(s)}_{\mathrm{Tree}}(\mathbf{6}) \ , \tag{36}$$

and imply relations similar to (33) and (34) in $B \to K^*\pi$ and $B \to \rho K$ decays.

It is convenient to study tree and EWP amplitudes for definite isospin states. We consider $\Delta I = 1$ amplitudes for final states $f = K\pi, K^*\pi, \rho K$ with isospin $I = 1/2, 3/2$, denoting corresponding tree and EWP contributions by $\mathcal{T}^f_I$, and $\mathcal{E}^f_I$, respectively,

$$A^f_I = \lambda^{(s)}_u \mathcal{T}^f_I + \lambda^{(s)}_t \mathcal{E}^f_I \ . \tag{37}$$

Thus one has for instance,

$$\begin{aligned} 6A^{K^*\pi}_{1/2} &= \sqrt{2}A(K^{*+}\pi^0) + 4A(K^{*0}\pi^+) - 3\sqrt{2}A(K^{*0}\pi^0) \\ &\equiv 6[\lambda_u^{(s)}\mathcal{T}^{K^*\pi}_{1/2} + \lambda_t^{(s)}\mathcal{E}^{K^*\pi}_{1/2}] \ , \end{aligned} \tag{38}$$

$$\begin{aligned} 3A^{K^*\pi}_{3/2} &= A(K^{*+}\pi^-) + \sqrt{2}A(K^{*0}\pi^0) = A(K^{*0}\pi^+) + \sqrt{2}A(K^{*+}\pi^0) \\ &\equiv 3[\lambda_u^{(s)}\mathcal{T}^{K^*\pi}_{3/2} + \lambda_t^{(s)}\mathcal{E}^{K^*\pi}_{3/2}] \ . \end{aligned} \tag{39}$$

We will apply flavor SU(3) to tree and EWP amplitudes in theses three classes of penguin-dominated decays. In the SU(3) limit both tree and EWP amplitudes may be expressed in terms of the same graphical contributions defined in Sections II [42] and III [44],

$$6\mathcal{T}^{K\pi}_{1/2} = -T + 2C + 3A \ , \qquad 3\mathcal{T}^{K\pi}_{3/2} = -T - C \ , \tag{40}$$

$$6\mathcal{E}^{K\pi}_{1/2} = \frac{\mathcal{K}}{2}(-6T + 3C - 9E) \ , \qquad 3\mathcal{E}^{K\pi}_{3/2} = \frac{3\mathcal{K}}{2}(T + C) \ , \tag{41}$$

$$6\mathcal{T}^{K^*\pi}_{1/2} = -T_P + 2C_V + 3A_P \ , \qquad 3\mathcal{T}^{K^*\pi}_{3/2} = -T_P - C_V \ , \tag{42}$$

$$6\mathcal{E}^{K^*\pi}_{1/2} = \frac{\mathcal{K}}{2}(-6T_V + 3C_P - 9E_P) \ , \qquad 3\mathcal{E}^{K^*\pi}_{3/2} = \frac{3\mathcal{K}}{2}(T_V + C_P) \ , \tag{43}$$

$$6\mathcal{T}^{\rho K}_{1/2} = -T_V + 2C_P + 3A_V \ , \qquad 3\mathcal{T}^{\rho K}_{3/2} = -T_V - C_P \ , \tag{44}$$

$$6\mathcal{E}^{\rho K}_{1/2} = \frac{\mathcal{K}}{2}(-6T_P + 3C_V - 9E_V) \ , \qquad 3\mathcal{E}^{\rho K}_{3/2} = \frac{3\mathcal{K}}{2}(T_P + C_V) \ . \tag{45}$$

We will make use of these expressions in the next section, assuming that the annihilation-like amplitudes $A_{(P,V)}$ and $E_{(P,V)}$ are negligible relative to the other amplitudes [25, 31].

One may use the above relations to estimate the magnitudes of EWP amplitudes relative to those of dominant penguin amplitudes. For instance, noting that the amplitude $T + C$ dominates $B^+ \to \pi^+\pi^0$ [21] with $\mathcal{B}(\pi^+\pi^0) = (5.59^{+0.41}_{-0.40}) \times 10^{-6}$ [4] , while $P_{tc}$ (or actually $P_{tc} - P^C_{EW}/3$) governs $B^+ \to K^0\pi^+$ in (3), Eq. (33) implies in the limit of flavor SU(3) [45]:

$$\frac{|P_{EW} + P^C_{EW}|}{|P_{tc}|} \simeq \frac{3|\mathcal{K}|}{\sqrt{2}}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(d)}|}\sqrt{\frac{\mathcal{B}(\pi^+\pi^0)}{\mathcal{B}(K^0\pi^+)}} = 0.10 \ , \tag{46}$$

We use values of CKM factors $\lambda_q^{(q')}$ quoted in Refs. [1, 2]. We have not introduced a flavor SU(3) breaking factor $f_K/f_\pi$ in $T+C$ because the color-suppressed tree amplitude $C$, which is comparable in magnitude to $T$ with a large relative strong phase, cannot be assumed to factorize.

A ratio including CKM factors of an EWP amplitude and a corresponding tree amplitudes in $B^+ \to K^+\pi^0$, which involve a common strong phase in the flavor SU(3) limit, is obtained directly from (33) [41, 42]:

$$\frac{\lambda_t^{(s)}(P_{EW} + P^C_{EW})}{\lambda_u^{(s)}(T + C)} = -\frac{3\mathcal{K}}{2}\frac{\lambda_t^{(s)}}{\lambda_u^{(s)}} = -0.61e^{-i\gamma} \ . \tag{47}$$

Deviations from this pure SU(3) limit have been estimated and were found to be at most at a level of ten percent in the magnitude of the right-hand-side and a few degrees in its strong phase [37, 46].

## VII. CP ASYMMETRY SUM RULES IN $B \to K\pi, K^*\pi, \rho K$

A rather precise sum rule among the four CP rate differences in $B \to K\pi$ decays has been proven in Ref. [20],

$$\Delta(K^+\pi^-) + \Delta(K^0\pi^+) - 2\Delta(K^+\pi^0) - 2\Delta(K^0\pi^0) \approx 0 \ . \tag{48}$$

Using the approximate relation (6) this implies a corresponding sum rule for CP asymmetries given in Eq. (1). This sum rule, which is based primarily on isospin symmetry, provides a prediction for $A_{CP}(K^0\pi^0)$ in terms of the other more precisely measured $B \to K\pi$ asymmetries. In this section we will study this sum rule and similar ones for $B \to K^*\pi$ and $B \to \rho K$ decays, comparing the precision of the three sum rules to one another.

We follow notations introduced in Section VI to denote by $\mathcal{T}^f_I$ and $\mathcal{E}^f_I$ $\Delta I = 1$ tree and EWP amplitudes for definite isospin states $I$, where $f = K\pi, K^*\pi, K\rho$. In addition, $\Delta I = 0$ amplitudes multiplying CKM factors $\lambda^{(s)}_t$ ($P_{tc}$ and EWP) and $\lambda^{(s)}_u$ ($P_{uc}$ and tree) will be denoted by $\mathcal{P}^f_{1/2}$ and $t^f_{1/2}$, respectively. Subscripts on amplitudes will refer the charges of the two mesons in the final state. Using isospin permits describing all twelve amplitudes for $B \to K\pi, K^*\pi, K\rho$, in the generic form,

$$-A^f_{+-} = \lambda^{(s)}_t\left(\mathcal{P}^f_{1/2} - \mathcal{E}^f_{1/2} - \mathcal{E}^f_{3/2}\right) + \lambda^{(s)}_u\left(t^f_{1/2} - \mathcal{T}^f_{1/2} - \mathcal{T}^f_{3/2}\right) \ , \tag{49}$$

$$A^f_{0+} = \lambda^{(s)}_t\left(\mathcal{P}^f_{1/2} + \mathcal{E}^f_{1/2} + \mathcal{E}^f_{3/2}\right) + \lambda^{(s)}_u\left(t^f_{1/2} + \mathcal{T}^f_{1/2} + \mathcal{T}^f_{3/2}\right) \ , \tag{50}$$

$$-\sqrt{2}A^f_{+0} = \lambda^{(s)}_t\left(\mathcal{P}^f_{1/2} + \mathcal{E}^f_{1/2} - 2\mathcal{E}^f_{3/2}\right) + \lambda^{(s)}_u\left(t^f_{1/2} + \mathcal{T}^f_{1/2} - 2\mathcal{T}^f_{3/2}\right) \ , \tag{51}$$

$$A^f_{00} = \lambda^{(s)}_t\left(\mathcal{P}^f_{1/2} - \mathcal{E}^f_{1/2} + 2\mathcal{E}^f_{3/2}\right) + \lambda^{(s)}_u\left(t^f_{1/2} - \mathcal{T}^f_{1/2} + 2\mathcal{T}^f_{3/2}\right) \ , \tag{52}$$

where $f = K\pi, K^*\pi, K\rho$.

Defining CP rate differences for each of these twelve processes (common phase space factors for a given $f$ are omitted) ,

$$\Delta^f_{ij} \equiv |\bar{A}^f_{ij}|^2 - |A^f_{ij}|^2 \ , \tag{53}$$

we consider the sums

$$\Delta(f) \equiv \Delta^f_{+-} + \Delta^f_{0+} - 2\Delta^f_{+0} - 2\Delta^f_{00} \ . \tag{54}$$

A generic amplitude of the form

$$A = \lambda^{(s)}_t P + \lambda^{(s)}_u T \ , \tag{55}$$

implies a CP rate difference (we omit a phase space factor)

$$\Delta \equiv |\bar{A}|^2 - |A|^2 = 4\mathrm{Im}(\lambda^{(s)}_t\lambda^{(s)*}_u)\mathrm{Im}(PT^*) \ . \tag{56}$$

Thus, using Eqs. (49)-(52) one finds

$$\Delta(f) = 24\mathrm{Im}[\lambda^{(s)}_t\lambda^{(s)*}_u]\mathrm{Im}\left(\mathcal{E}^f_{1/2}\mathcal{T}^{f*}_{3/2} + \mathcal{E}^f_{3/2}\mathcal{T}^{f*}_{1/2} - \mathcal{E}^f_{3/2}\mathcal{T}^{f*}_{3/2}\right) \ . \tag{57}$$

The dominant contributions to $\Delta(f)$, involving interference of the penguin amplitude (contained in $\mathcal{P}^f_{1/2}$) with tree amplitudes have cancelled. This is the essence of the three asymmetry sum rules, shown in Ref. [20] to follow from the isosinglet nature of the dominant penguin amplitude and an isospin quadrangle relation among the four $B \to f$ amplitudes [47, 48],

$$-A^f_{+-} + A^f_{0+} + \sqrt{2}A^f_{+0} - \sqrt{2}A^f_{00} = 0 \ . \tag{58}$$

A new $\Delta S = \Delta I = 0$ operator in the effective Hamiltonian would not affect this argument as such an operator could be absorbed in the penguin operator.

The remaining terms in the sum rules (57) involve second order interference terms of tree and EWP amplitudes. We will now estimate these remaining terms for each of the three cases, $f = K\pi, K^*\pi, K\rho$, or suggest methods for measuring these terms elsewhere. As we noted in Sec. IV, the potential asymmetries in $B \to K^{*+}\pi^-$ and $B^+ \to K^{*+}\pi^0$, or in $B^0 \to K^+\rho^-$ and $B^+ \to K^+\rho^0$, may be significantly larger than the 10% asymmetry measured in $B^0 \to K^+\pi^-$. Thus naively one would expect the remaining terms in the $B \to K^*\pi$ and $B \to K\rho$ sum rules to be correspondingly larger than in the $B \to K\pi$ sum rule.

**a.** $B \to K\pi$

Inserting (40) and (41) into the sum rule (57) for $f = K\pi$ and neglecting terms involving $A$ and $E$ one obtains

$$\Delta(K\pi) = -12\mathcal{K}\,\mathrm{Im}[\lambda^{(s)}_t \lambda^{(s)*}_u]\mathrm{Im}(CT^*) \ . \tag{59}$$

The sign of the remaining term in $\Delta(K\pi)$ is predicted to be negative because $\mathrm{Im}[\lambda^{(s)}_t \lambda^{(s)*}_u] = |V_{ts}||V_{cb}||V_{us}| \sin\gamma$ is positive while $\mathcal{K}$ and $\mathrm{Im}(CT^*)$ are negative. We have argued in Sec. II that the difference between $A_{CP}(K^+\pi^0)$ and $A_{CP}(K^+\pi^-)$ implies $\mathrm{Arg}(CT^*) < 0$ or $\mathrm{Im}(CT^*) < 0$ [22].

The remaining term in $\Delta(K\pi)$ should be compared with the negative CP rate difference in $B^0 \to K^+\pi^-$ which is dominated by an interference of $P$ and $T$,

$$\Delta(K^+\pi^-) \simeq 4\mathrm{Im}(\lambda^{(s)}_t \lambda^{(s)*}_u)\mathrm{Im}(PT^*) \ . \tag{60}$$

Using the numerical value of $\mathcal{K}$ in (32) one has

$$\frac{\Delta(K\pi)}{\Delta(K^+\pi^-)} \simeq -3\mathcal{K}\frac{\mathrm{Im}(CT^*)}{\mathrm{Im}(PT^*)} = 0.026\frac{|C|}{|P|}\frac{\sin[\mathrm{Arg}(CT^*)]}{\sin[\mathrm{Arg}(PT^*)]} \ . \tag{61}$$

The ratio $|C|/|P|$, where the numerator and denominator do not include CKM factors, may be evaluated by assuming that $|C| \le |T|$ and taking our estimate $|T'|/|P'| \sim 0.1$ in Sec. IV. Using numerical values of CKM factors [1, 2], this implies

$$\frac{|C|}{|P|} \le \frac{|V_{tb}V_{ts}|}{|V_{ub}V_{us}|}\frac{|T'|}{|P'|} \sim 4.6 \ . \tag{62}$$

One may further assume that the sine of the relative strong phase between $C$ and $T$ is not substantially larger than that of the relative strong phase between $P$ and $T$ (which we argued in Sec. IV to be around $30^\circ$ corresponding to $\sin[\mathrm{Arg}(PT^*)] = 0.5$). This implies

$$0 \le \frac{\Delta(K\pi)}{\Delta(K^+\pi^-)} \le 0.12 \ . \tag{63}$$

Although this positive upper bound is not rigid we consider it rather safe to conclude that the sum rule (1) for $B \to K\pi$ asymmetries holds within two or perhaps even one percent,

$$-0.02\ (-0.01) < A_{CP}(K^+\pi^-) + A_{CP}(K^0\pi^+) - A_{CP}(K^+\pi^0) - A_{CP}(K^0\pi^0) \leq 0\ . \qquad (64)$$

An important conclusion following from $\mathrm{Arg}(CT^*) < 0$ is that the very small remaining term in the sum rule must be negative. Using three of the $B \to K\pi$ asymmetries in Table I leads to a prediction for $A_{CP}(K^0\pi^0)$ which includes second order corrections in the asymmetry sum rule,

$$A_{CP}(K^0\pi^0) = -0.149 \pm 0.037 \pm 0.01\ . \qquad (65)$$

The first error is purely experimental while the second one is due to a possible small violation of the sum rule. We note that the current experimental errors in $A_{CP}(K^0\pi^+)$ and $A_{CP}(K^+\pi^0)$ dominate the uncertainty in this prediction.

**b.** $B \to K^*\pi$ **and** $B \to K\rho$

Substituting (42) and (43) in the sum rule (57) for $f = K^*\pi$ and neglecting small $A_P$ and $E_P$ terms one obtains,

$$\Delta(K^*\pi) = 6\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]\mathrm{Im}(T_V T_P^* + 2T_V C_V^* + C_P C_V^*)\ . \qquad (66)$$

As mentioned, expressions for $B \to K\rho$ amplitudes may be obtained from those for $B \to K^*\pi$ by interchanging subscripts $V \leftrightarrow P$. This applies also to $\Delta(K\rho)$ which can be obtained from $\Delta(K^*\pi)$ through the same transformation:

$$\Delta(K\rho) = 6\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]\mathrm{Im}(T_P T_V^* + 2T_P C_P^* + C_V C_P^*)\ . \qquad (67)$$

An interesting relation holds between the difference $\Delta(K\rho) - \Delta(K^*\pi)$ and a CP rate difference for the amplitude $3A_{3/2}^{K^*\pi} \equiv A(K^{*+}\pi^-) + \sqrt{2}A(K^{*0}\pi^0)$ already defined in (39),

$$\Delta\left((K^*\pi)_{3/2}\right) \equiv |3\bar{A}_{3/2}^{K^*\pi}|^2 - |3A_{3/2}^{K^*\pi}|^2\ . \qquad (68)$$

Using Eqs.(39), (42) and (43),

$$3A_{3/2}^{K^*\pi} = -\lambda_u^{(s)}(T_P + C_V) + \lambda_t^{(s)}\frac{3\mathcal{K}}{2}(T_V + C_P)\ , \qquad (69)$$

and applying (56) one has

$$\Delta\left((K^*\pi)_{3/2}\right) = 6\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]\mathrm{Im}[(T_P + C_V)(T_V^* + C_P^*)]\ , \qquad (70)$$

which implyes

$$\Delta(K\rho) - \Delta(K^*\pi) = 2\Delta\left((K^*\pi)_{3/2}\right)\ . \qquad (71)$$

Note that, while certain individual CP rate asymmetries in $B \to K^*\pi$ and $B \to K\rho$ (involving interference of penguin and tree amplitudes) are expected to be large as discussed in Sec IV, the asymmetry $\Delta((K^*\pi)_{3/2})$ involves interference of tree and EWP amplitudes and is considerably smaller.

The CP rate difference $\Delta((K^*\pi)_{3/2})$ can be measured in a Dalitz analysis of $B^0 \to K^+\pi^-\pi^0$ and its charge-conjugate [49, 50]. This analysis yields values for the magnitudes of $A(K^{*+}\pi^-)$, $A(K^{*0}\pi^0)$, their relative phase and their charge-conjugates which together fix $|3A^{K^*\pi}_{3/2}|$ and its charge conjugate thereby determining $\Delta((K^*\pi)_{3/2})$. This quantity is related to a ratio of amplitudes, $R_{3/2} \equiv \bar{A}^{K^*\pi}_{3/2}/A^{K^*\pi}_{3/2}$, which is measurable in $B^0 \to K^+\pi^-\pi^0$ and its charge conjugate:

$$\Delta\left((K^*\pi)_{3/2}\right) = |3A^{K^*\pi}_{3/2}|^2\left(|R_{3/2}|^2 - 1\right) \ . \tag{72}$$

The phase of $R_{3/2}$ has been proposed to give a new constraint on CKM parameters [51, 52]. Its magnitude $|R_{3/2}|$ and the magnitude $|3A^{K^*\pi}_{3/2}|$ determine $|\Delta\left((K^*\pi)_{3/2}\right)|$ which provides a good estimate for the combined violation of the $K^*\pi$ and $K\rho$ asymmetry sum rules.

Experimental errors using current data [50] are large and do not permit a useful constraint on $\Delta((K^*\pi)_{3/2})$. A four-fold ambiguity in the solution for $B \to K^*\pi$ amplitudes may be resolved by requiring destructive interference between the two penguin-dominated amplitudes $A(K^{*+}\pi^-)$ and $\sqrt{2}A(K^{*0}\pi^0)$ forming together the amplitude $3A^{K^*\pi}_{3/2}$ which involves smaller tree and EWP contributions. This requirement applies also to charge conjugate amplitudes, thus choosing solution IV among the four solutions in Table V of Ref. [50].

A somewhat less precise way for obtaining separate estimates for $\Delta(K^*\pi)$ and $\Delta(K\rho)$ requires using flavor SU(3) which relates $B \to K^*\pi$ and $B \to K\rho$ to $B \to \rho\pi$. In the SU(3) symmetry limit, the right-hand-side of (66) may be expressed in terms of amplitudes for $B \to \rho\pi$ decays which are dominated by $T_{V,P}$ and $C_{V,P}$ (compare the first two equations with expressions in Table II) [44]:

$$\begin{aligned}
-A(\rho^+\pi^-) &\simeq \lambda_u^{(d)}T_P + \lambda_t^{(d)}P_P \ , & (73)\\
-A(\rho^-\pi^+) &\simeq \lambda_u^{(d)}T_V + \lambda_t^{(d)}P_V \ , & (74)\\
-2A(\rho^0\pi^0) &\simeq \lambda_u^{(d)}(C_V + C_P) - \lambda_t^{(d)}(P_V + P_P) \ , & (75)\\
-\sqrt{2}A(\rho^+\pi^0) &\simeq \lambda_u^{(d)}(T_P + C_V) - \lambda_t^{(d)}(P_V - P_P) \ , & (76)\\
-\sqrt{2}A(\rho^0\pi^+) &\simeq \lambda_u^{(d)}(T_V + C_P) + \lambda_t^{(d)}(P_V - P_P) \ . & (77)
\end{aligned}$$

We neglect tiny EWP contributions and very small terms $E_{V,P} + PA_{V,P}$, just as we have neglected $E + PA$ in $A(B^0 \to \pi^+\pi^-)$ in Table II.

Working in the SU(3) symmetry limit, which is expected to introduce an uncertainty of $20 - 30\%$ in amplitudes, we will neglect contributions of penguin amplitudes in $B \to \rho\pi$ which will now be estimated to be at the same level. These contributions are measured by amplitudes for $B \to K^*\bar{K}, \bar{K}^*K$. Using the tree dominated branching ratio for $B^0 \to \rho^+\pi^-$ in Table II and the penguin dominated branching ratio for $B^+ \to \bar{K}^{*0}K^+$ quoted a line below Eq. (31), one has

$$\frac{|\lambda_t^{(d)}P_P|}{|\lambda_u^{(d)}T_P|} \simeq \sqrt{\frac{\mathcal{B}(\bar{K}^{*0}K^+)}{r_\tau\mathcal{B}(\rho^+\pi^-)}} = 0.20 \pm 0.03 \ . \tag{78}$$

This value is in agreement with the second range of values in Eq. (23), obtained for the same ratio using somewhat different considerations including an SU(3) breaking factor $f_{K^*}/f_\rho$.

In this approximation one finds

$$T_V T_P^* + 2T_V C_V^* + C_P C_V^* \simeq |\lambda_u^{(d)}|^{-2}[2A(\rho^0\pi^+)A^*(\rho^+\pi^0) + \sqrt{2}A(\rho^-\pi^+)A^*(\rho^+\pi^0) - \sqrt{2}A(\rho^0\pi^+)A^*(\rho^+\pi^-)] \ , \quad (79)$$

implying

$$\Delta(K^*\pi) \simeq \frac{6\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]}{|\lambda_u^{(d)}|^2}\,\mathrm{Im}[2A(\rho^0\pi^+)A^*(\rho^+\pi^0) + \sqrt{2}A(\rho^-\pi^+)A^*(\rho^+\pi^0) - \sqrt{2}A(\rho^0\pi^+)A^*(\rho^+\pi^-)] \ . \quad (80)$$

Charge conjugate modes may also be used on the right-hand-side to increase statistics.

The same quantity for $B \to K\rho$ may be obtained by interchanging subscripts $V \leftrightarrow P$ corresponding to interchanging the charges of $\rho$ and $\pi$ in $B \to \rho\pi$ amplitudes,

$$\Delta(K\rho) \simeq \frac{6\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]}{|\lambda_u^{(d)}|^2}\,\mathrm{Im}[-2A(\rho^0\pi^+)A^*(\rho^+\pi^0) + \sqrt{2}A(\rho^-\pi^+)A^*(\rho^+\pi^0) - \sqrt{2}A(\rho^0\pi^+)A^*(\rho^+\pi^-)] \ . \quad (81)$$

Comparing this expression with that of $\Delta(K^*\pi)$ in (80), we note that the only difference between the two expressions is the sign of the first term. Thus one may combine asymmetries as in (71) to obtain:

$$\Delta(K\rho) - \Delta(K^*\pi) \simeq \frac{24\mathcal{K}\,\mathrm{Im}[\lambda_t^{(s)}\lambda_u^{(s)*}]}{|\lambda_u^{(d)}|^2}\,\mathrm{Im}[A(\rho^+\pi^0)A^*(\rho^0\pi^+)] \ . \quad (82)$$

We now discuss a way by which the imaginary parts of products of $B \to \rho\pi$ amplitudes in Eqs. (80), (81) and (82) can be determined experimentally. Information about magnitudes and relative phases of the three $B^0$ decay amplitudes, $A(\rho^+\pi^-), A(\rho^-\pi^+)$ and $A(\rho^0\pi^0)$, is obtained in a time-dependent Dalitz analysis of $B^0 \to \pi^\pm\pi^\mp\pi^0$ [53, 54], $|A(\rho^0\pi^+)|$ is obtained in a Dalitz analysis of $B^+ \to \pi^+\pi^+\pi^-$ [55], while $|A(\rho^+\pi^0)|$ is measured directly in this quasi two-body mode [56, 57]. The five amplitudes obey an isospin pentagon relation [47, 48],

$$A(\rho^+\pi^-) + A(\rho^-\pi^+) + 2A(\rho^0\pi^0) = \sqrt{2}A(\rho^+\pi^0) + \sqrt{2}A(\rho^0\pi^+) = D \ , \quad (83)$$

where $D$ is a diagonal of the pentagon describing an $I = 2$ amplitude involving tree contributions but no penguin amplitude. A similar relation holds for $\bar{B}$ decay amplitudes.

The magnitude $|D|$ can be determined by studying the three $B^0$ amplitudes in a Dalitz analysis of $B^0 \to \pi^+\pi^-\pi^0$. The three magnitudes $|A(\rho^0\pi^+)|$, $|A(\rho^+\pi^0)|$ and $|D|$ fix the triangle for these amplitudes up to a two-fold ambiguity corresponding to flipping the triangle around $D$. This gives a value for $\mathrm{Im}[A(\rho^+\pi^0)A^*(\rho^0\pi^+)]$. Information gained in $B^0 \to \pi^+\pi^-\pi^0$ on magnitudes of $A(\rho^\pm\pi^\mp)$ and their phases relative to $D$ permits a determination of $\mathrm{Im}[A(\rho^-\pi^+)A^*(\rho^+\pi^0)]$ and $\mathrm{Im}[A(\rho^0\pi^+)A^*(\rho^+\pi^-)]$.

In the approximation of neglecting penguin amplitudes in Eqs. (73)-(77), (in which Eqs. (80)-(82) were written) or in the limit of vanishing strong phases between tree and

Table III: Branching fractions and CP asymmetries for $B \to \rho\pi$ [4].

| Mode | $\mathcal{B}$ $(10^{-6})$ | $A_{CP}$ |
|---|---|---|
| $B^+ \to \rho^+\pi^0$ | $10.9^{+1.4}_{-1.5}$ | $0.02 \pm 0.11$ |
| $B^+ \to \rho^0\pi^+$ | $8.3^{+1.2}_{-1.3}$ | $0.18^{+0.09}_{-0.17}$ |
| $B^0 \to \rho^0\pi^0$ | $2.0 \pm 0.5$ | $-0.30 \pm 0.38$[a] |

[a] We take an average of Belle [53] and Babar [54] results.

penguin amplitudes, all $B \to \rho\pi$ CP asymmetries vanish. Current experimental values of the five asymmetries quoted in Table II and III are consistent with zero within experimental errors. In this approximation the two pentagons for $B$ and $\bar{B}$ decays coincide, and Eq. (83) turns into a single pentagon relation for square roots of decay rates. A flat pentagon would correspond to

$$\sqrt{\mathcal{B}(\rho^+\pi^-)} + \sqrt{\mathcal{B}(\rho^-\pi^+)} + 2\sqrt{\mathcal{B}(\rho^0\pi^0)} = \sqrt{2}\sqrt{\mathcal{B}(\rho^+\pi^0)/r_\tau} + \sqrt{2}\sqrt{\mathcal{B}(\rho^0\pi^+)/r_\tau}\ . \tag{84}$$

Checking this possibility using branching ratios in Tables II and III, we find the following values (in units of $10^{-3}$) for the left and right-hand sides,

$$9.49 \pm 0.48 = 8.45 \pm 0.42\ , \tag{85}$$

which holds within $1.6\sigma$. In the limit of a flat pentagon the imaginary parts of all terms in (80) and (81) vanish implying $\Delta(K^*\pi) = \Delta(K\rho) = 0$. Current agreement with a flat pentagon may indicate that violation of the $K^*\pi$ and $K\rho$ asymmetry sum rules are strongly suppressed.

Current information on $B \to \rho\pi$ amplitudes, in particular that obtained from Dalitz analyses of $B^0 \to \pi^+\pi^-\pi^0$ is not sufficiently precise for a useful quantitative study of $\Delta(K^*\pi)$ and $\Delta(K\rho)$. Errors are large in relative phases between amplitudes and in CP asymmetries. In addition to a very large error in $A_{CP}(\rho^0\pi^0)$, Belle measured a large central value for $A_{CP}(\rho^+\pi^-)$ and a smaller value for $A_{CP}(\rho^-\pi^+)$ [53], $A_{CP}(\rho^+\pi^-) = 0.21 \pm 0.09, A_{CP}(\rho^-\pi^+) = 0.09 \pm 0.18$, whereas the situation in the Babar measurements was the opposite [54], $A_{CP}(\rho^+\pi^-) = 0.03 \pm 0.010, A_{CP}(\rho^-\pi^+) = -0.37 \pm 0.16$. Using the Babar data and performing a numerical study as described above, we obtain broad ranges for $\Delta(K^*\pi)$ and $\Delta(K\rho)$ which are consistent both with zero and with $\Delta(K^{*+}\pi^-)$ and $\Delta(K^+\rho^0)$, respectively.

A detailed study based on more precise data must also consider theoretical uncertainties in $\Delta(K^*\pi)$ and $\Delta(K\rho)$ caused by SU(3) breaking and by neglecting penguin amplitudes in $B \to \rho\pi$, which are given by decay amplitudes for $B \to K^*\bar{K}$ and $B \to \bar{K}^*K$. One may include these terms and smaller terms of the forms $E_{V,P} + PA_{V,P}$ by substituting into Eq. (57) the following expressions for the isospin amplitudes $\mathcal{T}_I^{K^*\pi}$ and $\mathcal{E}_I^{K^*\pi}$ in terms of $\Delta S = 0$ amplitudes [44]:

$$6\lambda_u^{(d)}\mathcal{T}_{1/2}^{K^*\pi} = 3A(\rho^+\pi^-) - 2\sqrt{2}A(\rho^+\pi^0) - 3A(K^{*+}K^-) + A(\bar{K}^{*0}K^+) + 2A(K^{*+}\bar{K}^0), \tag{86}$$

$$3\lambda_u^{(d)}\mathcal{T}_{3/2}^{K^*\pi} = \sqrt{2}A(\rho^+\pi^0) + A(\bar{K}^{*0}K^+) - A(K^{*+}\bar{K}^0)\,, \tag{87}$$

$$\lambda_u^{(d)}\mathcal{E}_{1/2}^{K^*\pi} = \frac{\mathcal{K}}{4}[3A(\rho^-\pi^+) - \sqrt{2}A(\rho^0\pi^+) + 3A(K^{*0}\bar{K}^0) + A(\bar{K}^{*0}K^+) - A(K^{*+}\bar{K}^0)]\ , \quad (88)$$

$$\lambda_u^{(d)}\mathcal{E}_{3/2}^{K^*\pi} = \frac{\mathcal{K}}{2}[-\sqrt{2}A(\rho^0\pi^+) - A(K^{*+}\bar{K}^0) + A(\bar{K}^{*0}K^+)]\ . \quad (89)$$

Corresponding expressions for the case $f = K\rho$ may be obtained from the above by interchanging the charges of $\rho$ and $\pi$ and of $K^*$ and $K$. We note that while both magnitudes and phases of $B \to \rho\pi$ amplitudes may be measured as discussed above, one can only determine experimentally the magnitudes for $B \to K^*\bar{K}$ and $B \to \bar{K}^*K$ amplitudes.

### VIII. CONCLUSION

A CP asymmetry of 10% has been measured in $B^0 \to K^+\pi^-$. We have argued that asymmetries in $B^0 \to K^{*+}\pi^-, K^+\rho^-$ and $B^+ \to K^{*+}\pi^0, K^+\rho^0$ may be two or three times larger because of their larger ratios of tree and penguin amplitudes. A previously proven approximate isospin sum rule for CP asymmetries in $B \to K\pi$ decays was generalized to $B \to K^*\pi$ and $B \to K\rho$. The residues of the three sum rules consist of contributions from interference of tree and electroweak penguin amplitudes. We have shown that the residue of the $K\pi$ sum rule is negative at a level of one or at most two percent. Combining the $K^*\pi$ and $K\rho$ asymmetries into a single sum rule, we have shown that its residue is given by a CP asymmetry in $B \to (K^*\pi)_{I=3/2}$, which may be measured in a Dalitz analysis of $B^0 \to K^+\pi^-\pi^0$. Using flavor SU(3), separate residues for the $K^*\pi$ and $K\rho$ asymmetry sum rules may be obtained by studying $B \to \rho\pi$ amplitudes in $B$ decays into three pions. Measuring asymmetries, at the LHCb detector or at a future Super Flavor Factory, in violation of these residues for $B \to K\pi$, $B \to K^*\pi$ or $B \to K\rho$ sum rules would provide evidence for a new $\Delta S = \Delta I = 1$ operator in the low energy effective Hamiltonian.

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