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# STRESSES AND DISPLACEMENTS IN A SEMIDIGGER MOULDBOARD AND A PLOUGHSHARE

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**Abstract.** The paper investigates stress-strain state in the most important parts of a plough - mouldboard and ploughshare. The mouldboard consists of three layers, while the ploughshare is homogeneous. Two software packets were used to solve the problem. AutoCAD enables us to create the quadrangle curvilinear surfaces of the mouldboard and the ploughshare. It has been achieved by using a special AutoLISP procedure, which must be run under AutoCAD release 15.0. In the environment of ANSYS 5.6 thickness of layers and material properties were added to these quadrangles. The loads according to Goryachev formulae have been added. This way we can find the most dangerous regions where the stress intensity reaches its maximum value. The dependence of displacements on thickness of mouldboard has also been found.

*Keywords*: Mouldboard, ploughshare, AutoCAD, AutoLISP, ANSYS

## 1. Symbols used

$a$ – processing depth of a plough
$b$ – grab width of a plough
$c$ – distance between the top working surface of mouldboard and the plane $YZ$
$d$ – distance between the lower edge of mouldboard and turned soil layer
$H$ – height of working surface at the top nearest to the field unploughed
$\Delta b$ – grab width overlap
$\alpha$, $\beta$, $\gamma$, $\varepsilon$ – angles to set the form of plough body
$x$ – horizontal co-ordinate, perpendicular to furrow
$y$ – horizontal co-ordinate towards the motion of plough
$z$ – vertical co-ordinate

## 2. Introduction

The textbook by A. Reintam [1] is devoted to the theoretical treatment of the issue how to design mouldboards and ploughshares by making use of graphical methods. These methods are quite troublesome and inaccurate. The main objective of the present paper is to show how to design mouldboards and ploughshares on the screen of a computer. The building process of the virtual models for a mouldboard and a ploughshare in the environment of the Finite Element Package ANSYS is bulky. The present paper uses an alternative possibility with more flexible modelling possibilities for creating virtual models in the environment provided by the package AutoCAD. The automatic procedure is written in the AutoCAD environment by using the special AutoLISP language for drawing a mesh of the plough body consisting of tetragonal elements.

The parameters are entered through a dialog box on the screen of the computer. Every parameter has its own interval for the numerical values to be entered and the default value is the mean value in this interval.

The plough body is divided into elements which can be used to calculate the strength of the working bodies of a plough. Using ANSYS, which is powerful software based on the finite element method, one can perform these calculations. It is expected that one can make some recommendations to choose the initial parameters for creating a mesh of the plough body. Here it is worthy of mention that the thicknesses of the tetragonal elements in AutoCAD environment are considered to be equal to zero: the thicknesses should be added under ANSYS only. The thickness of an element can either be constant or vary and the elements may consist of several layers.

## 3. Description of the drawing procedure

To design a mesh for a plough body, the AutoLisp procedure must be loaded and run under AutoCAD. A loaded procedure can be started repeatedly. A procedure can be loaded with aid of various menus or by entering text

(*load "plough"*)

(the brackets here are obligatory; *plough* is the filename of the corresponding procedure without the extension *lsp*). To run the procedure we enter *PLOUGH* (or *plough*). If the procedure *"plough.lsp"* should be loaded and/or run repeatedly, it is useful to create an icon under AutoCAD.

Now we describe the procedure *plough.lsp* and demonstrate how it functions. The theoretical base to draw a plough body is described by A. Reintam [1]. However, the graphical methods in the textbook [1] are not suitable for computer aided design, therefore all necessary co-ordinates of a plough body must be calculated prior to drawing.

In addition, the left-hand co-ordinates must be changed to right-hand co-ordinates, because AutoCAD uses only a right-hand co-ordinate system. Another reason is that in the left-hand co-ordinate system some parameters may have opposite signs (so-called pseudo-scalars and pseudo-vectors [2]). To adjust the theory [1] to a right-hand co-ordinate system it is sufficient to interchange the axes $X$ and $Y$.

The program we have developed for the design of a plough body performs seven steps:

1. Constructing the contours of the projection on the plane $XZ$ (a view of the plough body along furrow, as is described in [1] – see Figure 8a);
2. Formation of the three-dimensional directrix;
3. Drawing a set of horizontal straight lines through the points lying in the directrix;

4. Finding the intersection points of the lines with the cylinder which is parallel to the axis $Y$ and passes through the contours found in the $1^{st}$ step);
5. Using the points found in the previous step, the contours of the mouldboard and the ploughshare are formed (both closed contours should consist of exactly 4 edges);
6. Drawing the curvilinear meshes on these three-dimensional contours;
7. Exploding the meshes formed into single quadrangular elements. (The edges and the vertices of the neighboring elements should coincide.)

In fact the first four steps are executed without drawing anything. Only the corresponding calculations are accomplished and the results are saved into the main memory of the computer. However, the contour in the $5^{th}$ step must be drawn, because without a closed contour AutoCAD cannot execute the $6^{th}$ step. All steps must be performed both for the mouldboard and for the ploughshare, with a mutual unison in order to avoid a crack between mouldboard and ploughshare.

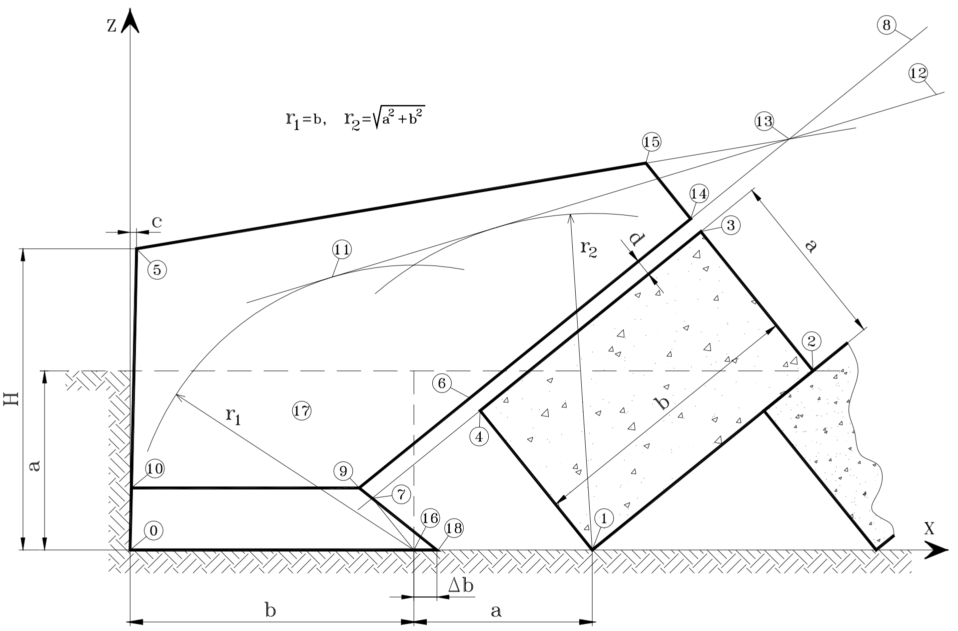


Figure 1. The front view of a plough body and two furrows

Now we discuss these steps in more details. The performance of the $1^{st}$ step is illustrated in Figure 1. The values of the parameters $a$, $b$, $c$, $d$, $\Delta b$ and $H$ are entered by the keyboard input, radii $r_1$ and $r_2$ are calculated by the formulae

$$r_1 = b, \quad r_2 = \sqrt{a^2 + b^2}.$$

The closed line that consists of straight segments through points 5, 10, 9, 14 and 15, is the projection of mouldboard on plane $XZ$. For the ploughshare this line passes through points 0, 18, 9 and 10. The rest of the numbered points in Figure 1 are auxiliary points for the computation process. The rectangle with vertices 1, 2, 3 and 4 is a turned non-disintegrated furrow (beneath it is a fragment of a previous furrow).

Our AutoLisp procedure uses two standard functions to find the co-ordinates. The standard function

$$(inters\ p_1\ p_2\ p_3\ p_4 nil)$$

finds the intersection point of two lines, where $p_1$ and $p_2$ are the end points of the first line, and $p_3$ and $p_4$ are the end points of the second line. If parameter *nil* exists, then *inters* returns the point where the lines intersect, even if that point is off the end of one or both lines. The standard function

(*polar starting-point angle distance*)

returns the point at a specified angle and distance from a starting-point. Here the angle is measured in radians relative to the axis $X$, with respect to the current construction plane in the counter-clockwise direction.

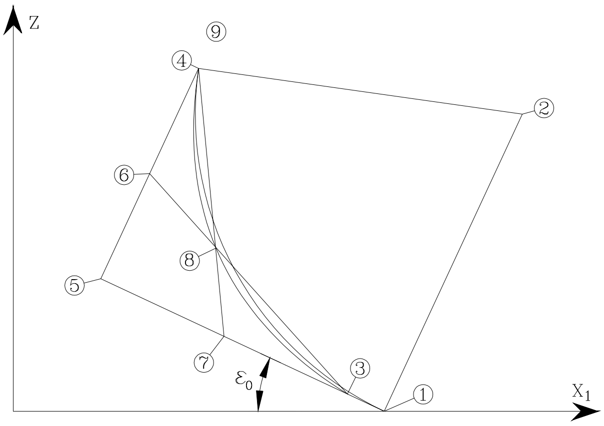


Figure 2. Parabolic directrix

Then the three-dimensional directrix is found, as is in the textbook [1], though the graphical methods are here replaced by appropriate analytical methods. It is known that a parabola is uniquely defined, when three points of the parabola and the slope angle of the axis of symmetry are given. In Figure 2 this angle is equal to $\varepsilon_0$ and the end points of the parabola are points 3 and 4. The intermediate point 8 can be found as the intersection of two lines. One of these lines connects point 3 with the midpoint 6 of the line passing through points 4 and 5; the other line connects point 4 with the midpoint 7 of the line passing through points 3 and 5. If we rotate now the parabola by the angle $\varepsilon_0$ (clockwise), the axis of symmetry of the parabola becomes parallel to the axis $X_1$ – see Figure 3 – and the equation of the parabola takes the form

$$x_1 = Az^2 + Bz + C,$$

where the coefficients $A$, $B$ and $C$ should be chosen in such a way that the parabola passes through points 3, 8 and 4. In this way we obtain a system of linear equations with $A$, $B$ and $C$ as unknowns. After finding $A$, $B$ and $C$ the parabola is rotated back into the previous position.

It is essential to say that the directrix lies in a vertical plane, the dihedral of this plane and the plane $XZ$ is equal to the known angle $\gamma_0$ – see Figure 3. As soon as the directrix has been formed, the horizontal generators are drawn. All the generators pass the directrix at different heights (the heights are measured along the axis $Z$). Besides, every generator lies under different angle $\gamma = f(z)$ towards the plane $YZ$. In the case of a semi-digger plough body it is recommended [1] that the angle $\gamma$ follow the parabolic law

$$f(z) = \frac{(z - z_c)^2}{2p} + \gamma_{min}\,. \tag{3.1}$$

Here $z_c$ is the height where the mouldboard and the plough body contact. The parameters $\gamma_{min}$ and $\gamma_{max}$ are then entered; the latter corresponds to the value of $\gamma$ at the maximum height $z_{max}$ of the mouldboard. These relations allow us to find an appropriate value for the parameter $p$ in equation (3.1).

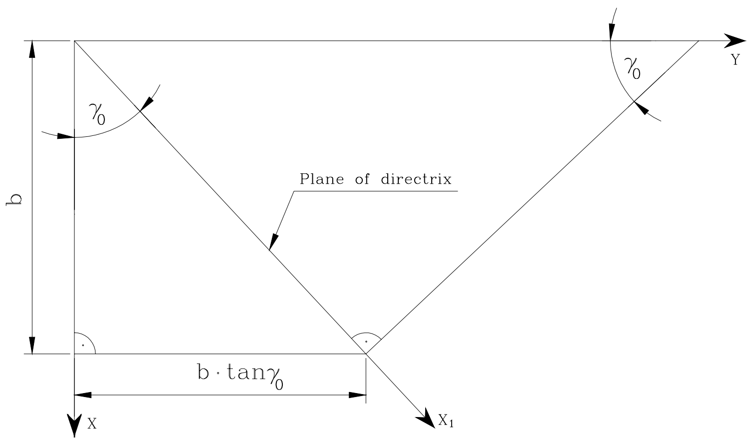


Figure 3. Top view of the directrix

In the next step the intersection points of the horizontal generators and the cylindrical surface (the latter was designed in the $1^{st}$ step), are found. Since the surface of mouldboard contains five planes each being parallel to axis $Y$ (see Figure 1 for details), every generator has up to five intersection points. However, we should use only those intersection points lying in the closed contour which passes through points 5, 10, 9, 14, 15 and 5, and should disregard the other intersection points (such as the points in the segment between points 15 and 12).

The intersection point of a three-dimensional plane and a line is given by a formula of the handbook [4]. Namely, let us assume, that the plane is given by the equation

$$Ax + By + Cz + D = 0 \tag{3.2}$$

and the line drawn through the points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ by the equation

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}. \tag{3.3}$$

Then the coordinates of the intersection point $(\bar{x}, \bar{y}, \bar{z})$ can be calculated as

$$\bar{x} = x_1 - l\rho, \quad \bar{y} = y_1 - m\rho, \quad \bar{z} = z_1 - n\rho,$$

where

$$l = \frac{x_2 - x_1}{d}, \quad m = \frac{y_2 - y_1}{d}, \quad n = \frac{z_2 - z_1}{d}$$

are the direction cosines,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

is the distance between two points and

$$\rho = \frac{Ax_1 + By_1 + Cz_1 + D}{Al + Bm + Cn} \tag{3.4}$$

is the distance of the intersection point $(\bar{x}, \bar{y}, \bar{z})$ from the point $(x_1, y_1, z_1)$.

The intersection point $(\bar{x}, \bar{y}, \bar{z})$ does not exist if the denominator in equation (3.4) is equal to zero. In such a case the plane (3.2) and the line (3.3) are mutually parallel. Incidentally, such is the case for the plane through points 10 and 9 – see Figure 1.

If we have found a sufficient number of intersection points, these must be ordered in such a manner that they form a closed contour consisting of exactly four (curvilinear) edges. It is worthy of mention that for the mouldboard one of these edges connects points 5, 15 and 14 – see again Figure 1.

Now it is necessary to determine the densities of meshes towards two alternative directions. These values must be set by two system variables of AutoCAD: *SURFTAB1* and *SURFTAB2*. The AutoLisp function

(command "EDGESURF" $e_1$ $e_2$ $e_3$ $e_4$)

constructs a three-dimensional polygon mesh approximating a Coons surface patch mesh from four adjoining edges $e_1$, $e_2$, $e_3$ and $e_4$. Here $e_1$ is an edge of the mouldboard in vertical direction; and $e_2$, $e_3$ and $e_4$ are the other three edges. A Coons surface patch mesh is a bicubic surface interpolated between four adjoining edges (which can be general space curves). The Coons surface patch mesh not only meets the corner of the defining edges, but also touches each edge, providing control over the boundaries of the generated surface patch. The same operations must be done for the ploughshare as well. We should ensure that the two densities along the joint edges of the mouldboard and the ploughshare are equal.

After forming the meshes we do not need the edges any more and they can be deleted. As a last step both two-dimensional meshes should be exploded into tetragonal surface elements. The edges and vertices of two neighbouring elements coincide. Exploding does not change the appearance of the mesh, but without exploding the formed mesh cannot be inserted into ANSYS to accomplish the finite element calculations.

In Figures 4 - 6 the same mouldboard and ploughshare are shown in three different views. The view direction in Figure 4 is determened by two angles: the angle in the

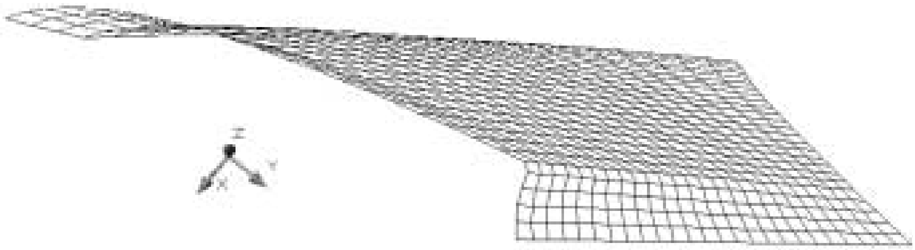

Figure 4. Slanting view of a plough

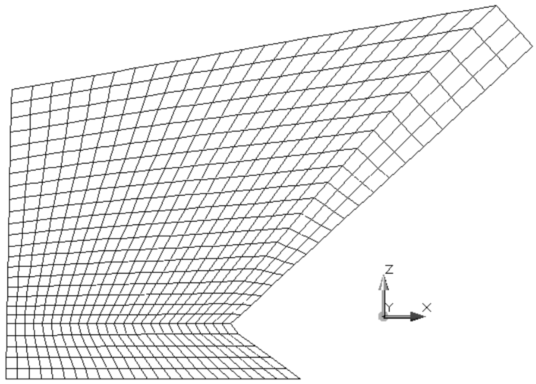

Figure 5. Front view of a plough

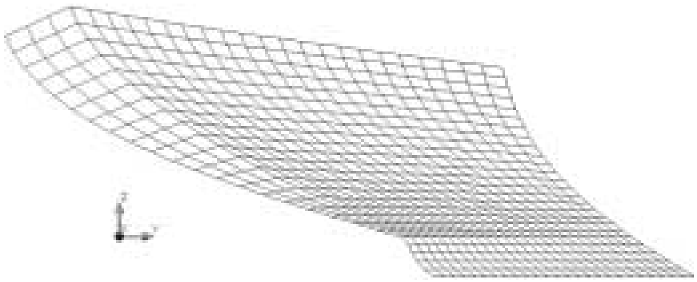

Figure 6. Right view of a plough

plane $XY$ from the axis $X$ is equal to 40°, the angle from the plane $XY$ is equal to 80°. The processing depth $a$ and the grab width $b$ are equal to 36 cm and 22 cm, respectively. The mesh densities in both directions are equal to 25. Consequently we have altogether 625 quadrangle elements.

## 4. Creation of finite elements

To create a finite element model of the plough body, the ANSYS5.6 linear layered structural shell elements SHELL99 have been used. In these applications up to 100

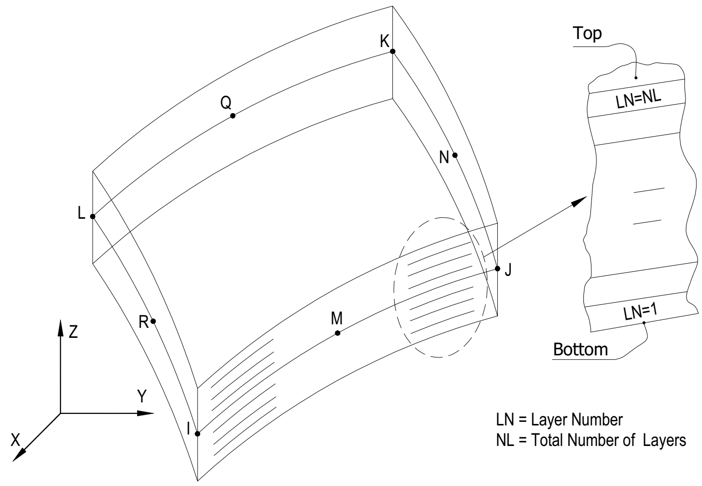


Figure 7. Up to 100-layer shell element SHELL99 with eight nodes

different layers are permitted. Every element has eight nodes: four ($I$, $J$, $K$ and $L$) at the vertices and four ($M$, $N$, $Q$ and $R$) at the midpoints of edges – see Figure 7. Naturally, the neighbouring elements must have common nodes - which insures against the disconnection of the virtual model of element SHELL99. If this property does not exist, the plough body disintegrates into single elements under the loads.

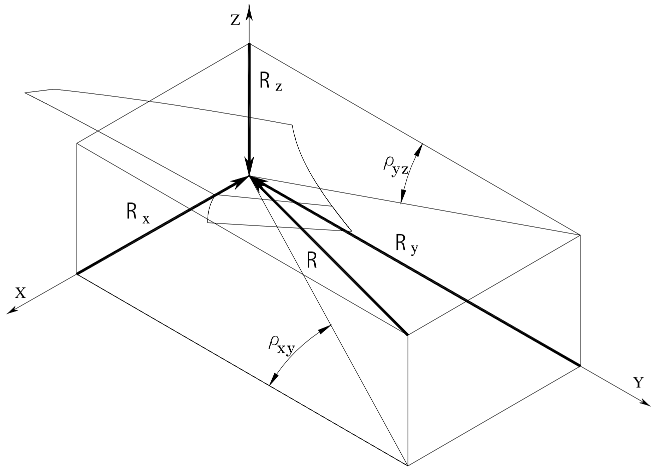


Figure 8. The soil reaction forces acting on the plough body

Every node of the element SHELL99 has six degrees of freedom: three displacements towards the coordinate axes $X$, $Y$ and $Z$ and three rotations around the same axes. The soil's influence on the plough body has been taken into account by the formulae of Goryachkin. The components of the resultant force $\mathbf{R}$ in the coordinate system $(X, Y, Z)$ – see Figure 8 for details – are given by

$$R_y = fG + (K_\partial + \varepsilon_o \nu_m^2)ab; \qquad R_x = R_y \tan\rho_{xy}; \qquad R_z = R_y \tan\rho_{yz}\,. \tag{4.1}$$

Table 1. The parameters in the formulae of Goryachkin

| Symbol | Meaning | Unit | Value |
|---|---|---|---|
| $K_\partial$ | specific resistance to deformation of furrow slice | kPa | 35 |
| $\varepsilon_0$ | factor, taking into account the form of the plough body and the type of soil | kg/m$^3$ | 250 |
| $v_m$ | velocity of moving the plough | m/s | 2 |
| $a$ | depth of the furrow | cm | 26 |
| $b$ | width of the furrow | cm | 40 |
| $f$ | summary friction coefficient | | 0.5 |
| $G$ | gravity of plough per one plough body | kN | 2 |
| $\rho_{xy}$ | inclination angles of the resultant force **R** (Figure 8) | deg | 16 |
| $\rho_{yz}$ | | | 13 |

The parameters in formula (4.1) are explained in Table 1 and Figure 8 (the plough moves towards the axis $Y$). The typical values for these parameters are also given there. The permitted values of these parameters can be found in the textbook [1]. We should mention here that these values may change over wide intervals, and it is difficult to get the exact value. Since the forces $R_x$, $R_y$ and $R_z$ are directed as shown in Figure 8, they are taken with the negative sign. These components should be distributed among the elements of SHELL99. As is well known, forces can be applied at the nodes only. For simplicity we assume that only the vertex nodes $I$, $J$, $K$ and $L$ are taken into account (Figure 7). As the actual distributions of the force components (3.1) along the plough body are unknown, we shall consider two possible distributions of the applied forces.

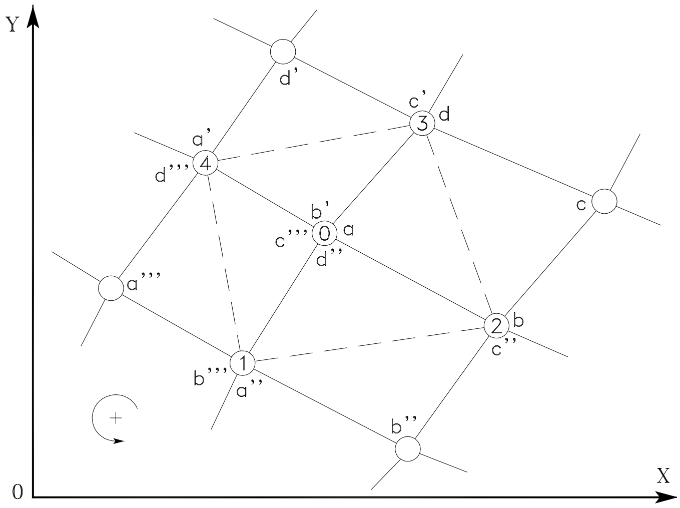


Figure 9. Projections of a quadrangle round a node

First, every force component (4.1) will be distributed uniformly along the projection of the plough body in the plane which is perpendicular to the selected coordinate axis. For instance, the component $R_y$ will be distributed along the projection of the plough body on the plane $XZ$. As is well known from mathematical statistics, uniform distribution is the best statistical distribution if the real distribution of some quantity is unknown. In the present case additional justification is given by the fact that real applied forces in a ploughing process are relatively incidental. Therefore, in this case no special cutting forces are applied along the edges of the plough body. This model is the first and best choice for soft soils (e.g. potato fields in autumn).

In the second case we shall try to take the cutting forces also into account. These are distributed along the cutting edge of the ploughshare and along the front edge of the plough body (only beneath the soil surface). At the other nodes the force components (4.1) are distributed uniformly (as in the previous case), although the intensities of the forces are different for these cases. To get a uniform distribution of forces, the weighting function for the force component applied at a node must be proportional to the area of quadrangle surrounding that node. For a node 0 this quadrangle is defined by the vertices 1, 2, 3 and 4 (Figure 9). The projections of the four neighboring elements on the plane $XY$ are also shown (one of these has vertices $a$, $b$, $c$ and $d$), surrounding the node 0. For the projections on coordinate planes $YZ$ and $ZX$ this will be done analogously. The projection of the area of a triangle with vertices 1, 2 and 3 (Figure 9) can be calculated by the formula [4]

$$S_{123} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} .$$

If we add the vertex 4, the last formula yields the projection of the quadrangle area in terms of the coordinates of the vertices 1, 2, 3 and 4:

$$S_0 = \frac{1}{2} \left[ (x_1 - x_3)(y_2 - y_4) - (x_2 - x_4)(y_1 - y_3) \right] .$$

If the vertices 1, 2, 3 and 4 are oriented towards the positive direction (as shown at the lower left corner in Figure 9), then the calculated area will be positive, otherwise it will be negative (or equal to zero). If the node 0 lays on the edge of the plough body (or at a vertex), one (or two) node(s) 1, 2, 3 or 4 coincide(s) with the central node 0.

The value of edge forces can be found by the following argumentation. In case of uniform distribution of forces based on formulae of theoretical mechanics it is possible to evaluate the centre of the parallel forces. In case of distributed edge forces for the axis $X$ we can use the following formulae

$$\left\{ \begin{array}{l} R'_x + R''_x = R_x \\ R'_x X'_C + R''_x X''_C = R_x X_C , \end{array} \right. \tag{4.2}$$

where the latter corresponds to the weighted average of the centre of forces. Here $R'_x$ and $R''_x$ are resultant edge force and remainder resultant uniform force, $X'_C$ and $X''_C$ are centres of distributed edge forces and remainder uniform forces, and $R_x$ is given by (4.1). According to the textbook [1] the coordinates of the application point of force $\mathbf{R}$ are equal to $X_C = 0.35b$ and $Z_C = 0.33a$. The system of equations (4.2) enables us to find the quantities of $R'_x$ and $R''_x$. Similar formulae are valid for axis Z. As the textbook [1] does not give $Y_C$, the distributed edge forces towards the axis $Y$ are not applied.

Figure 10 shows a typical uniform distribution of force projections $R_x$ (horizontal) and $R_z$ (vertical) (a view along the furrow). In the same figure we can see the six bolts where the plough body is fixed to the anchor of the plough [3].

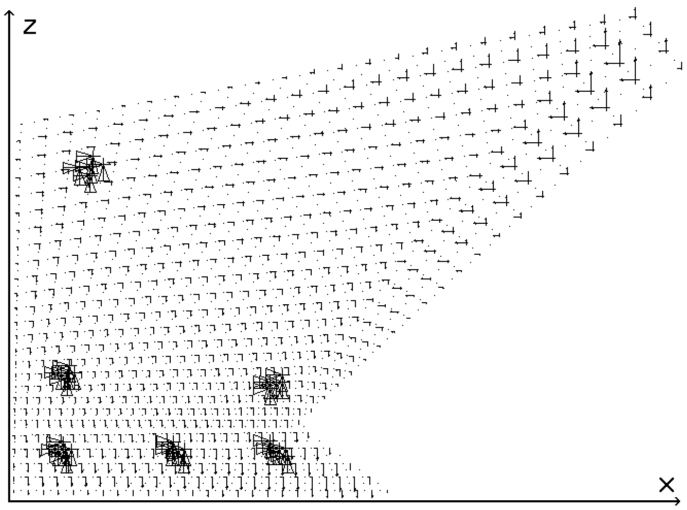


Figure 10. Applied forces and constrained elements (uniform distribution)

At every bolt the five neighboring nodes are fixed so that the number of degrees of freedom is equal to zero, which corresponds to rigid support. An analogous distribution of forces can be obtained for the case of edge forces. Although these two distributions are relatively different, the results of the calculations, as we can see later, are not as different.

## 5. Results of calculations and conclusions

Stress and displacement distributions were obtained through calculations using ANSYS 5.6. Due to the limited volume of this paper, we confine ourselves to one set of input parameters (velocity of plough movement and the properties of soil and materials of the plough body). Only the thickness of the mouldboard layers and ploughshare vary.

Table 2. The parameters of layers

| Location | Young's modulus (GPa) | Elastic limit (GPa) | Poisson's ratio |
|---|---|---|---|
| cutting edge of ploughshare | 210 | 0.471 | 0.29 |
| ploughshare | 175 | 0.373 | |
| bottom layer of mouldboard | 200 | 0.471 | |
| middle layer of mouldboard | 170 | 0.373 | |
| top layer of mouldboard | 210 | 0.471 | |

Parameters from Goryachkin formulae (4.1), which correspond to loam as a soil form, have been described in Table 1, properties of different materials in Table 2, and the thicknesses of plough body in the left five columns of Table 3. In reality the thickness of the ploughshare is uneven, but a thick rib behind the cutting edge has been ignored. It is not dangerous because by ignoring this only the calculated stress intensities increase. The model SHELL99 enables us to introduce only ribs with bilateral symmetry, but

Table 3. Parameters to design the surface of the plough body

| Thickness (mm) | | | | | Displacement (mm) | | Stress intensity (GPa) | |
|---|---|---|---|---|---|---|---|---|
| cutting edge | bottom layer | middle layer | top layer | sum total | uniform load | with edge loads | uniform load | with edge loads |
| 0.25 | 0.25 | 1.5 | 0.25 | 2.0 | 78.8 | 77.3 | 0.606 | 0.621 |
| 0.4 | 0.5 | 2.0 | 0.5 | 3.0 | 38.9 | 38.5 | 0.339 | 0.349 |
| 0.6 | 1.0 | 3.0 | 1.0 | 5.0 | 15.8 | 15.8 | 0.155 | 0.160 |
| 0.8 | 1.5 | 4.5 | 1.5 | 7.5 | 7.62 | 7.67 | 0.0825 | 0.0852 |
| 1.0 | 2.0 | 6.0 | 2.0 | 10.0 | 4.48 | 4.54 | 0.0529 | 0.0553 |

the upper rib is clearly inappropriate.

The model SHELL99 (part of ANSYS 5.6) enables us to find the distributions of stress intensities (naturally also that of the stress components) and the displacements (and the rigid body rotations about the coordinate axes) either as numerical values for all (or selected) nodes or in various graphical forms. These parameters can be found for every layer: by selecting the bottom, middle or top surface of a layer.

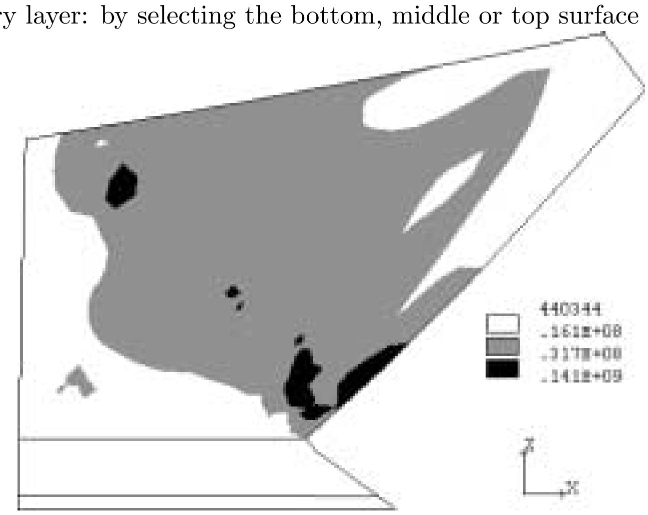


Figure 11. Stress intensities along the bottom surface of the bottom layer

Now we fix the thickness of the plough body at 5 mm (Table 3). Figure 11 illustrates the distribution of stress intensity along the bottom surface of the bottom layer of the plough body under uniform loading. Black areas represent the areas of high stress intensity (measured in pascals). The most dangerous areas are those surrounding the two fixed points of the mouldboard and the region between them, while the stresses inside the ploughshare are relatively small. Analogous distribution is illustrated in Figure 12. Here along the top surface of the top layer, the dangerous region is at the upper edge of the mouldboard. It appeared that the distributions of the stress components are clearly different from each other and also from the stress intensities. These distributions are not represented in these figures. As the maximum values of stress intensities occur at two fixed points of the mouldboard, it may be useful to increase the thickness of the lower layer of the mouldboard near these regions, or to use a complementary support under the free end of the mouldboard (some plough types have such support).

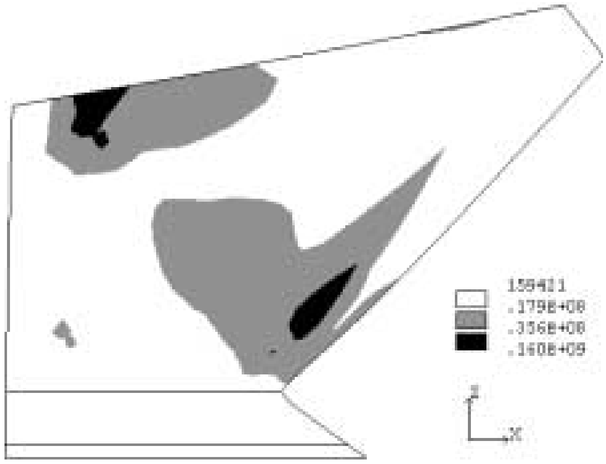


Figure 12. Stress intensities along the top surface of the top layer

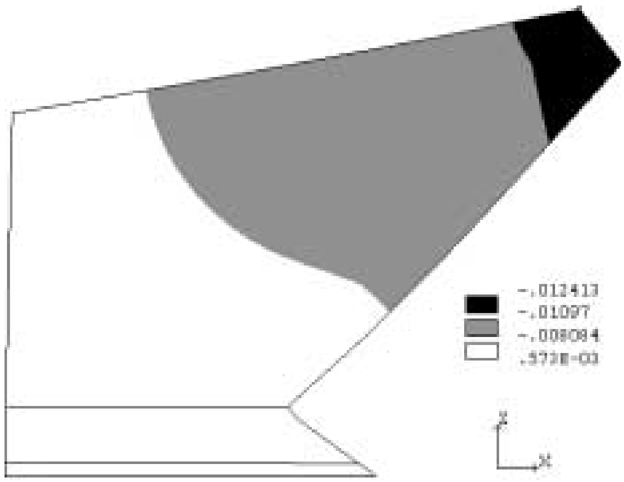


Figure 13. Displacements in the direction $X$

The distributions of displacements were found in the same manner – see Figure 13.

It is expected that the maximum displacement magnitudes occur at the free end of the mouldboard. Displacements are predominately negative (only vertical displacements towards the axis $Z$ at some nodes may be positive, compared with the distribution of vertical forces in Figure 10).

If we investigate the stress and displacement distribution in the case of distributed edge forces, we obtain similar pictures (see Figures 11 - 13). It is interesting to note that these distributed edge forces are insignificant for the distributions of stress intensities, while the stress components may be more different in comparison with the case where the edge forces are absent.

The four right columns of Table 3 give a numerical comparison of these two load cases. The table shows the maximum displacements and the stress intensities, which take place at the right-most point of the mouldboard (see Figure 13).

Table 3 shows also that if the thickness of the mouldboard is too small, the displacements and stress intensities can be quite large. According to formulae (4.1) the load of a plough body increases with the hardness of soil. Let us remember that our calculations are carried out for loam, which is not a very hard soil. Even though the elastic limit is not exceeded, such large deformations are not proper for a plough. Besides, there exists a risk to exceed the elastic limit, which gives rise to residual deformations. This happens when the total thickness of the mouldboard is equal to 2 mm only (see Table 3). The 3 mm thickness is also dangerous. However, such thin ploughs are not proper for the ploughing process.

Thus the creating of the virtual model in the AutoCAD environment, entering this model into an ANSYS environment for creating the finite element model and carrying the results of computations were successful. This alternative possibility is suitable for AutoCAD users, who wish to use the finite element method.

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