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# ON UNSATURATED INFILTRATION IN SOILS

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**Abstract.** The infiltration of water into the soil was investigated by many researchers using both theoretical and experimental approaches. While the ponded infiltration more or less can also be treated with analytical methods, the unsaturated infiltration can generally be solved only by numerical procedures. Unfortunately, the latter method provides no generally valid relatonships.
Based on the analysis of the wetting front and on experimental results, some general relatonships were established to calculate the equilibrium moisture content corresponding to a given water flux.

*Keywords*: Unsaturated infiltration, ponded infiltration, soil.

## 1. Introduction

Infiltration from rainfall or sprinkler irrigation is of great importance for the practice. An excess of rainfall over infiltration may cause severe overland flow and soil erosion. In the case of saturated or ponded infiltration all of the pores are filled with water and the hydraulic conductivity of the soil equals the saturated conductivity. If the rate of supply of water is limited in comparison to the maximum rate, then the water content at the surface and in the transmission zone can not reach the fully saturated value $W_s$.

In the wetting front of an infiltration process the matric potential of the soil predominates and, therefore, the absorption of water takes place. Behind the advancing wetting front, in which the water content is nearly constant (transmission zone), the gravitational potential becomes dominant.

If the water content of the transmission zone for any infiltration rate were known, then the wetting front velocity (or pore water velocity) could simply be determined. From the continuity equation of flow it follows that the average depth of the wetting front is given by

$$y = \frac{v \cdot t}{W - W_1} = w \cdot t \tag{1.1}$$

where

| | |
|---|---|
| $v$ | is the rain intensity, cm/h, |
| $W_1, W$ | are the initial and asymptotic volumetric water content, |

$t$ is the elapsed time,
$w$ is the velocity of the wetting front.

Unfortunately, the asymptotic water content for different infiltration conditions is generally not known. Therefore we have analyzed the wetting front, especially the distribution of the matric potential and the gradient of water content in it. We used also experimental results obtained on packed plexiglass columns.

## 2. Theoretical Considerations

In any infiltration process a wetting front starts as it is shown in Figure 1.

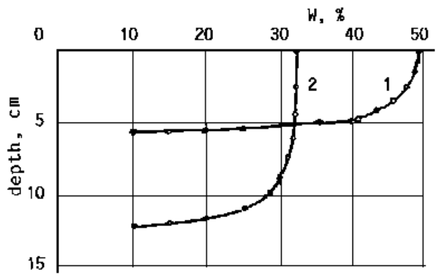



Figure 1. Wetting profiles for different water fluxes. $1 - v = K_s,\ 2 - v = 0.001 \times K_s$

The profile of water content generally shows a very steep wetting front and, therefore, in many cases a simplified approach may be used. Measurements on different types of soils have shown that the steepness of the water profile decreases with increasing hydraulic conductivity. The driving potential is the sum of the matric potential of the dry soil and the gravitational potential. In the wetting front the absorption of water occurs by tension forces while behind the wetting front the gravitational effect becomes more dominant. The steady-state one-dimensional motion of water can be described by Darcy's law as

$$-v = K(W) \cdot \frac{\mathrm{d}(\Psi + y)}{\mathrm{d}y} = K(W) \cdot \frac{\mathrm{d}\Psi}{\mathrm{d}y} + 1 \qquad (2.1)$$

where

$K(W)$ is the hydraulic conductivity of the soil,
$\Psi$ is the matric potential.

In order to use equation (1.1), the asymptotic water content must be known. In the case of ponded infiltration $W = W_s$, that is the saturated water content. Under unsaturated conditions the water content at the surface should approach a value appropriate for $K$ to equal $v$. Our main task is to find an appropriate relationship to describe the variation of the asymptotic water content in terms of soil physical properties such as the relative rain intensity $K_s/v$ or the soil matric potential.

The soil water diffusivity and the hydraulic conductivity are interrelated by the soil matric potential as follows

$$K\,(W) = D \cdot \frac{\mathrm{d}W}{\mathrm{d}\Psi}\;. \tag{2.2}$$

The soil matric potential is approximated by the following expression [1]:

$$MR = \frac{1}{\left[1 + (\alpha \cdot \Psi)^n\right]^m} \text{ and } m = 1 - \frac{1}{n} \tag{2.3}$$

where the moisture ratio is defined as

$$MR = \frac{W - W_r}{W_s - W_r}.$$

In this equation $W_r$ is a small moisture value and serves first of all to achieve a better fitting.

In order to use equation (2.2), the soil water diffusivity relationship is needed. Earlier experiments have shown that the diffusivity is influenced first of all by the degree of saturation [2]. Indeed, we have always found good correlation using the following expression

$$D = D_0 \cdot \exp\left[-A\,(1 - DS)\right] \tag{2.4}$$

where $A$ is a constant (for soils in question its value is 9.75 and 7.39 respectively) and the degree of saturation is given by

$$DS = \frac{W - W_{\min}}{W_s - W_{\min}} \tag{2.5}$$

$W_{\min}$ means a minimum water content at which the liquid diffusion coefficient drastically decreases and it is corresponding to pF-values between 4.2 and 4.5 (wilting point). For water contents less than $W_{\min}$ Darcy's law becomes no longer valid and the water movement occurs more and more in vapor phase.

Examining equation.(2.1), it is easy to realize that the ratio of infiltration rate to saturated conductivity has a fundamental influence on the equilibrium moisture content in the transmission zone. In the following we analyze the structure of wetting front for different flux conditions. Using the term saturated and relative hydraulic conductivity and, keeping in mind that $\mathrm{d}\Psi/\mathrm{d}y = (\mathrm{d}\Psi/\mathrm{d}W) \cdot (\mathrm{d}W/\mathrm{d}y)$, the moisture gradient in the wetting front is given by

$$\frac{\mathrm{d}W}{\mathrm{d}y} = \frac{v}{K_s} \cdot \frac{1}{K_r\left(\dfrac{\mathrm{d}\Psi}{\mathrm{d}W}\right)}\;. \tag{2.6}$$

This relationship for soil 1 is demonstrated in Figure 2. The curves decline below 15% moisture content because of the deviation from Darcy's law.

Knowing the moisture gradient, the potential gradient can also be calculated, which for different relative flux values is plotted in Figure 3. From this Figure the equilibrium

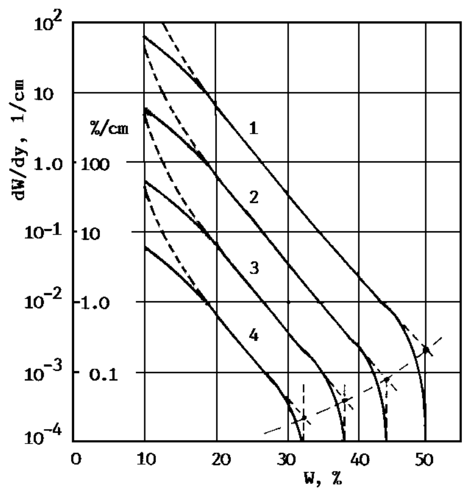



Figure 2. Moisture gradient in the wetting front as a function of moisture content. 1– $v = K_s$, 2–$v = 0.1K_s$, 3–$v = 0.01K_s$, 4– $v = 0.001K_s$.

moisture contents for any relative flux value can be read. Namely, in all cases the hydraulic potential gradient in the transmission zone is the unity given by the gravitational potential.

Figure 2 shows that the moisture gradient on the outer surface of the wetting front is the greatest and it rapidly decreases as the moisture content increases. A decreasing flux further decreases the moisture gradient at any moisture content and, therefore, the wetting profile changes from the steep shape to a gently sloping one. The measured wetting front profiles support this statement.

An integration of equation (2.6) gives the cumulative wetting front depth between two given moisture contents. We can write

$$\int \mathrm{d}y = \frac{K_s}{v} \int K_r\left(\Psi\right) \mathrm{d}\Psi\,. \tag{2.7}$$

It is interesting to note that the integral on the right side is the so-called Kirchhoff-potential often used in analytical solutions. The solution of equation (2.7) for soil 1 is given in Figure 4. It can clearly be seen that the cumulative wetting front thickness with increasing moisture content and with decreasing water flux rapidly increases.

A dimensionless equation describing the wetting front propagation can be derived in the following way. The moisture variation by time is expressed as

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{\mathrm{d}W}{\mathrm{d}y}\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}W}{\mathrm{d}y}w$$

where $w$ means the effective pore velocity of water. Using equations (1.1) and (2.6) we can write

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \frac{v^2}{W - W_1}\frac{1}{K_s K_r\left(\frac{\mathrm{d}\Psi}{\mathrm{d}W}\right)} \tag{2.8}$$

Keeping in mind equation (2.2), a formal integration of equations (2.6) and (2.8) supplies the following relationship:

$$\frac{vy}{D} = \frac{v^2 t}{\left(W - W_1\right) D} \tag{2.9}$$

which is equivalent to equation (1.1). In the above equation $y$ means the average depth of the wetting front corresponding to the mean water content $W$. The left and right sides of equation (2.9) can be used as dimensionless coordinates to represent measurement data.

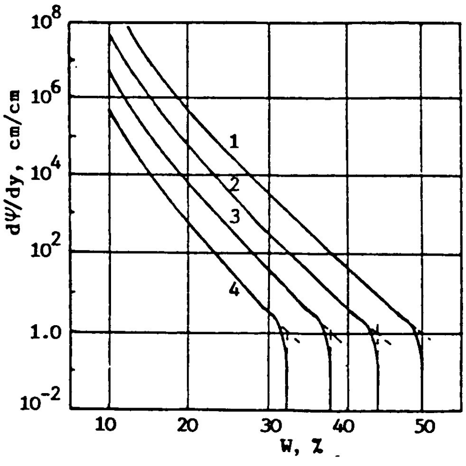



Figure 3. Matric potential gradient in the wetting front. Different relative fluxes from those in Figure 2.

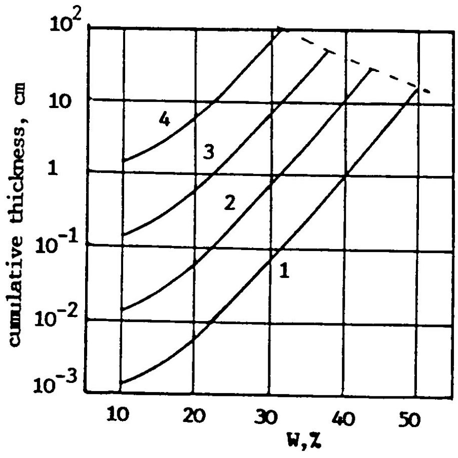



Figure 4. Cumulative wetting front thickness for different relative fluxes.

In all equations derived above the unknown water content $W$ appears. In order to calculate this water content either the diffusion function $D(W)$ and the matric potential curve $\Psi(W)$ or the relative hydraulic conductivity $K_r(W)$ is needed. In the first case, the solution of the following equation (keeping in mind that $\mathrm{d}\Psi/\mathrm{d}y = 1$):

$$\frac{v}{K_s} = K_r = \frac{D\,(W)}{K_s}\frac{\mathrm{d}W}{\mathrm{d}\Psi}$$

supplies the equilibrium moisture content. In the second case, if the $K_r(W)$ function is available (see Figure 5), then the moisture content value can simple be read from the curve.

## 3. Results

From the above discussion it is obvious that a generally valid closed form relation for calculating the asymptotic moisture content cannot be obtained. Nevertheless, an appropriate approximation would facilitate a rapid estimation of the expected value.

In order to establish such a relationship both theoretically derived asymptotic values and experimentally obtained values were used. For measurements two different soils were used with several initial moisture contents. The hydraulic properties of soils are given in Figure 5 and in Table 1.

| soil | porosity | $D_0$, $\frac{cm^2}{h}$ | $K_s$, $\frac{cm}{h}$ | $W_{\min}$ | $W_r$ | $\alpha$, $\frac{1}{cm}$ | n |
|---|---|---|---|---|---|---|---|
| silty loam | 50% | 2000 | 3.0 | 0.15 | 0.03 | 0.015 | 1.25 |
| loam | 40% | 400 | 0.4 | 0.12 | 0.02 | 0.018 | 1.25 |

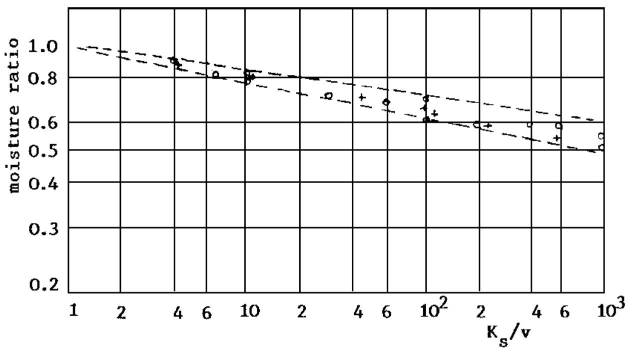
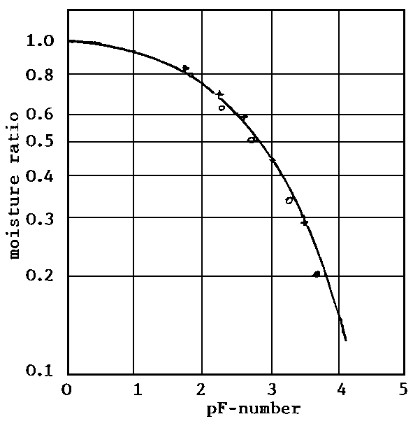



Figure 5. PF-curve and relative conductivity as a function of water content for soils. 1-silty loam, 2-loam



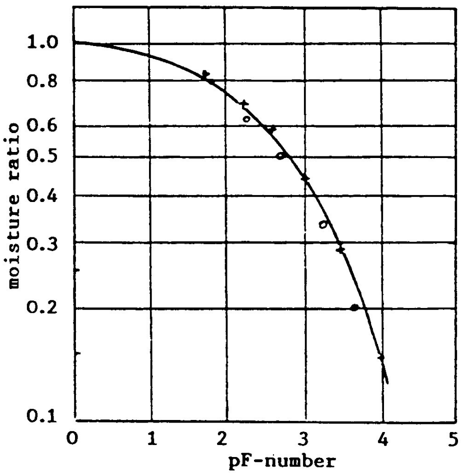




Figure 6. Equilibrium moisture content as a function of the relative water flux.

Figure 7. Equilibrium moisture content as a function of pF-number.

Measurements were carried out on packed plexiglass columns with $K_s/v$ ratios between 5 and 200 and with initial moisture contents of 10% and 20%.

The calculated and experimentally obtained results are plotted in Figure 6. using double logaritmic scale and dimensionless quantities. To describe the relationship the following simple equation is obtained:

$$\frac{W - W_1}{W_s - W_1} = \left(\frac{K_s}{v}\right)^{-n} \tag{3.1}$$

where the exponent $n$ varies between 0.08 and 0.1.

Using equation (2.9), equation (3.1) can be rewritten as

$$\frac{vy}{D} = \frac{v^2 t}{D} \frac{\left(\frac{K_s}{v}\right)^n}{W_s - W_1} . \tag{3.2}$$

In some cases it is interesting to express the equilibrium moisture content as a function of matric potential or pF-number. Such relationship for the two soils in question is to be seen in Figure 7. This curve can be approximated by the following empirical equation:

$$\frac{W - W_1}{W_s - W_1} = 1 - ApF^m$$

where the constants have the following values: $A = 0.08$ and $m = 1.75$. The $pF$-number corresponds to the equilibrium moisture content for a given flux.

## 4. Conclusion

Based on theoretical and experimental investigations the following conclusions can be drawn:

– the steepness of the wetting front is determined by the local moisture content and the relative flux ratio $v/K_s$,

– the equilibrium moisture content in all cases can be approximated as the intersection of the matric potential gradient and the $d\Psi/dy = 1.0$ lines,

– a simple dimensionless equation is obtained for determining the equilibrium moisture content which is in good correlation with the experimental results,

– to represent wetting front propagations the dimensionless quantities $vy/D$ and $v^2 t/[D(W - W_1)]$ can be used.

## REFERENCES

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