# Open charm meson spectroscopy: Where to place the latest piece of the puzzle 

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#### Abstract

We discuss how to classify the $c \bar{s}$ meson $D_{s J}(3040)$ recently discovered by the BaBar Collaboration. We consider four possible assignments, together with signatures useful to distinguish among them.


PACS numbers: 13.25.Ft, $12.39 . \mathrm{Fe}, 12.39 . \mathrm{Hg}$

In the infinite heavy quark mass limit, heavy-light $Q \bar{q}$ mesons can be conveniently classified in doublets labeled by the value of the total angular momentum $\vec{s}_{\ell}=\vec{s}_{\bar{q}}+\vec{\ell}$ of the light degrees of freedom (light antiquark and gluons) with respect to the heavy quark $Q$ [1]. $\vec{s}_{\bar{q}}$ is the spin of $\bar{q}$ and $\vec{\ell}$ the orbital angular momentum. For $\ell=0(s$-wave states in the constituent quark model), the doublet has $s_{\ell}=\frac{1}{2}$ and consists of two states (denoted as $\left(P, P^{*}\right)$, with $P$ a generic heavy meson) having spin-parity $J_{s \ell}^{P}=\left(0^{-}, 1^{-}\right)_{1 / 2} . p$-wave states, with $\ell=1$, form two doublets: $\left(P_{0}^{*}, P_{1}^{\prime}\right)$ with $J_{s_{\ell}}^{P}=\left(0^{+}, 1^{+}\right)_{1 / 2}$, and $\left(P_{1}, P_{2}^{*}\right)$ with $J_{s_{\ell}}^{P}=\left(1^{+}, 2^{+}\right)_{3 / 2}$. For $\ell=2, d$-wave states give rise to other two doublets: $\left(P_{1}^{*}, P_{2}\right)$ with $J_{s_{\ell}}^{P}=\left(1^{-}, 2^{-}\right)_{3 / 2}$, and $\left(P_{2}^{*}, P_{3}\right)$ with $J_{s_{\ell}}^{P}=\left(2^{-}, 3^{-}\right)_{5 / 2}$.

The construction proceeds further, and has been successfully applied to classify beauty mesons, as well as mesons with a charmed quark, albeit in the latter case the finite heavy quark mass corrections can be important.

In the case of $c \bar{s}$ mesons, both the lowest lying states, both the radial excitations (whose members of the various doublets we denote by a tilde) can be described using this classification scheme. Among the known $c \bar{s}$ hadrons, the two mesons $D_{s}(1969)$ and $D_{s}^{*}(2112)$ fill the doublet with $J_{s_{\ell}}^{P}=\left(0^{-}, 1^{-}\right)_{1 / 2}$, while one can identify $\left(D_{s 0}^{*}(2317), D_{s 1}^{\prime}(2460)\right)$ and $\left(D_{s 1}(2536), D_{s 2}^{*}(2573)\right)$ with the doublets $J_{s_{\ell}}^{P}=\left(0^{+}, 1^{+}\right)_{1 / 2}$ and $J_{s_{\ell}}^{P}=\left(1^{+}, 2^{+}\right)_{3 / 2}$ respectively (the discovery of $D_{s 0}^{*}(2317)$ and $D_{s 1}^{\prime}(2460)$ is rather recent 2], and their classification has been the subject of discussions [3]).

More recently, other candidates have been added to this list: $D_{s J}(2860)$, discovered by the BaBar Collaboration [4], and $D_{s J}(2710)$, discovered by the Belle and BaBar Collaborations [4, 5], both observed in the $D K$ final state. In the case of $D_{s J}(2710)$, it has been possible to determine the spin-parity: $J^{P}=1^{-}$, studying its production in $B$ decays. As for $D_{s J}(2860)$, the assignments $J^{P}=3^{-}$, with radial quantum number $n=1$ [6], and $J^{P}=0^{+}$, with $n=2$ (i.e. the first radial excitation of $D_{s J}(2317)$ ) 7] have been proposed soon after the discovery.

The latest piece of information comes from the BaBar Collaboration [8], thanks to a new analysis of $D K$ and $D^{*} K$ final states. $D_{s J}(2710)$ and $D_{s J}(2860)$ are seen decaying to both $D K$ and $D^{*} K$, hence they have natural parity $J^{P}=1^{-}, 2^{+}, 3^{-}, \cdots$. The decay into $D^{*} K$ ex-
cludes the assignment $J^{P}=0^{+}$for $D_{s J}(2860)$. Other information comes from the measurement of the ratios

$$
\begin{align*}
& \frac{B R\left(D_{s J}(2710) \rightarrow D^{*} K\right)}{B R\left(D_{s J}(2710) \rightarrow D K\right)}=0.91 \pm 0.13_{\text {stat }} \pm 0.12_{\text {syst }} \\
& \frac{B R\left(D_{s J}(2860) \rightarrow D^{*} K\right)}{B R\left(D_{s J}(2860) \rightarrow D K\right)}=1.10 \pm 0.15_{\text {stat }} \pm 0.19_{\text {syst }} \tag{1}
\end{align*}
$$

where $D^{(*)} K$ is the sum over the final states $D^{(*) 0} K^{+}$ and $D^{(*)^{+}} K_{S}^{0}$ [8]. Comparing these data with the predictions in [9], it seems very likely that $D_{s J}(2710)$ is the first radial excitation of $D_{s}^{*}(2112)$, while the case of $D_{s J}(2860)$ requires further study since the measured ratio in (11) is larger than the theoretical prediction $\frac{B R\left(D_{s J}(2860) \rightarrow D^{*} K\right)}{B R\left(D_{s J}(2860) \rightarrow D K\right)} \simeq 0.39 \quad$ 10].

In the same analysis, the BaBar Collaboration observed another broad structure in the $D^{*} K$ distribution, $D_{s J}(3040)$, with [8]

$$
\begin{align*}
M\left(D_{s J}(3040)\right) & =3044 \pm 8_{\text {stat }}\left({ }_{-5}^{+30}\right)_{\text {syst }} \mathrm{MeV} \\
\Gamma\left(D_{s J}(3040)\right) & =239 \pm 35_{\text {stat }}\left({ }_{-42}^{+46}\right)_{\text {syst }} \mathrm{MeV} \tag{2}
\end{align*}
$$

Studies of angular distributions for this state have not been attempted, due to the limited statistics. Here we discuss possible classifications for this new $c \bar{s}$ meson using the only information available, the measured mass, width and decay mode, together with the full set of information concerning the other levels/doublets.

Since $D_{s J}(3040)$ decays to $D^{*} K$ and not to $D K$, it has unnatural parity, with possible $J^{P}=1^{+}, 2^{-}, 3^{+}, \cdots$. The lightest not yet observed states with such quantum numbers are the the two $J^{P}=2^{-}$states belonging to the $\ell=2$ doublets, with $s_{\ell}=\frac{3}{2}$ and $s_{\ell}=\frac{5}{2}$ : we denote them as $D_{s 2}$ and $D_{s 2}^{* *}$, respectively (the case $J^{P}=3^{+}$corresponds to a doublet with $s_{\ell}=\frac{7}{2}$, which is expected to be heavier). In the case of radial excitations, the identification with the states with $n=2, J^{P}=1^{+}$, and $s_{\ell}=\frac{1}{2}$ (the meson $\tilde{D}_{s 1}^{\prime}$ ) or $s_{\ell}=\frac{3}{2}$ (the meson $\tilde{D}_{s 1}$ ) is possible.

These four assignments are indicated in Table which reports the classification of all the other known $c \bar{s}$ mesons

TABLE I: Known $c \bar{s}$ mesons organized according to $s_{\ell}^{P}$ and $J^{P}$; the measured masses are indicated. Allowed possibilities for $D_{s J}(3040)$ are displayed as $\mathbf{D}_{\mathbf{s J}}(\mathbf{3 0 4 0})$ ?.

| $s_{\ell}^{P}$ | $\frac{1}{2}^{-}$ | $\frac{1}{2}^{+}$ | $\frac{3}{2}^{+}$ | $\frac{3}{2}^{-}$ |
| :---: | :---: | :---: | :---: | :---: |
| $(n=1)$ |  |  |  |  |
| $J^{P}=s_{\ell}^{P}-\frac{1}{2}$ | $D_{s}(1965)\left(0^{-}\right)$ | $D_{s J}(2317)\left(0^{+}\right)$ | $D_{s 1}(2536)\left(1^{+}\right)$ | $\left(1^{-}\right)$ |
| $J^{P}=s_{\ell}^{P}+\frac{1}{2}$ | $D_{s}^{*}(2112)\left(1^{-}\right)$ | $D_{s J}(2460)\left(1^{+}\right)$ | $D_{s 2}^{*}(2573)\left(2^{+}\right)$ | $\mathbf{D}_{\mathbf{s J}}(\mathbf{3 0 4 0}) ?\left(2^{-}\right)$ |
| $(n=2)$ |  | $\left(0^{+}\right)$ | $\mathbf{D}_{\mathbf{s J}}(\mathbf{3 0 4 0}) ?\left(1^{+}\right)$ | $\mathbf{D}_{\mathbf{s J} J}(\mathbf{3 0 4 0}) ?\left(2^{-}\right)$ |
| $J^{P}=s_{\ell}^{P}-\frac{1}{2}$ | $\left(0^{-}\right)$ | $\left(2^{+}\right)$ | $\left(1^{-}\right)$ | $\left(2^{-}\right)$ |
| $J^{P}=s_{\ell}^{P}+\frac{1}{2}$ | $D_{s J}(2710)\left(1^{-}\right)$ | $\mathbf{D}_{\mathbf{s J} \mathbf{~}}(\mathbf{3 0 4 0}) ?\left(1^{+}\right)$ | $\left(2^{-}\right)$ | $\left(2^{-}\right)$ |

( $D_{s J}(2860)$ has been assigned to the $s_{\ell}=\frac{5}{2}$ doublet, with a question mark since confirmation is needed).

Some indications about the masses of these states come from potential model calculations. For example, in Ref. [11] the spectrum of heavy-light mesons is computed in the framework of a relativistic quark model (RQM), with results:

$$
\begin{array}{ll}
M\left(\tilde{D}_{s 1}\right)^{(R Q M)} & =3114 \mathrm{MeV} \\
M\left(\tilde{D}_{s 1}^{\prime}\right)^{(R Q M)} & =3165 \mathrm{MeV} \\
M\left(D_{s 2}\right)^{(R Q M)} & =2953 \mathrm{MeV}  \tag{3}\\
M\left(D_{s 2}^{* \prime}\right)^{(R Q M)} & =2900 \mathrm{MeV}
\end{array}
$$

Notice that, if the identification of $D_{s J}(2860)$ as the $J_{s_{\ell}}^{P}=3_{5 / 2}^{-}$state were experimentally confirmed, this would disfavor the assignment of $D_{s J}(3040)$ to its spin partner $D_{s 2}^{* \prime}$ with $J_{s_{\ell}}^{P}=2_{5 / 2}^{-}$, since a mass inversion in a spin doublet seems unlikely. For a similar reason, one would also disfavor the identification of $D_{s J}(3040)$ with $D_{s 2}$, although in that case the two mesons would belong to different doublets.

The four classifications for $D_{s J}(3040)$ in Table $\mathbb{\square}$ can be discussed computing the allowed strong decays. To this purpose, we work in the heavy quark limit in which the various spin doublets are described by effective fields: $H_{a}$ for $s_{\ell}^{P}=\frac{1}{2}^{-}(a=u, d, s$ is a light flavour index $) ; S_{a}$ and $T_{a}$ for $s_{\ell}^{P}=\frac{1}{2}^{+}$and $s_{\ell}^{P}=\frac{3}{2}^{+}$, respectively; $X_{a}$ and $X_{a}^{\prime}$ for the doublets corresponding to orbital angular momentum $\ell=2$, i.e. $s_{\ell}^{P}=\frac{3}{2}^{-}$and $s_{\ell}^{P}=\frac{5}{2}^{-}$:

$$
\begin{align*}
H_{a} & =\frac{1+\ngtr}{2}\left[P_{a \mu}^{*} \gamma^{\mu}-P_{a} \gamma_{5}\right] \\
S_{a} & =\frac{1+\ngtr}{2}\left[P_{1 a}^{\prime \mu} \gamma_{\mu} \gamma_{5}-P_{0 a}^{*}\right] \\
T_{a}^{\mu} & =\frac{1+\ngtr}{2}\left\{P_{2 a}^{\mu \nu} \gamma_{\nu}\right. \\
& \left.-P_{1 a \nu} \sqrt{\frac{3}{2}} \gamma_{5}\left[g^{\mu \nu}-\frac{1}{3} \gamma^{\nu}\left(\gamma^{\mu}-v^{\mu}\right)\right]\right\}  \tag{4}\\
X_{a}^{\mu} & =\frac{1+\ngtr}{2}\left\{P_{2 a}^{* \mu \nu} \gamma_{5} \gamma_{\nu}\right.
\end{align*}
$$

$$
\begin{aligned}
& \left.-P_{1 a \nu}^{* \prime} \sqrt{\frac{3}{2}}\left[g^{\mu \nu}-\frac{1}{3} \gamma^{\nu}\left(\gamma^{\mu}-v^{\mu}\right)\right]\right\} \\
X_{a}^{\prime \mu \nu} & =\frac{1+\ngtr}{2}\left\{P_{3 a}^{\mu \nu \sigma} \gamma_{\sigma}\right. \\
& -P_{2 a}^{*^{\prime} \alpha \beta} \sqrt{\frac{5}{3}} \gamma_{5}\left[g_{\alpha}^{\mu} g_{\beta}^{\nu}-\frac{1}{5} \gamma_{\alpha} g_{\beta}^{\nu}\left(\gamma^{\mu}-v^{\mu}\right)\right. \\
& \left.\left.-\frac{1}{5} \gamma_{\beta} g_{\alpha}^{\mu}\left(\gamma^{\nu}-v^{\nu}\right)\right]\right\}
\end{aligned}
$$

with the various operators annihilating mesons of fourvelocity $v$ which is conserved in strong interaction processes (the heavy field operators contain a factor $\sqrt{m_{P}}$ and have dimension $3 / 2$ ).

Let us consider decays with the emission of a light pseudoscalar meson. The octet of light pseudoscalar mesons is introduced considering the fields: $\xi=e^{\frac{i \mathcal{M}}{f_{\pi}}}$, $\Sigma=\xi^{2}$, and the matrix $\mathcal{M}$ containing $\pi, K$ and $\eta$ fields $\left(f_{\pi}=132 \mathrm{MeV}\right)$ :

$$
\mathcal{M}=\left(\begin{array}{ccc}
\sqrt{\frac{1}{2}} \pi^{0}+\sqrt{\frac{1}{6}} \eta & \pi^{+} & K^{+}  \tag{5}\\
\pi^{-} & -\sqrt{\frac{1}{2}} \pi^{0}+\sqrt{\frac{1}{6}} \eta & K^{0} \\
K^{-} & \bar{K}^{0} & -\sqrt{\frac{2}{3}} \eta
\end{array}\right)
$$

At the leading order in the heavy quark mass and light meson momentum expansion, the decays $F \rightarrow H M$ ( $F=H, S, T, X, X^{\prime}$ and $M$ a light pseudoscalar meson) are described by the Lagrangian interaction terms 12]:

$$
\begin{align*}
\mathcal{L}_{H} & =g \operatorname{Tr}\left[\bar{H}_{a} H_{b} \gamma_{\mu} \gamma_{5} \mathcal{A}_{b a}^{\mu}\right] \\
\mathcal{L}_{S} & =h \operatorname{Tr}\left[\bar{H}_{a} S_{b} \gamma_{\mu} \gamma_{5} \mathcal{A}_{b a}^{\mu}\right]+h . c . \\
\mathcal{L}_{T} & =\frac{h^{\prime}}{\Lambda_{\chi}} \operatorname{Tr}\left[\bar{H}_{a} T_{b}^{\mu}\left(i D_{\mu} \mathcal{A}+i D \mathcal{A}_{\mu}\right)_{b a} \gamma_{5}\right]+h . c . \\
\mathcal{L}_{X} & =\frac{k^{\prime}}{\Lambda_{\chi}} \operatorname{Tr}\left[\bar{H}_{a} X_{b}^{\mu}\left(i D_{\mu} \mathcal{A}+i D \mathcal{A}_{\mu}\right)_{b a} \gamma_{5}\right]+h . c .  \tag{6}\\
\mathcal{L}_{X^{\prime}} & =\frac{1}{\Lambda_{\chi}^{2}} \operatorname{Tr}\left[\overline { H } _ { a } X _ { b } ^ { \prime \mu \nu } \left[k_{1}\left\{D_{\mu}, D_{\nu}\right\} \mathcal{A}_{\lambda}\right.\right. \\
& \left.\left.+k_{2}\left(D_{\mu} D_{\lambda} \mathcal{A}_{\nu}+D_{\nu} D_{\lambda} \mathcal{A}_{\mu}\right)\right]_{b a} \gamma^{\lambda} \gamma_{5}\right]+h . c .
\end{align*}
$$

where $\mathcal{A}_{\mu b a}=\frac{i}{2}\left(\xi^{\dagger} \partial_{\mu} \xi-\xi \partial_{\mu} \xi^{\dagger}\right)_{b a} . \quad \Lambda_{\chi}$ is the chiral symmetry-breaking scale: we use $\Lambda_{\chi}=1 \mathrm{GeV} . \mathcal{L}_{S}$ and $\mathcal{L}_{T}$ describe transitions of positive parity heavy mesons with the emission of light pseudoscalar mesons in $s$ - and $d$ - wave, respectively, and $g, h$ and $h^{\prime}$ are the effective coupling constants. On the other hand, $\mathcal{L}_{X}$ and $\mathcal{L}_{X^{\prime}}$ describe the transitions of higher mass mesons of negative parity with the emission of light pseudoscalar mesons in $p-$ and $f$ - wave, with coupling constants $k^{\prime}, k_{1}$ and $k_{2}$.

At the same order in the expansion in the light meson momentum, the structure of the Lagrangian terms for radial excitations of the $H, S$ and $T$ doublets does not change, being only dictated by the spin-flavour and chiral symmetries, but the coupling constants $g, h$ and $h^{\prime}$ must be replaced by $\tilde{g}, \tilde{h}$ and $\tilde{h}^{\prime}$. The advantage of this formulation is that meson transitions into final states obtained by $S U(3)$ and heavy quark spin rotations can be related in a straightforward way.

With the effective Lagrangians in Eq.(77) we can evaluate the strong decays of $D_{s J}(3040)$ to a charmed meson and a light pseudoscalar one in correspondence to the four classifications in Table In particular, the ratio

$$
\begin{equation*}
R_{1}=\frac{\Gamma\left(D_{s J}(3040) \rightarrow D_{s}^{*} \eta\right)}{\Gamma\left(D_{s J}(3040) \rightarrow D^{*} K\right)} \tag{7}
\end{equation*}
$$

$\left(D^{(*)} K=D^{(*) 0} K^{+}+D^{(*)^{+}} K_{S}^{0}\right)$ can be computed, with results for the various assignments:

$$
\begin{align*}
R_{1}\left(\tilde{D}_{s 1}^{\prime}\right) & =0.34 \\
R_{1}\left(\tilde{D}_{s 1}\right) & =0.20 \\
R_{1}\left(D_{s 2}\right) & =0.245  \tag{8}\\
R_{1}\left(D_{s 2}^{* \prime}\right) & =0.143 .
\end{align*}
$$

It is important to notice that the dependence on the effective couplings cancels in the ratio. The spread among the various predictions permits to discriminate among the assignments, in particular between $\tilde{D}_{s 1}^{\prime}$ and $D_{s 2}^{* \prime}$.

The mass of $D_{s J}(3040)$ is large enough to allow other decay modes. Decays to the members of the doublets with $s_{\ell}^{P}=\frac{1}{2}^{+}$or $s_{\ell}^{P}=\frac{3}{2}^{+}$and a pseudoscalar meson are possible: these are the modes $\left(D_{0}^{*}, D_{1}^{\prime}\right) K$ and $\left(D_{1}, D_{2}^{*}\right) K$. As for channels with the $\eta$ in the final state, the only kinematically allowed mode is $D_{s 0}^{*} \eta$ (for $D_{s 1}^{\prime} \eta$ the available phase space is tiny). The features of such decay modes are different for the four considered assignments. At the leading order in $\frac{1}{m_{c}}$ expansion, the states $\tilde{D}_{s 1}^{\prime}$ and $\tilde{D}_{s 1}$ can decay to $D_{0}^{*} K, D_{s 0}^{*} \eta, D_{1}^{\prime} K, D_{1} K$ and $D_{2}^{*} K$ in $p-$ wave; $D_{s 2}$ and $D_{s 2}^{* \prime}$ both decay to $D_{0}^{*} K, D_{s 0}^{*} \eta, D_{1}^{\prime} K$ in $d-$ wave; $D_{s 2}^{* \prime}$ can decay to $D_{1} K$ and $D_{2}^{*} K$ in $d-$ wave, while in the case of $D_{s 2}$ the decay to $D_{2}^{*} K$ proceeds in $s-$ wave and the decay into $D_{1} K$ is allowed at $\mathcal{O}\left(\frac{1}{m_{c}}\right)$ in $d$ - wave.

Other kinematically allowed modes of $D_{s J}(3040)$ are those with the emission of a light vector meson, specifically decays into $D K^{*}$ or $D_{s} \phi$ 19]. It is possible to describe such decay modes using an approach based on
effective Lagrangian terms analogous to the one followed above. In the case of light vector mesons in the final state, the method has been developed in Ref. [13] on the basis on the hidden gauge symmetry idea [14]. We refer to the review [15] for a detailed description; here we only mention that in this approach the octet of light vector mesons is described by the field $\rho_{\mu}=i \frac{g_{V}}{\sqrt{2}} \hat{\rho}_{\mu}$, where $\hat{\rho}_{\mu}$ is a hermitian $3 \times 3$ matrix analogous to (5), containing the light vector meson fields $\rho^{ \pm, 0}, K^{* \pm}, K^{* 0}, \bar{K}^{* 0}, \omega_{8}$. The constant $g_{V}$ can be fixed to the value $g_{V}=5.8$ by the Kawarabayashi-Sukuzi, Riazuddin-Fayyazuddin relations [16].

In Ref. 15] the effective Lagrangian terms describing the transitions $S \rightarrow H V$ and $T \rightarrow H V$ are reported, $S$ and $H$ representing the heavy doublets $S_{a}$ and $H_{a}$ in Eq. (4) and $V$ denoting a generic light vector meson. They read:

$$
\begin{align*}
\mathcal{L}_{S H V} & =i \zeta_{S} \operatorname{Tr}\left[\bar{S}_{a} H_{b} \gamma_{\mu}\left(\mathcal{V}^{\mu}-\rho^{\mu}\right)_{b a}\right] \\
& +i \mu_{S} \operatorname{Tr}\left[\bar{S}_{a} H_{b} \sigma^{\lambda \nu} F_{\lambda \nu}(\rho)_{b a}\right] \\
\mathcal{L}_{T H V} & =i \zeta_{T} \operatorname{Tr}\left[\bar{H}_{a} T_{b}^{\mu}\left(\mathcal{V}_{\mu}-\rho_{\mu}\right)_{b a}\right]  \tag{9}\\
& +i \mu_{T} \operatorname{Tr}\left[\bar{H}_{a} T_{b}^{\mu} v^{\nu} F_{\mu \nu}(\rho)_{b a}\right]
\end{align*}
$$

where $\mathcal{V}_{\mu b a}=\frac{i}{2}\left(\xi^{\dagger} \partial_{\mu} \xi+\xi \partial_{\mu} \xi^{\dagger}\right)_{b a}$, and $\zeta_{S, T}, \mu_{S, T}$ are effective coupling constants, and the field strength tensor $F_{\mu \nu}$ is defined as $F_{\mu \nu}(\rho)=\partial_{\mu} \rho_{\nu}-\partial_{\nu} \rho_{\mu}+\left[\rho_{\mu}, \rho_{\nu}\right]$.

The calculation of all possible decay modes for each one of the four classifications of $D_{s J}(3040)$ would require the values of several coupling constants, which are unknown. Nevertheless, some conclusions can be drawn.

One can estimate the widths of the decays into $D K^{*}$ and $D_{s} \phi$ for the two $J^{P}=1^{+}$states by the effective Lagrangians in (10) using the values of $\zeta$ and $\mu$ : $\zeta_{S}=$ 0.10 and $\mu_{S}=-0.10 \mathrm{GeV}^{-1}$ computed for the analogous transitions involving the heavy mesons belonging to the lowest-lying doublet [15]. This provides us with hints about the contribution of such modes to the total width of $D_{s J}(3040)$ in the case it is either $\tilde{D}_{s 1}^{\prime}$ or $\tilde{D}_{s 1}$ (in the case of $D_{s 2}$ and $D_{s 2}^{* \prime}$ these decays proceed in $p$-wave and hence they are expected to contribute less significantly):

$$
\begin{align*}
\Gamma\left(\tilde{D}_{s 1}^{\prime} \rightarrow D K^{*}\right) & \simeq 95 \mathrm{MeV} \\
\Gamma\left(\tilde{D}_{s 1}^{\prime} \rightarrow D_{s} \phi\right) & \simeq 44 \mathrm{MeV}  \tag{10}\\
\Gamma\left(\tilde{D}_{s 1} \rightarrow D K^{*}\right) & \simeq 15 \mathrm{MeV} \\
\Gamma\left(\tilde{D}_{s 1} \rightarrow D_{s} \phi\right) & \simeq 6 \mathrm{MeV}
\end{align*}
$$

It can be noticed that the contribution of these modes to the full width should be sizable only in the case of $\tilde{D}_{s 1}^{\prime}$; therefore, this assignment can be disentangled through the experimental analysis of the $D K^{*}$ and $D_{s} \phi$ decay channels.

After having discussed four possible classifications for $D_{s J}(3040)$, it is interesting to consider some features of its spin partner in the various cases 20].

TABLE II: Features of the decay modes of $D_{s J}(3040)$ and of its spin partner at leading order in $\frac{1}{m_{c}}$ expansion for the four proposed assignments.

| decay modes | $\tilde{D}_{s 1}^{\prime}\left(n=2, J_{s_{\ell}}^{P}=1_{1 / 2}^{+}\right)$ | $\tilde{D}_{s 1}\left(n=2, J_{s_{\ell}}^{P}=1_{3 / 2}^{+}\right)$ | $D_{s 2}\left(n=1, J_{s_{\ell}}^{P}=2_{3 / 2}^{-}\right)$ | $D_{s 2}^{* \prime}\left(n=1, J_{s_{\ell}}^{P}=2_{5 / 2}^{-}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $D^{*} K, D_{s}^{*} \eta$ | $s$ - wave | $d$ - wave | $p$ - wave | $f$ - wave |
| $R_{1}$ | 0.34 | 0.20 | 0.245 | 0.143 |
| $D_{0}^{*} K, D_{s 0}^{*} \eta, D_{1}^{\prime} K$ | $p$ - wave | $p$ - wave | $d$ - wave | $d$ - wave |
| $D_{1} K$ | $p$ - wave | $p$ - wave | - | $d$ - wave |
| $D_{2}^{*} K$ | $p$ - wave | $p$ - wave | $s$ - wave | $d$ - wave |
| $D K^{*}, D_{s} \phi$ | $s$ - wave | $s$ - wave | $p$ - wave | $p$ - wave |
|  | $\Gamma \simeq 140 \mathrm{MeV}$ | $\Gamma \simeq 20 \mathrm{MeV}$ | negligible | negligible |
|  | spin partner |  |  |  |
|  | $\tilde{D}_{s 0}^{*}\left(n=2, J_{s_{\ell}}^{P}=0_{1 / 2}^{+}\right)$ | $\tilde{D}_{s 2}^{*}\left(n=2, J_{s_{\ell}}^{P}=2_{3 / 2}^{+}\right)$ | $D_{s 1}^{*}\left(n=1, J_{s_{\ell}}^{P}=1_{3 / 2}^{-}\right)$ | $D_{s 3}\left(n=1, J_{s_{\ell}}^{P}=3_{5 / 2}^{-}\right)$ |
| $D K, D_{s} \eta$ | $s$ - wave | $d$ - wave | $p$ - wave | $f$ - wave |
| $D^{*} K, D_{s}^{*} \eta$ | - | $d$ - wave | $p$ - wave | $f$ - wave |
| $D_{0}^{*} K, D_{s 0}^{*} \eta$ | - | - | $d$ - wave | - |
| $D_{1}^{\prime} K$ | $p$ - wave | $p$ - wave | $d$ - wave | $d$ - wave |
| $D_{1} K$ | $p$ - wave | $p$ - wave | $s$ - wave | $d$ - wave |
| $D_{2}^{*} K$ | - | $p$ - wave | - | $d$ - wave |

- If $D_{s J}(3040)$ is $\tilde{D}_{s 1}^{\prime}\left(s_{\ell}^{P}=\frac{1}{2}^{+}, J^{P}=1^{+}, n=2\right)$, its spin partner is $\tilde{D}_{s 0}^{*}$, a $J^{P}=0^{+}$state, the first radial excitation of $D_{s J}(2317)$. This state can decay to $D K$ and $D_{s} \eta$ in $s$-wave; $p$-wave decays to $D_{1}^{\prime} K$ and $D_{1} K$ are also allowed.
- If $D_{s J}(3040)$ is $\tilde{D}_{s 1}\left(s_{\ell}^{P}=\frac{3}{2}^{+}, J^{P}=1^{+}, n=2\right)$, its spin partner is $\tilde{D}_{s 2}^{*}$ with $J^{P}=2^{+}$. It is allowed to decay to $D K, D_{s} \eta, D^{*} K, D_{s}^{*} \eta, D K^{*}$ and $D_{s} \phi$ in $d$-wave, and to $D_{1}^{\prime} K, D_{1} K$ and $D_{2}^{*} K$ in $p$-wave.
- If $D_{s J}(3040)$ is $D_{s 2}\left(s_{\ell}^{P}=\frac{3}{2}^{-}, J^{P}=2^{-}, n=1\right)$, its spin partner is the vector meson $D_{s 1}^{*}$ with $J^{P}=1^{-}$. It can decay to $D K, D_{s} \eta, D^{*} K, D_{s}^{*} \eta, D K^{*}$ and $D_{s} \phi$ in $p$-wave, to $D_{0}^{*} K, D_{s 0}^{*} \eta$ and $D_{1}^{\prime} K$ in $d$ - wave and to $D_{1} K$ in $s$ - wave. The decay to $D_{2}^{*} K$ is allowed at $\mathcal{O}\left(\frac{1}{m_{c}}\right)$ in $d-$ wave.
- If $D_{s J}(3040)$ is $D_{s 2}^{* 1}\left(s_{\ell}^{P}=\frac{5}{2}^{-}, J^{P}=2^{-}, n=1\right)$, its spin partner is $D_{s 3}$ with $J^{P}=3^{-}$, decaying to $D K, D_{s} \eta, D^{*} K$ and $D_{s}^{*} \eta, D K^{*}, D_{s} \phi$ in $f-$ wave, and to $D_{1}^{\prime} K, D_{1} K$ and $D_{2}^{*} K$ in $d-$ wave.

Since $D_{s J}(3040)$ has a broad width, we expect that also its spin partner shares the same feature. Considering the previous list, we can argue that $\tilde{D}_{s 0}^{*}$ is broad due to its $s$-wave decays into $D K$ and $D_{s} \eta$. Also $D_{s 1}^{*}$ has allowed $s$-wave decays, but only to $D_{1} K$ which is suppressed by phase space effects.

The identification of $D_{s J}(3040)$ with $\tilde{D}_{s 1}^{\prime}$ is supported in Refs. 17, 18] on the basis of the $c \bar{s}$ mass spectrum [17] or of the decay widths computed in the ${ }^{3} P_{0}$ model
[18]. In the second case, the identification with $\tilde{D}_{s 1}^{\prime}$ and $\tilde{D}_{s 1}$ is discussed: the full widths of these two states are computed and compared to the experimental measurement of $\Gamma\left(D_{s J}(3040)\right)$, concluding that for $\tilde{D}_{s 1}^{\prime}$ the experimental width can be reproduced, with the predic-


FIG. 1: Spectrum of the $c \bar{s}$ system. All observed $D_{s J}$ states, with mass indicated on the $y$ axis, are assigned to a level with $J^{P}$ and proper name. The four assignments discussed for $D_{s J}(3040)$ are shown in correspondence to the mass value $M=3040 \mathrm{MeV}$.
tion: $\frac{\Gamma\left(D_{s J}(3040) \rightarrow D K^{*}+D_{s} \phi\right)}{\Gamma\left(D_{s J}(3040) \rightarrow D^{*} K+D_{s}^{*} \eta\right)} \simeq 0.79$. In our discussion we have examined other information that can be exploited.

To summarize the results of our considerations and compare the distinctive features of the four proposed assignments for $D_{s J}(3040)$ (illustrated in Fig 1 in which all the known $c \bar{s}$ states have been included), we collect our findings and observations in Table II. In particular, we emphasize the role of the final states $D K^{*}$ and $D_{s} \phi$, which deserve an experimental investigation. Moreover, although the identification with a $J^{P}=2^{-} s_{\ell}^{P}=\frac{3}{2}^{-}$
state seems less probable, this assignment can be discarded/confirmed studying the $D_{2}^{*} K s$-wave final state. Search of the spin partner in each doublet would provide further information, enriching the $c \bar{s}$ spectrum recently disclosed by experiments.

## Acknowledgments

We thank A. Palano for discussions. This work was supported in part by the EU contract No. MRTN-CT-2006035482, "FLAVIAnet".
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